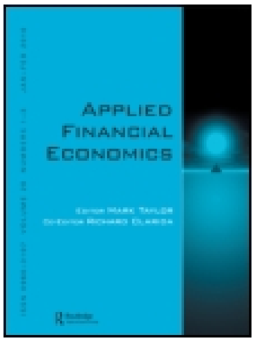

Applied Financial Economics

ISSN: 0960-3107 (Print) 1466-4305 (Online) Journal homepage: http://www.tandfonline.com/loi/rafe20

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To cite this article: Dima Alberg , Haim Shalit & Rami Yosef (2008) Estimating stock market volatility using asymmetric GARCH models, Applied Financial Economics, 18:15, 1201-1208, DOI: 10.1080/09603100701604225

To link to this article: https://doi.org/10.1080/09603100701604225

Published online: 21 Jul 2008.

Applied Financial Economics, 2008, **18**, 1201–1208

Routledge
Taylor & Francis Group

Estimating stock market volatility using asymmetric GARCH models

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A comprehensive empirical analysis of the mean return and conditional variance of Tel Aviv Stock Exchange (TASE) indices is performed using various GARCH models. The prediction performance of these conditional changing variance models is compared to newer asymmetric GJR and APARCH models. We also quantify the day-of-the-week effect and the leverage effect and test for asymmetric volatility. Our results show that the asymmetric GARCH model with fat-tailed densities improves overall estimation for measuring conditional variance. The EGARCH model using a skewed Student-*t* distribution is the most successful for forecasting TASE indices.

I. Introduction

Volatility clustering and leptokurtosis are commonly observed in financial time series (Mandelbrot, 1963). Another phenomenon often encountered is the so-called 'leverage effect' (Black, 1976), which occurs when stock prices change are negatively correlated with changes in volatility. Observations of this type in financial time series have led to the use of a wide range of varying variance models to estimate and predict volatility.

In his seminal paper, Engle (1982) proposed to model time-varying conditional variance with Auto-Regressive Conditional Heteroskedasticity (ARCH) processes using lagged disturbances. Empirical evidence based on his work showed that a high ARCH order is needed to capture the dynamic behaviour of conditional variance. The Generalized ARCH (GARCH) model of Bollerslev (1986) fulfills this requirement as it is based on an infinite ARCH specification which reduces the number of estimated parameters from infinity to two. Both the ARCH and GARCH models capture volatility clustering and leptokurtosis, but as their distribution is symmetric, they fail to model the leverage effect. To address this problem, many nonlinear extensions of GARCH have been proposed, such as the Exponential GARCH (EGARCH) model by Nelson (1991), the so-called GJR model by Glosten *et al*. (1993) and the Asymmetric Power ARCH (APARCH) model by Ding *et al*. (1993).

Another problem encountered when using GARCH models is that they do not always fully embrace the thick tails property of high frequency financial times-series. To overcome this drawback Bollerslev (1987), Baillie and Bollerslev (1989), and Beine *et al*. (2002) have used the Student's *t*-distribution. Similarly to capture skewness Liu and Brorsen (1995) used an asymmetric stable density. To model both skewness and kurtosis Fernandez and Steel (1998) used the skewed Student's *t*-distribution which was later extended to the GARCH framework by Lambert and Laurent (2000, 2001). To improve the fit of the GARCH and EGARCH models into international

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Applied Financial Economics ISSN 0960–3107 print/ISSN 1466–4305 online © 2008 Taylor & Francis
http://www.informaworld.com
DOI: 10.1080/09603100701604225

equity markets, Harris *et al*. (2004) used the skewed generalized Student's *t*-distribution to capture the skewness and leverage effects of daily returns.

Forecasting conditional variance with asymmetric GARCH models has been comprehensively studied by Pagan and Schwert (1990), Brailsford and Faff (1996) and Loudon *et al*. (2000). A comparison of normal density with nonnormal ones was made by Baillie and Bollerslev (1989), McMillan, *et al*. (2000), Lambert and Laurent (2001), Jun Yu (2002) and Siourounis (2002).

