In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
import itertools as itr
import time
import csv
import networkx as nx
```

This is supplementary code written by Barry O'Donnell in December 2021 for module AM4065.

The code supports a project focused on the Travelling Salesman Problem (TSP) and the algorithms of finding the shortest path.

In [2]:
```
def GetRandomArray(n, seed=None):
    """
    Creates weighted (n x n) array with no self loops
    IN:  n := int, amount of nodes
         seed := int, seed for random generation

    OUT: arr := numpy array, weighted network
    """

    if seed != None:
        np.random.seed(seed)

    arr = np.random.randint(1, 5, (n,n))
    np.fill_diagonal(arr, 0)

    return arr * arr.T


def GetCost(arr, i, j):
    """
    Return cost of travelling from 'i' node to 'j' node
    IN:  arr := numpy array, weighted network
         i := int, node i
         j := int, node j

    OUT: cost := int, weight of edge{i,j}
    """

    return arr[i][j]


def GetPathCost(arr, start, path):
    """
    Return cost of travelling along path [start, j, ... , k]
    IN:  arr := numpy array, weighted network
         start := int, start node for path
         path := list/numpy array of int, path along which you travel t

    OUT: cost := int, weight of path [start, j, .... , k]
    """

    cost = 0
    for i in range(len(path)):
        cost += GetCost(arr, start, path[i])
```

```
        start = path[i]

    return cost


def GetPathsCost(arr, start, paths):
    """
    Return cost of travelling along paths ie, [start, j, ... , k], [sta
    IN:  arr := numpy array, weighted network
         start := int, start node for path
         paths := list of lists/numpy arrays of int, paths along which

    OUT: costs := numpy array of int, weight of paths, [start, j, ... ,
    """

    costs = np.zeros(np.shape(paths)[0])
    for i in range(len(paths)):
        costs[i] = GetPathCost(arr, start, paths[i])

    return costs


def TSP_performance(algo, n, k=1):
    """
    Finds the computational time of a n node network by some algorithm
    IN:  algo := function, algorithm being analysed
         n := int, nodes
         k := int, for k-NN or ItF algorithm

    OUT: time := float, time elapsed while code ran
    """

    arr = GetRandomArray(n)
    if algo != TSP_KNN or algo != TSP_ItF:
        start = time.time()
        cost, path = algo(arr)
        end = time.time()

    else:
        start = time.time()
        cost, path = algo(arr, k)
        end = time.time()

    return end-start


def TSP_avgperform(algo, n=4, k=1, trials=100):
    """
    Finds the average computational time of a n node network by some al
    after some amount of trials.
    IN:  algo := function, algorithm being analysed
         n := int, nodes
         k := int, for k-NN or ItF algorithm
         trials := int, amount of trials

    OUT: average comp. time := float, average computational time
    """

    avg = 0
    for i in range(trials):
```

```
        avg += TSP_performance(algo, n, k)

    return avg/trials


def DrawNetwork(arr, save=False, fname="network.pdf", pos = None):
    """
    Draws network for user to see.
    IN:  arr := numpy array, an array representing a network
         save := bool, allows user to save the figure produced
         fname := str, saves the figure with a given name
         pos := dict, allows user to choose the positions of nodes
                in the figure

    OUT: figure representing the network arr
    """

    ## Creates network G from arr
    G = nx.from_numpy_matrix(arr)

    ## Checks if user put in positions,
    ##  If not, randomly generate some layout.
    if pos == None:
        pos =  nx.spring_layout(G)

    ## Begin figure description
    plt.figure(figsize=(10,10))

    ## Draws network
    nx.draw(G,pos,edge_color='black',width=1,linewidths=1,
            node_size=1000, node_color='black',
            labels={node:node for node in G.nodes()}, font_size=20, for

    ## Adds labels to edges
    edge_labels = dict([ ((n1, n2), d['weight'])
                        for n1, n2, d in G.edges(data=True) ])

    nx.draw_networkx_edge_labels(G, pos, edge_labels = edge_labels, for

    ## Removes axis
    plt.axis('off')

    ## if user made save = True, saves figure
    if save:
        plt.savefig(fname, dpi=300)

    ## SHows figure
    plt.show()


def DrawPath(arr, paths, save=False, fname="network.pdf", pos = None, l
    """
    Draws network for user to see.
    IN:  arr := numpy array, an array representing a network
         paths := list of lists / numpy arrays of ints, paths around th
         save := bool, allows user to save the figure produced
         fname := str, saves the figure with a given name
         pos := dict, allows user to choose the positions of nodes
                in the figure
         labels := list of str, adds labels to the paths (Up to 3)
```

