Planning Under Uncertainty

To decide among alternative goals and plans of action, agent's preferences between world states are captured by a real-valued utility function, which expresses the desirability of a state.

Decision theory

Decision theory = Probability theory + Utility theory

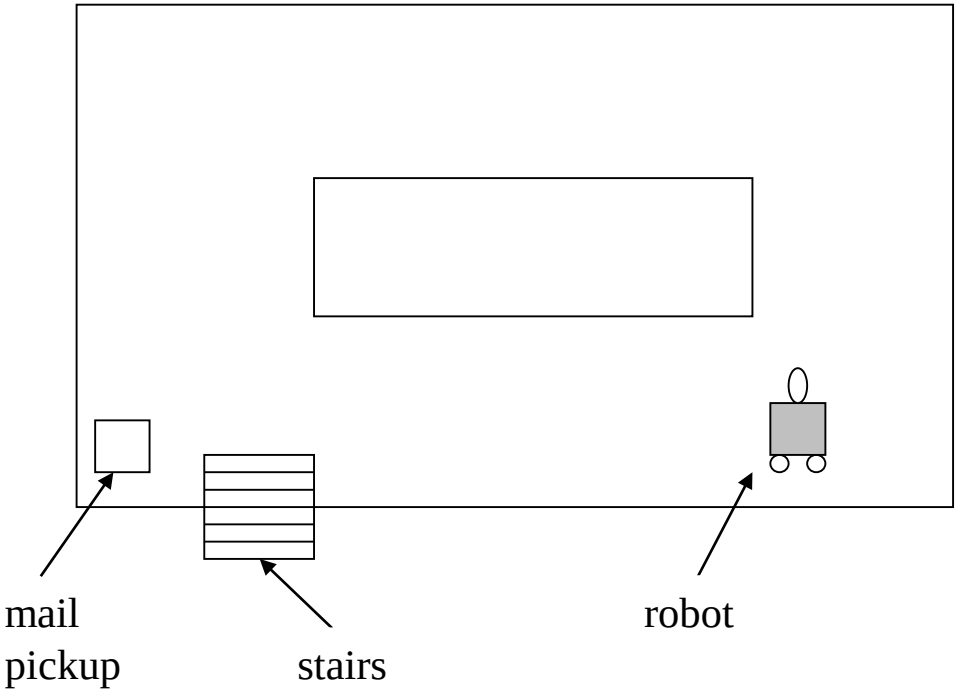
- Decision theory: describes what an agent should do
- Probability theory: describes what an agent should believe on the basis of the evidence
- Utility theory: describes what an agent wants

Decision networks

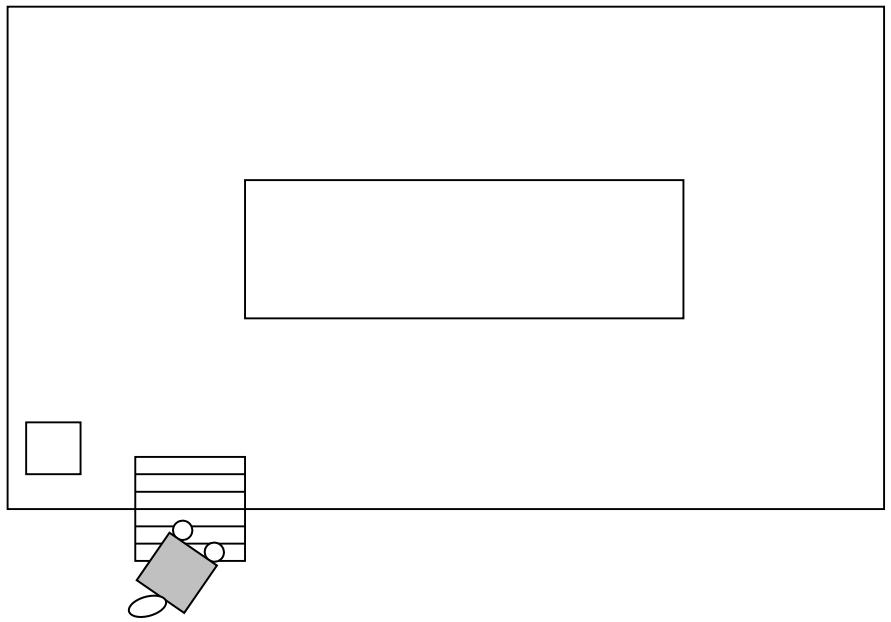
Decision network =

Bayesian network + actions + utilities

Consider a robot that delivers mail.