The purpose of this article is to characterize a volatility model by its ability to forecast and capture commonly held stylized facts about conditional volatility, such as persistence of volatility, mean reverting behaviour and asymmetric impacts of negative vs. positive return innovations. We investigate the forecasting performance of GARCH, EGARCH, GJR and APARCH models together with the different density functions: normal distribution, Student's *t*-distribution and asymmetric Student's *t*-distribution. We also compare between symmetric and asymmetric distributions using these three different density functions.

We forecast two major Tel-Aviv Stock Exchange (TASE) indices: TA100 and TA25. To compare the results, we use several standard performance measurements. Our results suggest that one can improve overall estimation by using the asymmetric GARCH model with fat-tailed densities for measuring conditional variance. For forecasting TASE indices, we find that the asymmetric EGARCH model is a better predictor than the asymmetric GARCH, GJR and APARCH models.

The article is structured as follows. Section II presents the data. In Section III, we present the methodology of the GARCH models used in the article. In Section IV, we describe the estimation procedures and present the forecasting results.

II. Data

The data consist of 3058 daily observations of the TA25[1] index from the period 20 october 1992 to 31 May 2005 and 1911 daily observations of the TA100[2] index from the period 2 July 1997 to 31 May 2005 that were obtained from finance.yahoo.com. To estimate and forecast these indices, we use G@RCH 2.0 by Laurent and Peters (2002), a package whose purpose is to estimate and forecast GARCH models and many of its extensions. The code written by Doornik (1999) in the Ox programming language provides a dialog-oriented interface with features that are not available in standard econometric software. To estimate the parameters, the G@RCH 2.0 package uses the Gaussian quasi-maximum likelihood (QML) model by Bollerslev and Wooldridge (1992). Although some of the models estimated are not Gaussian, the QML estimators were shown to be consistent assuming a correct specification of the conditional mean and the conditional variance. To maximize the quasi likelihood, the G@RCH 2.0 package uses numerical gradients and the Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization algorithm.[3]

III. Methodology

Conditional heteroskedastic models are the basic econometric tools used to estimate and forecast asset returns volatility. In this section we review succinctly the different ARCH models used in the article.

Let y_t be a time series of asset returns whose mean equation is given by $y_t = E(y_t|I_{t-1}) + \varepsilon_t$, here I_{t-1} is the information available at time $t-1$ and ε_t are the random innovations (surprises) with $E(\varepsilon_t) = 0$. To explain the conditional variance dynamics Engle (1982) proposed the autoregressive conditional heteroskedasticity (ARCH) model that estimates the variance of returns as a simple quadratic function of the lagged values of the innovations:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

where $\varepsilon_t = \sigma_t z_t$ and z_t is i.i.d. random variable with mean zero and variance one.

One of the weaknesses of the ARCH model is that it often requires many parameters and a high order q to capture the volatility process. To remedy this lacuna Bollerslev (1986) proposes the GARCH model, which is based on an infinite ARCH specification, that enables us to reduce the number of estimated parameters by imposing

[1] The TA25 Index is a value-weighted index of the shares of the 25 companies with the highest market capitalization that are traded on the TASE.
[2] The TA100 Index is a value-weighted index of the shares of the 100 companies with the highest market capitalization that are traded on the TASE.
[3] The BFGS method approximates the Hessian matrix by analyzing successive gradients vectors.

nonlinear restrictions. The standard GARCH (p, q) model expresses the variance at time t, σ_t^2, as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where α_i,β_j and ω are the parameters to be estimated. Using the lag operator L, the variance becomes:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

where $\alpha(L) = \sum_{i=1}^{q} \alpha_i L^i$ and $\beta(L) = \sum_{j=1}^{p} \beta_j L^j$.