```
        colors := list of str, adds colours to the paths (Up to 3)

    OUT: figure representing the network arr, with paths depicted on it
    """
    ## Creates network G from arr
    G = nx.from_numpy_matrix(arr)

    ## Checks if user put in positions,
    ##  If not, randomly generate some layout.
    if pos == None:
        pos =  nx.spring_layout(G)

    ## Begin figure description
    plt.figure(figsize=(10,10))

    ## Draws network
    nx.draw(G,pos,edge_color='black',width=1,linewidths=1,
            node_size=1000, node_color='black',
            labels={node:node for node in G.nodes()}, font_size=20, for

    ## Adds labels to edges
    edge_labels = dict([((n1, n2), d['weight'])
                        for n1, n2, d in G.edges(data=True)])

    nx.draw_networkx_edge_labels(G, pos, edge_labels = edge_labels, for

    ## Removes axis from figure
    plt.axis('off')

    ## Creates list to hold edge details
    edges = []

    ## Adds paths to network graph
    for path in paths:
        path_edges = [(path[n], path[n+1]) for n in range(len(path)-1)]
        G.add_edges_from(path_edges)
        edges.append(path_edges)

    ## linewidths to draw onto graph, first value being the 'bottom' pa
    linewidths = [13,9,3]
    for ctr, edgelist in enumerate(edges):
        nx.draw_networkx_edges(G, pos=pos, edgelist=edgelist, edge_colo
    plt.legend(loc='best')

    ## If sve = True, saves figure for user
    if save:
        plt.savefig(fname, dpi=300)

    ## Shows figure
    plt.show()
```

Brute Force Algorithm

In [30]:

```
def GetPaths(n):
    """
    Creates all paths for network of 'n' nodes
    IN:  n := int, amount of nodes

    OUT: paths := numpy array, array of all paths
```

```python
    """

    nodes = np.array([i for i in range(n)][1:])
    return np.array(list(itr.permutations(nodes)))


def TSP_BF_path(arr, path):
    """
    Calculates path cost
    IN:  arr := numpy array, weighted network
         path := numpy array, path through the network

    OUT: cost := int, cost of path
    """

    cost = 0
    start = 0
    for i in range(len(path)):
        cost += GetCost(arr, start, path[i])
        start = path[i]

    cost += GetCost(arr, start, 0)
    return cost


def TSP_BF(arr):
    """
    Brute Force Algorithm
    Finds shortest path through network
    IN:  arr := numpy array, weighted network

    OUT: cost := int, cost of the path
         path := numpy array, shortest path through network
    """

    paths = GetPaths(len(arr))
    costs = np.zeros(len(paths))

    for i in range(len(paths)):
        costs[i] += TSP_BF_path(arr, paths[i])


    path = np.insert( np.insert(paths[np.argmin(costs)],0,0), len(arr),
```

Nearest Neighbour Algorithm

In [31]:
```python
def GetMinCost(arr, start, nodes):
    """
    Finds the lowest weighted edge connected to 'start' node
    IN:  arr := numpy array, weighted network
         start := int, node i
         nodes := numpy array, nodes available to start node

    OUT: node := int, node with lowest weight connected to 'start'
         cost := int, cost of edge{i,j}
    """
```

```
    costs = GetCost(arr, start, nodes)

    return nodes[np.argmin(costs)], np.min(costs)


def TSP_NN(arr, start=0):
    """
    Nearest Neighbour algorithm.
    Finds path of shortest immediate edges through network
    IN:  arr := numpy array, weighted network
         start := int, chooses starting node i

