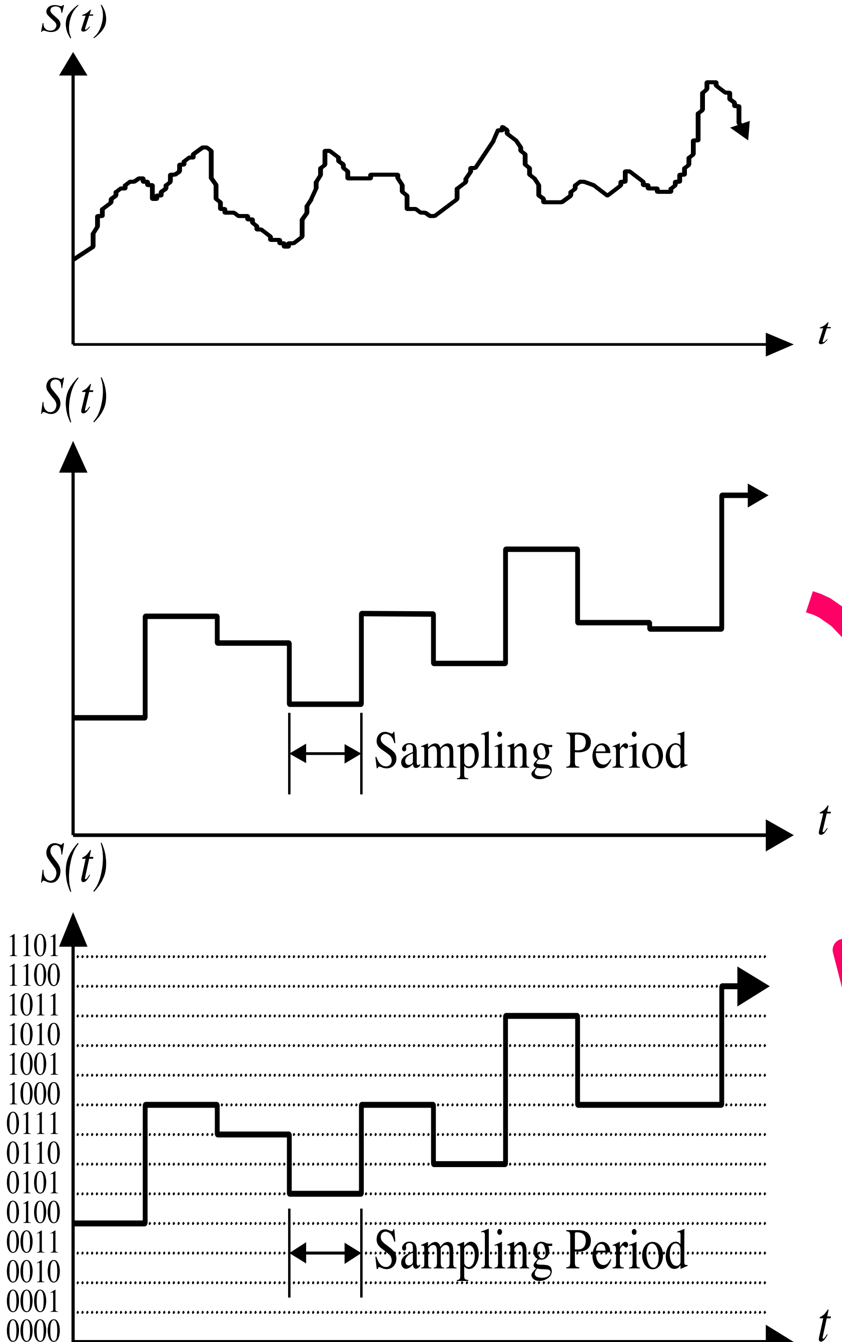
Lecture 1 (06/26)
Signal Modulation

- Introduction (Signals)
- Signal Analysis
- Advantages of Signal Modulation
- Amplitude Modulation
- Amplitude Modulation of Digital Signals
- Reducing Power and Bandwidth of AM Signals
- Frequency Modulation
- Power and Bandwidth of FM Signals
- Conclusion

Introduction

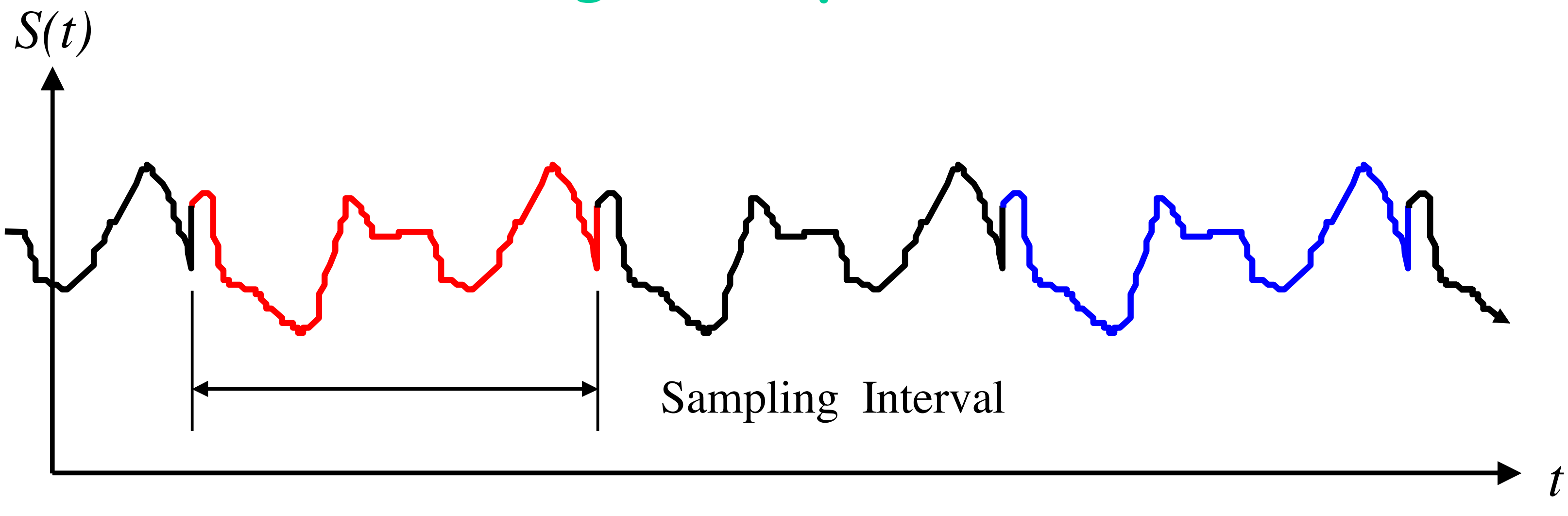
(Signals)

- The tasks of communication is to encode information as a **signal** level, transmit this **signal**, then decode the **signal** at the receiver.
- An **analog signal** varies continuously with time, and has an infinite number of possible signal levels.
- A **discrete signal** changes only once during a certain time interval. The signal value during this time interval (sampling period) is one sample. Each sample can occupy a infinite number of possible levels.
- A **digital signal** is discrete, but each sample has a finite number of possible signal levels. The limited number of levels means that each sample transmits a single information. It also means that each sample can be represented as digital data, a string of ones and zeroes.
- A digital signal is preferred in computer communications because computers already store and process information digitally.



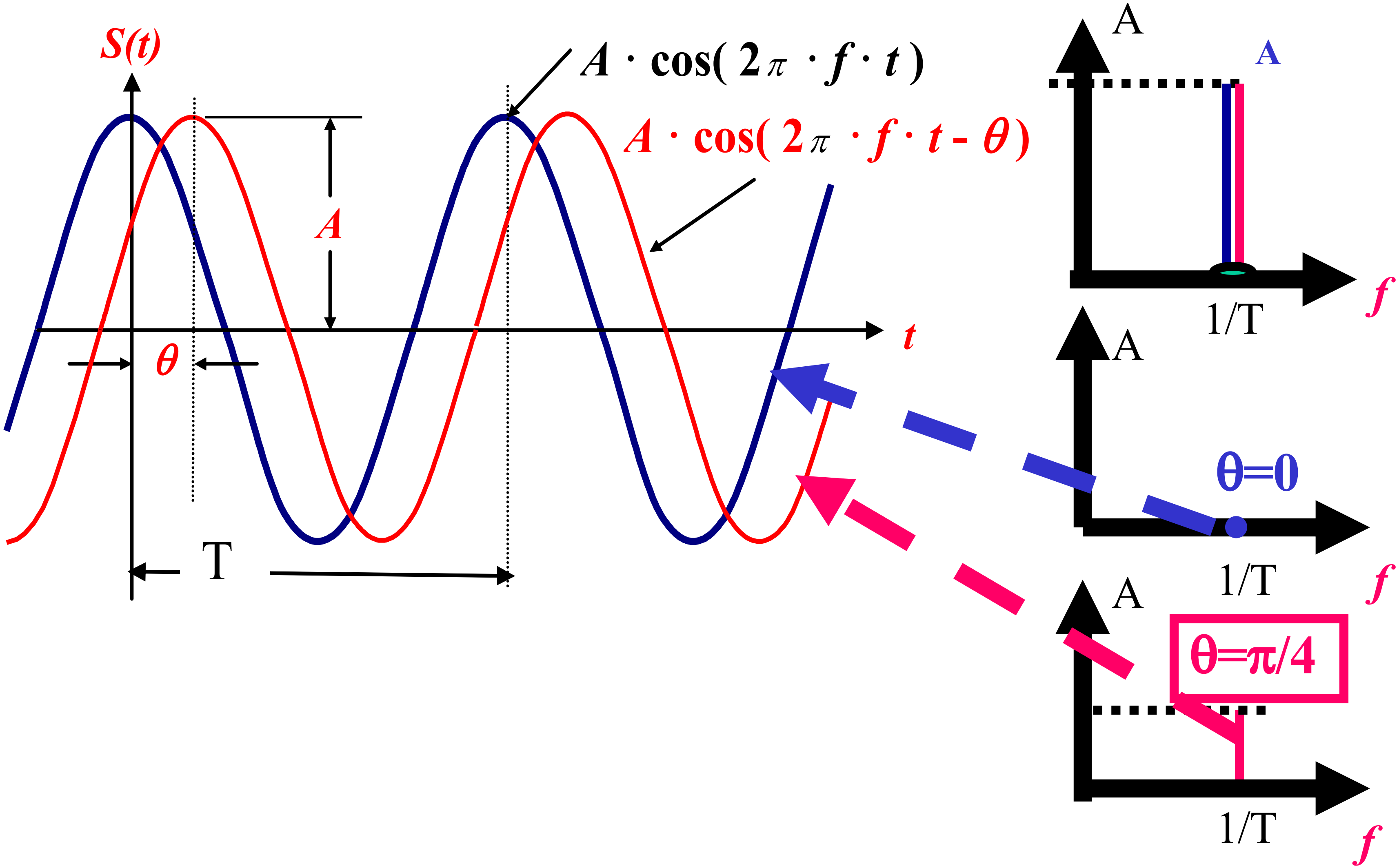
- In an **analog signal** noise are added to the signal during transmission, and signal processing. The received signal will never be identical to the original signal.
- A **discrete signal** changes only once during some time interval. The signal value during this time interval (sampling period) is one sample. Each sample can occupy a infinite number of possible levels.
- A **digital signal** will still be subject to noise, but the difference between signal levels (an "0101" and an "0110") will be sufficiently small, so that the receiver can always determine the original signal level. The regenerated, received digital data is an exact replica of the original digital data.
- **Analog to digital conversion** reduces the amount of information of the signal by approximating the analog signal with a digital signal. (Reading 1).

