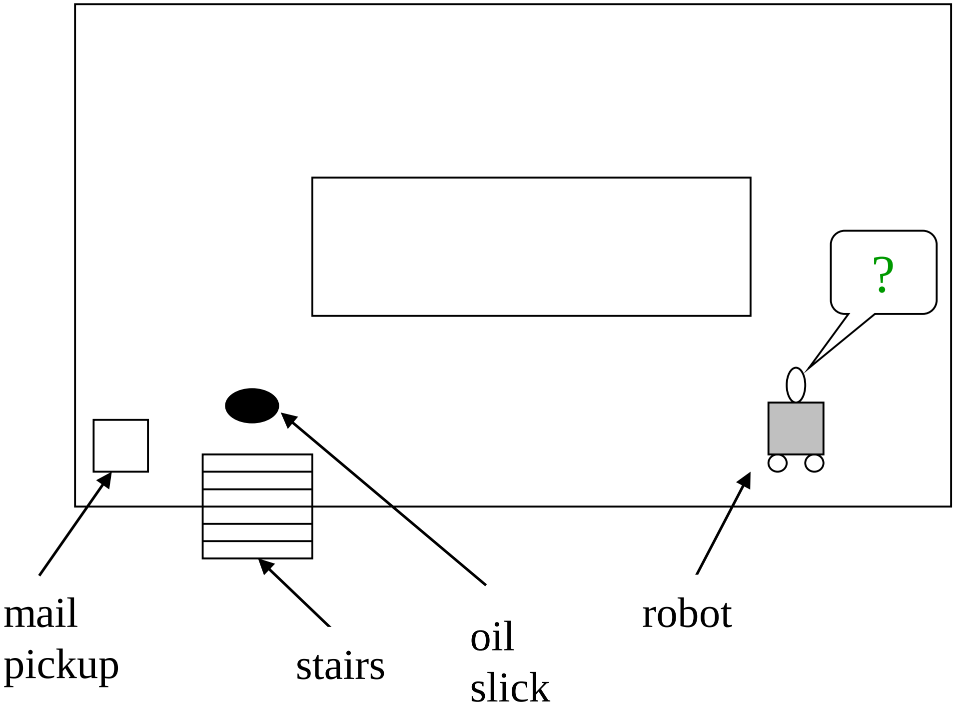
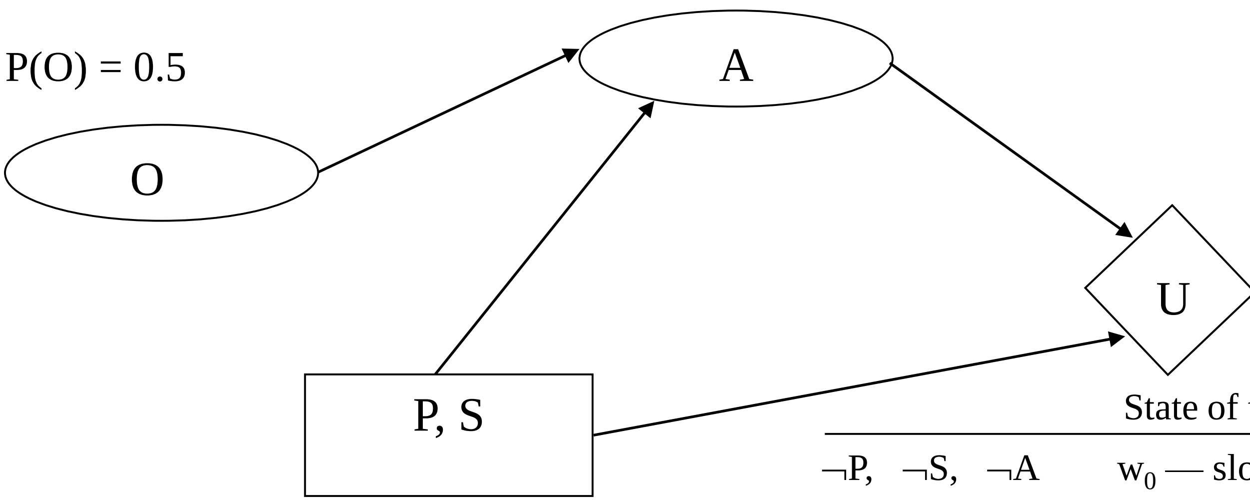
Robot example (expanded).



Suppose that sometimes there is a small oil slick at the top of the stairs and that the probability that the robot will have an accident is directly influenced by the presence or absence of an oil slick.