If all the roots of the polynomial $|1 - \beta(L)| = 0$ lie outside the unit circle, we have:

$$\sigma_t^2 = \omega[1 - \beta(L)]^{-1} + \alpha(L)[1 - \beta(L)]^{-1}\varepsilon_t^2.$$

This equation may be envisaged as an ARCH (∞) process since the conditional variance depends linearly on all previous squared residuals. As such, the conditional variance of y_t can become larger than the unconditional variance. Then, if past realizations of ε_t^2 are larger than σ^2 the variance is given by:

$$\sigma^2 \equiv E(\varepsilon_t^2) = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j}$$

Like ARCH, some restrictions are needed to ensure that σ_t^2 is positive for all t. Bollerslev (1986) shows that imposing $\omega > 0$, $\alpha_i \geq 0$ (for $i = 1, \ldots, q$) and $\beta_j \geq 0$ (for $j = 1, \ldots, p$) is sufficient for the conditional variance to be positive.

To capture the asymmetry observed in the data, a new class of ARCH models was introduced: the GJR by Glosten *et al.* (1993), the exponential GARCH (EGARCH) model by Nelson (1991) and the APARCH model by Ding, *et al.* (1993). This last model that has the feature to generate many ARCH models by varying the parameters is expressed as:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta$$

where $\delta > 0$ and $-1 < \gamma_i < 1$. The parameters allow us to capture the asymmetric effects. For example, if $\gamma_1 = 0$ a positive surprise $\varepsilon_t > 0$ has the same effect on volatility than a negative surprise $\varepsilon_t < 0$. The presence of a leverage effect can be investigated by testing the hypothesis that $\gamma_1 < 0$.

In our article we estimate conditional volatility using the three probability distributions that are available in the G@RCH package: the normal, the t-distribution and, the skewed t-distribution. The normal distribution was originally used by Engle (1982) in the ARCH model. Bollerslev (1987), on the other hand, proposed a standardized Student's t-distribution with $\nu > 2$ degrees of freedom whose density is given by:

$$D(z_t; \nu) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\pi(\nu-2)}} \left(1 + \frac{z_t^2}{\nu-2}\right)^{-(\nu+1)/2}$$

where $z_t = \varepsilon_t/\sigma_t$, $\Gamma(\nu) = \int_0^\infty e^{-x} x^{\nu-1} \mathrm{d}x$ is the gamma function and, ν is the parameter that measures the tail thickness. The Student's t-distribution is symmetric around mean zero. For $\nu > 4$, the conditional kurtosis equals $3(\nu-2)/(\nu-4)$, which exceeds the normal value of 3.

The common methodology for estimating ARCH is by maximum likelihood assuming i.i.d. innovations. For $D(z_t; \nu)$, the log-likelihood function of $\{y_t(\theta)\}$ for the Student's t-distribution is given by:

$$L_T(\{y_t\}, \theta) = T\left(\ln\Gamma\left(\frac{\nu+1}{2}\right) - \ln\left(\frac{\nu}{2}\right) - 1/2\ln(\pi(\nu-2))\right) - 1/2\sum_{t=1}^{T}\left(\ln(\sigma_t^2) + (1+\nu)\ln\left(1 + \frac{z_t^2}{\nu-2}\right)\right)$$

where θ is the vector of parameters to be estimated for the conditional mean, the conditional variance and the density function. When $\nu \to \infty$ we have a normal distribution, so that the lower ν is, the fatter are the tails. Recently, Lambert and Laurent (2000, 2001) extended the skewed Student's t-distribution proposed by Fernandez and Steel (1998) to the GARCH framework. Using $D(z_t;\nu)$, the log-likelihood function of $\{y_t(\theta)\}$ for the skewed Student's t-distribution is given by:

$$\begin{aligned} L_T(\{y_t\}; \theta) = T\Bigg(&\ln\Gamma\left(\frac{\nu+1}{2}\right) - \ln\left(\frac{\nu}{2}\right) - 1/2\ln(\pi(\nu-2)) \\ &+ \ln\left(\frac{2}{\xi + (1/\xi)}\right) + \ln(s)\Bigg) \\ &- 1/2\sum_{t=1}^{T}\Bigg(\ln\left(\sigma_t^2\right) + (1+\nu) \\ &\ln\left(1 + \frac{(sz_t + m)^2}{\nu-2}\xi^{-2I_t}\right)\Bigg) \end{aligned}$$

where ξ is the asymmetry parameter, ν the degree of freedom of the distribution and

$$I_t = \begin{cases} 1, & \text{if} \quad z_t \geq -\frac{m}{s} \\ -1 & \text{if} \quad z_t < -\frac{m}{s} \end{cases},$$

$$m = \frac{\Gamma((\nu+1/2))\sqrt{\nu-2}}{\sqrt{\pi}\Gamma(\nu/2)}\left(\xi - \frac{1}{\xi}\right) \quad \text{and}$$

$$s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$$

(See Lambert and Laurent (2001) for more details.)

Maximum likelihood estimates of parameters are obtained using the quasi-maximum likelihood estimator (QMLE). According to Bollerslev and Wooldridge (1992) this estimator is generally consistent, has a normal limiting distribution, and provides asymptotic standard errors that are valid under non-normality.

IV. Estimation Results

Descriptive statistics and stationarity

To obtain a stationary series, we use the returns $r_t = 100(\log(P_t) - \log(P_{t-1}))$ where P_t is the closing value of the index at date t. The sample statistics for the returns r_t are exhibited in Table 1. For the two indices, TA25 (sample 20 October 1992 to 31 May 2005) and TA100 (sample 2 July 1997 to 31 May 2005), the sample kurtosis is greater than three, meaning that return distributions have excess kurtosis. Excess skewness is also observed for the two indices, leading to high Jarque-Bera statistics indicating non-normality.

In addition to excess skewness and excess kurtosis as daily stock returns may be correlated with the day-of-the-week effect, we want to avoid the potential calendar effect on the volatility analysis. Indeed, many studies have documented the presence of calendar anomalies in financial markets (Cross (1973), French (1980), Alexakis and Xanthakis (1995) and Pena (1995) among others). Indeed, these studies have shown an increasing trading activity on Mondays and Fridays relative to the other days, leading researchers to check whether these patterns produce significantly negative return on Mondays as in the US stock markets data or a positive Monday effect seen with European shares. Contrary to the daily patterns observed in the European and American stock markets, the Israeli trading week lasts from Sunday till Thursday. Therefore the so-called Mondays and Fridays effects may translate to the Sunday and Thursday effects in the TASE. To isolate these seasonalities, we filter the daily means and variances using the following two regressions:

$$(1)\quad r_t = \alpha_1 \mathrm{SUN}_t + \alpha_2 \mathrm{MON}_t + \alpha_3 \mathrm{TUE}_t + \alpha_4 \mathrm{WED}_t + \alpha_5 \mathrm{THU}_t + \delta_t,$$

$$(2)\quad (r_t - \hat{r}_t)^2 = \beta_1 \mathrm{SUN}_t + \beta_2 \mathrm{MON}_t + \beta_3 \mathrm{TUE}_t + \beta_4 \mathrm{WED}_t + \beta_5 \mathrm{THU}_t + \varepsilon_t,$$

where $\mathrm{SUN_t}$, $\mathrm{MON_t}$, $\mathrm{TUE_t}$, $\mathrm{WED_t}$ and $\mathrm{THU_t}$ are the dummy variables for Sunday, Monday, Tuesday, Wednesday and Thursday, and $\hat{r}_t$ is the ordinary least squares (OLS) fitted value of r_t from regression (1) at date t.

The OLS estimates of the two regressions exhibited on Tables 2 and 3 show that the TA25 and TA100 indices have significantly positive daily means on Sunday and significant daily variations for Sunday through Thursday.