Signal Properties



$$S(t) = S(t+T); \quad -\infty < t < +\infty$$

- A periodic signal satisfies the condition:
- "**Period**" of the signal.
- "**Aperiodic**." signals
- An **even function** S *(t)* = S *(-t)*
- "**Symmetric**".
- The **phase** of the signal.


S(t)
A · cos(2π · f · t)
A · cos(2π · f · t - θ)
A
t
θ
T
A
A
1/T
f
A
θ=0
1/T
f
A
θ=π/4
1/T
f

Digital signals in the time and the frequency domain.

- The time domain is the signal level expressed as a function of time.
- The frequency domain is comprised of amplitude and a phase for an infinite number of sinusoidal functions. Signals in the time domain correspond to the superposition of an infinite number of sinusoidal waveforms in the time domain.
- To convert signal from the time domain to the frequency domain, we take the **Fourier transform** of the time domain representation.
- The frequency domain representation of a signal does not change with time, but the Fourier transform for the signal over an infinitely long time period would be impossible. Instead, we assume that the signal has some finite duration, over a **sampling interval**:

1. We assume that outside of the sampling interval, the signal repeats itself, so that the signal is **periodic.**
2. An alternative assumption is that the time-domain signal has a zero value outside of the sampling interval, so that the signal is **aperiodic**.

Fourier Series Expansion of Periodic Signals

a. **There are a finite number of discontinuities in the period T.**

b. It has a finite average value for the period T.

c. It has a finite number of posit. and negat. Max. in the period T.

$$S(t) = \sum_{n=0}^{\infty} a_n \cos(2\pi \cdot n \cdot f_0 \cdot t) + \sum_{n=1}^{\infty} b_n \sin(2\pi \cdot n \cdot f_0 \cdot t)$$

$$S(t) = a_0 + \sum_{n=1}^{\infty} c_n \cdot \cos\left(2\pi \cdot f_0 \cdot n \cdot t - \theta_n\right)$$

$$a_0 = \frac{1}{T_0} \int_0^{T_0} S(t)\,dt$$

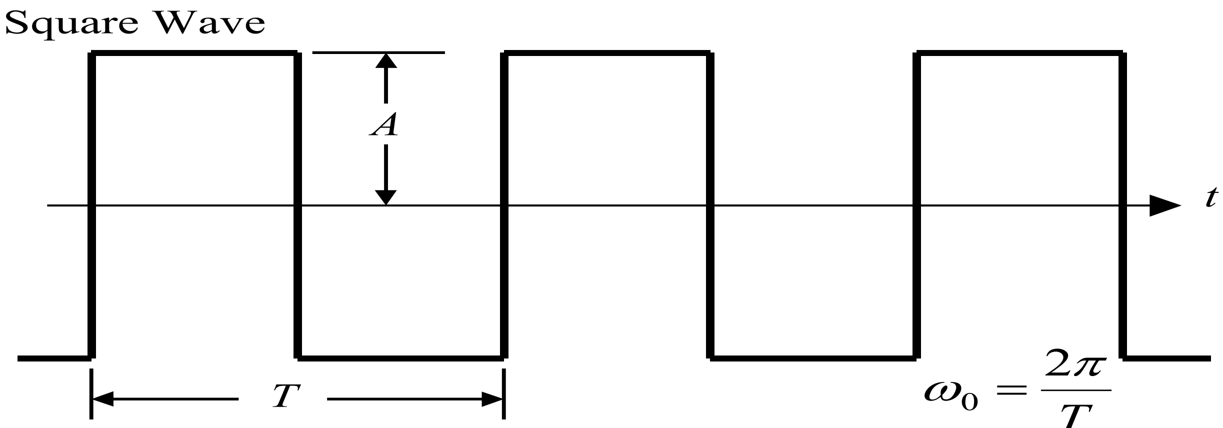
$$a_n = \frac{2}{T_0} \int_0^{T_0} S(t) \cos(2\pi \cdot f_0 \cdot n \cdot t)\,dt$$

$$b_n = \frac{2}{T_0} \int_0^{T_0} S(t) \sin(2\pi \cdot f_0 \cdot n \cdot t)\,dt$$

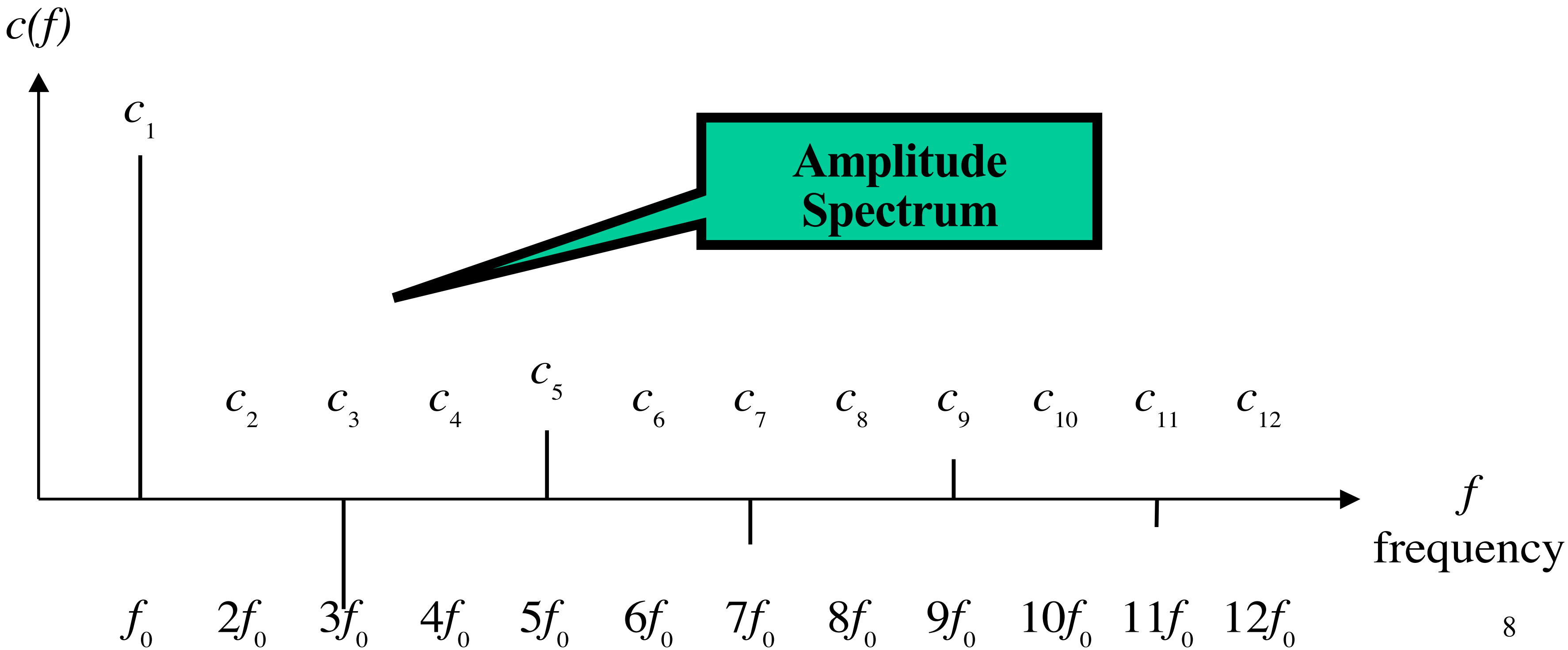
f_0 -Fundamental Frequency

$n \cdot f_0$ -*Frequency of each term*

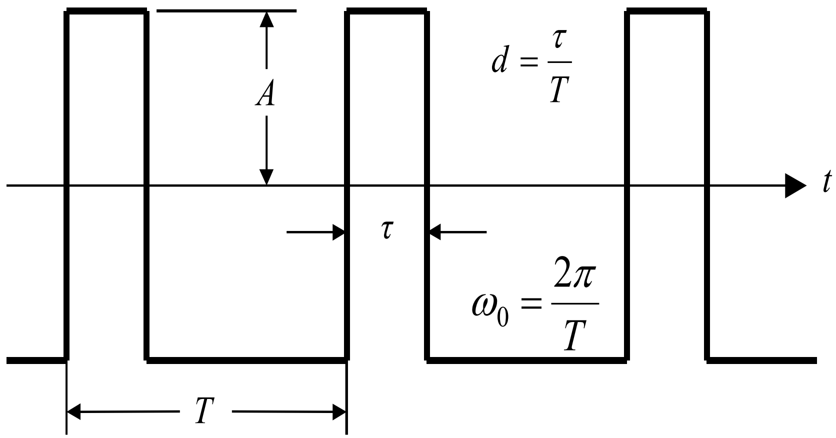
a_n, b_n- *Fourier series coefficients:*



$$S_m(t) = \frac{5\mathrm{V}\cdot 4}{\pi}\left(\cos\left(2\pi f_m t\right) - \frac{1}{3}\cos\left(6\pi f_m t\right) + \frac{1}{5}\cos\left(10\pi f_m t\right) - \ldots \right)$$