The robot must choose its route to pickup the mail. There is a short route and a long route. The long route is of course slower, but on the short route there are stairs and the robot might slip and fall down the stairs. The robot can put on pads. This won't change the probability of an accident, but it will make it less severe if it happens. Unfortunately, the pads add weight and slow the robot down.



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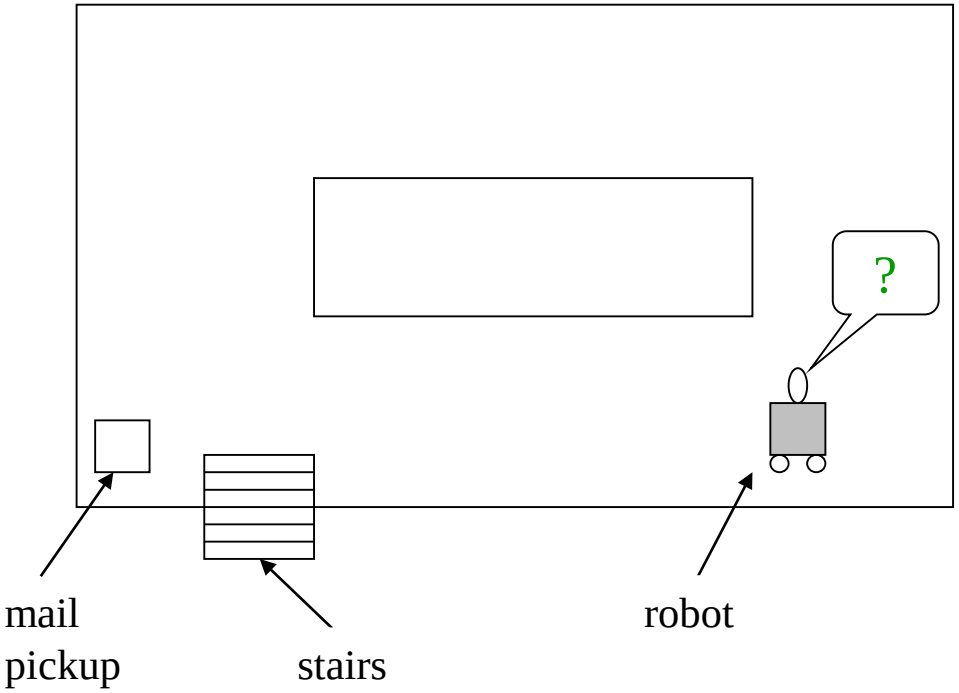

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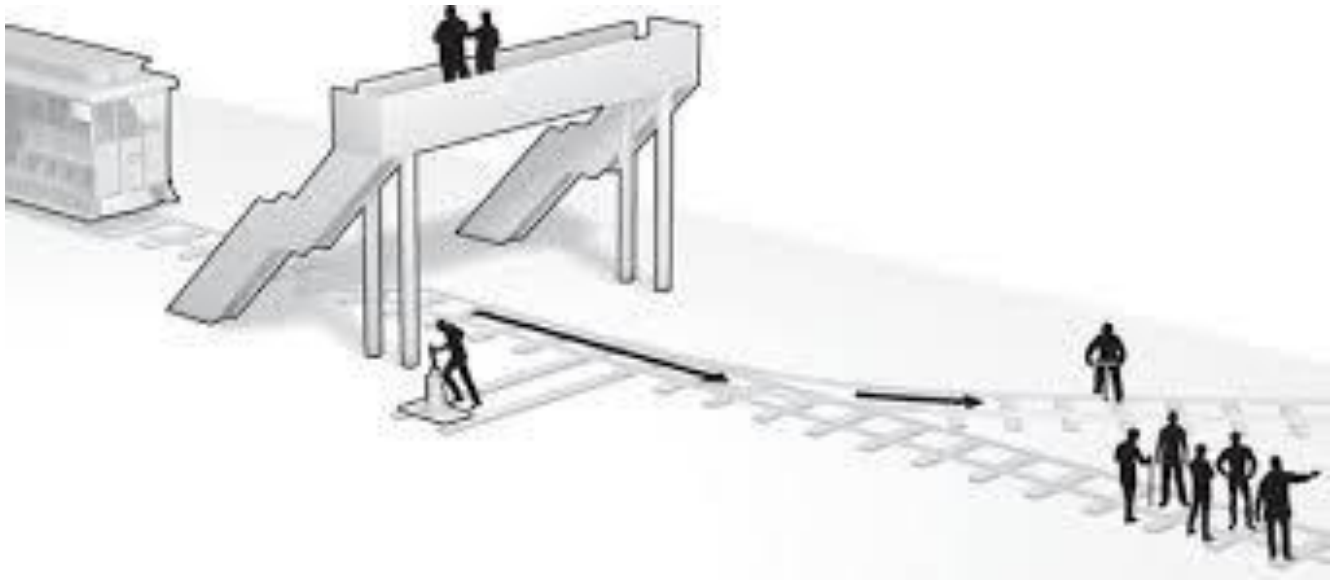
What should the robot do?



