Chemical Engineering 4G03

Tutorial 6 SOLUTION

Here for your own benefit and practice (best to do it individually)
Recommended completion: Before the 4G03 midterm.
Grading: 0% (Practice for assignments and tests)

Some people requested more practice problems for the upcoming midterm. I consider these tutorial questions, particularly the analysis of sensitivity reports, as EXCELLENT practice for the midterm and final.

PROBLEM 1

Background

Syncrudd, the company that you have been working for when blending their various gasoline products, has recently reported that although your optimization methods and results suggest significant increases in profitability, they want to see how your modeling assumptions (and their data) will change the expected outcomes of your optimization solutions. To see how these changes will affect the solution, let's consider **a modified scenario we solved for Syncrudd in Assignment 1**. The availabilities and qualities of our components are as follows (note they are different from A1):

Component	Availability (bbl/h)	Cost ($/bbl)	Octane No.	RVP (psi)	Volatility (%)
Reformate	12,000	68.00	90.5	4	17
LSR-Naphtha	6,500	52.00	68.0	10	96
***n*-Butane**	3,000	20.60	92.5	138	128
FCC Gasoline	4,500	63.60	80.0	6	22
Alkylate	7,000	69.00	95.0	7	34

The quality constraints and selling price are:

- The octane number of the blended product must be between **88.5** and **100.**
- The RVP must be between **4.5** and **10.8 psi.**
- The volatility must be between **0** and **48%**.
- You may sell your product for **$66 per barrel**.

Information and Resources

Before you get started on this tutorial, please download the `T06_Companion` folder from Avenue to Learn in the Tutorials section. In it you will find the GAMS solution the problem above. We want to use some sensitivity analysis tools to answer some probing questions about our model.

Moreover, from the GAMS resources page on A2L, download the file called `cplex.opt` and follow these instructions:

- Place the file in the directory you are working from (my documents, USB key, Desktop, whatever)

- Open GAMS and select FILE → PROJECT → NEW PROJECT. Name the project `T06` *in the same directory that you saved the file* `cplex.opt`. This is important because, much like saving a function on the correct file path, GAMS looks in its "project directory" to see if any options files are present. Options files like this one will invoke special rules, such as displaying the sensitivity range results for the constraints (`RHSRNG`) and cost coefficients (`OBJRNG`).
- Open the GAMS file corresponding to this scenario and ensure it contains the line `BLEND.OPTFILE = 1;`. If it does, and it is in the same directory as your PROJECT, you should receive the sensitivity analysis information we looked at in class this week.

Questions

1. Explain what the "`CURRENT`" values under the "`VARIABLE`" heading mean in the sensitivity output (you will notice they are not the costs of barrels nor the revenues of products).

   They are the marginal PROFITS of each component being used in the blend since GAMS organizes everything in the linear program into $\phi = c^T x$.

2. Your analyst projections for the coming period show that the demand for your product is going to increase to 11,000 barrels. Estimate the new profit using the sensitivity analysis output.

   We expect the profit to increase since this constraint has a marginal price of $\nu = 2.125$. Moreover, this change is in the range of allowable shifts that we can use the marginal price. The change in profit will therefore be:

$$\Delta\phi = \nu\Delta b$$
$$\Delta\phi = 2.125(4000)$$
$$\Delta\phi = 8{,}500$$

   The new profit should therefore be roughly \$23,377.50.

3. We have an opportunity to change suppliers entirely, which will modify the availabilities of a variety of our products. We want to know if these changes will be profitable if they happen **simultaneously**:

   Our reformate supply drops to 8000.
   Our butane supply drops to 1000.
   Our FCC gasoline supply drops to 4000.

   This clearly violates the rule of 100 so I have to be VERY CAREFUL about my analysis. However, it appears that all of these constraints are inactive at the current optimum and thus there is a strong likelihood that nothing will change.

4. Re-run the optimization problem from scratch by changing the maximum demands to correspond with the changes shown above. Does the output from GAMS match your expectations?

   No surprise – nothing changed. This indicates that the rule of 100 only applies to *active* constraints at the solution.

5. There was a sudden increase in LSR naphtha processing costs due to a change in the quality of crude oil coming into our plant. The *COST* of LSR naphtha has therefore gone UP by \$10 per bbl. Estimate the WORST CASE decrease in profit that is expected due to this cost increase.