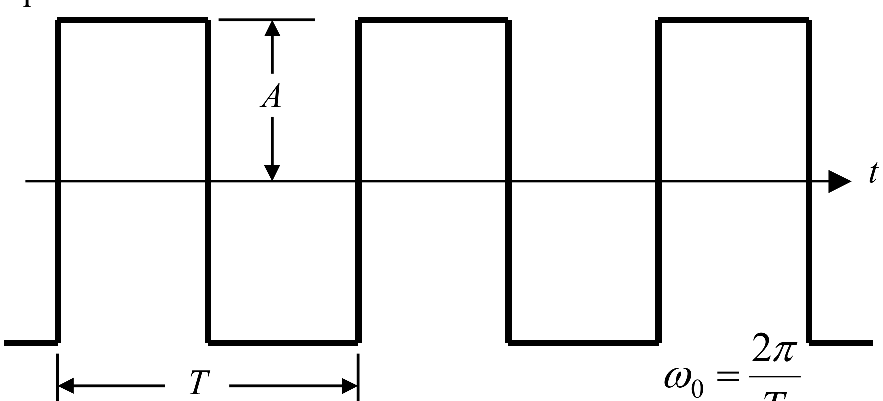
Rectangular wave with duty cycle d



$$d = \frac{\tau}{T} \qquad \omega_0 = \frac{2\pi}{T}$$

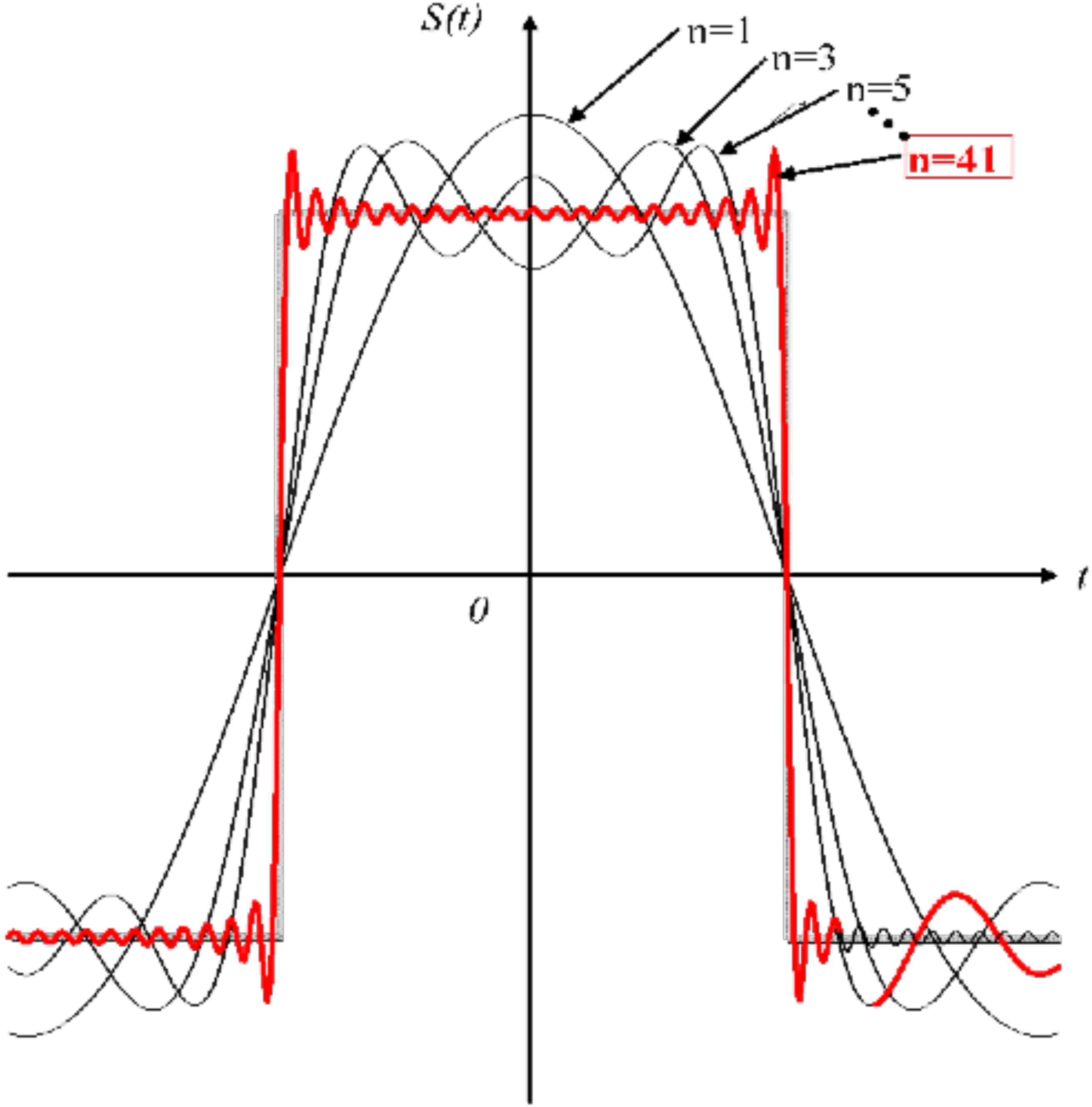
$$A\left(2d-1\right) + \frac{4A}{\pi}\left((\sin \pi d)\cos \omega_0 t + \frac{\sin 2\pi d}{2}\cos 2\omega_0 t + \frac{\sin 3\pi d}{3}\cos 3\omega_0 t + \ldots + \frac{\sin n\pi d}{n}\cos n\omega_0 t \right)$$

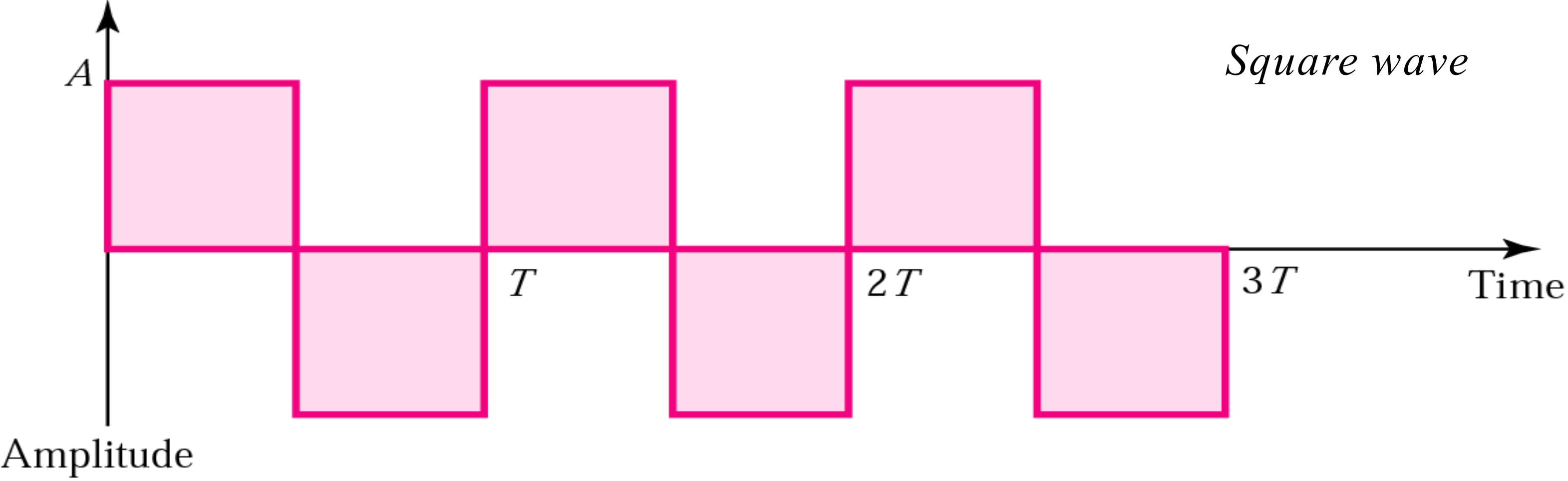
Square Wave

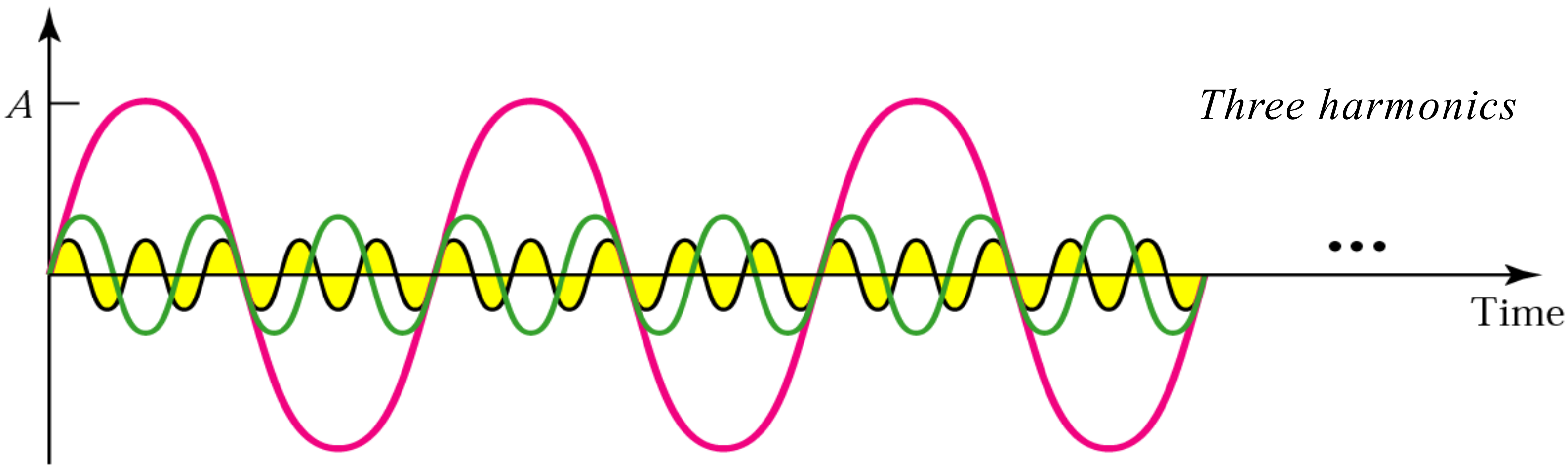


$$\omega_0 = \frac{2\pi}{T}$$

$$\frac{4A}{\pi}\left(\cos \omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos 5\omega_0 t - \frac{1}{7}\cos 7\omega_0 t + \frac{1}{9}\cos 9\omega_0 t + \ldots + \frac{1}{n}\cos n\omega_0 t \right)$$


S(t)
n=1
n=3
n=5
n=41
0
t


Square wave
A
T
2T
3T
Time
Amplitude


Three harmonics
A
Time


Signal corruption
Transmission medium
Input signal
Output signal

RMS Voltage Values and Power Spectrum

- By the definition RMS voltage is the square root of the average value of the voltage squared, taken over one period of the signal:

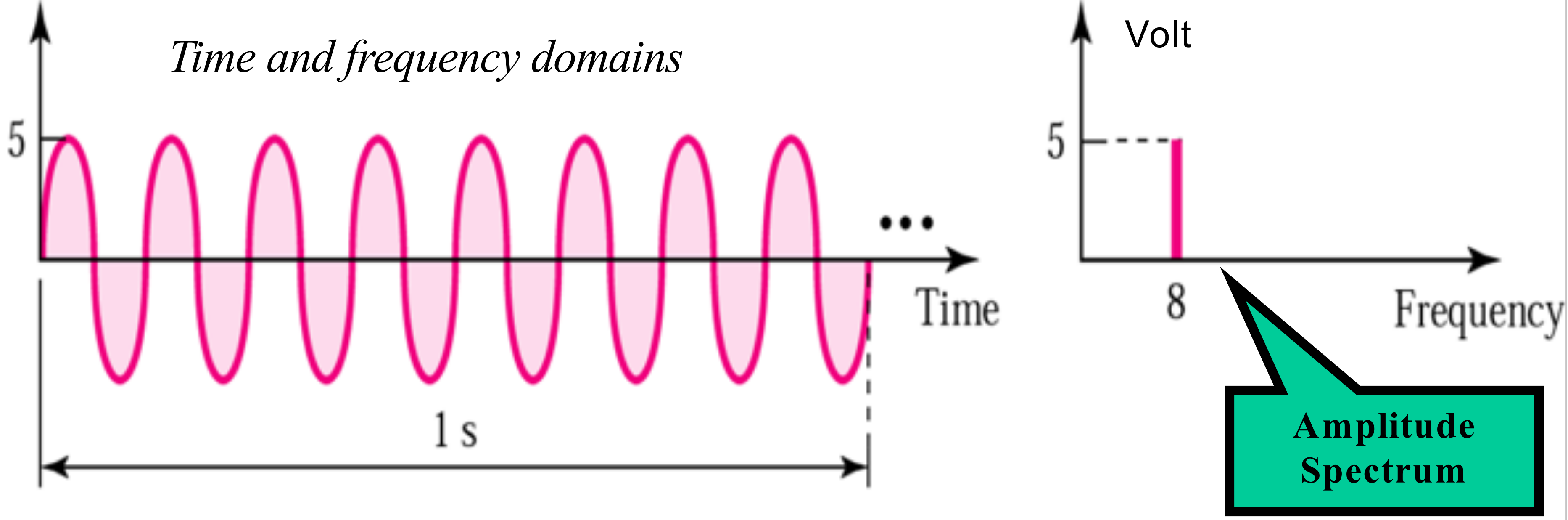
$$\boxed{V_{RMS} = \sqrt{\frac{1}{T}\int_0^T [S(t)]^2\, dt}}$$

