Networks.

CS4423.

Prof. Götz Pfeiffer
http://schmidt.nuigalway.ie/cs4423

School of Mathematics, Statistics and Applied Mathematics
NUI Galway

Semester 2 (2018/2019)

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Outline

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Useful Books.


Vito Latora, Vincenzo Nicosia and Giovanni Russo.
Complex Networks.
Cambridge UP 2017.


Albert-László Barabási.
Network Science.
Cambridge UP 2016.


Ernesto Estrada and Philip A. Knight.
A first course in network theory.
Oxford UP 2015.


Mark Newman.
Networks: an Introduction.
Oxford UP 2010.


David Easley and Jon Kleinberg.
Networks, Crowds and Markets.
Cambridge UP 2010.

Connections.

Modern societies are in many ways highly connected. Certain aspects of this phenomenon are frequently described as **networks**:

- **social** networks (of people linked by friendship or social interaction)
- **computer** networks (the Internet)
- **document** networks (World Wide Web)
- **academic** networks (collaborations, citations of scientific publications)
- **transportation** networks (road, rail, airline, subway)
- networks in **biology** (e.g., biochemical, ecological, or neural networks)
- **financial** networks (transactions between traders)
- . . .

What do these structures have in common? How can they be distinguished?

Simple Forms of Networks.

- In its simplest form, a network is just a collection of points (called **vertices** or **nodes**), some of which are joined in pairs (called **edges** or **links**).
- Many systems of interest are composed of individual parts that are in some way linked together.
- Such systems can be regarded as networks, and thinking about them in this way can often lead to new and useful **insights**.
- **Network science** studies the **pattern of connections** between the components of a system
- Naturally, the **structure** of the network can have a big impact on the **behavior** of a system.
- The connections in a **social network**, for example, affect how people learn, form opinions, or spread a disease.

Network Science

- A network is a **simplified representation** of a complex system by vertices and edges.
- The scientific study of networks is an **interdisciplinary** undertaking that combines ideas from
  - mathematics,
  - physics,
  - biology,
  - computer science,
  - the social sciences and
  - many other areas.
- Between these scientific fields, many **tools** have been developed for **analyzing**, **modeling** and **understanding** networks.

Network Measures

- An example of a useful and important type of network measures is that of **centrality**.
- It is concerned with the question of how **important** a particular vertex or edge is in a networked system.
- Different concepts have suggested for capturing **mathematically** what it means to be central.
- A simple measure of the centrality of a vertex is its **degree**, i.e., the number of edges it is part of.

Which measurements and calculations give **meaningful answers** for a particular system depends of course on the specific nature of the system and the questions one wants to ask.

Network Concepts

- Another interesting network concept is the **small-world effect**.
- It is concerned with the question of how far apart two randomly chosen points in a network typically are.
- Here,**distance** is usually measured by the number of edges one would need to cross over, when travelling along a **path** from one vertex to the other.
- In real world social networks the distance between people tends to be rather small.
- This observation and is known as the **six degrees of separation** in popular culture.

This course is meant as an **introduction to Network Science** where, within the given time and space constraints, some but certainly not all of its interesting aspects will be discussed.

Examples of Networks.

Newman broadly divides the most commonly studied real world networks into four classes:

