1. $y' = \sin(z)\, e^{-x} - x \sin(z)\, e^{-x} = \sin(z)\, e^{-x}\, (1 - x)$
2.
   a. $y^T z = (1 \quad 3) * \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 11$
   b. $xy = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} * \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ 10 \end{pmatrix}$
   c. detX = 2 ≠ 0. So, x is invertible.
   d. $X = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 4 \\ 0 & -5 \end{pmatrix}$ The rank of X is 2
3.
   a. mean = (1 + 1 + 0 + 1 + 0) / 5 = 0.6
   b. $s^2 = \frac{1}{n-1} \sum_i^n \quad (x_i - \overline{x})^2$ = 0.3
   c. $P = 0.5^5 = 0.03125$
   d. Assume $P(X_i = 1) = p$
      $P(\text{sample}) = p^3 * (1 - p)^2 = p^3 - 2p^4 + p^5$
      $P' = 3p^2 - 8p^3 + 5p^4 = 0$
      $5p^2 - 8p + 3 = 0$
      $(5p - 3)(p - 1) = 0$
      $p = \frac{3}{5}$
      $\text{So}, P(\text{sample}) = 0.6^3 * 0.4^2 = 0.03456$
   e. $P(X = T \mid Y = b) = \frac{0.1}{0.1 + 0.15} = 0.4$
4.
   a. False
   b. True
   c. False
   d. False
   e. True
5. (a) -> (v)
   (b) -> (iv)
   (c) -> (ii)
   (d) -> (i)
   (e) -> (iii)
6.

a. $E[X] = p$
   $Var(X) = p(1 - p)$
b. $Var(2X) = 4\ Var(X) = 4\sigma^2$
   $Var(X + 2) = Var(X) = \sigma^2$

7.
   a.
      i. Both
      ii. $g(n) = O(f(n))$
      iii. $g(n) = O(f(n))$
   b. Assume the name of array is a, and name of the function is transition

```
transition (start, end):
middle = (start + end) / 2
if a[middle] == 1:
        return transition (start, middle)
else if a[middle + 1] == 1:
        return middle
else:
        return transition (middle + 1, end)
```

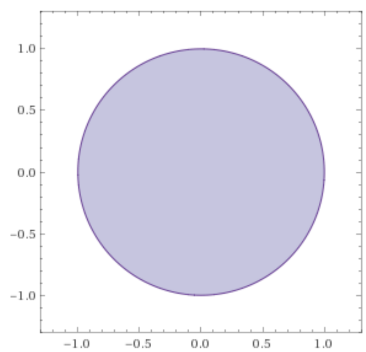
   The recursion stops when a[start] == 0 and a[start + 1] ==1. So, it fulfills the question.
   The running time $T(n) = T(n/2) + O(1)$, so the complexity is $O(logn)$

8.
   a. $$E(XY) = \int_{-\infty}^{\infty} XYf_{XY}(X,Y)dXdY$$
   $$= \int_{-\infty}^{\infty} f_X(X)f_Y(Y)XYdXdY$$
   $$= \int_{-\infty}^{\infty} f_X(X)XdX \int_{-\infty}^{\infty} f_Y(Y)YdY$$
   $$= E(X)E(Y)$$
   b.
      i. Due to the Law of Large Numbers, the number of times 3 shows up should be close to 6000 * 1/6 = 1000

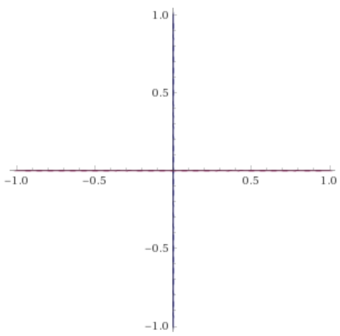
ii. According to Central Limit Theorem, $\sqrt{n}(S_n - \mu)$ converges in distribution to a Norm $N(0,\sigma^2)$

9.