For a sinusoidal signal:

$$\boxed{V_{RMS} = \sqrt{\frac{1}{T}\int_0^T [S(t)]^2\, dt} = \sqrt{\frac{1}{T}\int_0^T [V_m]^2[\cos(\omega t-\theta)]^2\, dt}}$$

$$\boxed{V_{RMS} = V_m\sqrt{\frac{1}{T}\int_0^T \frac{1+\cos(2\omega t-2\theta)}{2}\, dt} = \frac{V_m}{\sqrt{2}}}$$

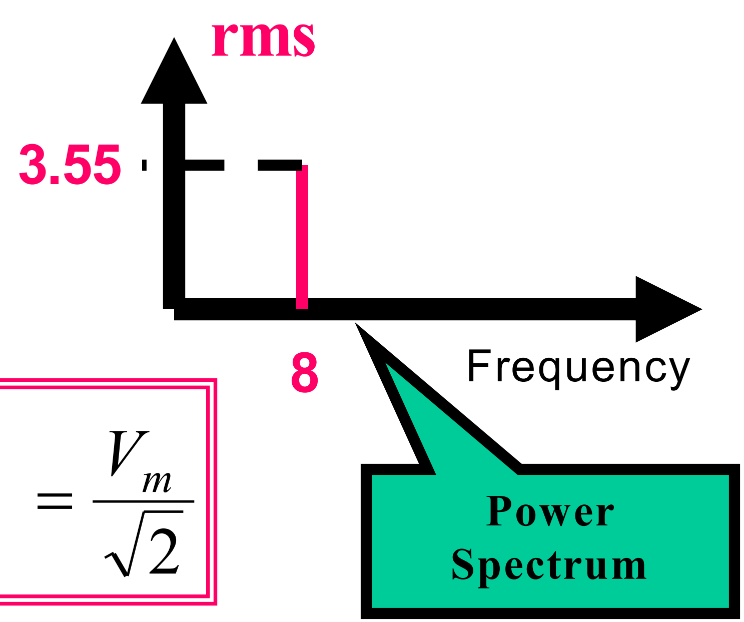
$$\left(\text{because } \int_0^T \frac{1}{2}\, dt = \frac{T}{2} \text{ and } \int_0^T \cos(2\omega t-2\theta)\, dt = 0\right)$$



b. A signal with frequency 8

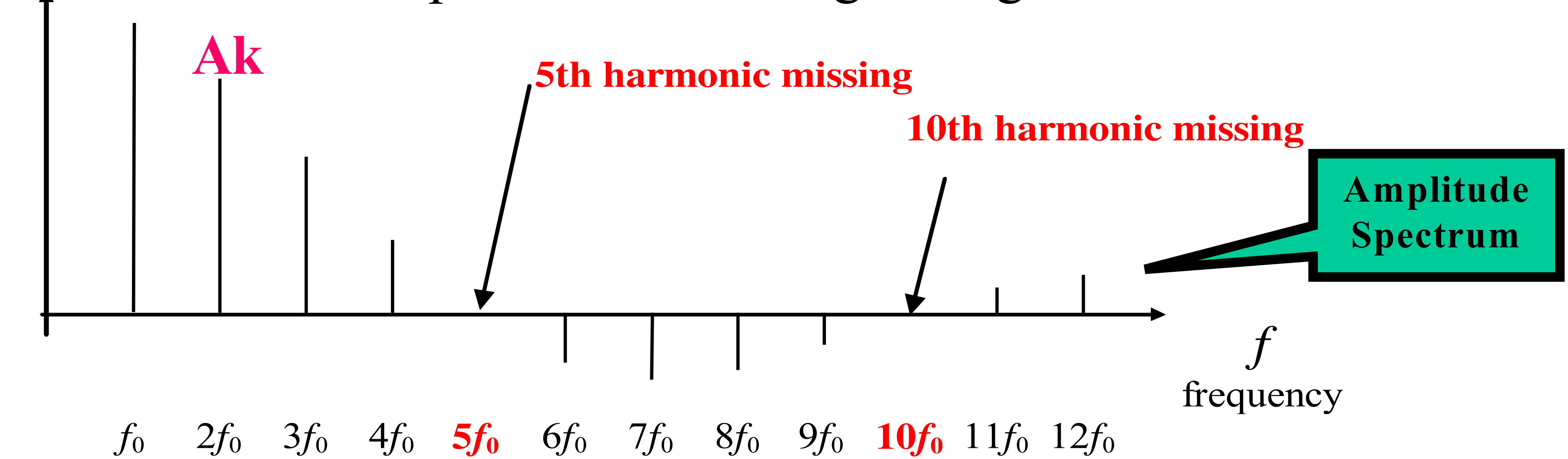
$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T [S(t)]^2 \, dt}$$

$$V_{RMS} = V_m \sqrt{\frac{1}{T}\int_0^T \frac{1+\cos(2\omega t - 2\theta)}{2} dt} = \frac{V_m}{\sqrt{2}}$$



c(f) V

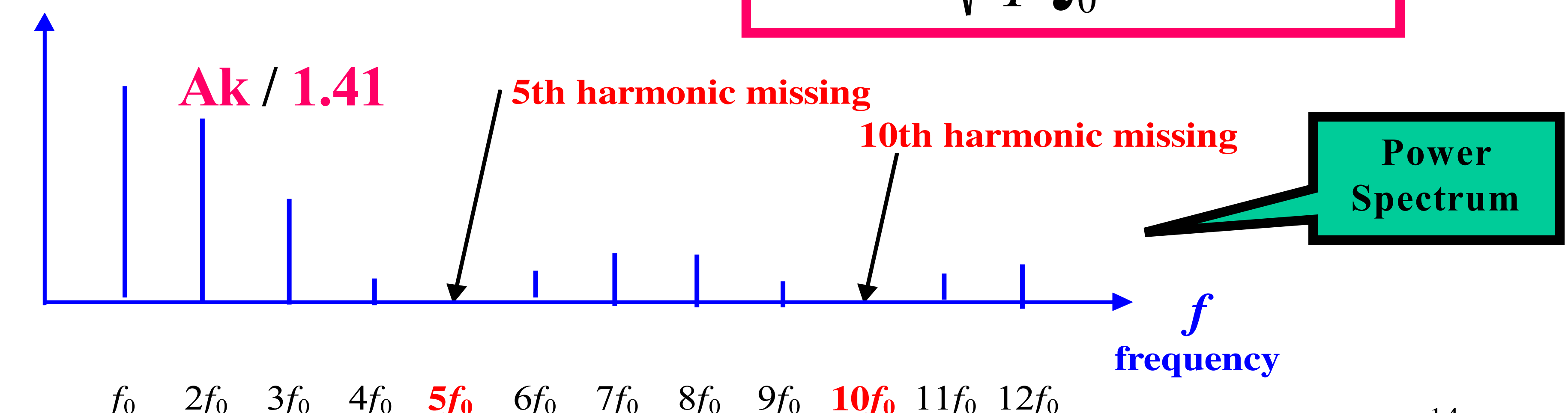
Fourier spectrum for rectangular signal is:



By the definition RMS

$$\boxed{V_{RMS} = \sqrt{\frac{1}{T}\int_0^T \left[S(t)\right]^2\, dt}}$$

c(f) rms



According to Fourier analysis, any composite signal can be represented as a combination of simple sine waves with different frequencies, phases, and amplitudes.

Introduction

A network and transport layers basic services comprise the **end-to-end transport** of the bit streams over a set of interconnected **routers (switches).**

They are produced using six basic mechanisms:

a. **Multiplexing**

b. Switching

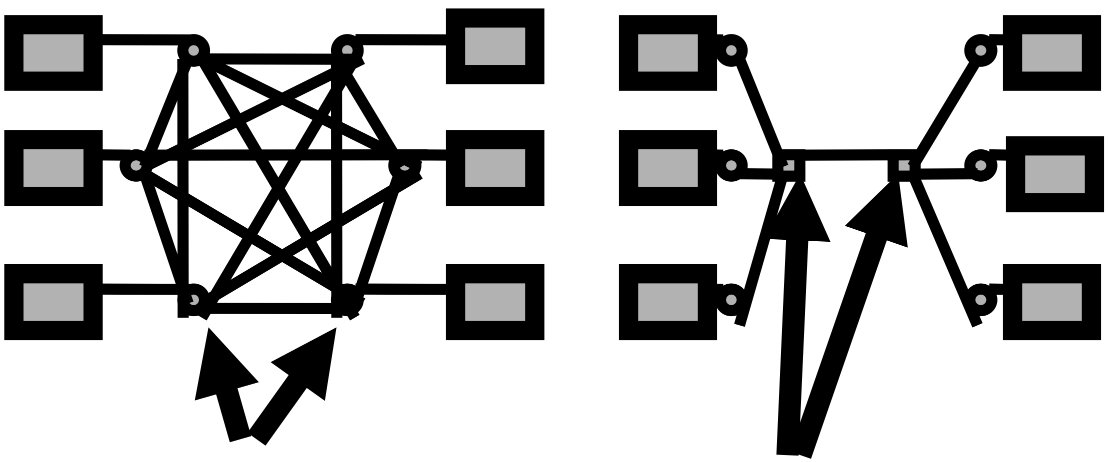
c. Error control

d. flow control

e. congestion control

f. and resource allocation.

Multiplexing Combines data streams of many users into one large bandwidth stream. Users can share communication medium. (Bandwidth is a frequency band that contains more than 50% power of the signal full power)



a. Switch b. Multiplexer/Demultiplexer

a. Fully connected network
b. network with shared links.

Signal Modulation is:

- The process of encoding a **baseband (data)**, source signal $S_m(t)$ onto a **carrier** signal.
- The carrier waveform is varied in a manner related to the baseband (Data) signal.
- The carrier can be a sinusoidal signal (or a pulse signal). The result of modulating the carrier signal is called the **modulated** signal.