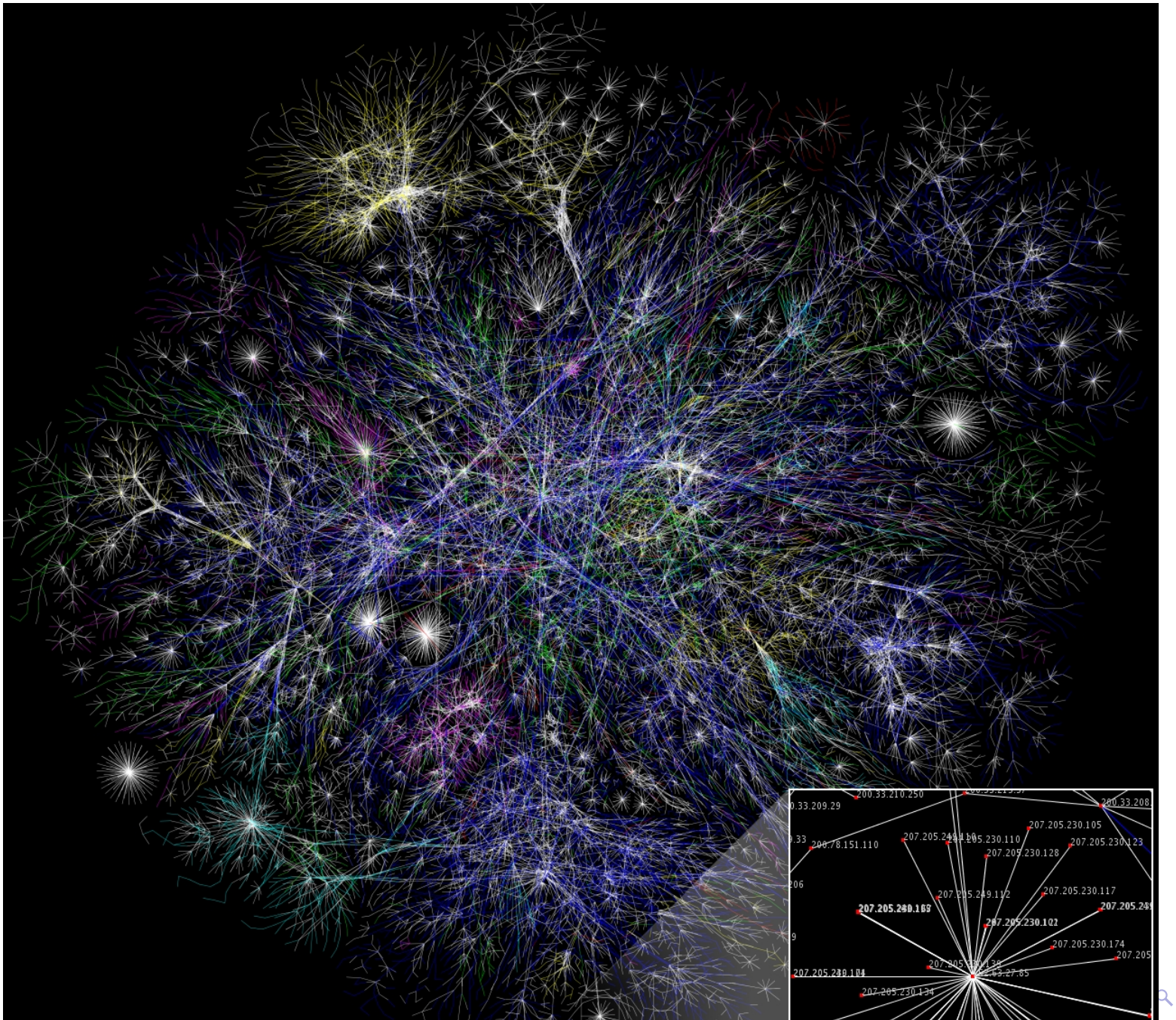
- **technological** networks,
- **social** networks,
- **information** networks and
- **biological** networks.

There is of course some **overlap** between these classes. But it is an interesting exercise to list important examples within each class, and to describe their **general structure**, and the techniques used to **discover and measure** this structure in each example.

Technological Networks.

- Technological networks rely on a **physical infrastructure**.
- In many cases, this infrastructure has been built over many decades
- It forms part of the **backbone** of modern societies.
- This includes
  - **road** and other **transportation networks**,
  - **power grids**,
  - the **telephone network**, and more recently
  - the **Internet**.