To eliminate the daily effects and obtain a stationary series, we 'standardize' the daily returns using $y_t = (r_t - \hat{r}_t)/\sqrt{\hat{\eta}_t}$, where $\hat{\eta}_t$ is the fitted value of $(r_t - \hat{r}_t)^2$ from regression (2) at date t. We now substitute the original daily return r_t with y_t and referred it to the daily return at date t. In Table 4, the returns y_t for the two indices are shown to be normalized to mean zero and variance one. By looking at their skewness, kurtosis and Jarque-Bera statistics we reject the null hypothesis that the returns y_t are normally distributed for the two indices.

Choosing a volatility model

The basic estimation model consists of two equations, one for the mean which is a simple autoregressive AR(1) model and another for the variance which is identified by a particular ARCH specification, i.e., GARCH(1,1), EGARCH(1,1), GJR(1,1) and, APARCH(1,1). For the two TASE indices the models are estimated using G@RCH 2.0 by the approximate quasi-maximum likelihood estimator assuming normal, Student-t or skewed Student-t errors. Note that it is quite evident that the recursive evaluation of maximum likelihood is conditional on unobserved values and therefore the estimation cannot be considered to be perfectly exact. To solve the problem of unobserved values, we set them to their unconditional expected values.[4]

For the TA25 index, convergence could not be reached with a EGARCH model and a Student's *t*-distribution. Therefore we use the other three models where all asymmetry coefficients are significant at standard levels. To compare the different models we apply several standard criteria: The $Q(20)$ and $Q^2(20)$ which are the Box-Pierce statistics at lag 20 of the standardized and squared standardized residuals, the $P(50)$ Pearson goodness-of-fit with 50 cells,[5] the *AIC*

[4] The estimated values for the four models are available from the authors.
[5] The Prob[1] and Prob[2] are the probability values for $P(50)$, the first using 49 degrees of freedom and the second 49 minus the number of estimated parameters.

Table 1. Descriptive statistics for logarithm differences 100 [ln(P_t) – ln(P_t – 1)]

Index	Average	Min	Max	SD	Kurtosis	Skewness	Jarque–Bera stat.
TA25	0.0433	−10.1555	7.1408	1.4895	3.4024	−0.2106	43.24
TA100	0.0452	−10.3816	7.6922	1.3198	5.0898	−0.4546	413.56

Table 2. Regression coefficients for day-of-the-week effect – TA25

Regression	Sunday	Monday	Tuesday	Wednesday	Thursday
Mean (1)	0.12**	0.020	0.041	−0.0395	0.074
SE	0.06	0.06	0.06	0.06	0.06
Variance (2)	3.41**	1.45**	2.19**	1.19**	2.12**
SE	0.205	0.205	0.207	0.208	0.207

Note: ** Indicates significance at 1%.

Table 3. Regression coefficients for day-of-the-week effect – TA100

Regression	Sunday	Monday	Tuesday	Wednesday	Thursday
Mean (1)	0.178**	0.089	−0.040	−0.097	0.090
SE	0.0663	0.0663	0.0673	0.0673	0.0671
Variance (2)	2.732**	0.878**	1.658**	1.757**	1.626**
SE	0.231	0.231	0.235	0.235	0.235

Note: See Table 2 footnote.

Table 4. Descriptive statistics for 'standardized' returns y_t

Index	Average	Min	Max	SD	Kurtosis	Skewness	Jarque–Bera stat.
TA25	0	−6.7131	5.30431	0.9999	3.1528	−0.2179	27.17
TA100	0	−8.0305	5.8756	1.00026	4.569	−0.4561	262.27

Akaike information criterion and the Log-Lik log-likelihood value. Tables 5 and 6 present only the most significant results. Indeed, the Akaike information criteria (AIC) and the log-likelihood values reveal that the EGARCH, APARCH and GJR models better estimate the series than the traditional GARCH.

