Taylor & Francis
Taylor & Francis Group

Journal of Business & Economic Statistics

ISSN: 0735-0015 (Print) 1537-2707 (Online) Journal homepage: http://amstat.tandfonline.com/loi/ubes20

# The Message in Daily Exchange Rates

Richard T Baillie & Tim Bollerslev

**To cite this article:** Richard T Baillie & Tim Bollerslev (2002) The Message in Daily Exchange Rates, Journal of Business & Economic Statistics, 20:1, 60-68, DOI: 10.1198/073500102753410390

**To link to this article:** https://doi.org/10.1198/073500102753410390

Published online: 01 Jan 2012.

Journal of Business & Economic Statistics, July 1989, Vol. 7, No. 3

# The Message in Daily Exchange Rates: A Conditional-Variance Tale

**Richard T. Baillie**
Department of Economics, Michigan State University, East Lansing, MI 48824

**Tim Bollerslev**
Department of Finance, Northwestern University, Evanston, IL 60208

Formal testing procedures confirm the presence of a unit root in the autoregressive polynomial of the univariate time series representation of daily exchange-rate data. The first differences of the logarithms of daily spot rates are approximately uncorrelated through time, and a generalized autoregressive conditional heteroscedasticity model with daily dummy variables and conditionally $t$-distributed errors is found to provide a good representation to the leptokurtosis and time-dependent conditional heteroscedasticity. The parameter estimates and characteristics of the models are found to be very similar for six different currencies. These apparent stylized facts carry over to weekly, fortnightly, and monthly data in which the degree of leptokurtosis and time-dependent heteroscedasticity is reduced as the length of the sampling interval increases.

KEY WORDS: Exchange rates; Generalized autoregressive conditional heteroscedasticity models; Intervention analysis; Leptokurtosis; Temporal aggregation; Unit roots.

## 1. INTRODUCTION

This article is concerned with modeling the dynamic and distributional properties of daily, weekly, fortnightly (i.e., every other week), and monthly foreign-exchange-market data. Many previous studies have noted how asset-price and foreign-exchange-market series are typically well described by a random-walk model. At the same time, Mandelbrot (1963) and Fama (1965) observed that price changes tended not to be independent over time but characterized by tranquil and volatile periods, and the unconditional distributions of the price changes were typically fat-tailed or leptokurtic. Consequently, many researchers attempted to describe price changes with nonnormal distributions; for example, the stable Paretian, the scaled $t$, and the lognormal normal models have all been proposed (see Blattberg and Gonedes 1974; Clark 1973; Mandelbrot 1963; Praetz 1972). Similar analysis, specifically for changes in the logarithms of spot-exchange rates, were performed by Westerfield (1977), Rogalski and Vinso (1978), McFarland, Pettit, and Sung (1982), and Boothe and Glassman (1987). The studies by McFarland et al. (1982) and Hsieh (1988) also drew attention to how the unconditional distributions of exchange-rate changes differ across different days of the week. These authors and others, such as Friedman and Vandersteel (1982), considered whether the data are independently and identically distributed or whether they are independently drawn from a normal distribution whose mean and variance are changing over time.

An alternative approach to these issues is the autoregressive conditional heteroscedasticity (ARCH) model framework of Engle (1982). If the conditional distribution is normal, then the unconditional distribution will be symmetric but leptokurtic. Hsieh (1988) and Milhøj (1987) applied ARCH models to daily exchange-rate series, and Diebold and Nerlove (1989) estimated ARCH models for weekly spot-exchange rates.

The contribution of this article is to extend the preceding analysis by considering conditional leptokurtic distributions, as in Bollerslev (1987), along with a parsimonious generalized ARCH (GARCH) parameterization. This model successfully accounts for the severe leptokurtosis present in the daily data for five of the six currencies considered. The analysis also allows interesting day-of-the-week and vacation effects to be discovered in the conditional mean and the conditional-variance part of the model.

The model that was successful in representing daily data is then modified and reestimated on weekly, fortnightly, and monthly data. The effect of increasing the length of the sampling interval is clearly seen on the persistence of the conditional variance and the reduction in kurtosis.

The plan of the rest of the article is as follows. In Section 2, recent tests of Phillips (1987), Perron (1988), and Phillips and Perron (1988) are implemented on daily spot-exchange-rate data to show that the hypothesis of a unit root cannot be rejected. Although the traditional random-walk model with drift and Gaussian errors appears to account for the serial correlation properties of the daily data, it does not adequately describe the heteroscedasticity or severe kurtosis present in daily data. Section 3, therefore, estimates GARCH models with conditionally $t$-distributed errors, including daily dummies in both the conditional-mean and the conditional-variance equations of the model. Section 4 describes estimation of similar models on weekly, fortnightly, and monthly data. The final section of the article presents a summary of the empirical findings, along with a few concluding remarks.

## 2. AUTOCORRELATION STRUCTURE OF DAILY EXCHANGE RATES

Daily spot-exchange rates were obtained from the New York Foreign Exchange Market between March 1, 1980, and January 28, 1985. The data were provided by Data Resources Incorporated, and are opening bid prices, composed of 1,245 observations for the currencies of France, Italy, Japan, Switzerland, the United Kingdom, and West Germany vis-à-vis the U.S. dollar. The data are available on request. To rigorously test the random-walk hypothesis, recent tests for the presence of unit roots by Phillips (1987) and Phillips and Perron (1988) were implemented on the logarithm of the spot-exchange rate series $s_t$. The tests involved computing the ordinary least squares regressions

$$\log s_t = \hat{\mu} + \hat{\beta}(t - T/2) + \tilde{\alpha} \log s_{t-1} + \tilde{u}_t \quad (1)$$