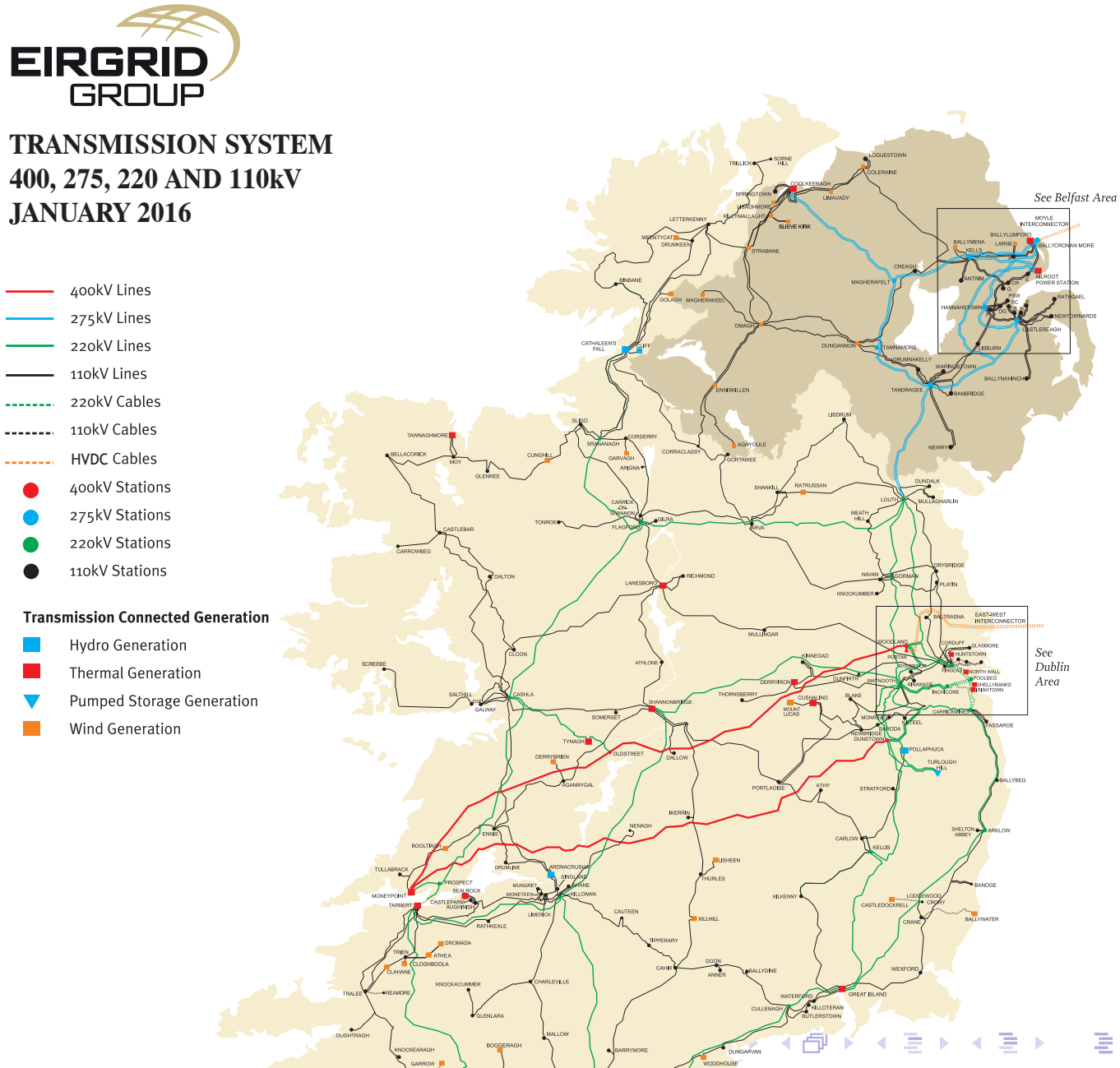
The Internet (Internet Mapping Project, 1998)



