Chemical Engineering 4G03

Tutorial 4 Solution

Here for your own benefit and practice (best to do it individually)
Recommended completion: Week 05.
Grading: 0% (Practice for assignments and tests)

Problem Adapted from Rardin (2017) Chapter 4

Objective Function

The objective in this problem to minimize net total cost (NTC) of interest and lost discounts over the eight-week period. This can be written as:

$$\min_{g,h,w,x,z,y} NTC = 0.002\sum_{t=1}^{8} y_t + 0.02\sum_{t=1}^{8} w_t - 0.001\sum_{t=1}^{8} x_t$$

Debt Balance

The debt balances can be written similarly to mass balances, as mentioned in the activity handout. Can write the following equation to relate our cumulative debt to previous debt along with any further amounts borrowed or paid off:

$$\boldsymbol{y_{t-1} + g_t - h_t = y_t}$$

Account Balance

The account balances can be written similarly to the debt balances, but more terms affect the cash on hand in a given week. Thus, once we have accounted for all the cash flows in a given week, we get the following:

$$\boldsymbol{z_{t-1} + g_t + (1 + 0.001)x_{t-1} + s_t + r_t - h_t - x_t - 0.002y_{t-1} - e_t - 0.98(p_t - w_t) - w_{t-3} = z_t}$$

Of special note here is that we are assuming we can go "negative" for the indices (for example, this balance would not work for $t = 0$ since we do not have an index for w_{-3}). GAMS has been updated to assume that all variables in negative indices are treated automatically as zero, and therefore the above formulation can be used. However, the coded solution and more mathematically complete way to write this constraint is to have two constraints valid over subsets of t. See the complete formulation below for this formulation strategy.

Account Balance Constraints

The minimum amount of cash on hand must meet 2 separate requirements (at least 20% of debt and at least $20k), which are expressed by the following constraints respectively:

$$\boxed{z_t \geq 0.2 y_t}$$
$$\boxed{z_t \geq 20}$$

Line of Credit Constraint

The following constraint dictates the maximum amount of debt from our line of credit:

$$\boxed{y_t \leq 4000}$$

Delayed Payables Constraint

The following constraint dictates that we can only delay an amount that is less than or equal to our payables for that week (*ie* you can't delay payment of more than what you have to pay).

$$\boxed{w_t \leq p_t}$$

When all is said and done, I have (trumpets please!):

$$\min_{g,h,w,x,y,z} NTC = 0.002 \sum_{t=1}^{8} y_t + 0.02 \sum_{t=1}^{8} w_t - 0.001 \sum_{t=1}^{8} x_t$$

Subject to

$$z_{t-1} + g_t + (1 + 0.001)x_{t-1} + s_t + r_t - h_t - x_t - 0.002 y_{t-1} - e_t - 0.98(p_t - w_t) - w_{t-3} = z_t \qquad \forall\, t : t > 0$$

$$y_{t-1} + g_t - h_t = y_t \qquad \forall\, t : t > 0$$

$$y_t \leq 4000 \qquad \forall\, t : t > 0$$

$$z_t \geq 0.2\ y_t \qquad \forall\, t : t > 0$$

$$w_t \leq p_t \qquad \forall\, t : t > 0$$

$$z_t \geq 20 \qquad \forall\, t : t > 0$$

$$g_t, h_t, w_t, x_t, y_t, z_t \geq 0 \qquad \forall\, t : t \geq 0$$

$$g_t, h_t, w_t, x_t, y_t, z_t = 0 \qquad \forall\, t : t < 0$$

Alternative formulation separating the balance constraint into two subsets to avoid negative indices:

$$\min_{g,h,w,x,y,z} NTC = 0.002\sum_{t=1}^{8} y_t + 0.02\sum_{t=1}^{8} w_t - 0.001\sum_{t=1}^{8} x_t$$

Subject to

$$z_{t-1} + g_t + (1+0.001)x_{t-1} + s_t + r_t - h_t - x_t - 0.002y_{t-1} - e_t - 0.98(p_t - w_t) = z_t \qquad \forall\, t = 1 \ldots 2$$

$$z_{t-1} + g_t + (1+0.001)x_{t-1} + s_t + r_t - h_t - x_t - 0.002y_{t-1} - e_t - 0.98(p_t - w_t) - w_{t-3} = z_t \qquad \forall\, t : t > 2$$

$$y_{t-1} + g_t - h_t = y_t \qquad \forall\, t : t > 0$$

$$y_t \leq 4000 \qquad \forall\, t : t > 0$$

$$z_t \geq 0.2\ y_t \qquad \forall\, t : t > 0$$

$$w_t \leq p_t \qquad \forall\, t : t > 0$$

$$z_t \geq 20 \qquad \forall\, t : t > 0$$

$$g_t, h_t, w_t, x_t, y_t, z_t \geq 0 \qquad \forall\, t : t \geq 0$$