and

$$\log s_t = \mu^* + \alpha^* \log s_{t-1} + u_t^*, \quad (2)$$

where $T$ denotes the sample size. The innovation sequences $\tilde{u}_t$ and $u_t^*$ have to satisfy a set of regularity conditions that ensure the existence of a nondegenerate asymptotic distribution for a suitably normalized function of their sums. One such set of $\alpha$-mixing conditions that permits many weakly dependent and heterogeneously distributed time series is stated by Phillips (1987). The two hypotheses on (1) given by $H_0^{(1)}$: $\tilde{\alpha} = 1$ and $H_0^{(2)}$: $\hat{\beta} = 0$, $\tilde{\alpha} = 1$ are tested by means of the statistics $Z(t_{\tilde{\alpha}})$ and $Z(\Phi_3)$, respectively. Similarly, a null hypothesis on (2) of the form $H_0^{(3)}$: $\alpha^* = 1$ is tested by the modified statistic $Z(t_{\alpha^*})$. These test statistics all involve long algebraic expressions and are omitted for reasons of space. The precise form of them is given in table 1 of Perron (1988). The statistics require consistent estimates of the variances of the sums of the innovations $\tilde{u}_t$ and $u_t^*$. The results given in this study were computed with a maximum lag of 22, or typically one month, on the autocovariances of the residuals, along with a Newey and West (1987) type of correction; see Perron (1988) for further details. The results are presented in Table 1 and indicate that the unit-root hypothesis cannot be rejected for any of the six currencies. This observation leads to a consideration of the martingale or random-walk model to explain short-run exchange-rate movements. Fama (1965) described an efficient market as having a "large number of rational profit maximizers actively competing with each other to predict future market values of individual securities and where important current information is almost freely available to all participants" (pp. 36–37). Under this weak form of market efficiency, all information is incorporated in past prices and the current price is a sufficient statistic for the distribution of future price movements. On denoting the one-period rate of return as $R_t$, the expected yield for period $t + 1$, conditional on the time $t$ information set $\Omega_t$, is then given by

$$E(R_{t+1} \mid \Omega_t) = \frac{E(s_{t+1} \mid \Omega_t) - s_t}{s_t}.$$

Assuming no transactions costs, no capital controls or taxes, no default risk, and no constraints on credit availability, it follows that $E(R_{t+1} \mid \Omega_t) = 0$ or, approximately, $E(\Delta \log s_{t+1} \mid \Omega_t) = 0$, where $\Delta$ denotes the first difference operator. This is known as the martingale property of log $s_t$ with respect to $\Omega$, which was formally derived by Samuelson (1965) and Mandelbrot (1963). Essentially, the condition states that the mean of the price change is unpredictable. Sargent (1987, pp. 93–97) also provided results based on asset-pricing theory for the validity of the martingale difference model. Typically, such theory says nothing about the relationship of the higher moments between successive price changes. This is in contrast to the random-walk model, originally due to Bachelier (1900), which states that yields or price changes are independent and identically distributed. Hence the presence of conditional heteroscedasticity is inconsistent with the random-walk model but does not violate the martingale difference hypothesis. Note that many previous studies of exchange-rate expectations or models of exchange-rate determination have ignored this form of heteroscedasticity (e.g., Baillie, Lippens, and McMahon 1983; Hakkio 1981). Moreover, Levich (1978) noted that some models of exchange-rate changes may imply autocorrelated errors, but no specific economic model appears to have been produced to explain the presence of heteroscedasticity.

Initially, the simple random-walk model with drift was estimated for all six currencies:

$$100\, \Delta \log s_t = b_0 + \varepsilon_t, \qquad \varepsilon_t \mid \Omega_{t-1} \sim N(0, \omega_0). \quad (3)$$

*Table 1. Phillips–Perron Tests for a Unit Root*

| Statistics | France | Italy | Japan | Switzerland | U.K. | West Germany |
|---|---|---|---|---|---|---|
| $Z(t_{\tilde{\alpha}})$ | −2.613 | −2.162 | −2.695 | −2.307 | −2.379 | −2.440 |
| $Z(\Phi_3)$ | 3.435 | 2.474 | 4.127 | 2.882 | 3.863 | 2.985 |
| $Z(t_{\alpha^*})$ | −.679 | −.968 | −1.935 | −.518 | .796 | −.921 |

NOTE: The 5% critical values for $Z(t_{\tilde{\alpha}})$, $Z(\Phi_3)$, and $Z(t_{\alpha^*})$ equal −3.41, 6.25, and −2.86, respectively. The truncation lag is equal to 22.

Table 2. Daily Normal Distributions

| Statistics | France | Italy | Japan | Switzerland | U.K. | West Germany |
|---|---|---|---|---|---|---|
| $b_0$ | −.067 | −.069 | −.002 | −.035 | −.056 | −.045 |
| | (.021) | (.017) | (.006) | (.021) | (.019) | (.020) |
| $\omega_0$ | .563 | .407 | .450 | .581 | .430 | .495 |
| | (.010) | (.012) | (.010) | (.019) | (.013) | (.016) |
| Log $L$ | −1,410.135 | −1,206.888 | −1,279.842 | −1,426.069 | −1,240.379 | −1,324.841 |
| $Q(15)$ | 19.319 | 22.406 | 24.987 | 19.025 | 18.012 | 13.483 |
| $Q^2(15)$ | 25.864 | 118.161 | 127.574 | 252.640 | 102.707 | 156.834 |
| $m_3$ | −.269 | .256 | .652 | .349 | −.274 | .345 |
| $m_4$ | 11.292 | 4.915 | 8.178 | 4.208 | 4.814 | 4.205 |

NOTE: Asymptotic standard errors are in parentheses. $100\ \Delta \log s_t = b_0 + \varepsilon_t, \varepsilon_t \mid \Omega_{t-1} \sim N(0, \omega_0)$ ($t = 1, \ldots, 1,244$).

The results are presented in Table 2, together with the Ljung and Box (1978) test statistic $Q(k)$ for $k$th-order serial correlation in $\hat{\varepsilon}_t$. In no case can the hypothesis of uncorrelated price changes be rejected. The statistics $m_3$ and $m_4$ are the standard measures of skewness and kurtosis. Under the assumptions of normality, they will have the asymptotic distributions of $m_3 \sim N(0, 6/T)$ and $m_4 \sim N(0, 24/T)$. Note that significant kurtosis is present in all of the residuals, confirming previous studies' findings of the inappropriateness of the assumption of normality for the unconditional distribution of $\Delta \log s_t$.