    OUT: cost := int, cost of the path
         path := numpy array, NN path through network
    """

    n = len(arr)
    costs = 0

    path = np.full((len(arr)+1),None)
    path[0] = start


    nodes = np.array([i for i in range(n)])
    nodes = nodes[nodes != start]

    for i in range(1,n):

        end, cost = GetMinCost(arr, path[i-1], nodes)

        path[i] = end
        nodes = nodes[nodes != end]
        costs += cost

    path[-1] = path[0]
    path = np.array(path, dtype='int')
    costs += GetCost(arr, end, path[0])

    return int(costs), path

def TSP_NN_mult(arr):
    """
    Multiple Nearest Neighbour algorithm.
    Finds paths of shortest immediate edges through network
    using multiple NN algorithms
    IN:  arr := numpy array, weighted network

    OUT: cost := int, cost of the path
         path := numpy array, NN path through network
    """

    ## Determine the amount of nodes in the network
    n = len(arr)

    ## Initialises cost
    cost = None

    ## Begins for loop iterating through the nodes of the network
    for i in range(n):
```

```
        ## Checks if the cost has a value / has smaller cost in using t
        if cost == None or TSP_NN(arr, start=i)[0] < cost:
            cost = TSP_NN(arr, start=i)[0]
            path = TSP_NN(arr, start=i)[1]

    return int(cost), path
```

k-Nearest Neighbour

In [32]:

```
def GetKPaths(nodes, node, k):
    """
    Creates all paths for network of 'n' nodes
    IN:  node := int, node i
         nodes := numpy array, nodes available to node i within k-steps
         k := int, amount of steps you are looking ahead

    OUT: paths := np.array(), array of all paths
    """

    nodes = nodes[nodes != node]

    ## Calculates permutations from node i to all available nodes withi
    if k >= len(nodes):
        ## If k >= nodes, then we just need a permutation of all the nc
        return np.array(list(itr.permutations(nodes, len(nodes))))
    else:
        ## If k < nodes, calculate every permutation of the nodes of le
        return np.array(list(itr.permutations(nodes, k)))


def TSP_KNN(arr, k=1):
    """
    k-Nearest Neighbour
    Finds shortest path through network by observing k-steps ahead
    IN:  arr := np.array(), weighted network
         k := int, amount of steps you are looking ahead.

    OUT: cost := int, cost of the path
         path := np.array(), k-NN path through network
    """

    ## Checks that k is >= 1
    if k<1:
        return print("Must enter a value of k >= 1")

    nodes = np.array([i for i in range(1, len(arr))])

    start = 0

    path = np.array([start])

    ## Begins while loop
    while len(nodes) != 0:

        ## Generates possible paths from node i to k-steps away
        kpaths = GetKPaths(nodes, start, k)

        ## Calculates cost of path
```

```
        costs = GetPathsCost(arr, start, kpaths)

        ## If k > the amount of remaining nodes, must account for retur
        if k > len(kpaths[0]):
            for i in range(len(kpaths)):
                zero_return = GetCost(arr, kpaths[i][-1], 0)
                costs[i] += zero_return

            ## Get minimum path
            minpath = kpaths[np.argmin(costs)]

            ## Construct the full path and end the loop
            path = np.append(path, minpath)
            break

        ## Get min path
        minpath = kpaths[np.argmin(costs)]

        ## Take 1 step forward
        path = np.append(path, minpath[0])
        nodes = nodes[nodes != path[-1]]
        start = path[-1]

    ## Append beginning node to path
    path = np.append(path, 0)
    costs = GetPathCost(arr, 0, path)
```

Into-the-Fray Algorithm

In [33]:
```
def GetKPaths(nodes, node, k):
    """
    Creates all paths for network of 'n' nodes
    IN:  node := int, node i
         nodes := numpy array, nodes available to node i within k-steps
         k := int, amount of steps you are looking ahead

    OUT: paths := np.array(), array of all paths
    """

    nodes = nodes[nodes != node]

    ## Calculates permutations from node i to all available nodes withi
    if k >= len(nodes):
        ## If k >= nodes, then we just need a permutation of all the no
        return np.array(list(itr.permutations(nodes, len(nodes))))
    else:
        ## If k < nodes, calculate every permutation of the nodes of le
        return np.array(list(itr.permutations(nodes, k)))


def TSP_ItF(arr, k=1):
    """
    Into-the-Fray
    Finds shortest path through network by observing k-steps ahead
        Is a worse version of KNN in every way
    IN:  arr := np.array(), weighted network
         k := int, amount of steps you are looking ahead.
```

```
    OUT: cost := int, cost of the path
         path := np.array(), k-NN path through network
    """