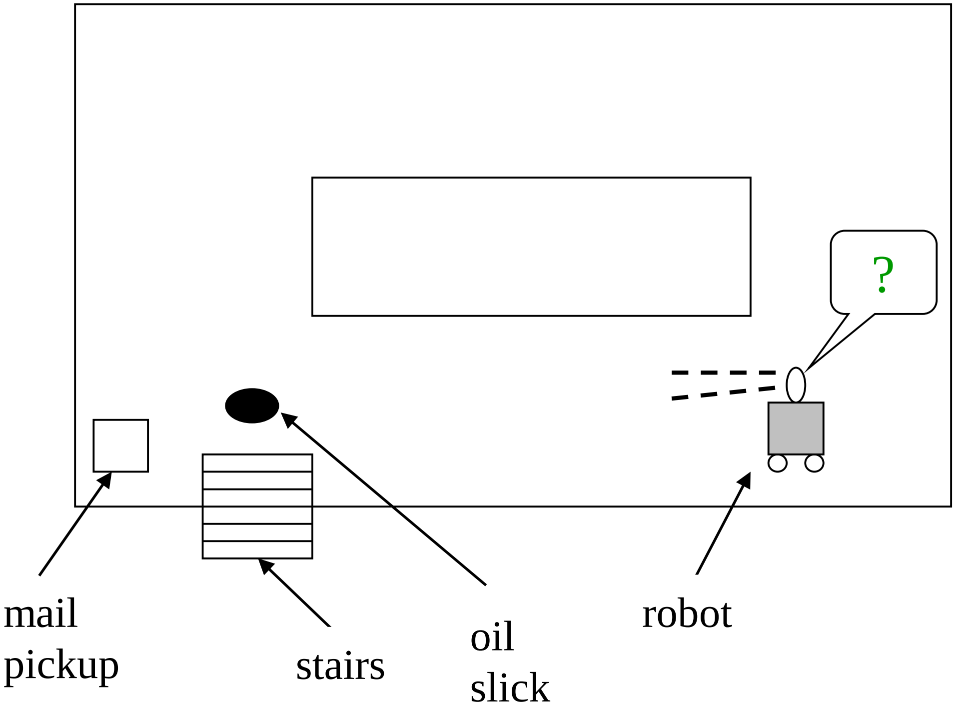
What should the robot do?

$$P(A \mid \neg P, \neg S, \neg O) = 0$$
$$P(A \mid \neg P, \neg S, O) = 0$$
$$P(A \mid \neg P, S, \neg O) = 0.1$$
$$P(A \mid \neg P, S, O) = 0.5$$
$$P(A \mid P, \neg S, \neg O) = 0$$
$$P(A \mid P, \neg S, O) = 0$$
$$P(A \mid P, S, \neg O) = 0.1$$
$$P(A \mid P, S, O) = 0.5$$



	State of the world	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 — slow, noweight	6
$\neg P, \neg S, A$	w_1 — impossible	–
$\neg P, S, \neg A$	w_2 — quick, noweight	10
$\neg P, S, A$	w_3 — severe damage	0
$P, \neg S, \neg A$	w_4 — slow, extra weight	4
$P, \neg S, A$	w_5 — impossible	–
$P, S, \neg A$	w_6 — quick, extra weight	8
P, S, A	w_7 — moderate damage	2

Robot example (expanded again).



Suppose the robot has to option of installing an “oil slick detection” sensor.

What should the robot do?

Value of information: the basic idea

One of the most important aspects of decision-making is knowing what information to gather, what questions to ask.

- e.g., A doctor decides which tests to perform
- e.g., Asking directions

How to decide? Evaluate by the effect on subsequent actions.

Value of information =

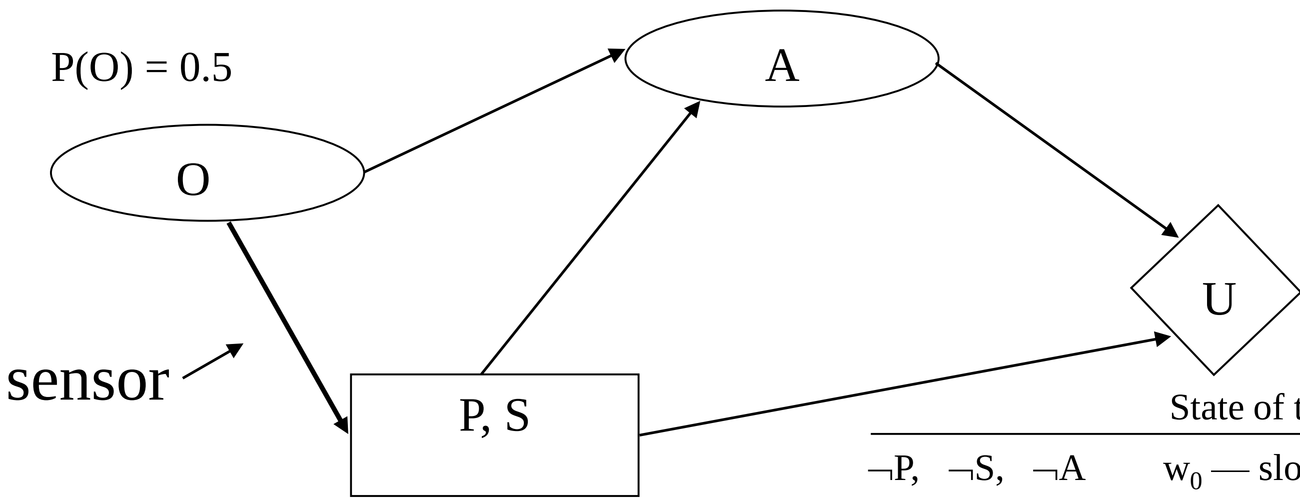
expected utility of optimal policy chosen using the new information