Table 2 also presents the Ljung–Box test statistic $Q^2(k)$ based on the squared residuals. Under the null hypothesis of conditional homoscedasticity, the statistic $Q^2(k)$ will have an asymptotic chi-squared distribution with $k$ df. The null hypothesis can be decisively rejected for five of the six currencies, while $Q^2(15)$ is only just significant at the .05 level for France. The extreme degree of kurtosis in the France/United States spot-exchange rate series, however, may well affect the power of the Ljung–Box test.

## 3. MODELS WITH TIME-DEPENDENT CONDITIONAL HETEROSCEDASTICITY

A model consistent with the results obtained in Section 2 is

$$100\ \Delta \log s_t = b_0 + \varepsilon_t, \qquad \varepsilon_t \mid \Omega_{t-1} \sim D(0, h_t),$$

$$h_t = \omega_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \tag{4}$$

where $D(0, h_t)$ represents some distribution with mean 0 and variance $h_t$. Following Bollerslev (1986), the conditional variance is given by the GARCH model of orders $p$ and $q$—that is, GARCH($p$, $q$). This model is a generalization of the pure ARCH($q$) model, originally due to Engle (1982), obtained when $p = 0$ in Equation (4). The conditional variance equation in (4) can also be expressed as

$$\varepsilon_t^2 = \omega_0 + \sum_{j=1}^{m} (\alpha_j + \beta_j)\varepsilon_{t-j}^2 - \sum_{j=1}^{p} \beta_j \eta_{t-j} + \eta_t,$$

where $\eta_t = \varepsilon_t^2 - h_t$, $m = \max(p, q)$, and $\eta_t$ is serially uncorrelated. Therefore, $\varepsilon_t^2$ will have the usual properties of an (autoregressive moving average) ARMA($m$, $p$) process, so standard identification procedures for the orders of $p$ and $m$ can be carried out on the $\hat{\varepsilon}_t^2$ series (see Bollerslev 1988).

An interesting aspect of daily exchange-rate data concerns the presence of significant autocorrelations around the seasonal lags in the $\hat{\varepsilon}_t^2$ series for all six currencies. For example, for West Germany the autocorrelations at lags 4 and 5 are .157 and .114, respectively, but the two-asymptotic-standard-error band is ±.057. One possibility would be to develop a multiplicative seasonal GARCH model in (4). In this study, however, the route of introducing seasonal dummy variables in the expressions for both the conditional mean and variance equations was followed. With the seasonal dummies included, it turned out that the simple GARCH(1, 1) model—that is, $p = q = 1$—provided a reasonably good fit for all the six currencies. Hence the estimated models were of the form

$$\begin{aligned} 100\ \Delta \log s_t &= b_0 + \sum_{j=1}^{5} b_j D_{jt} + b_8 W81_t + \varepsilon_t \\ \varepsilon_t \mid \Omega_{t-1} &\sim D(0, h_t) \\ h_t &= \omega_0(1 - \alpha_1 - \beta_1) + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \\ &\quad + (1 - (\alpha_1 + \beta_1)L) \\ &\quad \times \left[\sum_{j=1}^{5} \omega_j D_{jt} + (\omega_6 + \omega_7 L)V_t\right], \end{aligned} \tag{5}$$

where $L$ denotes the usual lag operator; $D_{1t}, \ldots, D_{5t}$ are day-of-the-week dummy variables for Monday through Friday, respectively; and $V_t$ is a vacation dummy that takes the value unity following the market being closed for any reason other than a weekend. In our sample of 1,245 daily observations, there are a total of 35 such vacations. Prior to October 1, 1981, transactions of foreign currencies on a Wednesday cleared on the Friday, while dollar transactions did not clear until the following Monday (see Levi 1983). Hence the potential loss of two days' interest reduces the demand for dollars relative to other currencies and, consequently, the value of other currencies vis-à-vis the U.S. dollar might be higher on Wednesdays, as noted by Levi

(1978), McFarland et al. (1982), and McCurdy and Morgan (1987). The $W81_t$ dummy variable in the mean which takes the value 1 for Wednesdays prior to October 1, 1981, allows for this effect.

The multiplicative parameterization in the conditional variance for the dummy variables has the advantage of allowing the effects to be interpreted as the response to a simple pulse as in standard intervention analysis (see Box and Tiao 1975). For instance, suppose that time $t$ refers to a Monday, in which case the immediate effect of $D_{1t}$ on $h_t$ will be $\omega_1$. Because of the GARCH(1, 1) parameterization, however, this Monday effect will lead to an expected increase/decrease in Tuesday's $h_{t+1}$ of $(\alpha_1 + \beta_1)\omega_1$, which consequently is offset by the $(\alpha_1 + \beta_1)L\omega_1 D_{1t+1}$ term.

Note also that in general neither $b_0$ nor $\omega_0$ can be estimated freely together with the five daily dummy-variable coefficients because of perfect multicolinearity. Only in the special situation in which $\alpha_1 + \beta_1 = 1$ [i.e., the model is integrated in variance; see Engle and Bollerslev (1986)] may a separate intercept term be estimated for the conditional variance.

Although the use of the conditional normal distribution in (5) will generate a leptokurtic unconditional distribution for $\Delta \log s_t$, initial estimation work revealed that it still did not adequately account for the degree of fat-tailedness in the unconditional distribution. Similar findings were reported by Hsieh (1988) and Milhøj (1987). Thus, to produce a more adequate representation of this data, an alternative methodology is to use a leptokurtic conditional distribution in (5) along with the GARCH conditional-variance model. Two possibilities for the conditional distribution were investigated, the $t$ distribution and the power exponential distribution.