    ## Checks that k is >= 1
    if k<1:
        return print("Must enter a value of k >= 1")

    nodes = np.array([i for i in range(1, len(arr))])

    start = 0

    path = np.array([start])

    ## Begins while loop
    while len(nodes) != 0:

        kpaths = GetKPaths(nodes, start, k)

        costs = GetPathsCost(arr, start, kpaths)

        ## If k > the amount of remaining nodes, must account for retur
        if k > len(kpaths[0]):
            for i in range(len(kpaths)):
                zero_return = GetCost(arr, kpaths[i][-1], 0)
                costs[i] += zero_return

            ## Get minimum path
            minpath = kpaths[np.argmin(costs)]

            ## Construct the full path and end the loop
            path = np.append(path, minpath)
            break

        minpath = kpaths[np.argmin(costs)]

        ## Jumps to the end of the min cost path
        path = np.append(path, minpath)

        for n in minpath:
            nodes = nodes[nodes != n]

        start = path[-1]

    ## Append beginning node to path
    path = np.append(path, 0)
    costs = GetPathCost(arr, 0, path)
```

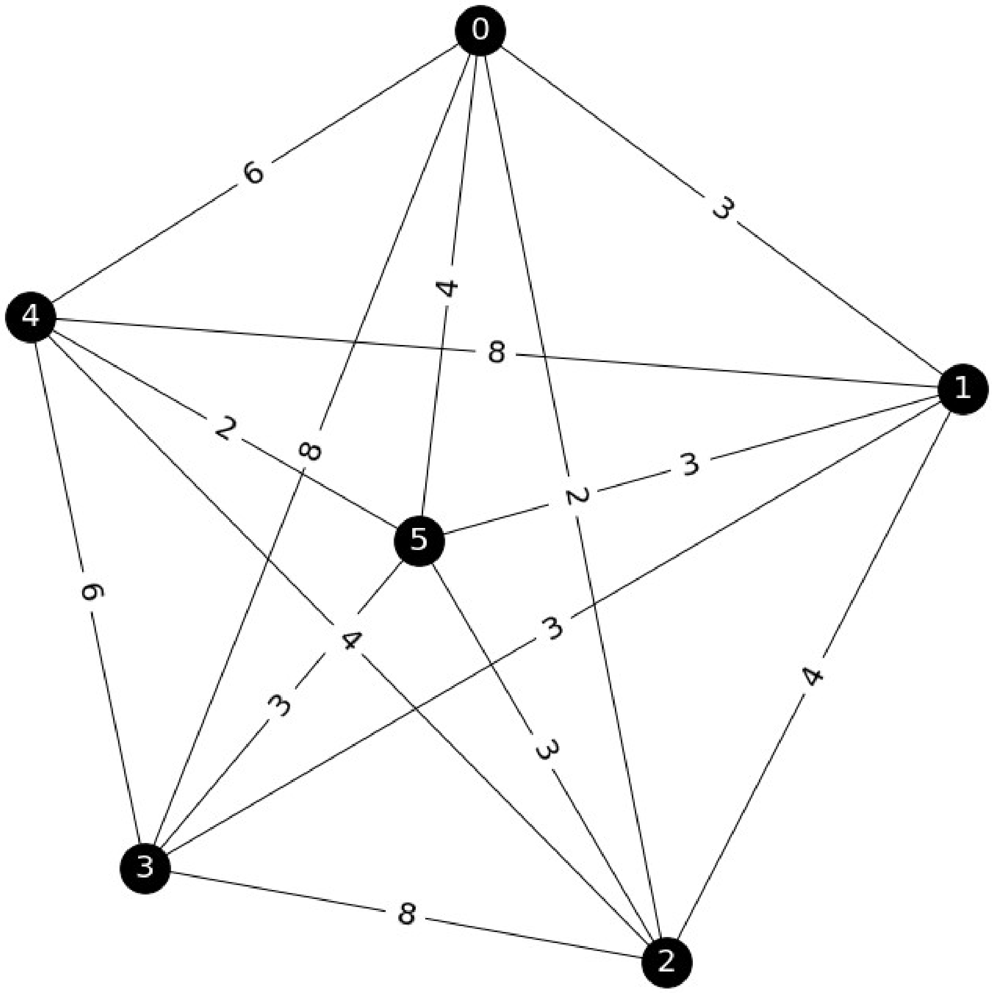
If-it-ain't-broke algorithm

In [34]:
```
def TSP_IiaB(arr):
    """
    If-it-ain't-Broke
    Very simplistic algorithm that travels through the network along [0
    IN:  arr := numpy array, weighted network

