

Tutorial 6 Practice: Sensitivity Analysis

Here for your own benefit and practice (best to do it individually)

Recommended completion: Before the 4G03 midterm.

Grading: 0% (Practice for assignments and tests)

Some people requested more practice problems for the upcoming midterm. I consider these tutorial questions, particularly the analysis of sensitivity reports, as EXCELLENT practice for the midterm and final.

PROBLEM 1

Background

Syncrudd, the company that you have been working for when blending their various gasoline products, has recently reported that although your optimization methods and results suggest significant increases in profitability, they want to see how your modeling assumptions (and their data) will change the expected outcomes of your optimization solutions. To see how these changes will affect the solution, let's consider **a modified scenario we solved for Syncrudd in Assignment 1**. The availabilities and qualities of our components are as follows (note they are different from A1):

Component	Availability (bbl/h)	Cost (\$/bbl)	Octane No.	RVP (psi)	Volatility (%)
Reformat	12,000	68.00	90.5	4	17
LSR-Naphtha	6,500	52.00	68.0	10	96
n-Butane	3,000	20.60	92.5	138	128
FCC Gasoline	4,500	63.60	80.0	6	22
Alkylate	7,000	69.00	95.0	7	34

The quality constraints and selling price are:

- The octane number of the blended product must be between **88.5** and **100**.
- The RVP must be between **4.5** and **10.8 psi**.
- The volatility must be between **0** and **48%**.
- You may sell your product for **\$66 per barrel**.

Information and Resources

Before you get started on this tutorial, please download the T06_Companion folder from Avenue to Learn in the Tutorials section. In it you will find the GAMS solution the problem above. We want to use some sensitivity analysis tools to answer some probing questions about our model.

Moreover, from the GAMS resources page on A2L, download the file called `cplex.opt` and follow these instructions:

- Place the file in the directory you are working from (my documents, USB key, Desktop, whatever)

- Open GAMS and select FILE → PROJECT → NEW PROJECT. Name the project `T06` *in the same directory that you saved the file `cplex.opt`*. This is important because, much like saving a function on the correct file path, GAMS looks in its “project directory” to see if any options files are present. Options files like this one will invoke special rules, such as displaying the sensitivity range results for the constraints (`RHSRNG`) and cost coefficients (`OBJRNG`).
- Open the GAMS file corresponding to this scenario and ensure it contains the line `BLEND.OPTFILE = 1;`. If it does, and it is in the same directory as your PROJECT, you should receive the sensitivity analysis information we looked at in class this week.

Questions

1. Explain what the “CURRENT” values under the “VARIABLE” heading mean in the sensitivity output (you will notice they are not the costs of barrels nor the revenues of products).
2. Your analyst projections for the coming period show that the demand for your product is going to increase to 11,000 barrels. Estimate the new profit using the sensitivity analysis output.
3. We have an opportunity to change suppliers entirely, which will modify the availabilities of a variety of our products. We want to know if these changes will be profitable if they happen **simultaneously**:
 - Our reformat supply drops to 8000.
 - Our butane supply drops to 1000.
 - Our FCC gasoline supply drops to 4000.
4. Re-run the optimization problem from scratch by changing the maximum demands to correspond with the changes shown above. Does the output from GAMS match your expectations?
5. There was a sudden increase in LSR naphtha processing costs due to a change in the quality of crude oil coming into our plant. The *COST* of LSR naphtha has therefore gone UP by \$10 per bbl. Estimate the WORST CASE decrease in profit that is expected due to this cost increase.
6. Estimate the BEST CASE decrease in profit for the increase in LSR Naphtha cost discussed above.
7. Rerun the GAMS file with the increase in Naphtha cost and verify that it is in-between your best- and worst-case scenarios you answered above.
8. The availability of our most valuable constituent, alkylate, has suddenly decreased due to an urgent need elsewhere in our processing plant. It is going to decrease to 4000 bbl! Estimate the BEST CASE decrease in profit resulting from this shortage.
9. Re-run your GAMS file with the maximum availability of alkylate being restrained to 4000 bbl and confirm that the resulting decrease in profit is *at least* as bad as you expected.

PROBLEM 2 – Practice for Midterm (Adapted from 2015 MT)

Your company is implementing an engineering design project and three types of employees are available:

- Outsourced workers that can be hired at \$14 per hour.
- Newly graduated students from McMaster University can be hired at a rate of \$24 per hour.
- Professional engineers (P.Eng) can be contracted in at a rate of \$64 per hour.

The full project would take the P.Eng 1000 hours to complete (this is an approximation, of course). Graduates from McMaster are inexperienced and are only about 40% as productive as P.Eng workers, and outsourced workers, due to a lack of familiarity of the company and increased training required, are only 25% as productive as a P.Eng.

To further complicate things, you have a project supervisor (a sunk cost, so their wage is irrelevant) who has to monitor the work of each employee. **The supervisor only has 160 supervisory hours available.** Naturally, the experienced P.Eng requires less supervision than the inexperienced graduates and outsourced workers. The required supervisory times are:

- Outsourced workers require 0.2 supervisory hours per hour of work.
- Newly graduated students require 0.15 supervisory hours per hour of work.
- Professional engineers require only 0.05 supervisory hours per hour of work.

On top of all of this, you only have one graduate available to work 40-hour weeks, and the project must be completed in 12 weeks, meaning that there are 480 hours of graduate time available. You can hire multiple outsourced workers or P.Eng certified engineers at a time, so there is no effective limit on their availability.

1. Formulate this problem as a linear program.
2. Describe, in plain language as if you were explaining your formulation to your supervisor (a civil engineer from Queen's who didn't take optimization), what we are trying to maximize/minimize in this problem and why.
3. Code the scenario into GAMS and solve it using CPLEX. Extract the optimal allocation of worker hours and the total labour cost of the project (in the midterm, you might have the code given to you on the page).

For the following questions, consider each scenario **completely independently** of the others. You are welcome to postulate what would happen if you combine scenarios (and try it out!) if you want.

4. In the meeting with your supervisor, you do not have access to your laptop (and thus GAMS). Your supervisor asks "actually, if I reschedule a couple of other projects, we can bump the total number of available supervisor hours up to 210. Will this save us any money?" What do you say?
5. In the same meeting, your supervisor also mentions that "we can re-assign one of our recent graduates to this project, bringing the total number of available graduate hours up to 960." Estimate the savings and explain whether or not this represents a best-case or worst-case scenario.

6. The estimate for total project time (1000 hours) is argued by a lead engineer to be "way too low, in my opinion." He thinks it will actually take at least 300 more professional-equivalent hours to complete. However, if this is the case, your supervisor is willing to allocate another supervisor to the project, thus doubling the number of supervisor hours required. What is your new estimate of the cost of the project?
7. The cost of outsourced labour was undervalued and is actually \$18 per hour. If you were to use the sensitivity analysis to report a change in project cost, would you be reporting a worst-case or best-case scenario?
8. Estimate your worst- or best-case change in the cost of the project. Will the number of hours for each type of worker change at the optimal solution? Why or why not?
9. Re-run the optimization for the scenario in question (7) and confirm your response to the answer above.
10. Your supervisor says that the original cost of a P.Eng working on the project was grossly underestimated, and he fears that the number of hours allocated to a P.Eng will surely change once the real cost per hour is considered. Explain why you doubt this is the case, and give a convincing reason as to why.
11. Re-run the GAMS file with a P.Eng worker cost MUCH HIGHER than the current \$64 per hour estimate to prove your point.