For the standardized $t$ distribution with $v$ df, the log-likelihood function is given by

$$\log L = T\left[\log \Gamma\left(\frac{v+1}{2}\right) - \log \Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\log(v-2)\right] - \frac{1}{2}\sum_{t=1}^{T}[\log h_t + (v+1)\log(1 + \varepsilon_t^2 h_t^{-1}(v-2)^{-1})],$$

where $\Gamma(\cdot)$ denotes the usual gamma function (see Bollerslev 1987). For the power exponential distribution with indexing parameter $\lambda$, the log-likelihood can be expressed as

$$\log L = T \log \omega(\lambda) - \sum_{t=1}^{T}\left[\frac{1}{2}\log h_t - c(\lambda)(\varepsilon_t^2 h_t^{-1})^{1/(1+\lambda)}\right],$$

where

$$\omega(\lambda) = \{\Gamma(3(1+\lambda)/2)\}^{1/2}(1+\lambda)^{-1}\{\Gamma((1+\lambda)/2)\}^{-3/2}$$

and

$$c(\lambda) = \{\Gamma(3(1+\lambda)/2)/\Gamma((1+\lambda)/2)\}^{1/(1+\lambda)}$$

(e.g., see Box and Tiao 1973; Poirier, Tello, and Zin 1986). Even though the corresponding two likelihood functions are not nested within each other, it is interesting to note that for all of the models estimated here the maximized value of the log-likelihood was substantially higher for the $t$ distribution rather than the power exponential distribution. The estimates for the parameters in the conditional mean and variance, however, were very similar for the two different distributions. This suggests the possibility of calculating a Hausman-type specification test for model adequacy although it was not explicitly done in this study. For reasons of space, only the results obtained using the conditional $t$ distribution $td(0, h_t, v)$ in the place of $D(0, h_t)$ are reported.

The maximum likelihood estimates for the parameters in (5), including the degrees-of-freedom parameter $v$, are given in Table 3 for all six currencies. The estimates are obtained by the Berndt, Hall, Hall, and Hausman (1974) algorithm with numerical derivatives being used. To allow for easier day-of-the-week comparisons, the reported estimates for $b_1, \ldots, b_5$ and $\omega_1, \ldots, \omega_5$ have been standardized to sum to 0. Thus, for instance, the effect each week for Monday in the mean is given by $b_1$ plus the implied constant $b_0$.

Remarkable similarities are seen to exist between the six estimated models. The coefficient estimates of the dummy variables in both the conditional mean and the conditional variance part of the model generally take the same sign and are also of comparable relative magnitude across the different currencies. The estimated coefficients indicate that Tuesday-to-Wednesday price changes tend to be positive, whereas Friday-to-Monday, Wednesday-to-Thursday, and Thursday-to-Friday changes all tend to be negative, even when corrected for the appreciation of the U.S. dollar over the sample period. Interestingly, the Wednesday effect is not restricted to the period before October 1, 1981. Although the effect is higher before the acceleration of the clearing process for all the currencies as indicated by the positive $b_8$ coefficients, the difference is not significant at conventional levels. Mondays and Thursdays tend to differ because of the rate of flow of information arriving in the market. Monday's price reflects the accumulation of news that has occurred since the market closed on Friday, and money-supply figures are generally announced on Thursday. The Monday effect is particularly pronounced in the conditional-variance part of the model and is associated with a positive increase in the conditional variance for each currency. The increase in volatility is not proportional to the time elapsed between Friday's and Monday's price, however. A reduction in volatility occurs on Thursday. It is also interesting to note that, except for Monday's high, all of the variances are at a maximum on Tuesday. This

*Table 3. Daily GARCH Models*

| Parameter estimates and statistics | France | Italy | Japan | Switzerland | U.K. | West Germany |
|---|---|---|---|---|---|---|
| $b_0$ | −.073 | −.082 | −.027 | −.056 | −.051 | −.063 |
| | (—) | (—) | (—) | (—) | (—) | (—) |
| $b_1$ | −.025 | −.031 | −.040 | −.046 | −.015 | −.080 |
| | (.048) | (.041) | (.042) | (.046) | (.042) | (.045) |
| $b_2$ | −.006 | .011 | .051 | .007 | −.012 | .015 |
| | (.036) | (.033) | (.033) | (.038) | (.040) | (.038) |
| $b_3$ | .081 | .072 | .058 | .072 | .048 | .068 |
| | (.042) | (.037) | (.040) | (.057) | (.029) | (.052) |
| $b_4$ | −.032 | −.036 | −.010 | −.028 | −.012 | .004 |
| | (.032) | (.030) | (.027) | (.038) | (.033) | (.034) |
| $b_5$ | −.018 | −.015 | −.058 | −.005 | −.008 | −.007 |
| | (.035) | (.033) | (.035) | (.040) | (.036) | (.037) |
| $b_8$ | .130 | .117 | .018 | .120 | .092 | .114 |
| | (.084) | (.079) | (.078) | (.098) | (.072) | (.080) |
| $\omega_0$ | .505 | .403 | .402 | .537 | .404 | .448 |
| | (—) | (—) | (—) | (—) | (—) | (—) |
| $\omega_1$ | .183 | .102 | .120 | .099 | .069 | .121 |
| | (.105) | (.094) | (.159) | (.144) | (.082) | (.092) |
| $\omega_2$ | −.013 | −.008 | .002 | −.008 | .041 | .005 |
| | (.092) | (.086) | (.143) | (.134) | (.076) | (.081) |
| $\omega_3$ | −.023 | −.025 | −.020 | −.015 | −.037 | −.021 |
| | (.088) | (.085) | (.144) | (.127) | (.072) | (.074) |
| $\omega_4$ | −.103 | −.060 | −.097 | −.052 | −.062 | −.076 |
| | (.086) | (.086) | (.141) | (.127) | (.067) | (.076) |
| $\omega_5$ | −.044 | −.008 | −.005 | −.023 | −.011 | −.028 |
| | (.088) | (.089) | (.142) | (.126) | (.070) | (.077) |
| $\omega_6$ | .202 | .083 | .090 | .318 | .350 | .186 |
| | (.181) | (.121) | (.143) | (.237) | (.209) | (.187) |
| $\omega_7$ | .260 | .197 | .125 | .335 | .206 | .299 |
| | (.161) | (.112) | (.141) | (.195) | (.165) | (.151) |
| $\alpha_1$ | .114 | .113 | .049 | .073 | .061 | .085 |
| | (.025) | (.023) | (.011) | (.017) | (.017) | (.018) |
| $\beta_1$ | .829 | .848 | .941 | .907 | .910 | .881 |
| | (.035) | (.031) | (.013) | (.020) | (.025) | (.026) |
| $1/\nu$ | .140 | .093 | .159 | .054 | .109 | .054 |
| | (.017) | (.022) | (.020) | (.026) | (.022) | (.023) |
| Log $L$ | −1,249.475 | −1,104.247 | −1,154.179 | −1,337.186 | −1,167.155 | −1,236.408 |
| $Q(15)$ | 10.908 | 9.922 | 17.297 | 9.222 | 10.692 | 8.445 |
| $Q^2(15)$ | 7.579 | 20.702 | 26.625 | 21.805 | 9.012 | 21.417 |
| $m_3$ | −.069 | .105 | .425 | .152 | −.302 | .245 |
| $m_4$ | 7.372 | 3.873 | 4.909 | 3.395 | 4.904 | 3.535 |
| $3(\hat{\nu} - 2)(\hat{\nu} - 4)^{-1}$ | 4.915 | 3.890 | 5.621 | 3.413 | 4.160 | 3.414 |
| $LR_{1/\nu=0}$ | 71.640 | 19.416 | 57.382 | 5.373 | 36.895 | 7.717 |