Power Grids

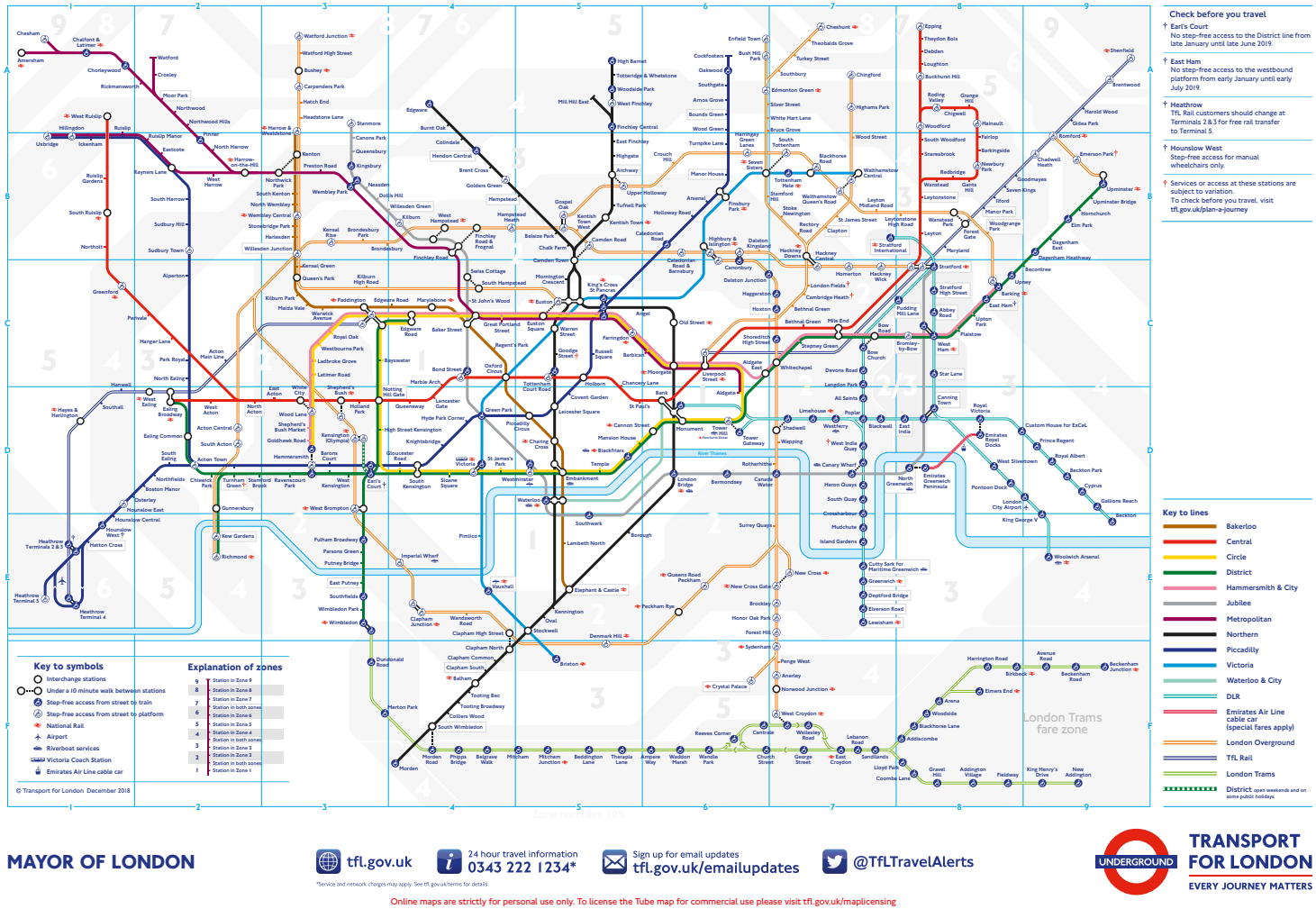
EIRGRID GROUP

TRANSMISSION SYSTEM
400, 275, 220 AND 110kV
JANUARY 2016

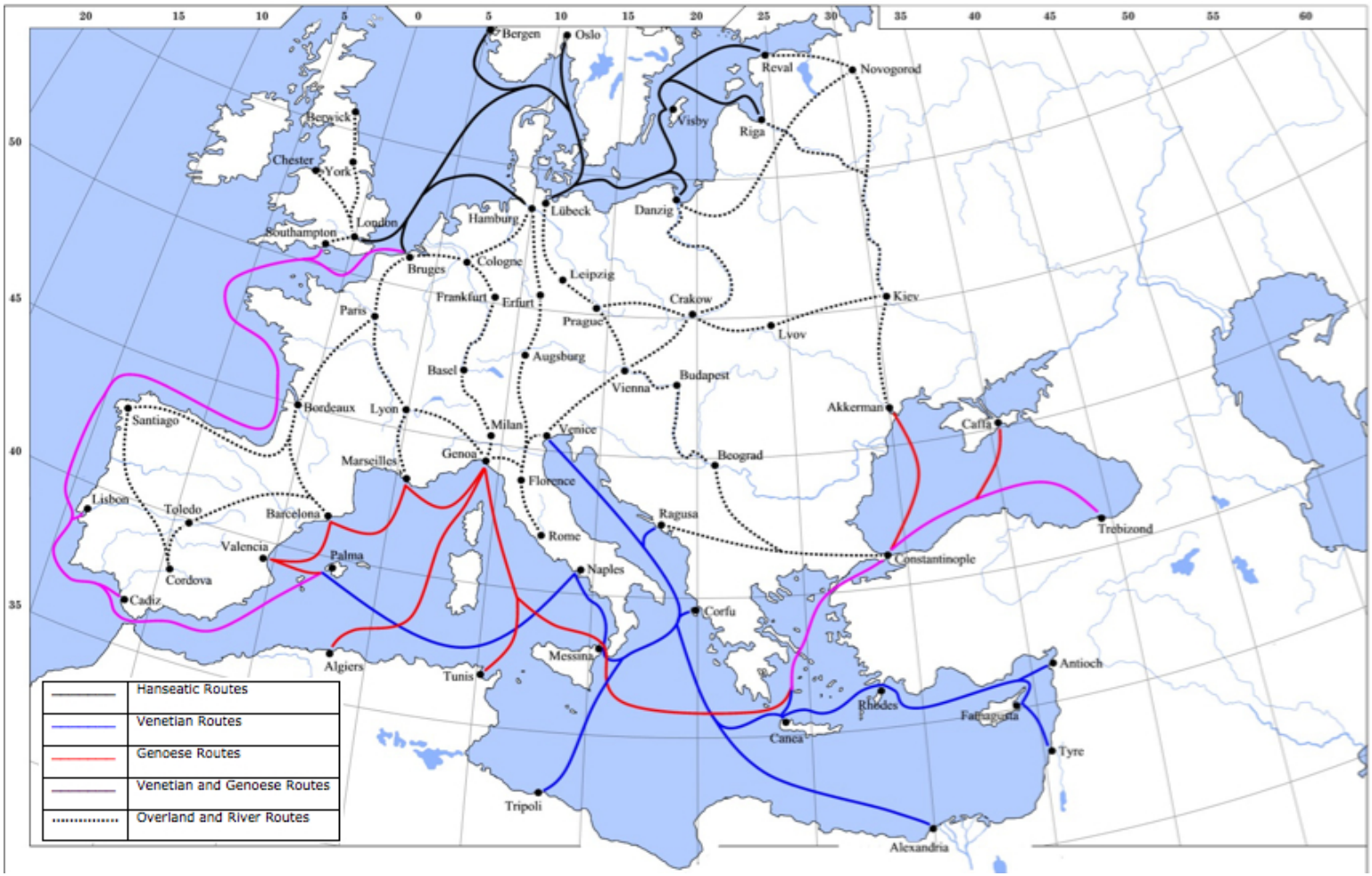


Transportation networks.

Tube map



Mediaval Trade Routes



Social Networks.

- The vertices of a social network are **people**, with edges representing some sort of **social interaction**.
- A social network's **existence** does not depend on social networking sites like `facebook` or `twitter`.
- Sociologists have studied social networks long before people started exhibiting their relations to others online.
- An **affiliation network** has two types of nodes, one type representing people and another type representing **groups of people**.
- In mathematical terms, an affiliation network is an example of a **bipartite graph**.
- The affiliation network can be **projected** onto the set of people, defining a new network which has only the people as vertices ...

Information Networks.

- An information network consists of data items which are linked to each other in some way.
- Example: relational databases.
- Sets of information (like scientific publications) have been linking to each other (e.g., through citations) long before computers were invented.
- However, links in digital form are easier to follow

The (World Wide) Web.

- The WWW is probably the most wide spread and best known examples of an information network.
- Its nodes are **web pages** containing information in form of text and pictures, and its edges are the **hyperlinks**.
- Hyperlinks run in one direction only, from the page that contains the hyperlink to the page that is referenced.
- Therefore, the WWW is a **directed network**, a graph where each edge has a direction.

Citation Networks

homomorphism for the rational double Burnside algebra $\mathbb{Q}B(G,G)$ can be constructed in a similar way. This will be the subject of future research.

