

# Holistic Mathematics: An Exploration into Category Theory

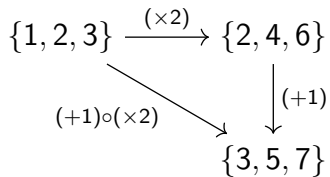
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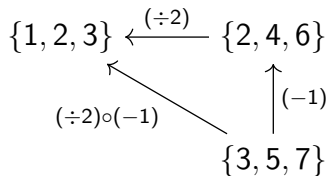




**Discrete category for the set  $\{a, b, c\}$ .**



**Composition of arrows in Set**



**Composition of arrows in  $\text{Set}_{op}$**



$$\begin{array}{ccc}
 A & \xrightarrow{f} & B \\
 & \searrow g \circ f & \downarrow g \\
 & & C
 \end{array}$$

$$\begin{array}{ccc}
 F A & \xrightarrow{F(f)} & F B \\
 & \searrow F(g) \circ F(f) & \downarrow F(g) \\
 & & F C
 \end{array}$$

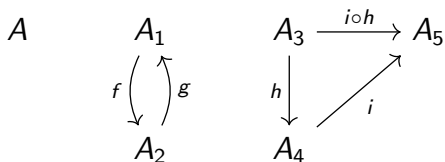
$$\begin{array}{ccc}
 F' A & \xleftarrow{F'(f)} & F' B \\
 & \nwarrow F'(f) \circ F'(g) & \uparrow F'(g) \\
 & & F' C
 \end{array}$$

**Covariant and Contravariant Functors**

$$\begin{array}{ccc}
 A & & A \\
 \downarrow f & & \uparrow g \\
 B & & B
 \end{array}$$

**Functor between a category and its opposite category**





**Functor between the category of one object and any category**



$$\begin{array}{ccc}
 Sa & \xrightarrow{\tau a} & Ta \\
 \downarrow S(f) & & \downarrow T(f) \\
 Sb & \xrightarrow{\tau b} & Tb
 \end{array}$$

**Commutative diagram for natural transformations**

$$\begin{array}{ccc}
 I_C a & \xleftarrow{\tau a} & OP' \circ OP a \\
 \downarrow I_C(f) & & \downarrow OP' \circ OP(f) \\
 I_C b & \xleftarrow{\tau b} & OP' \circ OP b
 \end{array}$$

$$\begin{array}{ccc}
 I_{C_{op}} a & \xleftarrow{\theta a} & OP \circ OP' a \\
 \downarrow I_{C_{op}}(f) & & \downarrow OP \circ OP'(f) \\
 I_{C_{op}} b & \xleftarrow{\theta b} & OP \circ OP' b
 \end{array}$$

$$\begin{aligned}
 \forall a, \quad \tau a &= a \\
 \theta a &= a
 \end{aligned}$$

**Every category is equivalent to its opposite category**



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