NOTE: $100\,\Delta \log s_t = b_0 + \sum_{i=1}^{5} b_i D_{it} + b_8 W81_t + \varepsilon_t$, $\varepsilon_t \mid \Omega_{t-1} \sim td(0, h_t, \nu)$ $(t = 1, \ldots, 1{,}244)$. $h_t = \omega_0(1 - \alpha_1 - \beta_1) + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + (1 - (\alpha_1 + \beta_1)L)[\sum_{i=1}^{5} \omega_i D_{it} + (\omega_6 + \omega_7 L)V_t]$.

could be because the data are opening bid prices, so the full "weekend effect" does not reveal itself in Monday's price. Similarly, the dummy variables measuring vacation effects are associated with an increase in volatility that lasts for at least two days. We also calculated a series of Lagrange multiplier (LM) tests for the inclusion of a second lag of the vacation dummy $V_{t-2}$ in the conditional variance. The test statistics ranged from a high of 2.177 for Italy to a low of .274 for the United Kingdom, and none of the test statistics are significant at conventional levels in the asymptotic $\chi_1^2$ distribution. Moreover, the LM test statistic for the inclusion of the vacation dummy $V_t$ in the conditional mean ranges from 2.451 for West Germany to .087 for Italy, and all are insignificant at the 10% level.

To test for the overall effect of the different dummy variables, five separate likelihood ratio tests are presented in Table 4. Note that, although only a few of the dummy variables $b_1, \ldots, b_5, b_8$ are individually significant at the usual 5% level, only for the United Kingdom is there little evidence that the seasonal dummies are not worth incorporating in the conditional mean. The vacation dummies in the conditional variance, $\omega_6$ and $\omega_7$, add only a little explanatory power for Japan. At the same time, all of the six currencies show clear evidence of a systematic pattern in the daily movements in the conditional variances. Some of these effects have been noted by other authors; for example, Keim and Stambaugh (1984) described increased volatility around weekends in other financial markets, whereas McFarland et al. (1982, 1987), So (1987), and Hsieh (1988) discussed individual day-of-the-week effects in foreign-exchange-rate data.

The estimates of the GARCH parameters in Table 3

Table 4. Likelihood Ratio Tests for Daily Models

| Tests | France | Italy | Japan | Switzerland | U.K. | West Germany |
|---|---|---|---|---|---|---|
| $b_1 = \cdots = b_5 = b_8 = 0, b_0 \neq 0$ | 14.790 (.011) | 15.520 (.008) | 9.122 (.104) | 9.482 (.091) | 6.606 (.252) | 11.800 (.038) |
| $\omega_6 = \omega_7 = 0$ | 8.122 (.017) | 5.592 (.061) | 2.366 (.306) | 9.764 (.008) | 10.128 (.006) | 8.906 (.012) |
| $\omega_1 = \cdots = \omega_7 = 0, \omega_0 \neq 0$ | 34.908 (.000) | 22.310 (.001) | 18.842 (.004) | 19.092 (.004) | 22.156 (.001) | 29.812 (.000) |
| $b_1 = \cdots = b_5 = b_8 = 0, b_0 \neq 0$ $\omega_1 = \cdots = \omega_7 = 0, \omega_0 \neq 0$ | 48.598 (.000) | 36.056 (.000) | 29.244 (.002) | 27.918 (.003) | 27.586 (.004) | 41.370 (.000) |
| $b_1 = \cdots = b_5 = b_8 = 0, b_0 \neq 0$ $\omega_1 = \cdots = \omega_7 = 0, \omega_0 \neq 0$ $\alpha_1 = \beta_1 = 0$ | 133.828 (.000) | 139.750 (.000) | 123.742 (.000) | 141.728 (.000) | 92.709 (.000) | 143.512 (.000) |

NOTE: Asymptotic $p$ values are in parentheses.

are also extremely similar across currencies with the estimated value of $\alpha_1 + \beta_1$ being close to unity in all cases. This indicates the possible existence of an integrated GARCH process (see Engle and Bollerslev 1986). In fact, for none of the six currencies does a conventional $t$-type test on $\hat{\alpha}_1 + \hat{\beta}_1$ reject the null of a unit root in the conditional variance.