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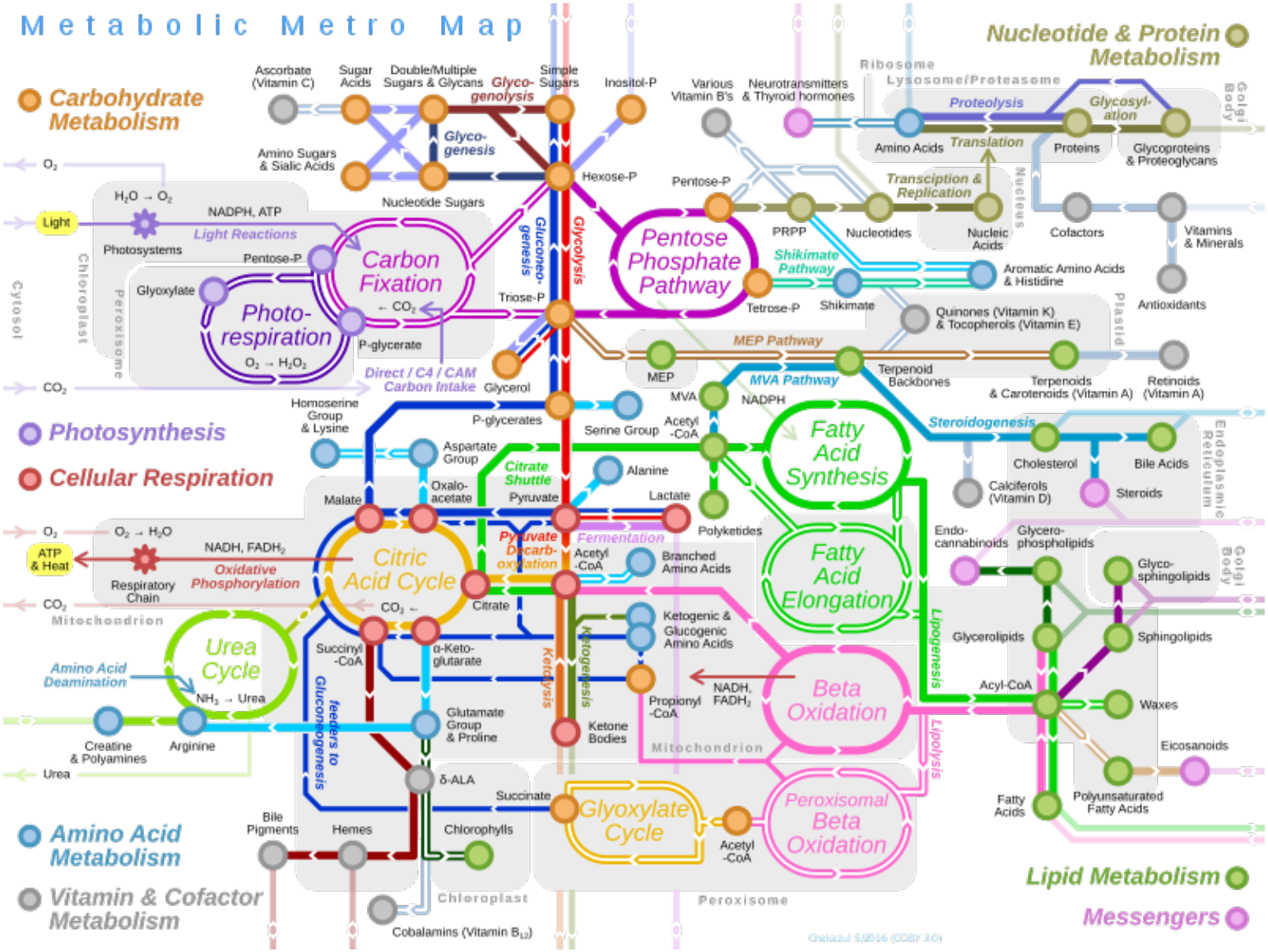