Decision networks: Nodes

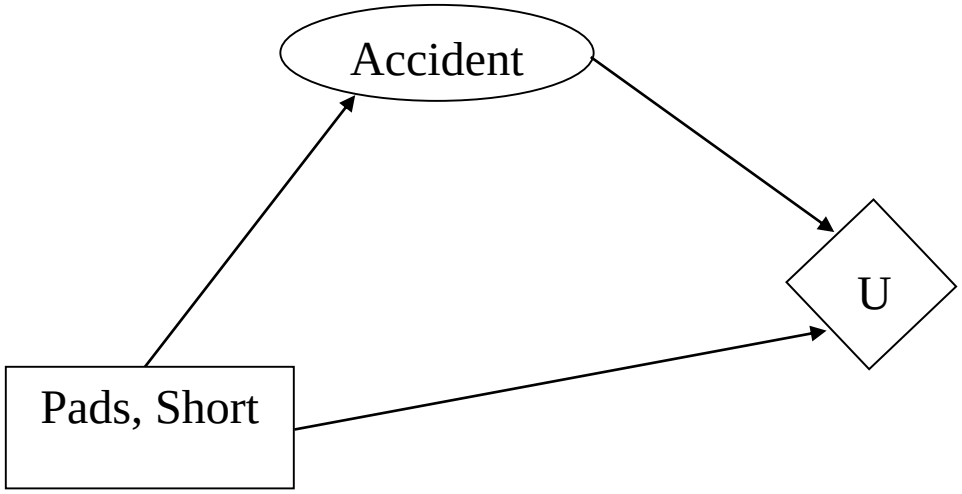
- Three types of nodes:
  1. Chance nodes (ovals)
     - represent random variables (as in Bayesian nets)
  2. Decision nodes (rectangles)
     - represent decision variables (choice of action)
  3. Utility node (diamond)
     - represents agent's utility function on states

Decision networks: Arcs

- Directed acyclic graph
- Arcs into Chance nodes
  - a chance node has as parents all those chance and decision variables that directly influence it
- Arcs into Decision nodes
  - a decision node has as parents all those chance and decision variables whose values will be known when the decision is made
- Arcs into Utility node
  - utility node has as parents all those chance and decision variables describing the outcome state that directly affect utility

Robot decision network

$P(A \mid \neg P, \neg S) = 0$
$P(A \mid \neg P, S) = q$
$P(A \mid P, \neg S) = 0$
$P(A \mid P, S) = q$



Utility function for robot example

	State of the world	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 — slow, noweight	
$\neg P, \neg S, A$	w_1 — impossible	–
$\neg P, S, \neg A$	w_2 — quick, noweight	
$\neg P, S, A$	w_3 — severe damage	
$P, \neg S, \neg A$	w_4 — slow, extra weight	
$P, \neg S, A$	w_5 — impossible	–
$P, S, \neg A$	w_6 — quick, extra weight	
P, S, A	w_7 — moderate damage	

