Chemical Engineering 4G03

Tutorial 8 Practice Problem

Discrete Programming and Formulation Practice – Optimizing Process Improvement

Here for your own benefit and practice (best to do it individually)
Recommended completion: Week 09.
Grading: 0% (Practice for assignments and tests)

A microelectronics manufacturing facility is considering six projects to improve operations as well as profitability. However, not all of these projects can be implemented due to budget limitations and engineering personnel constraints.

Each project requires a certain amount of money in each stage, and each stage has a well-defined budget from the government. In the **table below** is a list of prospective projects and their anticipated cost at each stage of the budgeting plan. Furthermore, in the table is the expected NPV "benefit" (an aggregate score based on likelihood of success and information gathered as a result of the mission) of each project. Below the table is a list of projects that depend on or are mutually exclusive to other projects. Note that if you choose a project in the table below, you **must pay for all costs in each stage for that project**.

i	Project	First Year Cost	Second Year Cost	Engineer Hours	Net Present Value
1	Upgrade & automate existing production line	\$300,000	\$0	4000	\$100,000
2	Build new production line	\$100,000	\$200,000	5000	\$120,000
3	Automate new production line	\$0	\$200,000	2500	\$40,000
4	Install plating line	\$50,000	\$100,000	4000	\$80,000
5	Build waste recovery plant	\$50,000	\$300,000	3000	\$80,000
6	Subcontract waste disposal	\$100,000	\$200,000	600	\$60,000
7	Dump waste illegally	\$200,000	\$200,000	4000	\$100,000
	Budget	**\$450,000**	**\$400,000**	**10,000**	

A new or modernized production line MUST be chosen (project 1 or 2). Automation of the new line is only feasible if a new line is built. Only one of projects 5, 6, and 7 can be selected, but no more.

Questions

1. Define the variables for this problem, clearly indicating if they are continuous or binary.

   This problem is about selecting projects that would be funded from a budget such that the NPV is maximized. Thus, binary variables should be defined. Moreover, once a project is chosen, it must be funded in all stages. This indicates that the binary variables have a single index defined over the projects. Thus, define the index $i = 1 \ldots 7$ to represent the prospective projects, and y_i is the binary variable that will correspondingly represent whether or not project i will be funded.

2. Define the objective for this problem. You may write this out in terms of hard-coded numbers or define new parameters as you see fit.

   The objective of this problem is to maximize the total expected NPV. This can be mathematically represented as follows

$$\max_{y} \phi = \sum_{i} NPV_i \cdot y_i$$

   where NPV_i represents the expected net present value of project i.

3. Define the constraint(s) that correspond(s) to the available budget and engineering hours for each stage in each for this problem. Try to be as concise as possible.

   Define FYC_i to be the anticipated cost of project i in the first year, SYC_i to be the anticipated cost of project i in the second year, and EH_i to represent the engineering hours required for each project i. Thus, the constraint that corresponds to respecting the first-year budget is:

$$\sum_{i} FYC_i \cdot y_i \;\; \leq \;\; 450{,}000$$

   The constraint that corresponds to respecting the second-year budget is:

$$\sum_{i} SYC_i \cdot y_i \;\; \leq \;\; 400{,}000$$

   The constraint that corresponds to respecting the availability of engineering hours is:

$$\sum_{i} EH_i \cdot y_i \;\; \leq \;\; 10{,}000$$

   Alternatively, let set j to be all the specifications for each project (i.e., first-year cost, second-year cost and engineering hours) and define matrix $S_{i,j}$ to be the spec j for each project i and vector B_j to be the budget limitation corresponding to each spec j. With this, we can now define a single constraint that corresponds to the available budget and engineering hours to be:

$$\sum_{i} S_{i,j} \cdot y_i \;\; \leq \;\; B_j \qquad (\forall j)$$

4. Code your formulation *so far* (that is, DO NOT include the mutual exclusivity or dependence constraints) in GAMS and find the solution.

   The optimal NPV returned for this problem is $\phi = \$240{,}000$ and corresponds to funding projects 1, 4 and 6. The GAMS code and solution report are given below.

```
SETS
i   projects    / 1 * 7 /
j   specs       / FYC, SYC, EH / ;


PARAMETERS
```

```
NPV(i)  expected NPV of project i
/ 1     100000
  2     120000
  3     40000
  4     80000
  5     80000
  6     60000
  7     100000 /

B(j)    limitations of spec j
/ FYC   450000
  SYC   400000
  EH    10000 /
;

TABLE
S(i,j)  spec j of project i
     FYC       SYC       EH
1    300000    0         4000
2    100000    200000    5000
3    0         200000    2500
4    50000     100000    4000
5    50000     300000    3000
6    100000    200000    600
7    200000    200000    4000  ;


VARIABLES
PHI     total expected NPV
y(i)    whether or not project i is funded ;
BINARY VARIABLES y(i);


EQUATIONS
OBJECTIVE   objective function
BUDGETS     budget constraints
;

OBJECTIVE..   PHI =E= SUM(i, NPV(i)*y(i))  ;
BUDGETS(j)..  SUM(i, S(i,j)*y(i)) =L= B(j) ;


MODEL PROJECT /ALL/;
OPTION MIP = CPLEX;

SOLVE PROJECT MAXIMIZING PHI USING MIP;
DISPLAY PHI.L, y.L;
```

```
Proven optimal solution
MIP Solution:       240000.000000    (3 iterations, 0 nodes)
Final Solve:        240000.000000    (0 iterations)

---- EQU BUDGETS  budget constraints

           LOWER          LEVEL          UPPER         MARGINAL
FYC        -INF        450000.0000    450000.0000         .
SYC        -INF        300000.0000    400000.0000         .
EH         -INF          8600.0000     10000.0000         .


                          LOWER          LEVEL          UPPER         MARGINAL
---- VAR PHI              -INF        240000.0000       +INF             .
```

```
        PHI  total expected NPV


---- VAR y  whether or not project i is funded

        LOWER          LEVEL          UPPER         MARGINAL
1         .           1.0000         1.0000     100000.0000
2         .             .            1.0000     120000.0000
3         .             .            1.0000      40000.0000
4         .           1.0000         1.0000      80000.0000
5         .             .            1.0000      80000.0000
6         .           1.0000         1.0000      60000.0000
7         .             .            1.0000     100000.0000
```

5. Define the mandatory selection constraint(s) for projects 1 and 2 and the mutual exclusivity constraint(s) for projects 5, 6 & 7 in this problem. Note that these should look slightly different from each other (Think about why).
   The mutual exclusivity constraints are given below. Note the difference in operator to say the either project 1 or 2 **must** be pursued, while **at most one** of projects 5, 6 and 7 can be funded.

$$\begin{aligned} y_1 + y_2 &= 1 \\ y_5 + y_6 + y_7 &\leq 1 \end{aligned}$$

6. Add the above constraints to GAMS and re-run the problem. Comment on the results and how adding the mutual exclusivity constraints affect your solution.

7. Define the dependency constraints for projects 2 & 3 in this problem.
   The dependence of project 3 on project 2 can be expressed via the following constraint:

$$y_3 \leq y_2$$

8. Re-run your GAMS code with *all* the mutual exclusivity *and* dependency constraints included. Comment on *this* solution.

9. Your company's CEO has decided that project 7 is unethical and has decided to veto it. Re-run your GAMS code without this project option and comment on *this* solution.