Holistic Mathematics: An Exploration into Category Theory

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$$\begin{tikzcd}
a \arrow[loop above] & b \arrow[loop above] & c \arrow[loop above]
\end{tikzcd}$$

Discrete category for the set $\{a, b, c\}$.

$$\begin{tikzcd}
\{1,2,3\} \arrow[r, "(\times 2)"] \arrow[dr, "(+1)\circ(\times 2)"'] & \{2,4,6\} \arrow[d, "(+1)"] \\
& \{3,5,7\}
\end{tikzcd}$$

Composition of arrows in Set

$$
\begin{tikzcd}
\{1,2,3\} & \{2,4,6\} \arrow[l, "(\div 2)"'] \\
 & \{3,5,7\} \arrow[u, "(-1)"'] \arrow[ul, "(\div 2)\circ(-1)"]
\end{tikzcd}
$$

Composition of arrows in $\mathbf{Set}_{op}$

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[dr, "g \circ f"'] & B \arrow[d, "g"] \\ & C \end{tikzcd}$$

$$\begin{tikzcd} F\ A \arrow[r, "F(f)"] \arrow[dr, "F(g) \circ F(f)"'] & F\ B \arrow[d, "F(g)"] \\ & F\ C \end{tikzcd} \qquad \begin{tikzcd} F'\ A & F'\ B \arrow[l, "F'(f)"'] \\ & F'\ C \arrow[ul, "F'(f) \circ F'(g)"] \arrow[u, "F'(g)"'] \end{tikzcd}$$

Covariant and Contravariant Functors

$$\begin{tikzcd} A \arrow[d, "f"] & A \\ B & B \arrow[u, "g"] \end{tikzcd}$$

Functor between a category and its opposite category

$$
\begin{tikzcd}
A & A_1 \arrow[d, bend right, "f"'] & A_3 \arrow[r, "i \circ h"] \arrow[d, "h"'] & A_5 \\
 & A_2 \arrow[u, bend right, "g"'] & A_4 \arrow[ur, "i"'] &
\end{tikzcd}
$$

Functor between the category of one object and any category

$$\begin{tikzcd} Sa \arrow[r, "\tau a"] \arrow[d, "S(f)"] & Ta \arrow[d, "T(f)"] \\ Sb \arrow[r, "\tau b"] & Tb \end{tikzcd}$$

Commutative diagram for natural transformations

$$
\begin{tikzcd}
I_C \; a \arrow[r, leftrightarrow, "\tau a"] \arrow[d, "I_C(f)"] & OP' \circ OP \; a \arrow[d, "OP' \circ OP(f)"] \\
I_C \; b \arrow[r, leftrightarrow, "\tau b"] & OP' \circ OP \; b
\end{tikzcd}
\qquad
\begin{tikzcd}
I_{C_{op}} \; a \arrow[r, leftrightarrow, "\theta a"] \arrow[d, "I_{C_{op}}(f)"] & OP \circ OP' \; a \arrow[d, "OP \circ OP'(f)"] \\
I_{C_{op}} \; b \arrow[r, leftrightarrow, "\theta b"] & OP \circ OP' \; b
\end{tikzcd}
$$

$$
\forall a, \qquad \begin{aligned} \tau a &= a \\ \theta a &= a \end{aligned}
$$

Every category is equivalent to its opposite category

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