Utility function for robot example

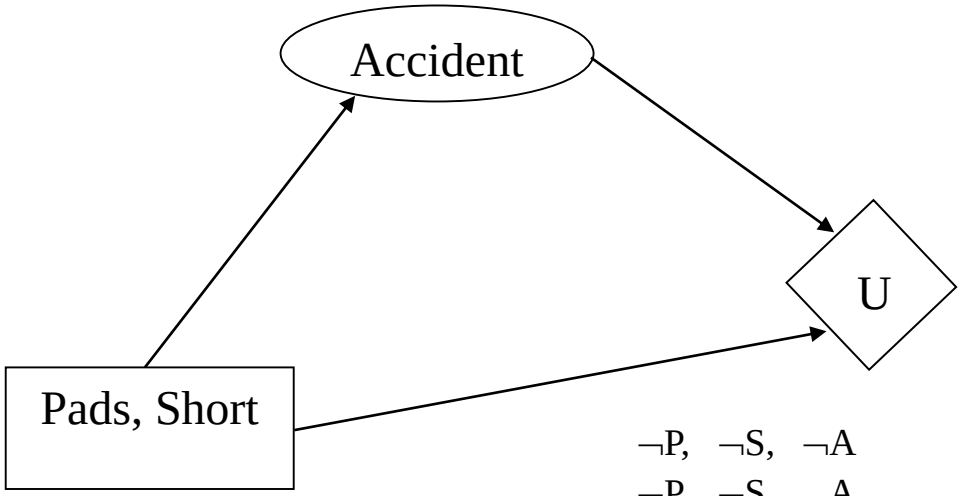
	State of the world	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 — slow, noweight	6
$\neg P, \neg S, A$	w_1 — impossible	–
$\neg P, S, \neg A$	w_2 — quick, noweight	10
$\neg P, S, A$	w_3 — severe damage	0
$P, \neg S, \neg A$	w_4 — slow, extra weight	4
$P, \neg S, A$	w_5 — impossible	–
$P, S, \neg A$	w_6 — quick, extra weight	8
P, S, A	w_7 — moderate damage	2

Evaluating a decision network

- Choose an action by:
  1. Set evidence variables for current state
  2. For each possible value of decision node
     (a) set decision node to that value
     (b) calculate posterior probability for parent nodes of the utility node
     (c) calculate expected utility for the action
  3. Return action with highest expected utility

Robot decision network

$P(A \mid \neg P, \neg S) = 0$
$P(A \mid \neg P, S) = q$
$P(A \mid P, \neg S) = 0$
$P(A \mid P, S) = q$



			State of the world	$U(w_i)$
$\neg P$,	$\neg S$,	$\neg A$	w_0 — slow, noweight	6
$\neg P$,	$\neg S$,	A	w_1 — impossible	–
$\neg P$,	S,	$\neg A$	w_2 — quick, noweight	10
$\neg P$,	S,	A	w_3 — severe damage	0
P,	$\neg S$,	$\neg A$	w_4 — slow, extra weight	4
P,	$\neg S$,	A	w_5 — impossible	–
P,	S,	$\neg A$	w_6 — quick, extra weight	8
P,	S,	A	w_7 — moderate damage	2

General case: Given evidence, action

w_i			$P(w_i \mid \text{evidence, action})$	×	$U(w_i)$	=	$EU(w_i \mid \text{evidence, action})$
¬P,	¬S,	¬A	$P(w_0 \mid \text{evidence, action})$	×	6	=	
¬P,	¬S,	A	$P(w_1 \mid \text{evidence, action})$	×	−	=	
¬P,	S,	¬A	$P(w_2 \mid \text{evidence, action})$	×	10	=	
¬P,	S,	A	$P(w_3 \mid \text{evidence, action})$	×	0	=	
P,	¬S,	¬A	$P(w_4 \mid \text{evidence, action})$	×	4	=	
P,	¬S,	A	$P(w_5 \mid \text{evidence, action})$	×	−	=	
P,	S,	¬A	$P(w_6 \mid \text{evidence, action})$	×	8	=	
P,	S,	A	$P(w_7 \mid \text{evidence, action})$	×	2	=	
							$EU(\text{action} \mid \text{evidence})$ =