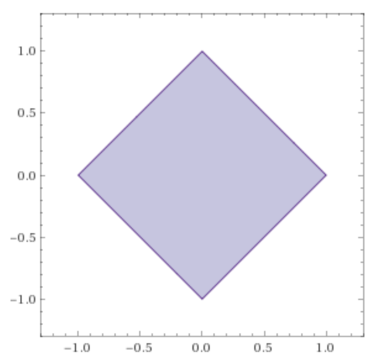
a.



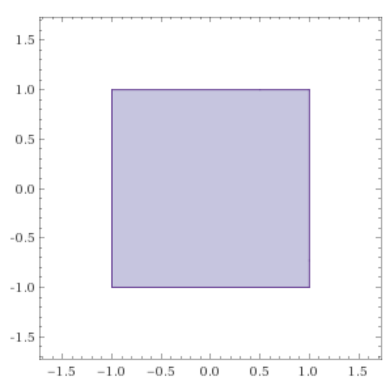
i.



ii.



iii.



iv.

b.

i. Assume A is a n x n matrix, if there exists a vector X such that
$AX = \lambda X$, for some scalar λ,
then λ is called the eigenvalue of A with corresponding eigenvector X.

ii. Set $|M - \lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 3 - 4\lambda + \lambda^2 = 0$
$\lambda = 1 \text{ or } \lambda = 3$
$v = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ or } v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

iii. By induction,
Hypothesis: $A^k x = \lambda^k x$ for some positive integer k
Assume the hypothesis is true:
$A^{k+1}x = A(A^k x) = A(\lambda^k)x = \lambda^k(Ax) = \lambda^{k+1}x$
Conclude that $A^k x = \lambda^k x$

c.

i. $\frac{\partial(a^T x)}{\partial x} = a^T$

ii. $\frac{\partial(x^T A x)}{\partial x} = x^T(A + A^T)$
$$\frac{\partial(x^T(A + A^T))}{\partial x} = (A + A^T)$$

d.

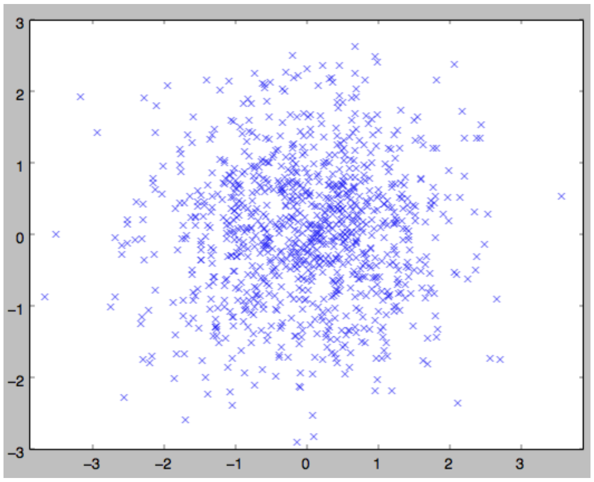
i. $w^T x_1 + b = 0 = w^T x_2 + b$
$w^T(x_1 - x_2) = 0$
Recall that $x_1 - x_2$ is parallel to the line, w is orthogonal to the line

ii. Suppose origin is o and a is a point on the plane, so it satisfies $w^T a + b = 0$
$$d = \left\|\text{proj}_{w^T}(o - a)\right\| = \left\|\frac{(o-a) * w^T}{w^T * w^T} * w^T\right\|$$
$$= \left|o * w^T - a * w^T\right| * \frac{\|w\|_2}{\|w\|_2^2}$$
$$= \frac{\left|o * w^T - a * w^T\right|}{\|w\|_2}$$