When we analyze the densities we find that the two Student's *t*-distributions (symmetric and skewed) clearly outperform the normal distribution. Indeed, the log-likelihood function increases when using the skewed Student's *t*-distribution, leading to AIC criteria of 2.701 and 2.730 for the normal density vs. 2.665 and 2.697 for the non–normal densities, for the TA100 and the TA25 indices, respectively.

For all the models, the dynamics of the first two moments of the series are tested with the Box-Pierce statistics for residuals and squared residuals which reject no serial correlation at the 5% level. Furthermore, stationary is satisfied for every model and for every density.

Forecasting

The forecasting ability of GARCH models has been comprehensively discussed by Poon and Granger (2003). We use the G@RCH 2.0 package to evaluate 30 one-step-ahead forecasts using a rolling window of 3058 and 1911 observations for TA25 and TA100. This is done for both the mean equation and the variance equation. The forecasts we obtained are evaluated using five different measures since the squared daily returns may not be the proper measure to assess the forecasting performance of the different

Table 5. Comparison between the models for the TA100

TA100	Normal APARCH	Student's t APARCH	Skewed t EGARCH
$Q(20)$	20.739	20.731	20.313
$Q^2(20)$	24.051	27.762	24.630
$P(50)$	72.909	48.472	46.326
Prob[1]	0.015	0.494	0.582
Prob[2]	0.002	0.168	0.196
AIC	2.730	2.697	2.697
Log-lik	−2600.690	−2568.060	−2566.940

Table 6. Comparison between the models for the TA25

TA25	Normal GJR	Student's t APARCH	Skewed t GJR	APARCH	EGARCH
0(20)	32.290	32.363	32.080	32.157	32.689
$Q^2(20)$	14.875	17.177	18.705	26.229	24.349
$P(50)$	58.710	65.806	46.055	50.241	45.564
Prob[1]	0.161	0.055	0.593	0.424	0.613
Prob[2]	0.045	0.008	0.271	0.129	0.218
AIC	2.701	2.699	2.669	2.665	2.665
Log-lik	−4122.670	−4118.770	−4073.290	−4065.650	−4065.430

GARCH models for conditional variance according to Andersen and Bollerslev (1998). The advantage of using many forecasting measures resides in the robustness in choosing an optimal predictor model.[6] We consider the following measures:

(1) Mean squared error (MSE):

$$\mathrm{MSE} = \frac{1}{h+1}\sum\nolimits_{t=S}^{S+h} (\hat{\sigma}_t^2 - \sigma_t^2)^2$$

(2) Median squared error (MedSE):

$$\mathrm{MedSE} = \mathrm{Inv}(f_{\mathrm{Med}}(e_t)),$$
$$\text{where } e_t = (\hat{\sigma}_t^2 - \sigma_t^2)^2 \text{ and } t \in [S, S+h]$$

(3) Mean absolute error (MAE):

$$\mathrm{MAE} = \frac{1}{h+1}\sum\nolimits_{t=S}^{S+h} |\hat{\sigma}_t^2 - \sigma_t^2|$$

(4) Adjusted mean absolute percentage error (AMAPE):

$$\mathrm{AMAPE} = \frac{1}{h+1}\sum\nolimits_{t=S}^{S+h} \left|\frac{\hat{\sigma}_t^2 - \sigma_t^2}{\hat{\sigma}_t^2 - \sigma_t^2}\right|,$$

(5) Theil's inequality coefficient (TIC):

$$\mathrm{TIC} = \frac{\sqrt{\frac{1}{h+1}\sum_{t=S}^{S+h}\left(\hat{Y}_t - Y_t\right)^2}}{\sqrt{\frac{1}{h+1}\sum_{t=S}^{S+h}(\hat{y}_t)^2} - \sqrt{\frac{1}{h+1}\sum_{t=S}^{S+h}(y_t)^2}}$$

where h is the number of lead steps, S is the sample size, $\hat{\sigma}_t^2$ is the forecasted variance and σ_t^2 is the 'actual' variance.

