Chemical Engineering 4G03

Tutorial 4 Practice Activity

Here for your own benefit and practice (best to do it individually)
Recommended completion: Week 05.
Grading: 0% (Practice for assignments and tests)
Based on Rardin (ch. 4)

Objective

Time-phased models are very important for optimization applications because they allow us to perform optimization over a time horizon (*i.e.*, making multiple decision every hour, day *etc.*) Moreover, time-phased models can be modeled as linear programs! The objective of this tutorial activity is to get to know what time-phased models can do for us and how we can program them into `GAMS`.

THIS PROBLEM IS TRICKY – DON'T WORRY IF YOU FIND IT ABSTRACT. IT'S GREAT PRACTICE!

Problem Background

You are working for a large intermediate chemical company that receives various intermediate products and combines them into high-value finished products for sale. In this example, we want to manage our resources and cash flows for the next 8 weeks. We thus have the following set:

- $t \triangleq$ the week number ($t = 1 \ldots 8$). Note that it may also be convenient to consider $t = 0$ as your initial condition (this is optional and depends on how you want to formulate the problem).

Your company has some projections for the following accounts over the next eight weeks:

- $s_t \triangleq$ Projected sales revenue in week t from cash sales to smaller customers.
- $r_t \triangleq$ Accounts receivable revenue due in week t from large customers purchasing on credit.
- $p_t \triangleq$ Accounts payable from your company to your suppliers in week t.
- $e_t \triangleq$ Expenses to be paid by your company (salaries, operating costs, utilities) in week t. *No exceptions*.

The projected figures for each of the above parameters for the next 8 weeks are as follows (high variabilities due to forecasted cyclical demand):

Item	Projected Weekly Amount ($000's) for Week							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Sales s_t	600	750	1200	2100	2250	180	330	540
Receivables r_t	770	1260	1400	1750	2800	4900	5250	420
Payables p_t	3200	5600	6000	480	880	1440	1600	2000
Expenses e_t	350	400	550	940	990	350	350	410

Since it is very unlikely that we will be able to handle all these cash flows while maintaining a positive cash balance, we have other options available:

- Accounts payable p_t are not due until three weeks later ($t+3$), but if they are paid immediately your company receives a 2% discount.
- Your company has a $4 million line of credit with the bank that pays 0.2% interest per week. However, the bank requires that at least 20% of the total debt balance is reserved as cash on hand in your company's chequing account. Interest cannot accumulate and must be paid in cash each week.
- Excess cash on hand can be invested in a short-term money market that earns 0.1% per week.
- You must always reserve at least $20,000 in your chequing account for emergencies starting at time $t = 1$.

THE PLAN for this problem is to minimize net total cost in interest and lost discounts at the end of the eight-week period. To do this, you will need to perform a *cash balance* on your bank account at each time t based on inflows and outflows. You currently have a chequing account balance of $0. You are to use the following variables:

- $g_t \triangleq$ Amount borrowed in week t against your line of credit.
- $h_t \triangleq$ Amount of your line of credit you pay off in week t.
- $w_t \triangleq$ Amount of accounts payable in week t that you choose to delay payment until $t+3$ (at a loss of the 2% discount).
- $x_t \triangleq$ Amount invested in the short-term money market in week t. Note that these investments are one week long and are returned to you in the next time period.

It may also be convenient to define the following **dependent** variables:

- $y_t \triangleq$ Cumulative line of credit debt in week t.
- $z_t \triangleq$ Cash on hand (chequing balance) during week t.

ALL BALANCES (chequing z_0, debt y_0, and investments x_0) are zero at $t = 0$. You have all parameters and variables you need for this problem – but it is up to you to determine the constraints and objective.

Advice

I strongly suggest you employ a "mass balance" approach to this problem. That is:

$$\{\text{starting level in period } t\} + \{\text{impacts of period } t \text{ decisions}\} = \{\text{starting level in period } t+1\}$$

To help you out, refer to the following table of cash inflows and outflows for a given week t:

Cash Inflows	**Cash Outflows**
Funds borrowed in week t	Debt paid off in week t
Investment principal returned from week $t-1$	Amount invested in week t
Investment interest from week $t-1$	Interest owed on debt in week $t-1$
Cash sales in week t	Expenses paid in week t
Accounts receivable in week t	Accts payable paid with discounts in week t
	Accts payable paid without discounts from week $t-3$

GAMS tools

You may want to consider this GAMS help page that details the `ord()` command. The `ord()` command stands for "order" and allows you to control the index of a set of equations to prevent it from violating the equation bounds. For example, if I have the set $t = 0 \ldots 8$ and I want to write a simple equation like:

$$z_t = z_{t-1} - e_t \; \forall t = \mathbf{1} \ldots \mathbf{8}$$

I notice that I can't use $t = 0$ in this equation since that would request z_{-1} which (according to my set) does not exist! As such I only want this equation to work for $t = 1 \ldots 8$. I achieve that in GAMS by using `ord()` command like this:

```
Equation(t) $ (ord(t) > 1)..  z(t) =e= z(t-1) - e(t);
```

Basically, I am saying "make this equation happen for all values of t greater than the first one." Note here that > 1 is referring to the ELEMENT number, not the number in the actual set (for example, $t = 1$ is element 2 since the set starts at $t = 0$). Remember that sets can be names or other codified entries, and therefore the `ord()` command asks you to identify the numerical index location of the set.

I would then have to make sure that the FIRST condition (for example, $z_0 = 100$) using another constraint:

```
Equation2.. z("0") =e= 100;
```

It is worth checking out this help page from GAMS to work on sorting through this. I do not expect you to get it immediately, but MANY projects and other assignment problems require the use of time-phased methods.

https://www.gams.com/latest/docs/UG_OrderedSets.html