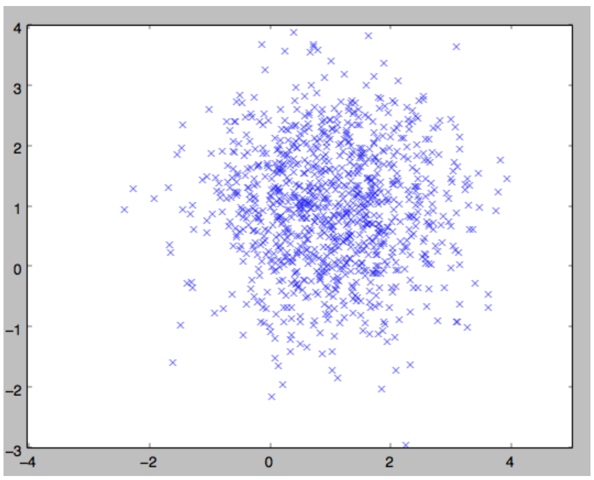
because $o * w^T = 0, a * w^T = -b,$

$$d = \frac{|b|}{\|w\|_2}$$

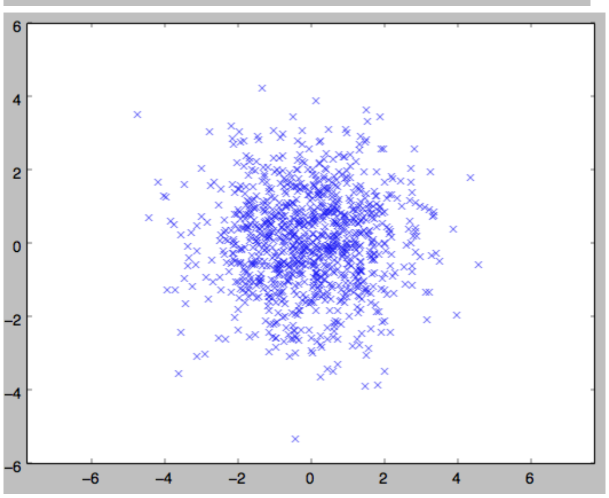
10.



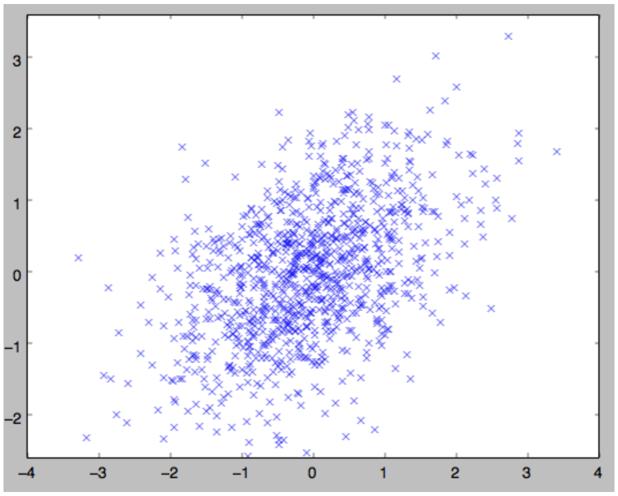
a.

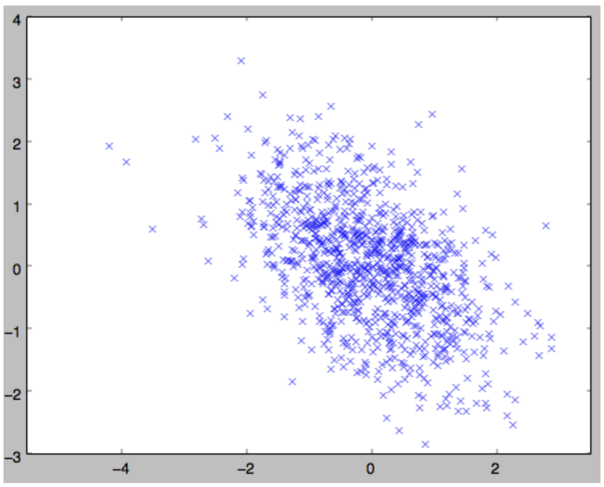


b.



c.

d. 

e. 

11. [0.   0.89442719]

12.

   a. Service requests received by the Oakland Call Center
   b. https://data.oaklandnet.com/Infrastructure/Service-requests-received-by-the-Oakland-Call-Cent/quth-gb8e/data
   c. The datasets include the information of the requests, such as source, addresses and the predicted value is the request is open, referred or pending.
   d. 481K
   e. 12