Special case: Given *no* evidence, action

w_i	$P(w_i \mid \text{action}) \times U(w_i) = EU(w_i \mid \text{action})$
$\neg P, \neg S, \neg A$	$P(w_0 \mid \text{action}) \times 6 =$
$\neg P, \neg S, A$	$P(w_1 \mid \text{action}) \times - =$
$\neg P, S, \neg A$	$P(w_2 \mid \text{action}) \times 10 =$
$\neg P, S, A$	$P(w_3 \mid \text{action}) \times 0 =$
$P, \neg S, \neg A$	$P(w_4 \mid \text{action}) \times 4 =$
$P, \neg S, A$	$P(w_5 \mid \text{action}) \times - =$
$P, S, \neg A$	$P(w_6 \mid \text{action}) \times 8 =$
P, S, A	$P(w_7 \mid \text{action}) \times 2 =$
	$EU(\text{action}) =$

Suppose: *no* evidence, action is ¬P, ¬S

w_i	$P(w_i \mid \neg P, \neg S)$	×	$U(w_i)$	=	$EU(w_i \mid \neg P, \neg S)$
¬P, ¬S, ¬A	$P(w_0 \mid \neg P, \neg S)$	×	6	=	
¬P, ¬S, A	$P(w_1 \mid \neg P, \neg S)$	×	−	=	
¬P, S, ¬A	$P(w_2 \mid \neg P, \neg S)$	×	10	=	0
¬P, S, A	$P(w_3 \mid \neg P, \neg S)$	×	0	=	0
P, ¬S, ¬A	$P(w_4 \mid \neg P, \neg S)$	×	4	=	0
P, ¬S, A	$P(w_5 \mid \neg P, \neg S)$	×	−	=	0
P, S, ¬A	$P(w_6 \mid \neg P, \neg S)$	×	8	=	0
P, S, A	$P(w_7 \mid \neg P, \neg S)$	×	2	=	0
			$EU(\neg P, \neg S)$	=	

Suppose: *no* evidence, action is ¬P, ¬S

w_i	$P(w_i \mid \neg P, \neg S)$	×	$U(w_i)$	=	$EU(w_i \mid \neg P, \neg S)$
¬P, ¬S, ¬A	$P(w_0 \mid \neg P, \neg S)$	×	6	=	$1 \times 6 = 6$
¬P, ¬S, A	$P(w_1 \mid \neg P, \neg S)$	×	−	=	$0 \times - = 0$
¬P, S, ¬A	$P(w_2 \mid \neg P, \neg S)$	×	10	=	0
¬P, S, A	$P(w_3 \mid \neg P, \neg S)$	×	0	=	0
P, ¬S, ¬A	$P(w_4 \mid \neg P, \neg S)$	×	4	=	0
P, ¬S, A	$P(w_5 \mid \neg P, \neg S)$	×	−	=	0
P, S, ¬A	$P(w_6 \mid \neg P, \neg S)$	×	8	=	0
P, S, A	$P(w_7 \mid \neg P, \neg S)$	×	2	=	0
				=	$EU(\neg P, \neg S) = 6$

Suppose: *no* evidence, action is ¬P, S

w_i	$P(w_i \mid \neg P, S)$	×	$U(w_i)$	=	$EU(w_i \mid \neg P, S)$
¬P, ¬S, ¬A	$P(w_0 \mid \neg P, S)$	×	6	=	0
¬P, ¬S, A	$P(w_1 \mid \neg P, S)$	×	−	=	0
¬P, S, ¬A	$P(w_2 \mid \neg P, S)$	×	10	=	
¬P, S, A	$P(w_3 \mid \neg P, S)$	×	0	=	
P, ¬S, ¬A	$P(w_4 \mid \neg P, S)$	×	4	=	0
P, ¬S, A	$P(w_5 \mid \neg P, S)$	×	−	=	0
P, S, ¬A	$P(w_6 \mid \neg P, S)$	×	8	=	0
P, S, A	$P(w_7 \mid \neg P, S)$	×	2	=	0
	$EU(\neg P, S)$	=			

Suppose: *no* evidence, action is ¬P, S

w_i	$P(w_i \mid \neg P, S)$	×	$U(w_i)$	=	$EU(w_i \mid \neg P, S)$
¬P, ¬S, ¬A	$P(w_0 \mid \neg P, S)$	×	6	=	0
¬P, ¬S, A	$P(w_1 \mid \neg P, S)$	×	−	=	0
¬P, S, ¬A	$P(w_2 \mid \neg P, S)$	×	10	=	$(1 - q) \times 10$
¬P, S, A	$P(w_3 \mid \neg P, S)$	×	0	=	$q \times 0$
P, ¬S, ¬A	$P(w_4 \mid \neg P, S)$	×	4	=	0
P, ¬S, A	$P(w_5 \mid \neg P, S)$	×	−	=	0
P, S, ¬A	$P(w_6 \mid \neg P, S)$	×	8	=	0
P, S, A	$P(w_7 \mid \neg P, S)$	×	2	=	0
					$EU(\neg P, S) = 10 - 10q$