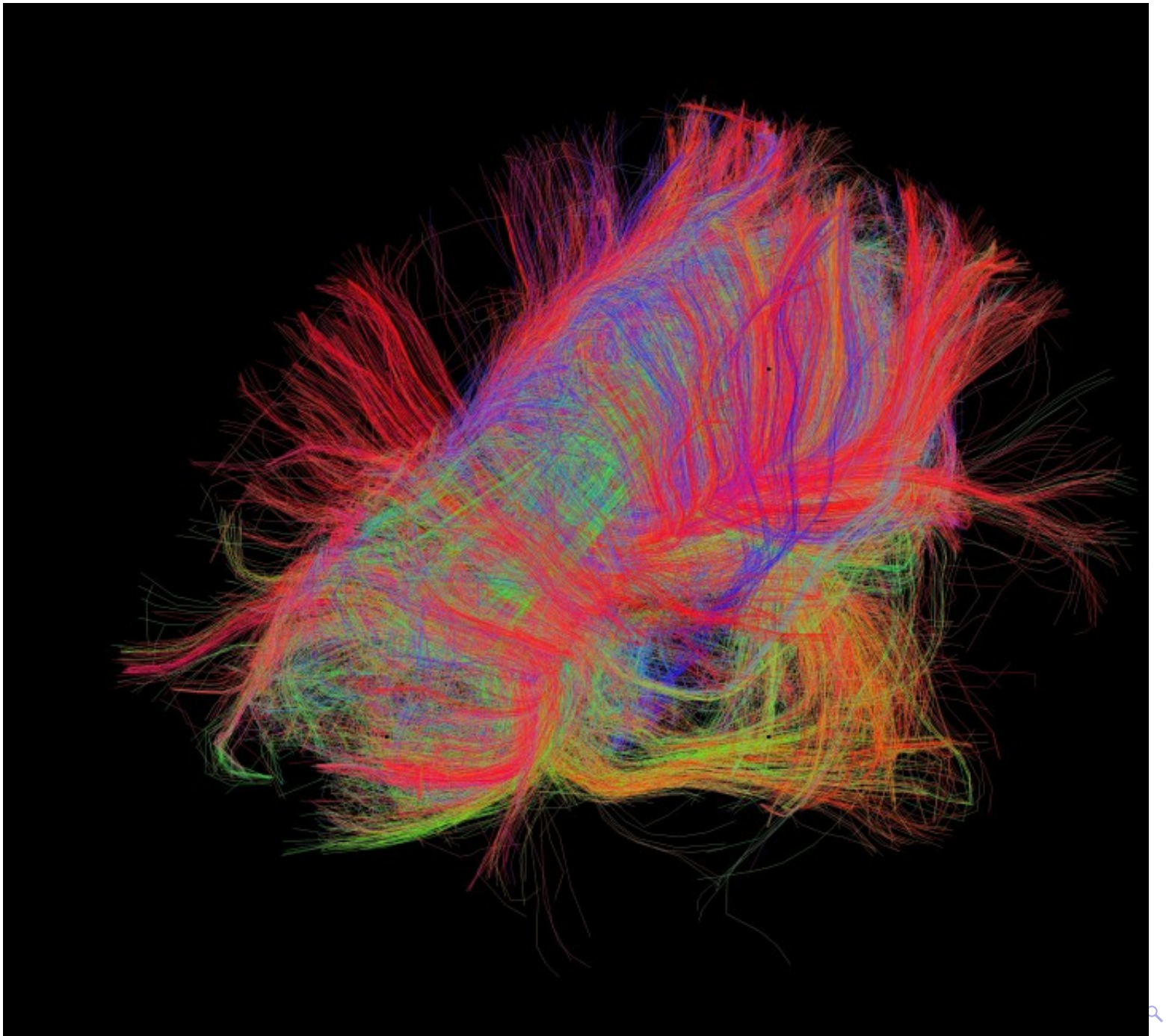
Biological Networks.