    OUT: cost := int, cost of the path
```

```
        path := numpy array, NN path through network
    """

    path = np.array([i for i in range(len(arr))])
    cost = GetPathCost(arr, 0, path[1:])

    cost += GetCost(arr, path[-1], 0)
    path = np.append(path, 0)
```

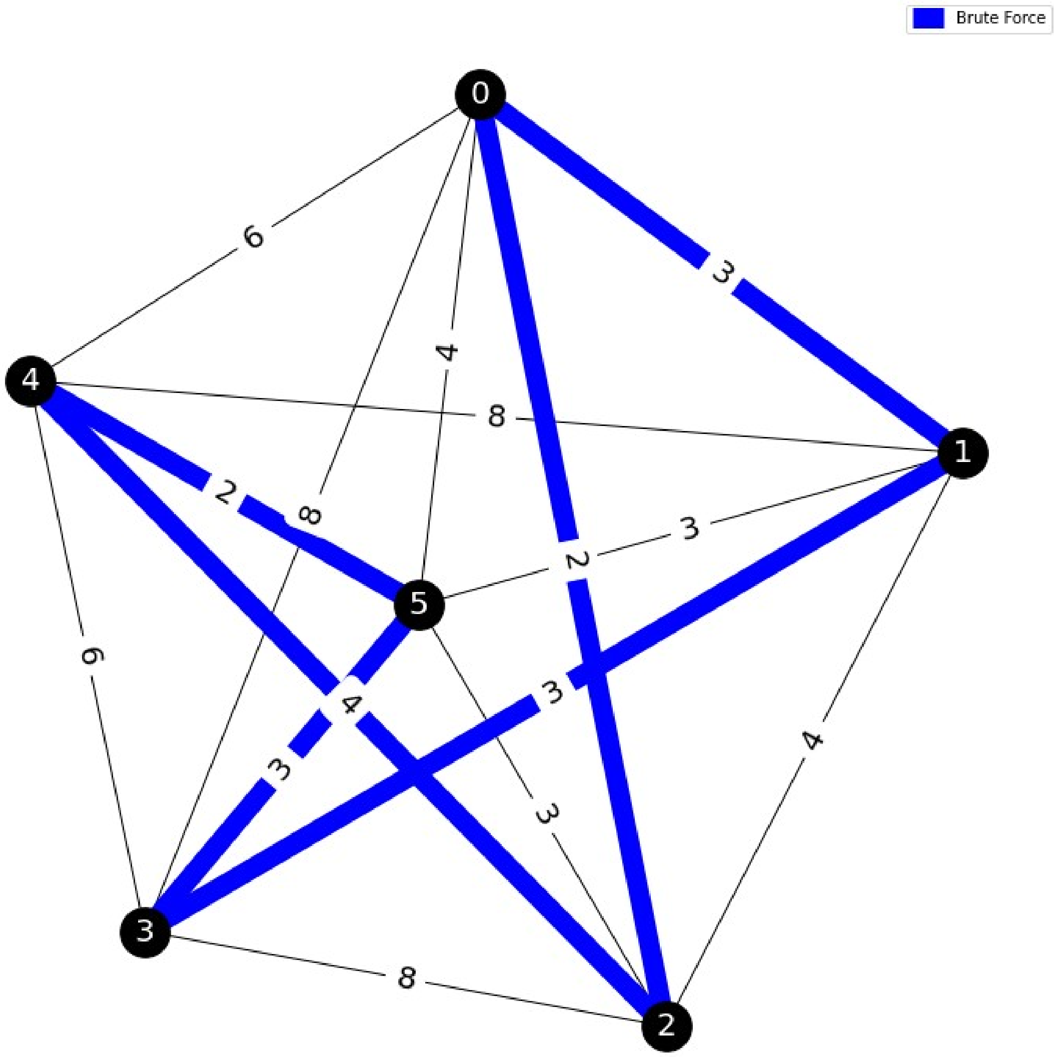
Example

In [35]:
```
## Toy example using network with n=6 nodes
arr = np.array([[0, 3, 2, 8, 6, 4],
 [3, 0, 4, 3, 8, 3],
 [2, 4, 0, 8, 4, 3],
 [8, 3, 8, 0, 6, 3],
 [6, 8, 4, 6, 0, 2],
 [4, 3, 3, 3, 2, 0]])