Signal Modulation

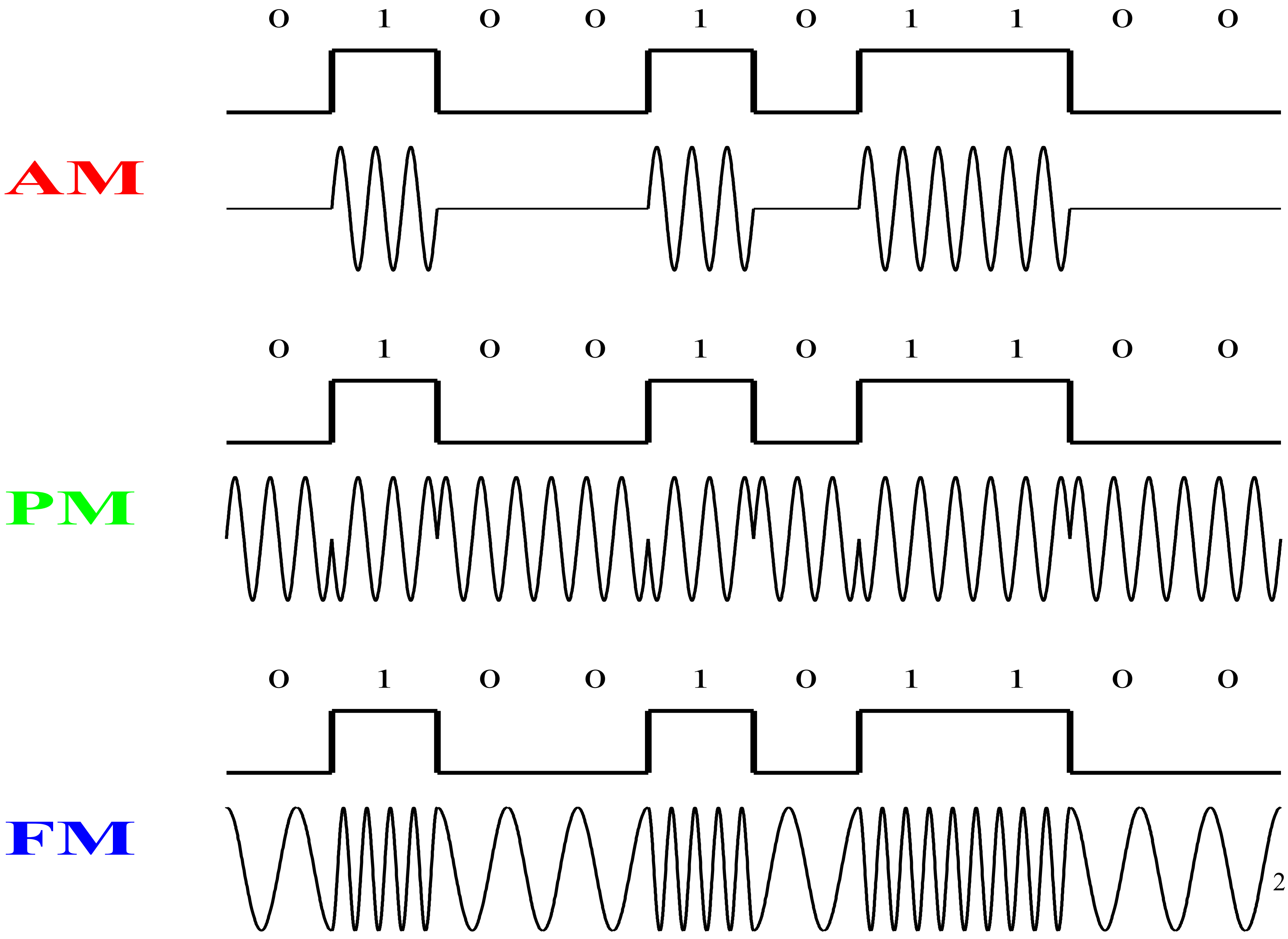
$$S_c(t) = A \cdot \cos(2\pi f t + \phi)$$

A. Amplitude Modulation **(AM),** or Amplitude Shift Keying **(ASK):** The amplitude **A** of the carrier signal changes in proportion to the baseband signal.

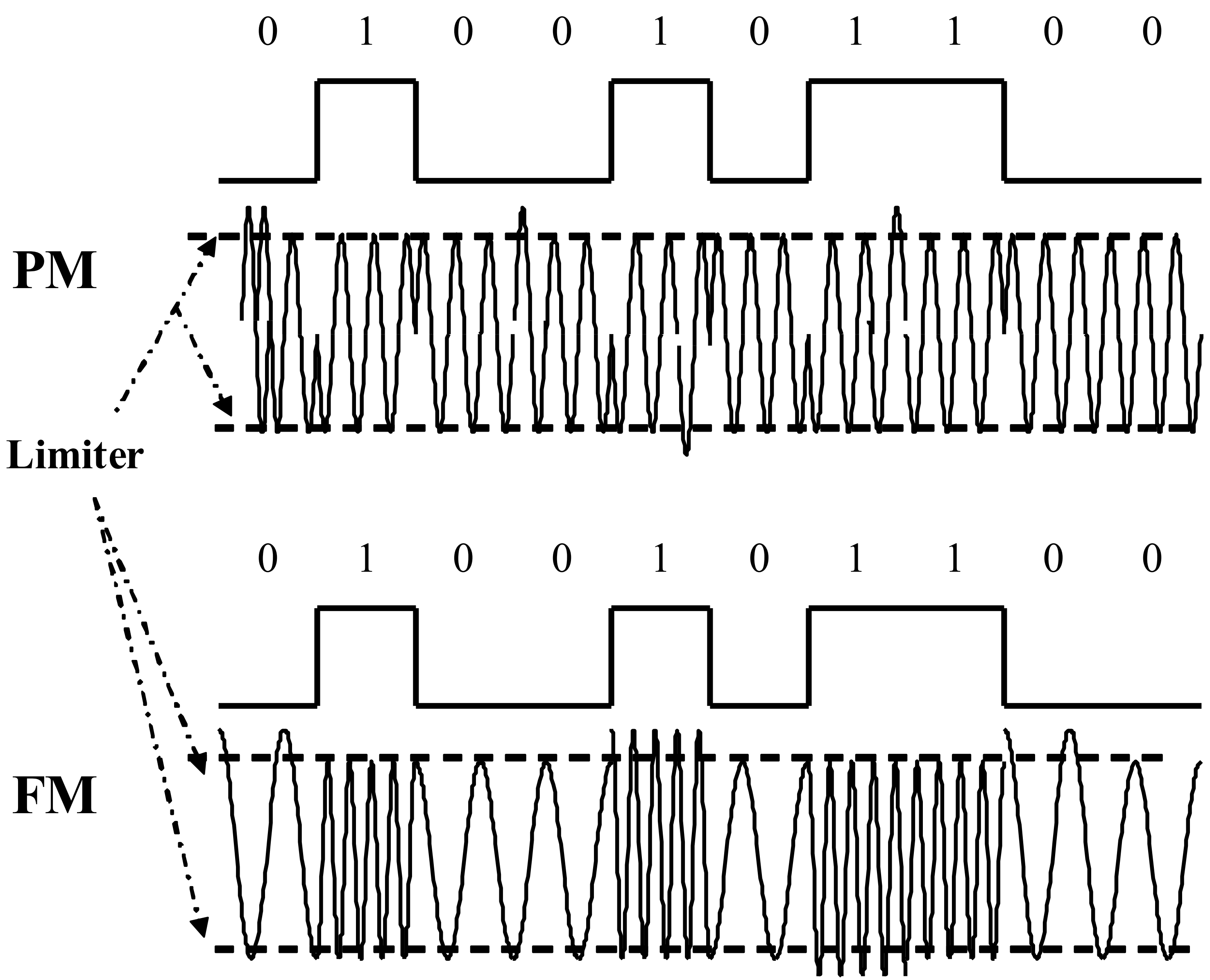
B. Frequency Modulation **(FM),** or Frequency Shift Keying **(FSK):** The frequency f of the carrier changes in proportion to the baseband signal.

C. Phase Modulation **(PM),** or Phase Shift Keying **(PSK):** The phase ϕ of the carrier signal changes in proportion to the baseband signal.

Amplitude, Phase, and Frequency Modulation by digital baseband signal



PM and FM with additive noise



Advantages of Signal Modulation

- Radio transmission of the signal.

$$f = 1000\frac{1}{\text{s}} \; ; \quad \lambda = \frac{c}{f} = \frac{3\times10^{8}\,\dfrac{\text{m}}{\text{s}}}{1000\,\dfrac{1}{\text{s}}} = 3\times10^{5}\,\text{m}$$

$$Sc\,(\,t\,) = A\cdot \cos\,(\,2\pi fc\,)$$

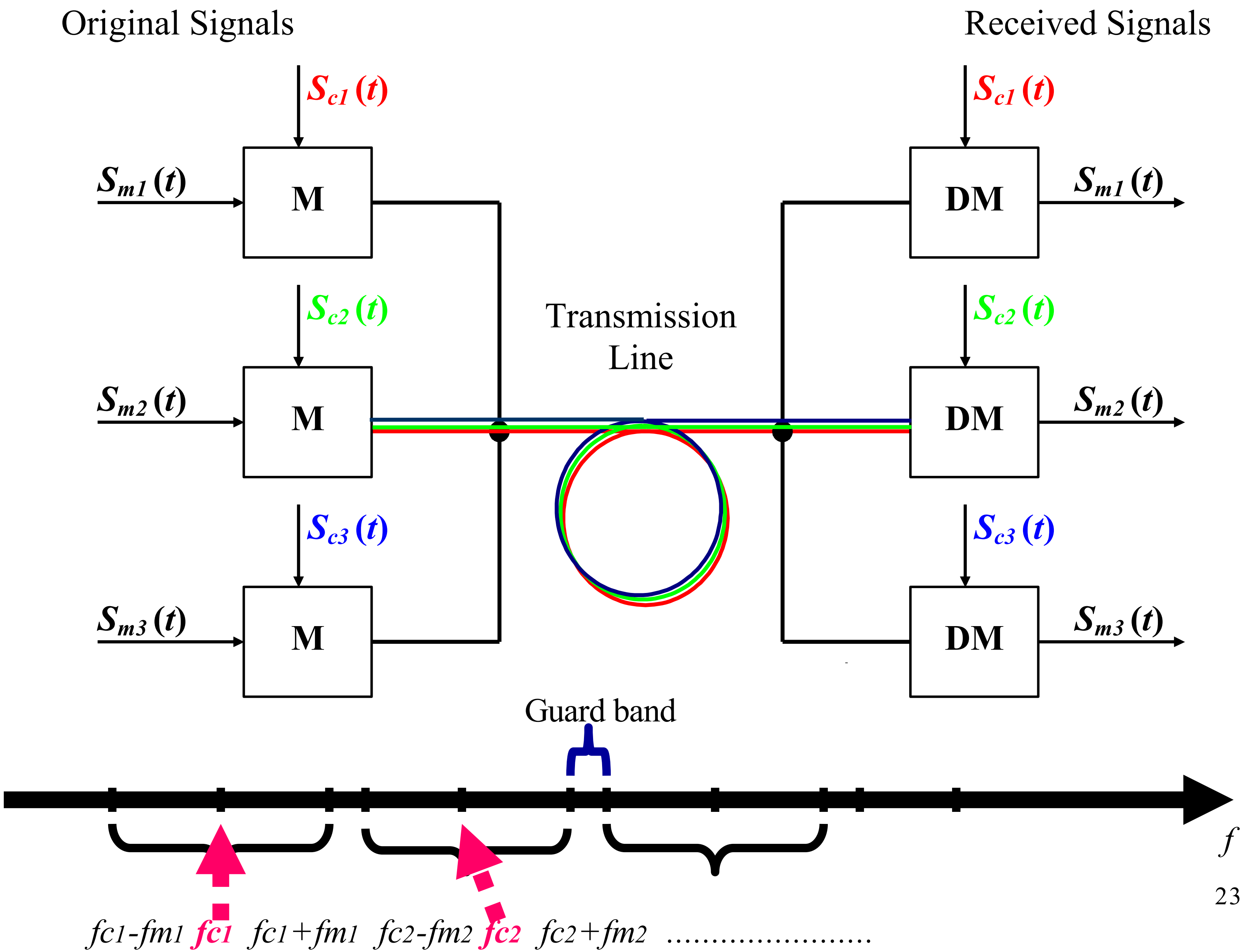
Frequency range of modulated is (fc + f) and (fc - f).