Suppose: *no* evidence, action is P, ¬S

w_i	$P(w_i \mid P, \neg S)$	×	$U(w_i)$	=	$EU(w_i \mid P, \neg S)$
¬P, ¬S, ¬A	$P(w_0 \mid P, \neg S)$	×	6	=	0
¬P, ¬S, A	$P(w_1 \mid P, \neg S)$	×	–	=	0
¬P, S, ¬A	$P(w_2 \mid P, \neg S)$	×	10	=	0
¬P, S, A	$P(w_3 \mid P, \neg S)$	×	0	=	0
P, ¬S, ¬A	$P(w_4 \mid P, \neg S)$	×	4	=	
P, ¬S, A	$P(w_5 \mid P, \neg S)$	×	–	=	
P, S, ¬A	$P(w_6 \mid P, \neg S)$	×	8	=	0
P, S, A	$P(w_7 \mid P, \neg S)$	×	2	=	0
			$EU(P, \neg S)$	=	

Suppose: *no* evidence, action is P, ¬S

w_i	$P(w_i \mid P, \neg S)$	×	$U(w_i)$	=	$EU(w_i \mid P, \neg S)$
¬P, ¬S, ¬A	$P(w_0 \mid P, \neg S)$	×	6	=	0
¬P, ¬S, A	$P(w_1 \mid P, \neg S)$	×	–	=	0
¬P, S, ¬A	$P(w_2 \mid P, \neg S)$	×	10	=	0
¬P, S, A	$P(w_3 \mid P, \neg S)$	×	0	=	0
P, ¬S, ¬A	$P(w_4 \mid P, \neg S)$	×	4	=	1 × 4
P, ¬S, A	$P(w_5 \mid P, \neg S)$	×	–	=	0 × –
P, S, ¬A	$P(w_6 \mid P, \neg S)$	×	8	=	0
P, S, A	$P(w_7 \mid P, \neg S)$	×	2	=	0
				=	$EU(P, \neg S)$ = 4

Suppose: *no* evidence, action is P, S

w_i	$P(w_i \mid P, S)$	×	$U(w_i)$	=	$EU(w_i \mid P, S)$
¬P, ¬S, ¬A	$P(w_0 \mid P, S)$	×	6	=	0
¬P, ¬S, A	$P(w_1 \mid P, S)$	×	−	=	0
¬P, S, ¬A	$P(w_2 \mid P, S)$	×	10	=	0
¬P, S, A	$P(w_3 \mid P, S)$	×	0	=	0
P, ¬S, ¬A	$P(w_4 \mid P, S)$	×	4	=	0
P, ¬S, A	$P(w_5 \mid P, S)$	×	−	=	0
P, S, ¬A	$P(w_6 \mid P, S)$	×	8	=	
P, S, A	$P(w_7 \mid P, S)$	×	2	=	
			$EU(P, S)$	=	

Suppose: *no* evidence, action is P, S

w_i	$P(w_i \mid P, S)$	×	$U(w_i)$	=	$EU(w_i \mid P, S)$
¬P, ¬S, ¬A	$P(w_0 \mid P, S)$	×	6	=	0
¬P, ¬S, A	$P(w_1 \mid P, S)$	×	−	=	0
¬P, S, ¬A	$P(w_2 \mid P, S)$	×	10	=	0
¬P, S, A	$P(w_3 \mid P, S)$	×	0	=	0
P, ¬S, ¬A	$P(w_4 \mid P, S)$	×	4	=	0
P, ¬S, A	$P(w_5 \mid P, S)$	×	−	=	0
P, S, ¬A	$P(w_6 \mid P, S)$	×	8	=	$(1 - q) \times 8$
P, S, A	$P(w_7 \mid P, S)$	×	2	=	$q \times 2$
			$EU(P, S)$	=	$8 - 6q$