Maths Problems: Triangle

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March 30, 2009

Find x:

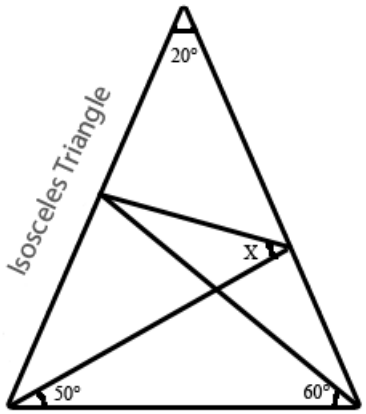

Figure 1: Isosceles Triangle

Solution on next page.

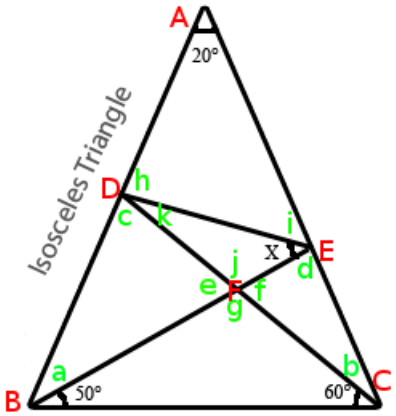

Figure 2: Isosceles Triangle

First, I wrote down everything obvious about the angles:

$$
\begin{aligned}
g &= 70^\circ \\
j &= 70^\circ \\
e &= 110^\circ \\
f &= 110^\circ \\
a &= b + 10^\circ \\
50 &= a + b \\
\therefore b &= 20^\circ \\
\therefore a &= 30^\circ \\
d &= 50^\circ \\
c &= 40^\circ \\
k &= 110^\circ - x \\
h &= 30^\circ + x \\
i &= 130^\circ - x
\end{aligned}
$$

Then, I worked out all the sides using the sine and cosine rules

$$
\begin{aligned}
BC &= \sqrt{2 - 2\cos 20^\circ} \\
BE &= \frac{BC\sin 80^\circ}{\sin 50^\circ} \\
BF &= \frac{BC\sin 60^\circ}{\sin 70^\circ} \\
CD &= \frac{BC\sin 80^\circ}{\sin 40^\circ} \\
CF &= \frac{BC\sin 50^\circ}{\sin 70^\circ} \\
CE &= \frac{CF\sin 110^\circ}{\sin 50^\circ} \\
BD &= \frac{BF\sin 110^\circ}{\sin 40^\circ} \\
DF &= \frac{BF\sin 30^\circ}{\sin 40^\circ} \\
EF &= \frac{CF\sin 20^\circ}{\sin 50^\circ} \\
DE &= \frac{DF\sin 70^\circ}{\sin x} \\
DA &= \frac{DE\sin(130^\circ - x)}{\sin 20^\circ} \\
EA &= \frac{DE\sin(30^\circ + x)}{\sin 20^\circ}
\end{aligned}
$$

I then took a closer look at triangle DFE:

$$
\begin{aligned}
DE &= \sqrt{DF^2 + FE^2 - 2 \times DF \times FE \times \cos 70^\circ} \\
DF &\approx 0.24897 \\
FE &\approx 0.12641 \\
\therefore DE &\approx 0.23757
\end{aligned}
$$

$$
\begin{aligned}
DF^2 &= FE^2 + DE^2 - 2 \times FE \times DE \times \cos x \\
\cos x &\approx 0.1737 \\
x &\approx 79.996^\circ \\
x &= 80.0^\circ \text{ (3sf)}
\end{aligned}
$$

Quod erat faciendum.