The use of the conditional $t$ distribution combined with the time-dependent conditional-variance equation can also be seen to have been quite successful in explaining leptokurtosis in the unconditional distribution of the exchange-rate changes. The estimated value of $\nu^{-1}$, the inverse of the degrees-of-freedom parameter in the $t$ distribution, lies between .054 and .159 with highly significant associated likelihood ratio (LR) test statistics $LR_{1/\nu=0}$. The estimated value of $\nu$ implies a conditional kurtosis equal to $3(\hat{\nu} - 2)(\hat{\nu} - 4)^{-1}$ (see Bollerslev 1987; Kendall and Stuart 1969). This estimated value of the conditional kurtosis might be compared with the sample analog of the standardized residuals $\hat{\varepsilon}_t^4\, \hat{h}_t^{-2} = m_4$. To a first-order approximation, the asymptotic variance of the estimated conditional kurtosis is given by $36[1 - 4/\hat{\nu}^{-1}]^{-4}\, \mathrm{var}(\hat{\nu}^{-1})$. It is unclear, however, as to how the likelihood behaves for small positive values of $\nu^{-1}$, and for this reason standard errors of $3(\hat{\nu} - 2)(\hat{\nu} - 4)^{-1}$ are not reported. Informal examination, however, indicates that only the standardized residuals for France exhibit unaccounted-for kurtosis. It is also interesting to note the reduction in the conditional kurtosis when compared to Table 2. Similarly, the implied values of $\omega_0$ given in Table 3 are all less than the corresponding sample variances in Table 2.

## 4. ANALYSIS OF EXCHANGE-RATE SERIES SAMPLED LESS FREQUENTLY

Tables 5, 6, and 7 present the results of estimating model (5) on weekly, fortnightly, and monthly (or, more precisely, four-weekly) data. The series are constructed by taking every Wednesday, every other Wednesday, and an observation every four Wednesdays, respectively. In the event of the market being closed, an observation on the following Thursday was used. Experiments using days other than Wednesdays or different Wednesdays for the fortnightly and monthly models did not reveal any markedly different results. For ease of exposition, in each case we report the most parsimonious model compatible with the data in terms of a likelihood ratio test versus the GARCH(1, 1) model with conditionally $t$-distributed errors, where the size of the test is equal to 5%.

For weekly data, the GARCH effects remain very pronounced (see also Diebold and Nerlove 1989). The models exhibit substantially less excess kurtosis in the residuals than before, however. Only France and Japan require the use of the conditional $t$ distribution; the normal appears adequate for the remainder. Note also that the significant $Q^2(10)$ statistics for Italy and West Germany are almost entirely due to significant autocorrelations of the squared residuals at lag 7 for Italy and lags 7 and 8 for West Germany. No explanation of this phenomenon was readily available.

For the fortnightly data, significant ARCH(1) effects are still visible for France, Italy, and Japan, but they are not significant for the other three currencies. Conditional normality is appropriate for all six series.

The details of analyzing the monthly series are given in Table 7. It can now be seen that with the exception of France there are no significant ARCH effects nor any substantial departures from normality. The excess kurtosis for France, however, is almost entirely due to the observation for August 1982 corresponding to the implementation of the austerity program by the Mauroy government. The August 1982 observation is about four standard deviations from the mean, and with this observation omitted, $m_4$ for France reduces to 3.469.

How much of this "convergence" to normality is due to the smaller sample size remains an open question. Since all of the daily models are random walks except for day-of-the-week dummies, however, increasing the sampling frequency beyond one week or longer will have the same effect as temporal aggregation, but, as noted by Diebold (1987), a standard central limit theorem type of argument implies that temporal aggregates

Table 5. Weekly GARCH Models

| Parameter estimates and statistics | France | Italy | Japan | Switzerland | U.K. | West Germany |
|---|---|---|---|---|---|---|
| $b_0$ | −.338 | −.356 | −.105 | −.226 | −.258 | −.289 |
| | (.087) | (.078) | (.078) | (.096) | (.083) | (.085) |
| $\omega_0$ | .488 | .281 | .006 | .245 | .202 | .295 |
| | (.442) | (.166) | (.037) | (.176) | (.325) | (.168) |
| $\alpha_1$ | .144 | .187 | .072 | .121 | .049 | .249 |
| | (.099) | (.097) | (.037) | (.062) | (.048) | (.107) |
| $\beta_1$ | .655 | .658 | .927 | .784 | .842 | .636 |
| | (.228) | (.146) | (.041) | (.106) | (.205) | (.125) |
| $1/\nu$ | .147 | — | .177 | — | — | — |
| | (.038) | | (.007) | | | |
| Log $L$ | −461.569 | −428.181 | −439.541 | −476.218 | −439.671 | −457.861 |
| $Q(10)$ | 11.494 | 9.423 | 7.683 | 8.317 | 10.354 | 9.246 |
| $Q^2(10)$ | 11.973 | 26.131 | 2.310 | 13.587 | 8.447 | 23.194 |
| $m_3$ | −.317 | .118 | 1.244 | .219 | .048 | .096 |
| $m_4$ | 4.671 | 3.280 | 7.759 | 2.998 | 3.485 | 3.007 |
| $3(\hat{\nu} - 2)(\hat{\nu} - 4)^{-1}$ | 5.132 | — | 6.653 | — | — | — |
| $LR_{1/\nu=0}$ | 10.496 | — | 10.764 | — | — | — |

NOTE: $100\,\Delta \log s_t = b_0 + \varepsilon_t$, $\varepsilon_t \mid \Omega_{t-1} \sim td(0, h_t, \nu)$, $h_t = \omega_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}$ $(t = 1, \ldots, 255)$.

of stable ARCH or GARCH processes tend to normality. Other studies, such as Domowitz and Hakkio (1985) and Kaminsky and Perugia (1987), have also reported finding only minimal ARCH effects on monthly exchange-rate data. This particular characteristic of exchange-rate data is in marked contrast to other types of financial-market data. For example, Engle, Lilien, and Robins (1987) reported significant ARCH effects for quarterly U.S. interest-rate data, while Morgan and Morgan (1987) and Chou (1988) found similar effects on monthly and less frequently sampled stock returns.