$$fc = 10\times10^{6}\,\frac{1}{\text{s}} \; ; \quad \lambda = \frac{c}{f} = \frac{3\times10^{8}\,\dfrac{\text{m}}{\text{s}}}{10\times10^{6}\,\dfrac{1}{\text{s}}} = 30\,\text{m}$$

For 5GHz* frequency *Wavelength =6 cm

FDM WDM

FDM


Original Signals
Received Signals
$S_{c1}(t)$
$S_{m1}(t)$
M
$S_{c2}(t)$
$S_{m2}(t)$
M
$S_{c3}(t)$
$S_{m3}(t)$
M
Transmission Line
$S_{c1}(t)$
DM
$S_{m1}(t)$
$S_{c2}(t)$
DM
$S_{m2}(t)$
$S_{c3}(t)$
DM
$S_{m3}(t)$
Guard band
f
fc1-fm1 fc1 fc1+fm1 fc2-fm2 fc2 fc2+fm2 ....

Amplitude Modulation by Digital Signals

AM is multiplication of the baseband signal with the carrier signal.

$$\boxed{S_c(t) = A_c \cdot \cos(2\pi f_c t)} \quad \mathbf{X} \quad \boxed{\mathbf{Sm}(t)}$$

Baseband Digital Signal

$$\boxed{S_m(t) = \frac{5\text{V} \cdot 4}{\pi}\left(\cos\left(2\pi f_m t\right) - \frac{1}{3}\cos\left(6\pi f_m t\right) + \frac{1}{5}\cos\left(10\pi f_m t\right) - \ldots \right)}$$

Sinusoidal Carrier Signal

$$\boxed{S_c(t) = 5\text{V} \cdot \cos\left(2\pi f_c t\right)}$$

Baseband Digital Signal with DC shift

$$\boxed{S_m(t) = 5\text{V} + \frac{5\text{V} \cdot 4}{\pi}\left(\cos\left(2\pi f_m t\right) - \frac{1}{3}\cos\left(6\pi f_m t\right) + \frac{1}{5}\cos\left(10\pi f_m t\right) - \ldots \right)}$$

Transmitted Signal

$$S(t) = \frac{25V^2}{k}\cos(2\pi f_c t)$$

carrier frequency

$$+\frac{15.92V^2}{k}\left[\cos\left(2\pi\left(f_c - f_m\right)t\right) - \frac{1}{3}\cos\left(2\pi\left(f_c - 3f_m\right)t\right) + \frac{1}{5}\cos\left(2\pi\left(f_c - 5f_m\right)t\right) - \ldots\right]$$

lower sideband

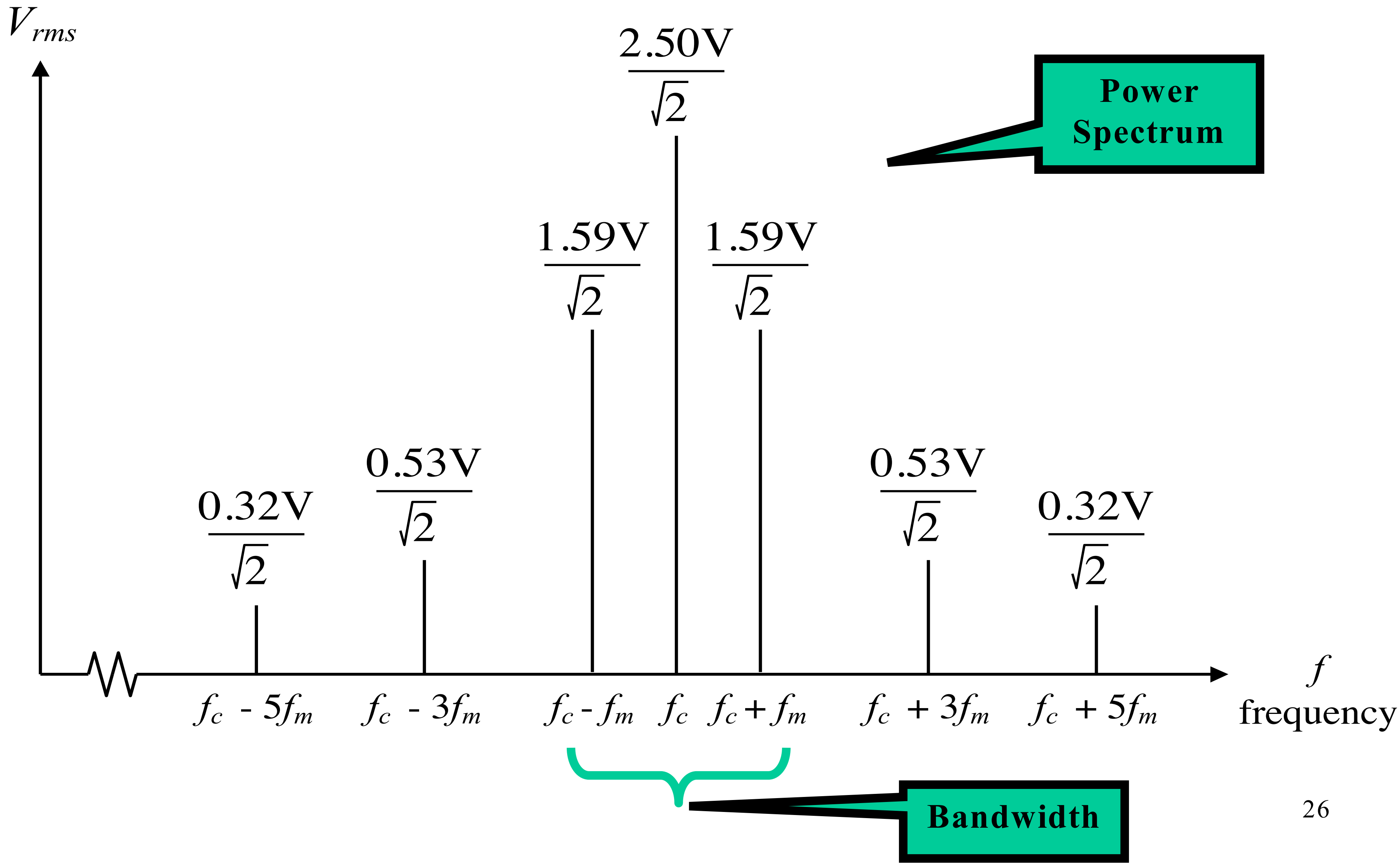
$$+\frac{15.92V^2}{k}\left[\cos\left(2\pi\left(f_c + f_m\right)t\right) - \frac{1}{3}\cos\left(2\pi\left(f_c + 3f_m\right)t\right) + \frac{1}{5}\cos\left(2\pi\left(f_c + 5f_m\right)t\right) - \ldots\right]$$

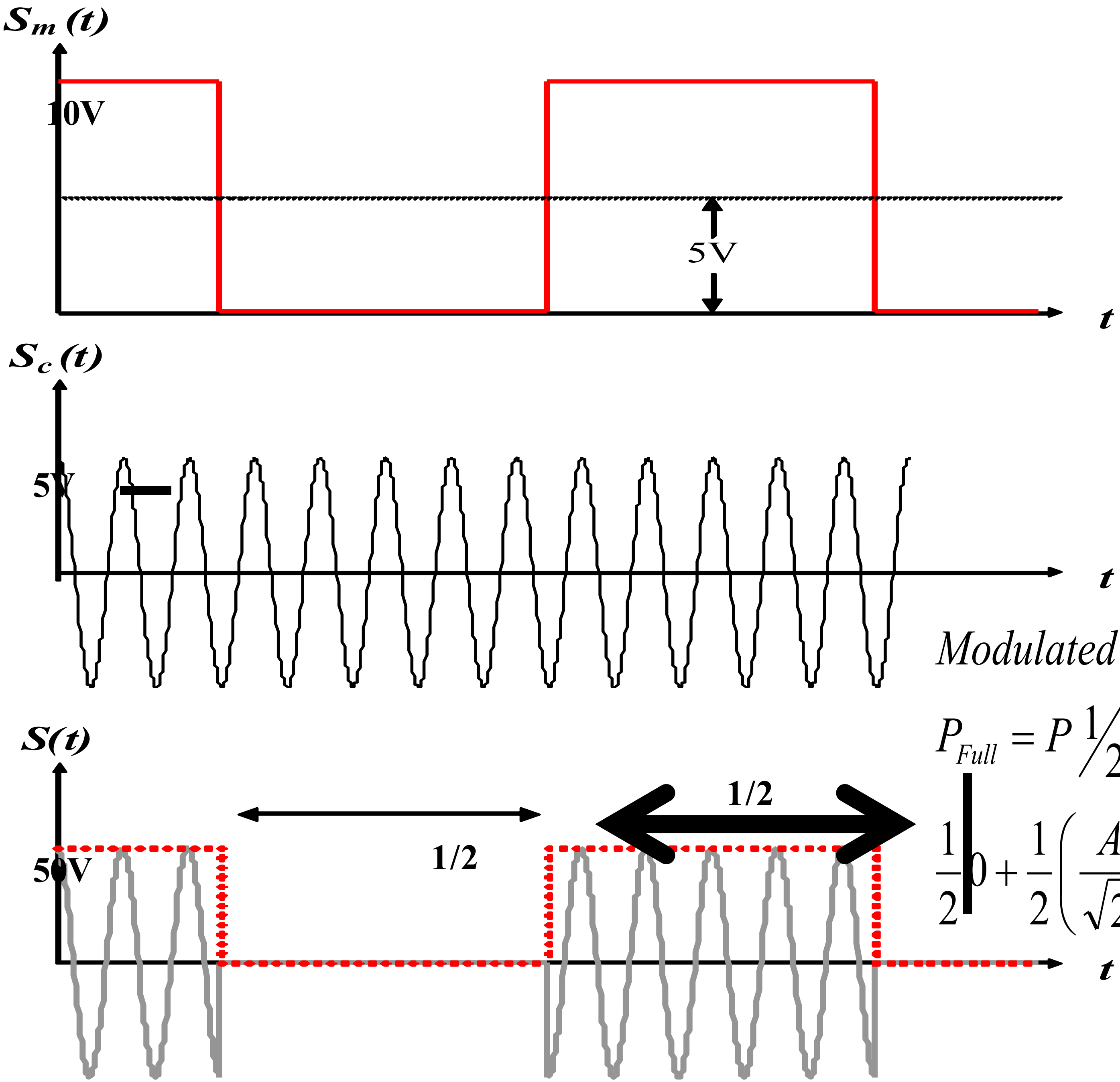
upper sideband

In the above formula is used the trigonometrical identity:

$$\boxed{\cos(x)\cdot\cos(y) = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]}$$

DSBTC AM signal with square wave baseband signal, i = 100%, k = 10.