pos = {0: np.array([0.05763901, 0.85264863]),
 1: np.array([1.        , 0.22539623]),
 2: np.array([ 0.42116974, -0.77775065]),
 3: np.array([-0.59645827, -0.61144073]),
 4: np.array([-0.82176077,  0.35161524]),
 5: np.array([-0.0605897 , -0.04046871])}

## Displays network for user
```



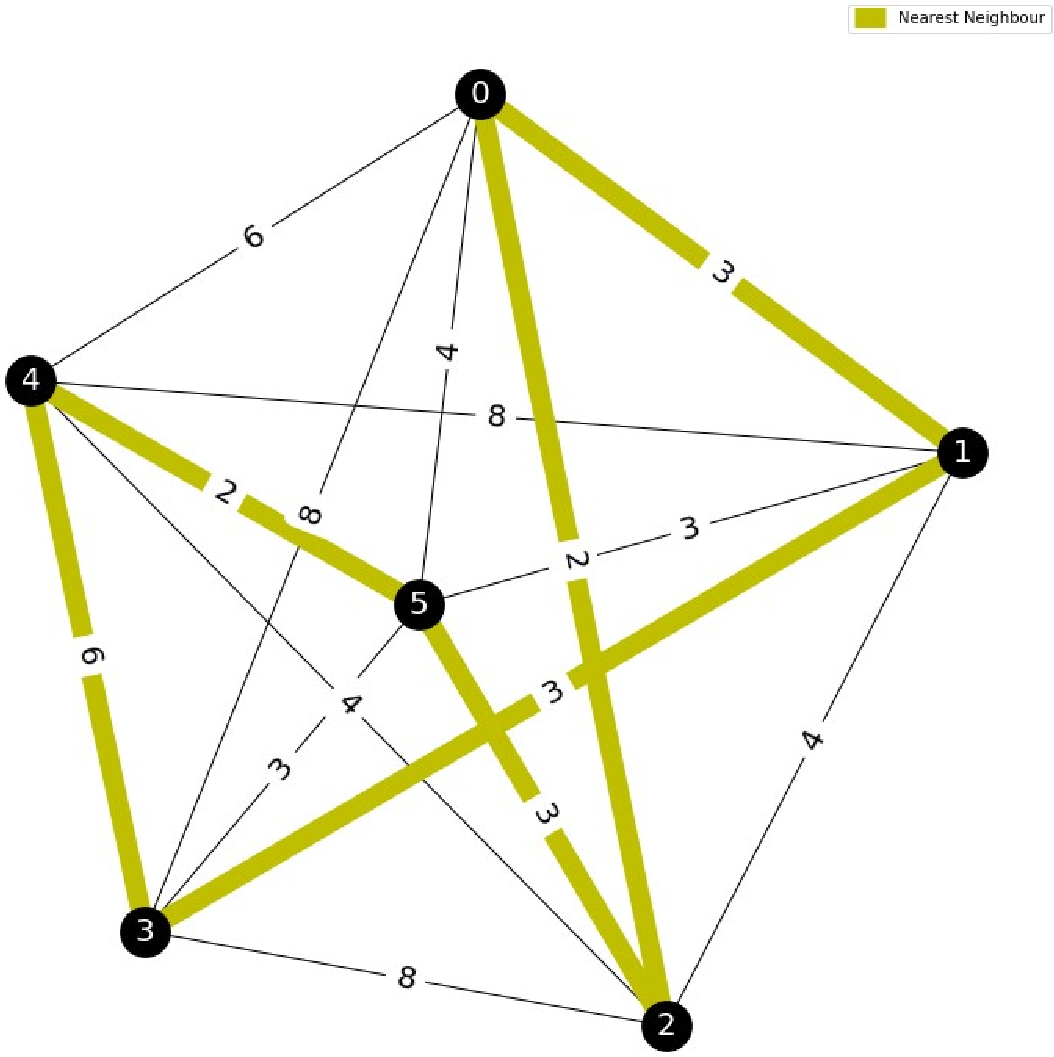
In [36]:
```
BF = TSP_BF(arr)
print("Brute force algorithm: The cost of the shortest path {} is {}".f
```

