Multiagent Systems

To cooperate or compete with other agents, an agent must reason about other agents' preferences and knowledge of the world, as well as its own preferences and knowledge.

Multiagent Framework

- Multiple agents, each with
  - its own information about the world and other agents
  - its own utility function
- Each agent decides what to do
  - outcome depends on the actions of all agents

Multiagent Framework

- Two ends of the spectrum:
  - fully cooperative: agents have same utility function
  - fully competitive: win-lose (zero-sum games)
- Most frameworks are between these two extremes
  - agent's utilities are synergistic in some aspects, competing in some, independent in others

Applications of Multiagent Systems

- Board games: checkers, chess, backgammon, Go
- Card games: Poker
- Interactive computer games
- Planning and control
- Bankruptcy proceedings
- Bidding for wireless spectrum rights
- ...

Game Theory: Framework

- Studies what agents should do in multi-agent setting
- Strategic (normal) form of a game
  - a finite set of agents $I = \{1, \ldots, n\}$
  - a set of actions A_i, for each agent $i \in I$
  - a utility function u_i, for each agent $i \in I$
- Each agent chooses an action without knowing what the other agents choose
- Joint action of all the agents produces the outcome

Game Theory: Strategies

- A **strategy** for an agent is either pure or stochastic:
  - pure strategy: one action
  - stochastic (mixed) strategy: probability distribution over the actions for this agent (more than one action has a non-zero probability)
- A **strategy profile** is an assignment of a strategy to each agent
  - if σ is a strategy profile, let σ_i be the strategy of agent i in σ, and let σ_{-i} be the strategies of the other agents; i.e., σ is $\sigma_i\sigma_{-i}$

Game Theory: Nash Equilibrium

- A strategy profile σ has a utility for each agent
  - let *utility(*σ*, i)* be the (expected) utility of strategy profile σ for agent i
- A **best response** for an agent i to the strategies σ_{-i} of the other agents is a strategy σ_i that has maximal utility for agent i
  - i.e., $utility(\sigma_i \sigma_{-i}, i) \geq utility(\sigma'_i \sigma_{-i}, i)$, for all other strategies σ'_i
- A strategy profile σ is a **Nash equilibrium** if, for each agent i, strategy σ_i is a best response to σ_{-i}
  - every finite game has at least one Nash equilibrium
  - not necessarily pure strategy though

Game Theory: Pareto Optimal

- An outcome is **Pareto optimal** if there is no other outcome that makes every player at least as well off and at least one player strictly better off
  - every finite game has at least one Pareto optimal outcome
  - not necessarily a Nash equilibrium though

Game Theory: Dominated Strategies

- Strategy σ_i **strictly dominates** strategy σ'_i for agent i if, for all strategy profiles σ_{-i} of the other agents,

$$utility(\sigma_i \sigma_{-i}, i) > utility(\sigma'_i \sigma_{-i}, i)$$

(I.e., σ_i is better than σ'_i for agent i, no matter how the other agents play)

Computing Nash Equilibriums

Iterated elimination of dominated strategies

- eliminate any pure strategy dominated by another strategy
- the dominating strategy can be a randomized strategy
- this is done repeatedly

Computing Nash Equilibriums: Stochastic (mixed) strategies

- If a stochastic (mixed) strategy is a best response, then each of the pure strategies involved in the mix must itself be a best response; i.e., each must yield the same expected utility
- Forms a set of constraints that can (sometimes) be solved to give a Nash equilibrium

Stochastic (mixed) strategies

Consider Fred and Barney. They each would prefer to be in the same place (the swim or the hike), but their preferences differ about which it should be. Fred would rather go swimming, and Barney would rather go hiking. Here is the utility matrix:

		Barney	
		swim	hike
Fred	swim	(2, 1)	(0, 0)
	hike	(0, 0)	(1, 2)

Consider the stochastic (mixed) strategies:

$\sigma_{Fred} = [p, \text{swim}; 1-p, \text{hike}]$

$\sigma_{Barney} = [q, \text{swim}; 1-q, \text{hike}]$

Are there values for p and q such that this pair of strategies is a Nash equilibrium?

Let us first consider the expected utilities for these strategies:

		Barney		
		swim	hike	$[q, \text{swim}; 1-q, \text{hike}]$
Fred	swim	(2, 1)	(0, 0)	$(2q, q)$
	hike	(0, 0)	(1, 2)	$(1\text{-}q, 2(1\text{-}q))$
	$[p, \text{swim}; 1-p, \text{hike}]$	$(2p, p)$	$(1\text{-}p, 2(1\text{-}p))$	$(2pq + (1\text{-}p)(1\text{-}q),$ $pq + 2(1\text{-}p)(1\text{-}q))$

Now, consider Fred. Both swim and hike must be best responses from Fred's point of view to Barney's strategy. For both swim and hike to be best responses, they must give the same expected utility (they must be equal):

$EU_{Fred}(\text{swim}) = EU_{Fred}(\text{hike})$

$2q = 1 - q$

So, $q = 1/3$

(Suppose, for example, that hike was not a best response for Fred, given Barney's strategy; i.e., it had a lower expected utility. Then Fred could drop hike from his stochastic (mixed) strategy and improve his expected utility. Therefore, each of the pure strategies in the stochastic strategy must be a best response; i.e., have the same expected utility.

To summarize, if Fred is mixing on both swim and hike in a Nash equilibrium, then both swim and hike must yield the same expected utility. Thus, Barney must be following the mixed strategy:

σ_{Barney} = [1/3, swim; 2/3, hike]

Now, consider Barney. Both swim and hike must be best responses from Barney's point of view to Fred's strategy. For both swim and hike to be best responses, they must give the same expected utility (they must be equal):

$EU_{\text{Barney}}(\text{swim}) = EU_{\text{Barney}}(\text{hike})$

$p = 2(1-p)$

So, $p = 2/3$

Thus, Fred must be following the mixed strategy:

σ_{Fred} = [2/3, swim; 1/3, hike]

Given this pair of mixed strategies, the expected utility for Fred is:

$$2pq + (1-p)(1-q) = (2)(2/3)(1/3) + (1/3)(2/3) = 6/9 = 2/3$$

Given this pair of mixed strategies, the expected utility for Barney is:

$$pq + 2(1-p)(1-q) = (2/3)(1/3) + 2(1/3)(2/3) = 6/9 = 2/3$$

We can verify that this is a Nash equilibrium:

		Barney		
		swim	hike	[1/3, swim;2/3, hike]
	swim	(2, 1)	(0, 0)	(2/3, 1/3)
Fred	hike	(0, 0)	(1, 2)	(2/3, 4/3)
	[2/3, swim; 1/3, hike]	(4/3, 2/3)	(1/3, 2/3)	(2/3, 2/3)