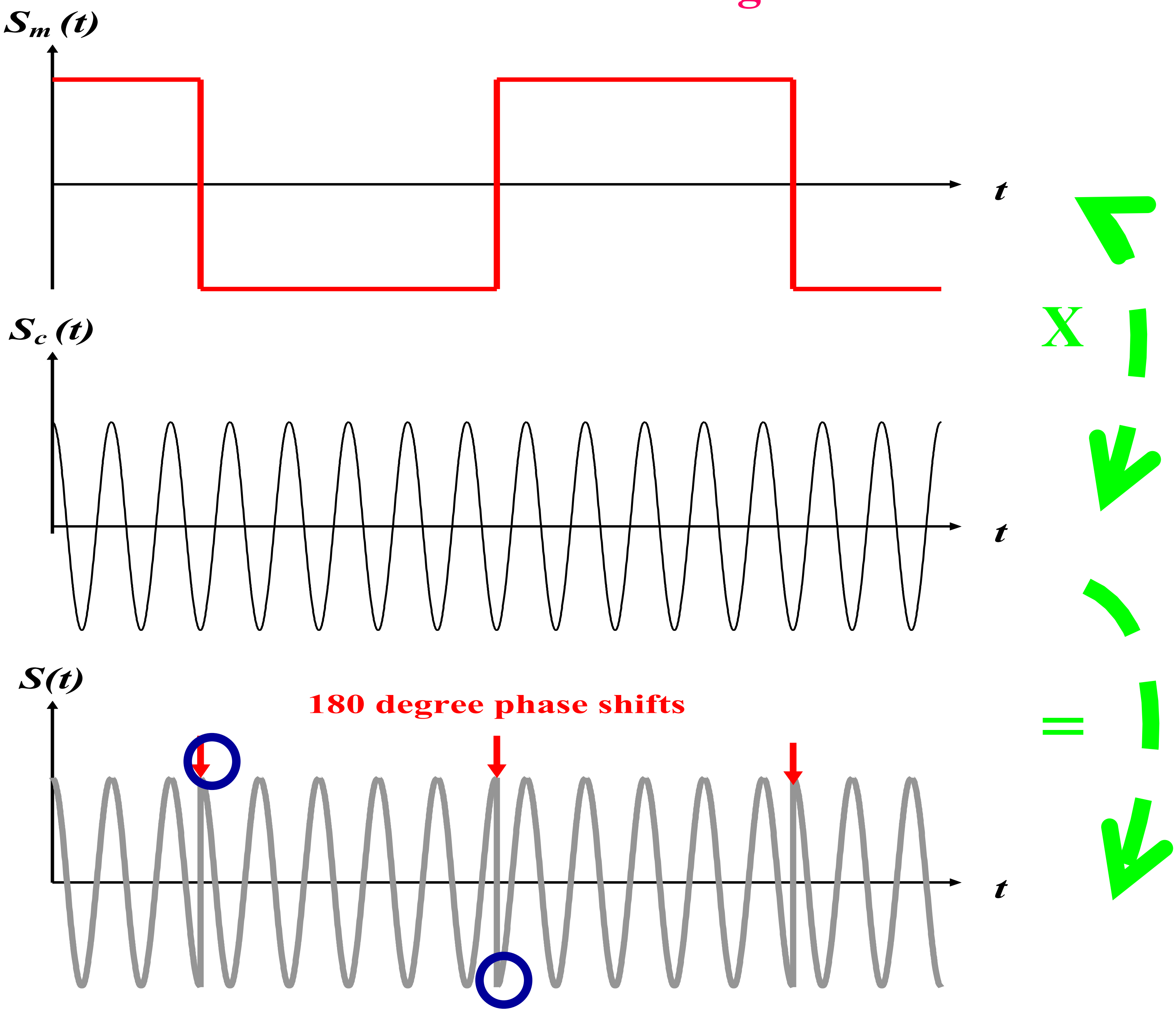


Modulated signal Power

$$P_{Full} = P\,{}^{1}\!/_{2} + P\,{}^{1}\!/_{2} =$$

$$\frac{1}{2}0 + \frac{1}{2}\left(\frac{A}{\sqrt{2}}\right)^2 = 0 + \frac{1}{4}(50)^2 = 625$$

If DC offset of the data signal =0



Binary Phase Shift Keying and Quadrature Phase Shift Keying

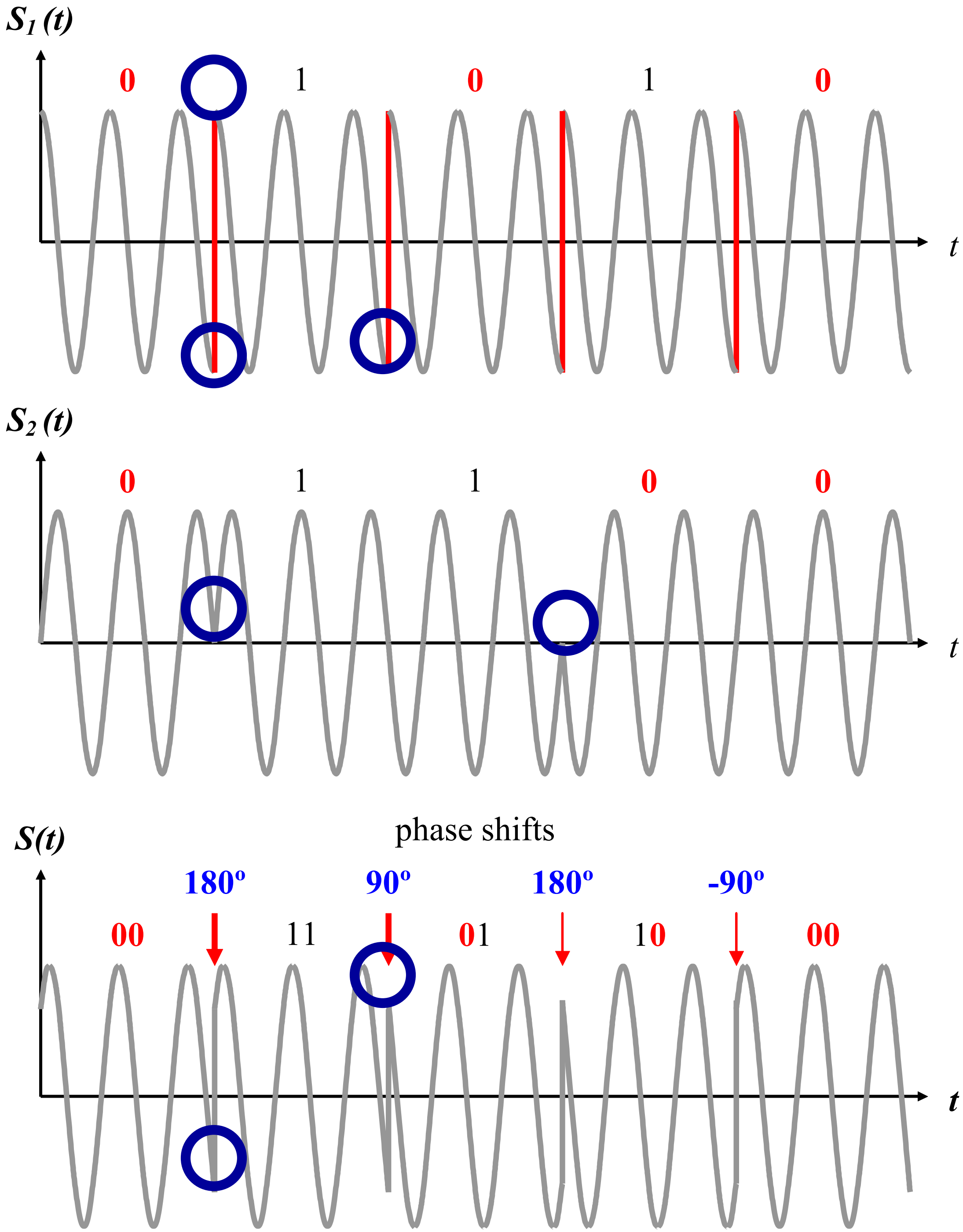
$$\boxed{\begin{aligned} S(t) &= \frac{25\text{V}}{k} \cdot \cos\left(2\pi f_c t + \phi(t)\right) \\ \phi(t) &= \begin{cases} 0^\circ & \text{if } S_m(t) = -5\text{V} \\ 180^\circ & \text{if } S_m(t) = +5\text{V} \end{cases} \end{aligned}}$$

Since the phase of the transmitted signal can take one of two values (phase jumping), this type of modulation is called binary phase-shift keying (BPSK). It is a constant-amplitude method of modulation.

If we add together two BPSK signals that are offset by a 90 degree phase shift, there are four possible phase jumping during every sampling period. Instead of one possible shift of 180º, there are three possible transitions: +90º, 180º, and -90º (+270º). The signal can be represented by the following formula:

$$S(t) = \frac{25\text{V}}{k} \cdot \cos\left(2\pi f_c t - \phi(t)\right)$$

$$\phi(t) = \begin{cases} 45^\circ & \text{if } S_{m1}(t) = -5\text{V},\ S_{m2}(t) = -5\text{V} \\ 135^\circ & \text{if } S_{m1}(t) = +5\text{V},\ S_{m2}(t) = -5\text{V} \\ 225^\circ & \text{if } S_{m1}(t) = +5\text{V},\ S_{m2}(t) = +5\text{V} \\ 315^\circ & \text{if } S_{m1}(t) = -5\text{V},\ S_{m2}(t) = +5\text{V} \end{cases}$$



Time domain representation of QPSK signal, created: by combining BPSK signals *S1* and *S2* .

Concept 1:

The total bandwidth required for AM can be determined from the bandwidth of the audio signal:

$$BWt = 2 \times BWm.$$

Example

We have an audio signal with a bandwidth of 4 KHz. What is the bandwidth needed if we modulate the signal using AM? Ignore FCC regulations.