   This is a bad thing since it is making my cost coefficient WORSE (down by \$10 per bbl) and it is outside of the allowable range. Since changing the active set will result in a new objective function location, it is safe to claim that the WORST thing that could happen is that I continue to use the same amount of LSR Naphtha. The $\Delta\phi$ would thus be (in the worst case):

$$\Delta\phi = \Delta c x^{\star}$$
$$\Delta\phi = -10(1527.486)$$
$$\boxed{\boldsymbol{\Delta\phi = -\$15,275}}$$

6. Estimate the BEST CASE decrease in profit for the increase in LSR Naphtha cost discussed above.

   The best case is that, after an active set change, we end up using no LSR Naphtha at all! Thus, the change in ϕ will be limited to just the change permitted before the active set change:

$$\Delta\phi = \Delta c^{ALLOWED} x^{\star}$$
$$\Delta\phi = -3.43(1527.486)$$
$$\boxed{\boldsymbol{\Delta\phi = -\$5,240}}$$

7. Rerun the GAMS file with the increase in Naphtha cost and verify that it is in-between your best- and worst-case scenarios you answered above.

   Increasing the cost of LSR-Naphtha by \$10 per barrel results in a new profit of $\phi = 7996.17$, which is \$6,880 lower than the original profit of \$14,877. Unsurprisingly, this is in the middle of the ranges identified above. It is also worth noting here that, as expected, no LSR-Naphtha is used in the final blend!

8. The availability of our most valuable constituent, alkylate, has suddenly decreased due to an urgent need elsewhere in our processing plant. It is going to decrease to 4000 bbl! Estimate the BEST CASE decrease in profit resulting from this shortage.

   Again (going back to the base problem), this result is going to cause a change in the active set. The BEST case scenario is that the shadow price for alkylate ν_{ALK} is applicable even after the active set changes. The decrease in profit should be AT LEAST:

$$\Delta\phi = \nu_{ALK}\Delta b_{ALK}$$
$$\Delta\phi = 0(-3000)$$
$$\Delta\phi = \$0$$

   In reality, it will likely not be so pretty. In the WORST possible scenario, I would make the problem infeasible.

9. Re-run your GAMS file with the maximum availability of alkylate being restrained to 4000 bbl and confirm that the resulting decrease in profit is *at least* as bad as you expected.

   Re-running gives a profit of \$14,578 – certainly worse than a zero-change, but not all that bad!

PROBLEM 2 –Practice for Midterm (Adapted from 2015 MT)

Your company is implementing an engineering design project and three types of employees are available:

- Outsourced workers that can be hired at $14 per hour.
- Newly graduated students from McMaster University can be hired at a rate of $24 per hour.
- Professional engineers (P.Eng) can be contracted in at a rate of $64 per hour.

The full project would take the P.Eng 1000 hours to complete (this is an approximation, of course). Graduates from McMaster are inexperienced and are only about 40% as productive as P.Eng workers, and outsourced workers, due to a lack of familiarity of the company and increased training required, are only 25% as productive as a P.Eng.

To further complicate things, you have a project supervisor (a sunk cost, so their wage is irrelevant) who has to monitor the work of each employee. **The supervisor only has 160 supervisory hours available**. Naturally, the experienced P.Eng requires less supervision than the inexperienced graduates and outsourced workers. The required supervisory times are:

- Outsourced workers require 0.2 supervisory hours per hour of work.
- Newly hired graduated students require 0.15 supervisory hours per hour of work.
- Professional engineers require only 0.05 supervisory hours per hour of work.

On top of all of this, you only have one graduate available to work 40-hour weeks, and the project must be completed in 12 weeks, meaning that there are 480 hours of graduate time available. You can hire multiple outsourced workers or P.Eng certified engineers at a time, so there is no effective limit on their availability.

1. Formulate this problem as a linear program.

   The problem can be formulated as follows:

$$\min_{x} \phi = \sum_{i} c_i x_i$$

   Subject to the constraints:

$$\sum_{i} p_i x_i \geq 1000$$
$$\sum_{i} s_i x_i \leq 160$$
$$x_{GRAD} \leq 480$$
$$x_i \geq 0 \; \forall \, i$$

   Where I can define my variables and parameters as:
   - $x_i = [x_{OUT} \; x_{GRAD} \; x_{PRO}]^T \triangleq$ the number of hours for which each type of worker is hired.
   - $p_i = [0.25 \; 0.4 \; 1.0]^T \triangleq$ the productivity (in project hours per hour worked) of each worker.
   - $s_i = [0.2 \; 0.15 \; 0.05]^T \triangleq$ the number of supervisor hours required per hour of work for each worker.
   - $c_i = [14 \; 24 \; 64]^T \triangleq$ the cost per hour to hire each type of worker.