## 5. CONCLUSIONS

From subjecting daily foreign-exchange-market data to a close statistical examination, several stylized facts emerge:

1. Apart from similar day-of-the-week effects across currencies, the short-run movements in daily log spot rates are well approximated by a martingale difference model with severe excess kurtosis and time-dependent heteroscedasticity.

2. The conditional heteroscedasticity in daily spot rates is well represented by a GARCH(1, 1) process with near unit roots. Distinctive daily seasonality and vacation effects are present in the conditional variance, which can be partly explained by differing information flows.

3. ARCH effects are still very strong in weekly data, less so on fortnightly data, and minimal on monthly data.

4. After taking account of any ARCH effects, the assumption of conditional normality is a reasonable approximation on monthly and fortnightly data, whereas for weekly data the validity of the assumption seems to vary across currencies. With daily data, conditional normality is quite inappropriate, but can be replaced by the assumption of conditionally $t$ distributed errors.

As discussed earlier, many standard-asset pricing models imply the martingale difference model in which price changes are an uncorrelated process and hence unpredictable in the mean. The models are often uninformative about higher moments of such price

Table 6. Fortnightly ARCH Models

| Parameter estimates and statistics | France | Italy | Japan | Switzerland | U.K. | West Germany |
|---|---|---|---|---|---|---|
| $b_0$ | −.769 | −.764 | −.214 | −.361 | −.547 | −.457 |
| | (.180) | (.178) | (.202) | (.228) | (.176) | (.201) |
| $\omega_0$ | 3.096 | 2.562 | 3.236 | 5.766 | 3.665 | 4.989 |
| | (.672) | (.568) | (.590) | (.682) | (.473) | (.701) |
| $\alpha_1$ | .478 | .423 | .346 | — | — | — |
| | (.184) | (.211) | (.171) | | | |
| Log $L$ | −279.834 | −265.923 | −276.107 | −291.666 | −262.770 | −282.187 |
| $Q(10)$ | 18.438 | 14.680 | 7.272 | 9.309 | 6.437 | 12.088 |
| $Q^2(10)$ | 10.721 | 11.479 | 5.447 | 12.352 | 7.026 | 16.162 |
| $m_3$ | −.107 | −.030 | .559 | .592 | .325 | .231 |
| $m_4$ | 2.926 | 2.855 | 2.839 | 3.585 | 2.989 | 2.653 |

NOTE: $100\,\Delta \log s_t = b_0 + \varepsilon_t$, $\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$, $h_t = \omega_0 + \alpha_1 \varepsilon_{t-1}^2$ $(t = 1, \ldots, 127)$.

Table 7. Monthly Normal Distributions

| Parameter estimates and statistics | France | Italy | Japan | Switzerland | U.K. | West Germany |
|---|---|---|---|---|---|---|
| $b_0$ | −1.333 | −1.358 | −.035 | −.684 | −1.068 | −.916 |
| | (.452) | (.406) | (.394) | (.501) | (.377) | (.459) |
| $\omega_0$ | 12.935 | 10.159 | 10.453 | 13.890 | 8.199 | 12.521 |
| | (1.703) | (1.574) | (1.785) | (2.245) | (1.487) | (2.076) |
| Log $L$ | −170.158 | −162.528 | −163.390 | −172.279 | −155.640 | −169.112 |
| $Q(10)$ | 16.158 | 13.449 | 12.021 | 9.796 | 4.745 | 8.578 |
| $Q^2(10)$ | 10.684 | 10.969 | 6.192 | 16.807 | 5.927 | 13.108 |
| $m_3$ | −.097 | .208 | .528 | .601 | .416 | .375 |
| $m_4$ | 4.643 | 3.670 | 3.447 | 3.790 | 3.109 | 3.434 |

NOTE: $100\,\Delta \log s_t = b_0 + \varepsilon_t,\ \varepsilon_t \mid \Omega_{t-1} \sim N(0, \omega_0)$ $(t = 1, \ldots, 63)$.

changes, however. To this extent, our results are consistent with many standard-asset pricing models and show interesting characteristics of the price-change distribution. Furthermore, in models of currency-option pricing, the mean price change is related to its own variance and/or the covariance with other assets. Thus analysis of the time series properties of the own conditional variances of exchange rates may prove particularly helpful in the future analyses and understanding of such option pricing models.

## ACKNOWLEDGMENTS

We thank Robert Hodrick, Mark Watson, two anonymous referees, and an associate editor for useful suggestions and Lori Austin and Maxine Schafer for their excellent typing of a difficult manuscript.

[*Received August 1987. Revised September 1988.*]

## REFERENCES

Bachelier, L. J. B. A. (1900), *Théorie de la Speculation,* Paris: Gauthier-Villars.

Baillie, R. T., Lippens, R. E., and McMahon, P. C. (1983), "Testing Rational Expectations and Efficiency in the Foreign Exchange Market," *Ecônometrica,* 51, 553–563.

Berndt, E. K., Hall, B. H., Hall, R. E., and Hausman, J. A. (1974), "Estimation and Inference in Nonlinear Structural Models," *Annals of Economic and Social Measurement,* 4, 653–665.

Blattberg, R. C., and Gonedes, N. J. (1974), "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices," *Journal of Business,* 47, 244–280.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics,* 31, 307–327.

——— (1987), "A Conditional Heteroskedastic Time Series Model for Speculative Prices and Rates of Return," *Review of Economics and Statistics,* 69, 542–547.

——— (1988), "On the Correlation Structure for the Generalized Autoregressive Conditional Heteroskedastic Process," *Journal of Time Series Analysis,* 9, 121–131.

Boothe, P., and Glassman, D. (1987), "The Statistical Distribution of Exchange Rates," *Journal of International Economics,* 22, 297–319.

Box, G. E. P., and Tiao, G. C. (1973), *Bayesian Inference in Statistical Analysis,* Reading, MA: Addison-Wesley.