- **Biochemical networks** represent molecular level patterns of interaction and control mechanisms in the biological cell:
  - metabolic networks,
  - protein-protein interaction networks and
  - genetic regulatory networks.
- A **neural network** can be represented as a set of vertices, the neurons, connected by two types of directed edges, one for excitatory inputs and one for inhibiting inputs.
- **Ecological networks** are networks of ecological interactions between species.

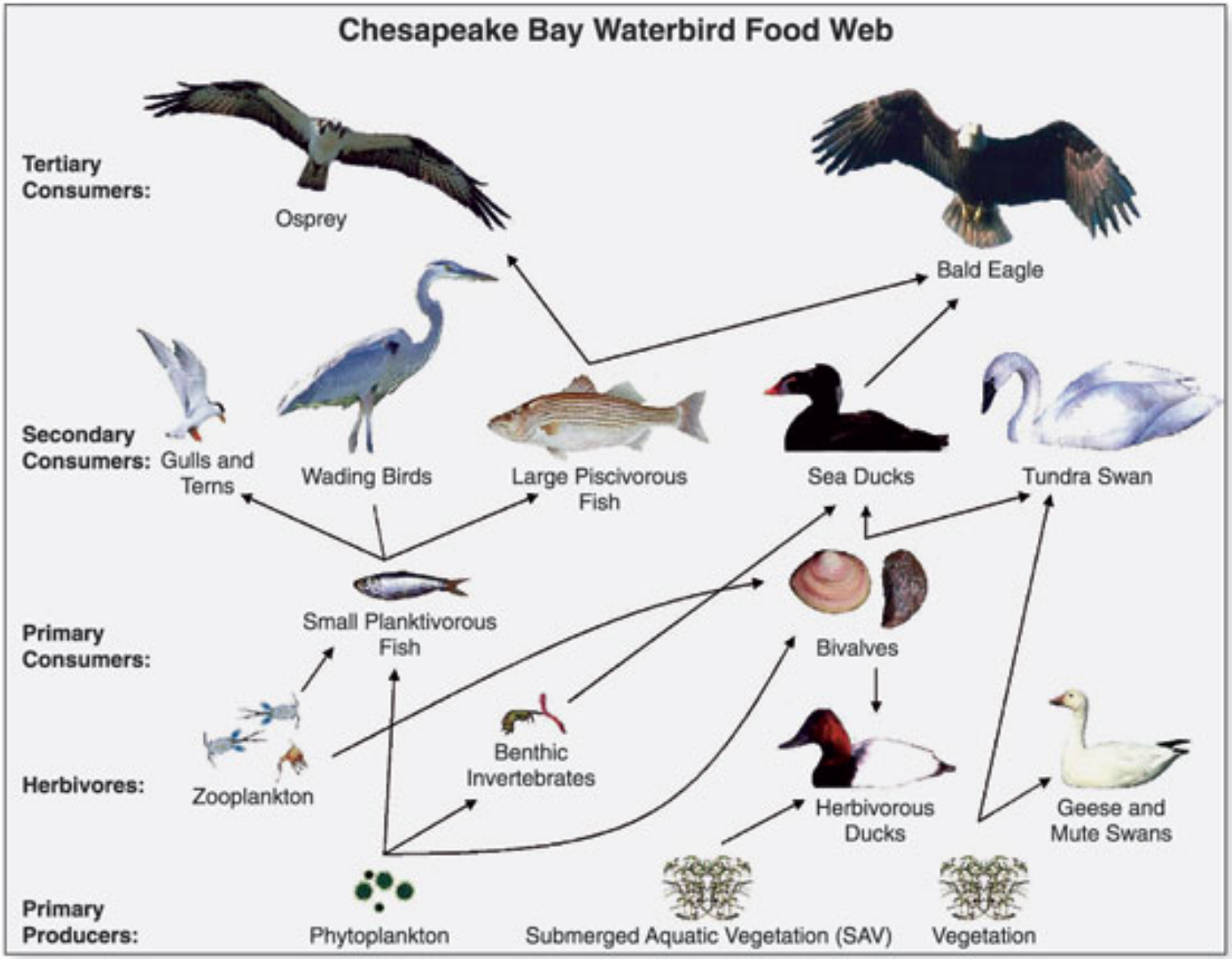
Metabolic Networks



Human Brain as Neural Network



Food Web

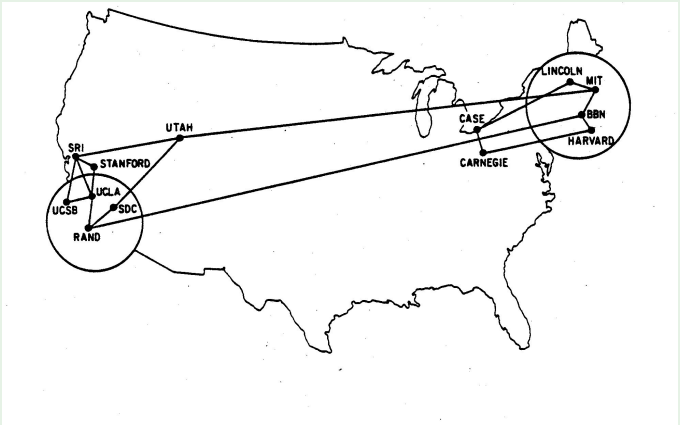


Outline

Graphs.

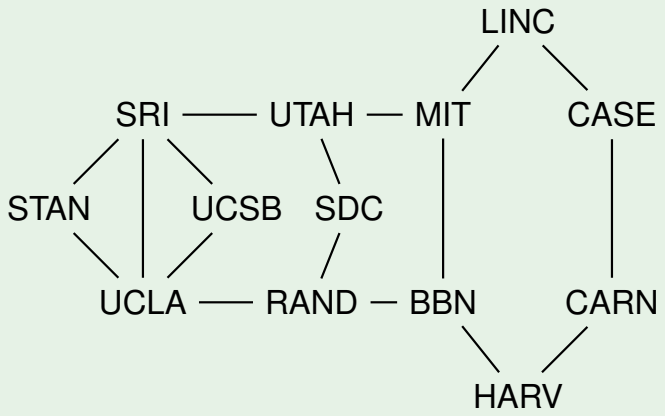
A **graph** can serve as a **mathematical model** of a **network**.

Example



The **internet** in December 1970. Nodes are computers, connected by a link if they can directly communicate with each other. At the time, only 13 computers participated in that network.

As far as the network structure is concerned, this diagram contains the same information, without the distracting details of the US geography.



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