2. Describe, in plain language as if you were explaining your formulation to your supervisor (a civil engineer from Queen's who didn't take optimization), what we are trying to maximize/minimize in this problem and why.

   In this problem we are trying to minimize the total COST of labour of completing our project by using a variety of workers with different wages and skill expertise.

3. Code the scenario into GAMS and solve it using CPLEX. Extract the optimal allocation of worker hours and the total labour cost of the project (in the midterm, you might have the code given to you on the page).

   See the solution file `project_allocation.gms`. The solution for the number of hours each employee must be hired for and the total project labour cost are:

$$x^\star = [253.4 \;\; 480 \;\; 744.5]^T$$
$$\phi^\star = \$62{,}724$$

For the following questions, consider each scenario **completely independently** of the others. You are welcome to postulate what would happen if you combine scenarios (and try it out!) if you want.

4. In the meeting with your supervisor, you do not have access to your laptop (and thus GAMS). Your supervisor asks "actually, if I reschedule a couple of other projects, we can bump the total number of available supervisor hours up to 210. Will this save us any money?" What do you say?

   Absolutely. It will save us *exactly*

$$\Delta\phi = \Delta b v = (50)(-10.667) = -\$\mathbf{533}$$

5. In the same meeting, your supervisor also mentions that "we can re-assign one of our recent graduates to this project, bringing the total number of available graduate hours up to 960." Estimate the savings and explain whether or not this represents a best-case or worst-case scenario.

   Increasing the grad limit to 960 takes us outside of the range. Thus the BEST CASE SCENARIO is if the shadow price remains constant:

$$\Delta\phi = \Delta b v = (480)(-0.213) = -\$\mathbf{102}$$

   However, I would carefully suggest that the supervisor make sure the grad student is not best used elsewhere since this decrease in cost is not very much (and subject to uncertainty, of course!)

6. The estimate for total project time (1000 hours) is argued by a lead engineer to be "way too low, in my opinion." He thinks it will actually take at least 300 more professional-equivalent hours to complete. However, if this is the case, your supervisor is willing to allocate another supervisor to the project, thus doubling the number of supervisor hours required. What is your new estimate of the cost of the project?

   This series of changes abides by the rule of 100 (verify!), so we are good to use the shadow prices to determine the overall cost *increase* for the project as follows:

$$\Delta\phi = (300)(64.533) + (160)(-10.667) = \$\mathbf{17,653}$$

7. The cost of outsourced labour was undervalued and is actually \$18 per hour. If you were to use the sensitivity analysis to report a change in project cost, would you be reporting a worst-case or best-case scenario?

   If we were to use the sensitivity analysis, it would be a *worst case scenario* since the cost we are trying to minimize is increasing with an increasing cost of outside labour, and this increase exceeds the allowable range to assume the solution point will not change.

8. Estimate your worst- or best-case change in the cost of the project. Will the number of hours for each type of worker change at the optimal solution? Why or why not?

   The worst case *increase* in the cost of the project will be:

$$\Delta\phi = \Delta cx = (4)(253.867) = \$\mathbf{1015}$$

   It is likely that we will actually use less outsourced worker hours at this solution, but it is not guaranteed.

9. Re-run the optimization for the scenario in question (7) and confirm your response to the answer above.

   After re-running, the cost is \$63,232, which is an increase of \$508. This is better than our worst-case scenario as defined above.

10. Your supervisor says that the original cost of a P.Eng working on the project was grossly underestimated, and he fears that the number of hours allocated to a P.Eng will surely change once the real cost per hour is considered. Explain why you doubt this is the case, and give a convincing reason as to why.

    This won't change the solution at all because the cost of P.Eng workers could increase to ∞ and there would be no change in the solution. This is likely because of the supervisory hours limit combined with the maximum number of graduate hours available. We just can't help but hire the P.Eng for the allotted time, no matter the cost.

11. Re-run the GAMS file with a P.Eng worker cost MUCH HIGHER than the current \$64 per hour estimate to prove your point.

    Re-running with a P.Eng cost of \$1000 per hour gives the **exact same** solutions for x^*. The system works!