——— (1975), "Intervention Analysis With Applications to Economic and Environmental Problems," *Journal of the American Statistical Association,* 70, 70–79.

Chou, R. Y. (1988), "Persistent Volatility and Stock Returns—Some Empirical Evidence Using GARCH," *Journal of Applied Econometrics,* 3, 279–294.

Clark, P. (1973), "A Subordinate Stochastic Process Model With Finite Variance for Speculative Prices," *Econometrica,* 50, 987–1008.

Diebold, F. X. (1987), "Temporal Aggregation of ARCH Processes and the Distribution of Asset Returns," Special Studies Paper 200, Federal Reserve Board, Washington, DC.

Diebold, F. X., and Nerlove, M. (1989), "The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model," *Journal of Applied Econometrics,* 4, 1–21.

Domowitz, I., and Hakkio, C. S. (1985), "Conditional Variance and the Risk Premium in the Foreign Exchange Market," *Journal of International Economics,* 19, 47–66.

Engle, R. F. (1982), "Autoregressive Conditional Heteroskedasticity With Estimates of the Variance of U.K. Inflation," *Econometrica,* 50, 987–1008.

Engle, R. F., and Bollerslev, T. (1986), "Modelling the Persistence of Conditional Variances," *Econometric Reviews,* 5, 1–50.

Engle, R. F., Lilien, D. M., and Robins, R. P. (1987), "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model," *Econometrica,* 55, 391–407.

Fama, E. F. (1965), "The Behavior of Stock Market Prices," *Journal of Business,* 38, 34–105.

Friedman, D., and Vandersteel, S. (1982), "Short-Run Fluctuations in Foreign Exchange Rates: Evidence From the Data 1973–1979," *Journal of International Economics,* 13, 171–186.

Hakkio, C. S. (1981), "Expectations and the Forward Exchange Rate," *International Economic Review,* 22, 663–678.

Hsieh, D. A. (1988), "The Statistical Properties of Daily Foreign Exchange Rates: 1974–1983," *Journal of International Economics,* 24, 129–145.

Kaminsky, G., and Perugia, R. (1987), "Risk Premium and the Foreign Exchange Market," unpublished manuscript, University of California, San Diego, Dept. of Economics.

Keim, D. B., and Stambaugh, R. F. (1984), "A Further Investigation of the Weekend Effect in Stock Returns," *Journal of Finance,* 39, 819–837.

Kendall, M. G., and Stuart, A. (1969), *The Advanced Theory of Statistics* (Vol. 3, 2nd ed.), London: Charles Griffin.

Levi, M. D. (1978), "The Weekend Game: Clearing House Versus Federal Funds," *Canadian Journal of Economics,* 11, 750–757.

——— (1983), *International Finance, Financial Management and the International Economy,* New York: McGraw-Hill.

Levich, R. (1978), "On the Efficiency of Markets for Foreign Exchange," in *International Economic Policy, Theory and Evidence,* eds. R. Dornbusch and J. Frenkel, Baltimore: Johns Hopkins University Press, pp. 246–267.

Ljung, G. M., and Box, G. E. P. (1978), "On a Measure of Lack of Fit in Time Series Models," *Biometrika,* 65, 297–303.

Mandelbrot, B. (1963), "The Variation of Certain Speculative Prices," *Journal of Business,* 36, 394–419.

McCurdy, T., and Morgan, I. G. (1987), "Tests of the Martingale Hypothesis for Foreign Currency Futures With Time Varying Volatility," *International Journal of Forecasting,* 3, 131–148.

McFarland, J. W., Pettit, R. R., and Sung, S. K. (1982), "The Distribution of Foreign Exchange Price Changes: Trading Day Effects and Risk Measurement," *Journal of Finance,* 37, 693–715.

——— (1987), "The Distribution of Foreign Exchange Price Changes: Trading Day Effects and Risk Measurement: A Reply," *Journal of Finance,* 42, 189–194.

Milhøj, A. (1987), "A Conditional Variance Model for Daily Deviations of an Exchange Rate," *Journal of Business and Economic Statistics,* 5, 99–103.

Morgan, A., and Morgan, I. G. (1987), "Measurement of Abnormal Returns From Small Firms," *Journal of Business and Economic Statistics,* 5, 121–129.

Newey, K. N., and West, K. D. (1987), "A Simple, Positive Semidefinite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica,* 55, 703–708.

Perron, P. (1988), "Trends and Random Walks in Macroeconomic Time Series: Further Evidence From a New Approach," *Journal of Economic Dynamics and Control,* 12, 297–332.

Phillips, P. C. B. (1987), "Time Series Regression With a Unit Root," *Econometrica,* 55, 277–301.

Phillips, P. C. B., and Perron, P. (1988), "Testing for a Unit Root in Time Series Regression," *Biometrika,* 75, 335–346.

Poirier, D. J., Tello, M. D., and Zin, S. E. (1986), "A Diagnostic Test for Normality Within the Power Exponential Family," *Journal of Business and Economic Statistics,* 4, 359–373.

Praetz, P. D. (1972), "The Distribution of Share Price Changes," *Journal of Business,* 45, 49–55.

Rogalski, R. J., and Vinso, J. D. (1978), "Empirical Properties of Foreign Exchange Rates," *Journal of International Business Studies,* 9, 69–79.

Samuelson, P. A. (1965), "Proof That Properly Anticipated Prices Fluctuate Randomly," *Industrial Management Review,* 6, 41–49.

Sargent, T. J. (1987), *Dynamic Macroeconomic Theory,* Cambridge, MA: Harvard University Press.

So, J. C. (1987), "The Distribution of Foreign Exchange Price Changes: Trading Day Effects and Risk Measurement—a Comment," *Journal of Finance,* 42, 181–188.

Westerfield, J. (1977), "An Examination of Foreign Exchange Risk Under Fixed and Floating Rate Regimes," *Journal of International Economics,* 7, 181–200.