− expected utility of optimal policy chosen without the new information

Notes:

1. *Information has value to the extent that it is likely to cause a change of plan, and to the extent that the new plan is better than the old plan.*
2. *Usually assume that exact information is obtained about the value of some random variable. May need to average over all possible values of the random variable.*

$P(A \mid \neg P, \neg S, \neg O) = 0$
$P(A \mid \neg P, \neg S, O) = 0$
$P(A \mid \neg P, S, \neg O) = 0.1$
$P(A \mid \neg P, S, O) = 0.5$
$P(A \mid P, \neg S, \neg O) = 0$
$P(A \mid P, \neg S, O) = 0$
$P(A \mid P, S, \neg O) = 0.1$
$P(A \mid P, S, O) = 0.5$



	State of the world	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 — slow, noweight	6
$\neg P, \neg S, A$	w_1 — impossible	–
$\neg P, S, \neg A$	w_2 — quick, noweight	10
$\neg P, S, A$	w_3 — severe damage	0
$P, \neg S, \neg A$	w_4 — slow, extra weight	4
$P, \neg S, A$	w_5 — impossible	–
$P, S, \neg A$	w_6 — quick, extra weight	8
P, S, A	w_7 — moderate damage	2

Evaluating a decision network

- Choose an action by:

1. Set evidence variables for current state
2. For each possible value of decision node
   (a) set decision node to that value
   (b) calculate posterior probability for parent nodes of the utility node
   (c) calculate expected utility for the action
3. Return action with highest expected utility

General case: Given evidence, action

w_i	$P(w_i \mid \text{evidence, action})$	$\times$	$U(w_i)$	$=$	$EU(w_i \mid \text{evidence, action})$
$\neg P, \neg S, \neg A$	$P(w_0 \mid \text{evidence, action})$	$\times$	6	$=$	
$\neg P, \neg S, A$	$P(w_1 \mid \text{evidence, action})$	$\times$	–	$=$	
$\neg P, S, \neg A$	$P(w_2 \mid \text{evidence, action})$	$\times$	10	$=$	
$\neg P, S, A$	$P(w_3 \mid \text{evidence, action})$	$\times$	0	$=$	
$P, \neg S, \neg A$	$P(w_4 \mid \text{evidence, action})$	$\times$	4	$=$	
$P, \neg S, A$	$P(w_5 \mid \text{evidence, action})$	$\times$	–	$=$	
$P, S, \neg A$	$P(w_6 \mid \text{evidence, action})$	$\times$	8	$=$	
P, S, A	$P(w_7 \mid \text{evidence, action})$	$\times$	2	$=$	
			$EU(\text{action} \mid \text{evidence})$	$=$	

Basis of utility theory

- Notation

A, B, C	states of the world
[p, A; 1-p, B]	a lottery with two possible outcomes: state A with probability p, state B with probability 1-p
A > B	outcome A is preferred to B
A ~ B	agent is indifferent between A and B

Axioms of utility theory

- Orderability $(A > B) \vee (A < B) \vee (A \sim B)$
- Transitivity $(A > B) \wedge (B > C) \Rightarrow (A > C)$
- Continuity $A > B > C \Rightarrow \exists p\ [p, A; 1\text{-}p, C] \sim B$
- Substitutability $A \sim B \Rightarrow [p, A; 1\text{-}p, C] \sim [p, B; 1\text{-}p, C]$
- Monotonicity $A > B \Rightarrow$

  $(p > q \Leftrightarrow [p, A; 1\text{-}p, B] > [q, A; 1\text{-}q, B]$
- Decomposability Compound lotteries can be defined as one lottery and vice-versa

Consequences of axioms

- If an agent's preferences obey the axioms of utility, there exists a real-valued function U such that

  $$U(A) > U(B) \Leftrightarrow A > B$$

  $$U(A) = U(B) \Leftrightarrow A \sim B$$

- Scale is arbitrary

  Given two agents with utility functions U_1 and U_2,
  if $U1(S) = k_1 + k_2\, U_2(S)$, for all S and for all k_1, $k_2 > 0$,
  the agents will behave identically