Solution

An AM signal requires twice the bandwidth of the original signal:

$$BW = 2 \times 4 \text{ KHz} = 8 \text{ KHz}$$

Frequency Modulation

- Frequency modulation is a special case of phase modulation, **where the transmitted signal phase accumulation changes frequency in accordance with the amplitude of the baseband signal:**

$$S(t) = A_c \cdot \cos\left(2\pi f_c t + \boxed{\int_{-\infty}^{t} K_{FM} \cdot S_m(\xi)\, d\xi} \right)$$

K_{FM}: Constructional coefficient of the modulator

- Additive noise

To find the maximum possible frequency deviation of an FM signal, we can assume the baseband signal is a constant voltage of **+*Am*** , then the integral will reduced to a linear function of *t* :

$$S(t) = A_c \cdot \cos\left(2\pi f_c t + k \cdot A_m \cdot t\right) = A_c \cdot \cos\left[2\pi \cdot \left(f_c + \frac{k \cdot A_m}{2\pi}\right) \cdot t\right]$$

$$(\Delta f_c)_{\max} = \frac{k \cdot A_m}{2\pi}$$

$$\boldsymbol{Sm(t) = Am \cdot \sin(2\pi fm\, t)}$$

$$\phi(t) = \int_{-\infty}^{t} k \cdot S_m(\xi) d\xi = -k \cdot A_m \int_{0}^{t} \sin(2\pi f_m \xi) d\xi = \frac{k \cdot A_m}{2\pi f_m} \cos(2\pi f_m t)$$

$$k_f = \frac{k \cdot A_m}{2\pi \cdot f_m}$$

$$S(t) = A_c \cdot \cos\left[2\pi f_c t + k_f \cos(2\pi f_m t)\right]$$

- Equation can be expanded using Bessel's trigonometric identities into a series of phase-shifted sinusoidal terms.
- The amplitude of each term is determined by the Bessel function of the first kind $J_n(k_f)$, where k_f is the modulation index.

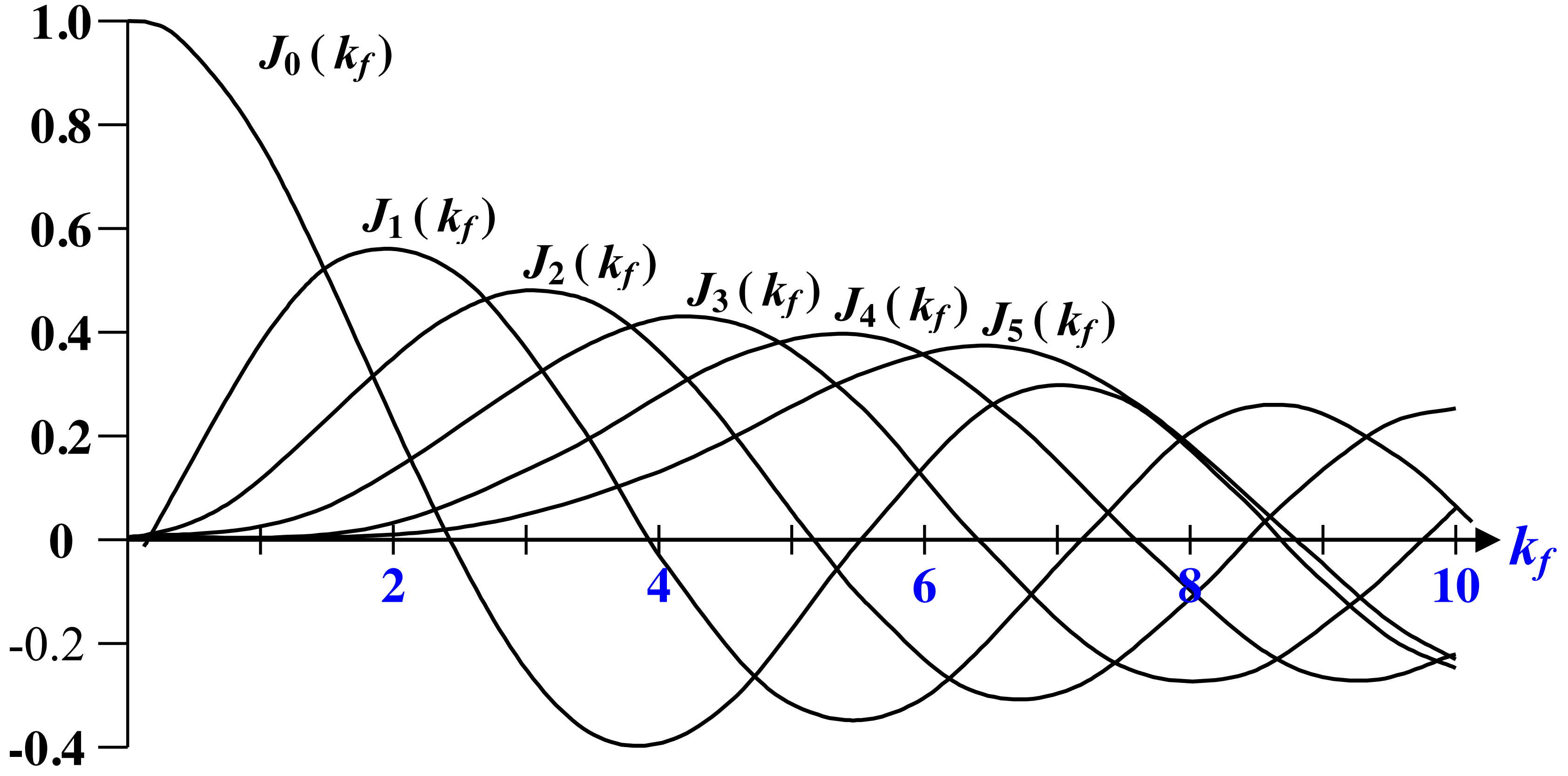
$$\boxed{\cos\left[2\pi f_c t + k_f \cos(2\pi f_m t)\right] = \sum_{n=-\infty}^{\infty} J_n(k_f)\cdot \cos\left(2\pi t\left(f_c + n f_m\right) + \frac{n\pi}{2}\right)}$$

$$\boxed{J_{-n}(x) = (-1)^n \cdot J_n(x)}$$

$$\boxed{S(t) = A_c \cdot \sum_{n=-\infty}^{\infty} J_n(k_f)\cdot \cos\left(2\pi t\left(f_c + n f_m\right) + \frac{n\pi}{2}\right)}$$

$$\boxed{\begin{aligned} S(t) &= A_c \cdot J_0(k_f)\cdot \cos\left(2\pi f_c t\right) \\ &+ A_c \cdot \sum_{n=1}^{\infty} J_n(k_f)\cdot \left[\cos\left(2\pi\cdot\left(f_c - n f_m\right)\cdot t + \frac{n\pi}{2}\right) \right. \\ &\qquad \left. + \cos\left(2\pi\cdot\left(f_c + n f_m\right)\cdot t + \frac{n\pi}{2}\right)\right] \end{aligned}}$$

First five Bessel coefficients for varying values of k_f .



Power and Bandwidth of FM Signals

- Calculating the power of an FM signal is much simpler in the time domain.

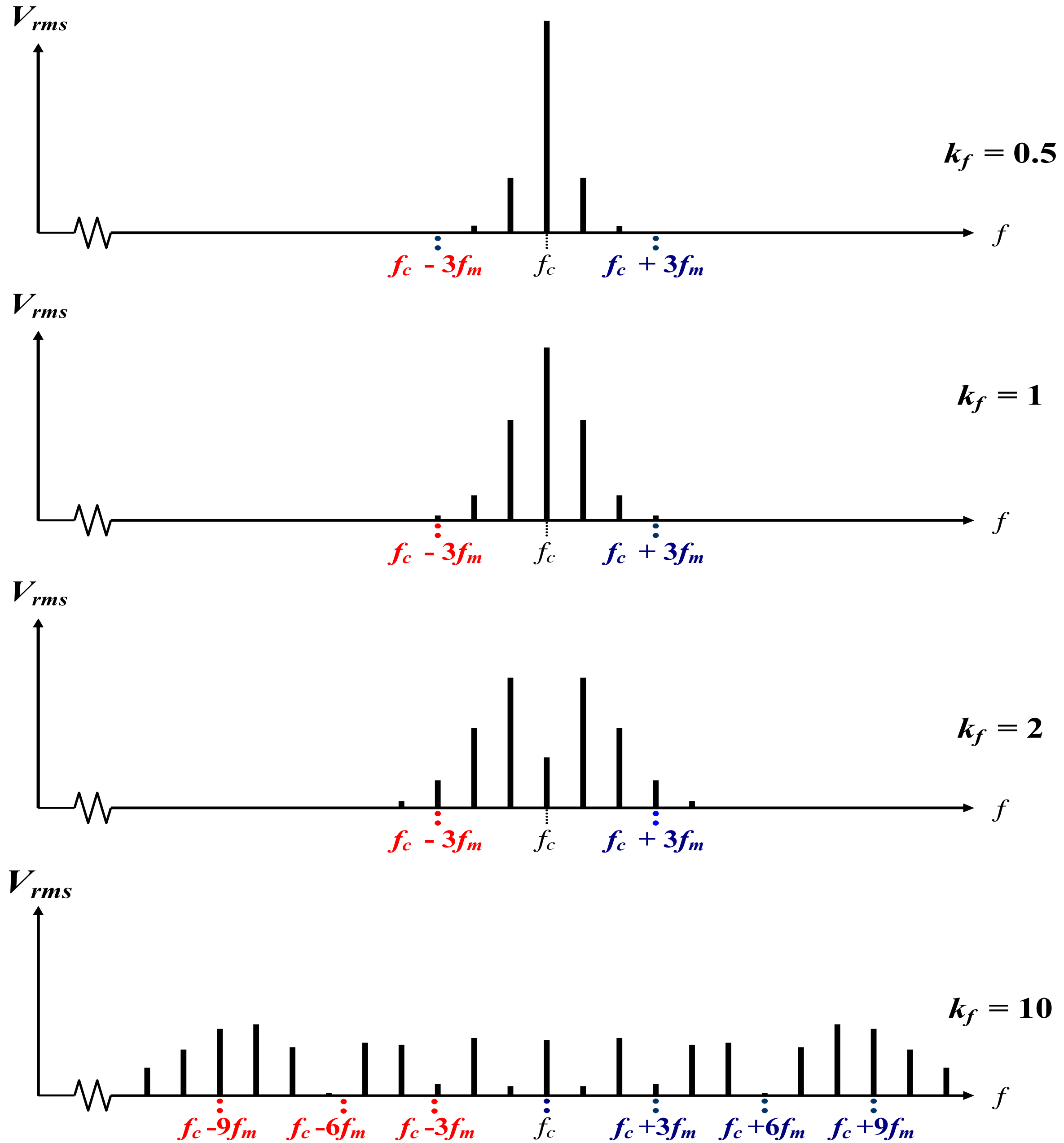
$$\boxed{S(t) = A_c \cdot \cos\left[\, 2\pi f_c t + k_f \cos(2\pi f_m t) \,\right]}$$

$$\boxed{P_{FM} = \frac{A_c^2}{2} = \frac{1}{2} A_c^2 \cdot \left[\, j_0^2(k_f) + 2 \cdot \sum_1^{\infty} j_n^2(k_f) \,\right]}$$

$$\boxed{B_T = 2 \cdot B \cdot \left(k_f + 1\right)}$$

In this equation **B is effective bandwidth of baseband signal**, but **BT is absolute bandwidth of FM signal**

$Ac \cdot \cos(2\pi fc + k_f \cos(2\pi fm))$ at various values of k_f.



Concept 2:

The total bandwidth required for FM can be determined from the bandwidth of the audio signal:
$BWt = 10 \times BWm$.

Example 1

We have an audio signal with a bandwidth of 4 MHz. What is the bandwidth needed if we modulate the signal using FM? Ignore FCC regulations.

Solution

An FM signal requires 10 times the bandwidth of the original signal:

$$BW = 10 \times 4\ \text{MHz} = 40\ \text{MHz}$$

THANK YOU!