The forecasting ability is reported by ranking the different models with respect to the five measures. This is done in Tables 7 and 8 that compare the distributions for the TA25 and TA100 indexes.

For the TA25 index the results support the use of the asymmetric EGARCH model. For most measures in the variance equation, the EGARCH model outperforms the APARCH model. The GARCH model provides much less satisfactory results and the GJR model provides the poorest forecasts.

For the TA100 index, the EGARCH model gives better forecasts than the GARCH model while the APARCH and GJR models give the poorest forecasts. The skewed Student-t distribution is the most successful in forecasting the TA100 conditional variance, contrary to the TA25 where the

[6] MSE and MAE are generally affected by larger errors such as in the case of outliers. MedSE and AMAPE have the advantage of reducing the effect of outliers.

Table 7. Forecasting analysis for the TA25 index: comparing between densities

	GARCH		EGARCH	GJR		APARCH
TA25	Student-t	Skewed-t	Skewed-t	Student-t	Skewed-t	Skewed-t
MSE(1)	0.187	0.187	0.188	0.187	0.187	0.188
MSE(2)	0.344	0.343	**0.269**	0.407	0.415	0.347
MedSE(1)	0.029	0.029	0.033	0.033	0.033	0.033
MedSE(2)	0.223	0.227	**0.128**	0.309	0.316	0.224
MAE(1)	0.274	0.274	0.272	0.273	0.272	0.272
MAE(2)	0.500	0.499	**0.397**	0.568	0.575	0.501
RMSE(1)	0.432	0.432	0.433	0.433	0.433	0.434
RMSE(2)	0.587	0.586	**0.519**	0.638	0.644	0.589
AMAPE(2)	0.782	0.782	**0.758**	0.795	0.796	0.781
TIC(1)	0.938	0.944	0.983	0.968	0.976	0.981
TIC(2)	0.559	0.559	0.565	0.565	0.566	0.560

Notes: (1) – mean equation, (2) – variance equation. The bold figures show the best result for the forecasting measures.

Table 8. Forecasting analysis for the TA100 index: comparing between densities

	GARCH	EGARCH	GJR	APARCH
TA100	Skewed-t	Skewed-t	Student-t	Student-t
MSE(1)	0.218	0.220	0.219	0.220
MSE(2)	0.366	**0.275**	0.444	0.393
MedSE(1)	0.093	0.092	0.093	0.092
MedSE(2)	0.376	**0.292**	0.466	0.409
MAE(1)	0.379	0.381	0.38	0.381
MAE(2)	0.562	**0.480**	0.624	0.586
RMSE(1)	0.466	0.469	0.468	0.469
RMSE(2)	0.605	**0.525**	0.666	0.627
AMAPE(2)	0.648	**0.621**	0.664	0.654
TIC(1)	0.954	0.969	0.965	0.970
TIC(2)	0.538	**0.509**	0.561	0.547

Notes: (1) – mean equation, (2) – variance equation. The bold figures show the best result for the forecasting measures.

results conflict. As such we were unable to draw a general conclusion. The skewed Student-*t* distribution seems to be the best for forecasting series showing higher skewness. In fact, Lambert and Laurent (2001) found that the skewed Student-*t* density is more appropriate for modelling the NASDAQ index than symmetric densities.

V. Conclusion

We compared the forecasting performance of several GARCH models using different distributions for two Tel Aviv stock index returns. We found that the EGARCH skewed Student-t model is the most promising for characterizing the dynamic behaviour of these returns as it reflects their underlying process in terms of serial correlation, asymmetric volatility clustering and leptokurtic innovation. The results also show that asymmetric GARCH models improve the forecasting performance. Among the forecasts tested, the EGARCH skewed Student-t model outperformed GARGH, GJR and APARCH models. This result later further implies that the EGARCH model might be more useful than the other three models when implementing risk management strategies for Tel Aviv stock index returns.

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