

Linearization:

Lateral dynamic:

$$\begin{cases} \ddot{y} = -\dot{\psi}\dot{x} + \frac{1}{m}(F_{yf}\cos\delta + F_{yr}) \\ F_{yf} = 2C_{\alpha}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) \\ F_{yr} = 2C_{\alpha}\left(-\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \end{cases}$$

$$\begin{aligned} \ddot{y} &= -\dot{\psi}\dot{x} + \frac{2C_{\alpha}}{m}\left(\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right)\cos\delta\right) - \frac{2C_{\alpha}}{m}\left(\frac{\dot{y}}{\dot{x}} + \frac{l_r\dot{\psi}}{\dot{x}}\right) \\ &= -\dot{\psi}\dot{x} + \frac{2C_{\alpha}}{m}\delta - \frac{2C_{\alpha}\dot{y}}{m\dot{x}} + \frac{2C_{\alpha}l_f\dot{\psi}}{m\dot{x}} - \frac{2C_{\alpha}\dot{y}}{m\dot{x}} - \frac{2C_{\alpha}l_r\dot{\psi}}{m\dot{x}} \\ &= \dot{\psi}\left(-\dot{x} + \frac{2C_{\alpha}l_r}{m\dot{x}} + \frac{2C_{\alpha}l_f}{m\dot{x}}\right) + \delta\left(\frac{2C_{\alpha}}{m}\right) + \dot{y}\left(-\frac{4C_{\alpha}}{m\dot{x}}\right) \end{aligned}$$

$$\ddot{\psi}I_z = l_f F_{yf} - l_r F_{yr}$$

$$\ddot{\psi} = \frac{l_f F_{yf}}{I_z} - \frac{l_r F_{yr}}{I_z}$$

$$= \dot{\psi}\left(-\frac{2l_f^2 C_{\alpha} + 2l_r^2 C_{\alpha}}{I_z \dot{x}}\right) + \delta\left(2\frac{l_f C_{\alpha}}{I_z}\right) + \dot{y}\left(\frac{2l_r C_{\alpha} - 2l_f C_{\alpha}}{I_z \dot{x}}\right)$$

$$\dot{S}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_{\alpha}}{m\dot{x}} & 0 & -\dot{x} + \frac{2C_{\alpha}(l_r + l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_{\alpha}(l_r l_f)}{I_z \dot{x}} & 0 & -\frac{2C_{\alpha}(l_r + l_f)}{I_z \dot{x}} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2C_{\alpha}}{m} & 0 \\ 0 & 0 \\ \frac{2l_f C_{\alpha}}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} + \dot{\psi}\dot{y} - Fg$$

$$\ddot{x} = \dot{\psi}\dot{y} + \frac{F}{m} - fg$$