```
Brute force algorithm: The cost of the shortest path [0 1 3 5 4 2 0]
is 17
```



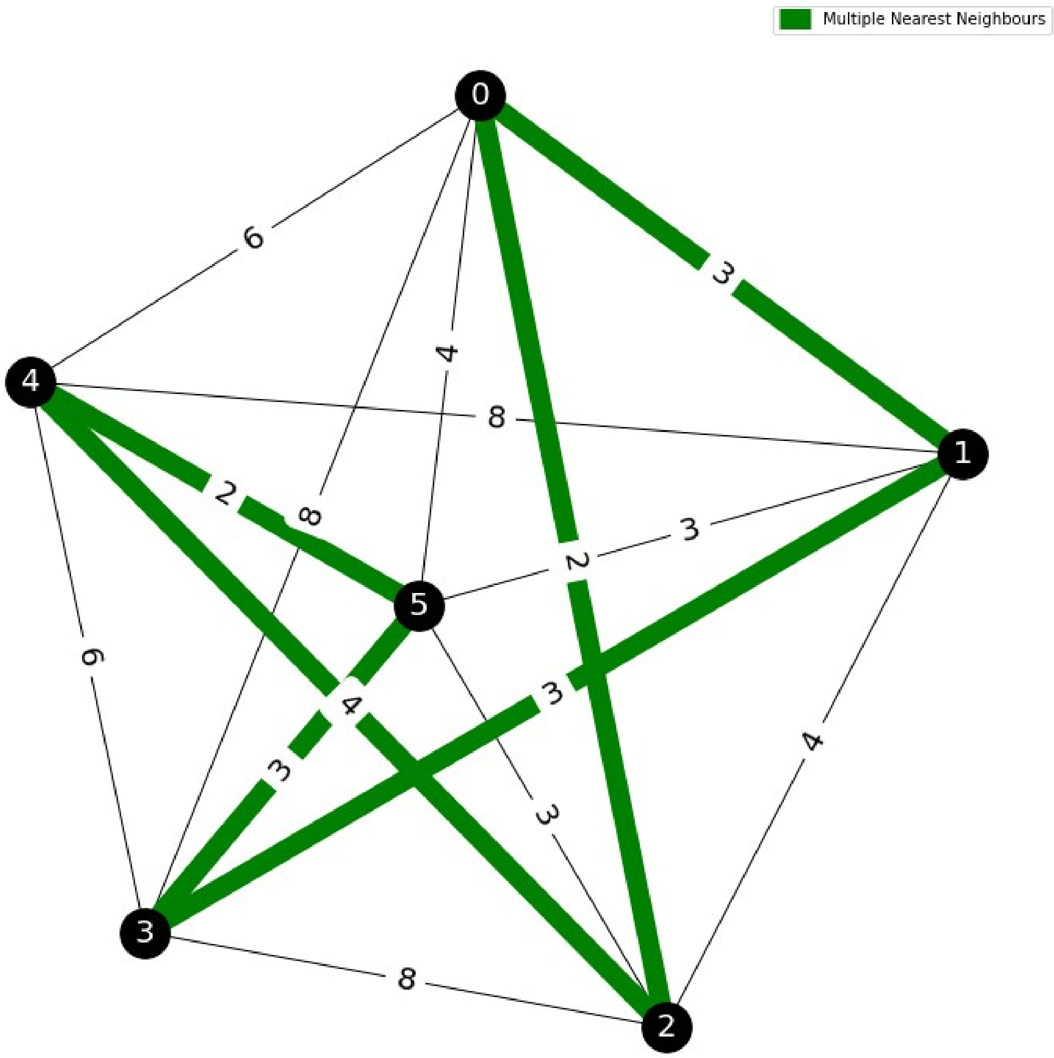
In [37]:
```
NN = TSP_NN(arr)
print("Nearest neighbour algorithm: The cost of the path {} is {}".form
```

```
Nearest neighbour algorithm: The cost of the path [0 2 5 4 3 1 0] is
19
```



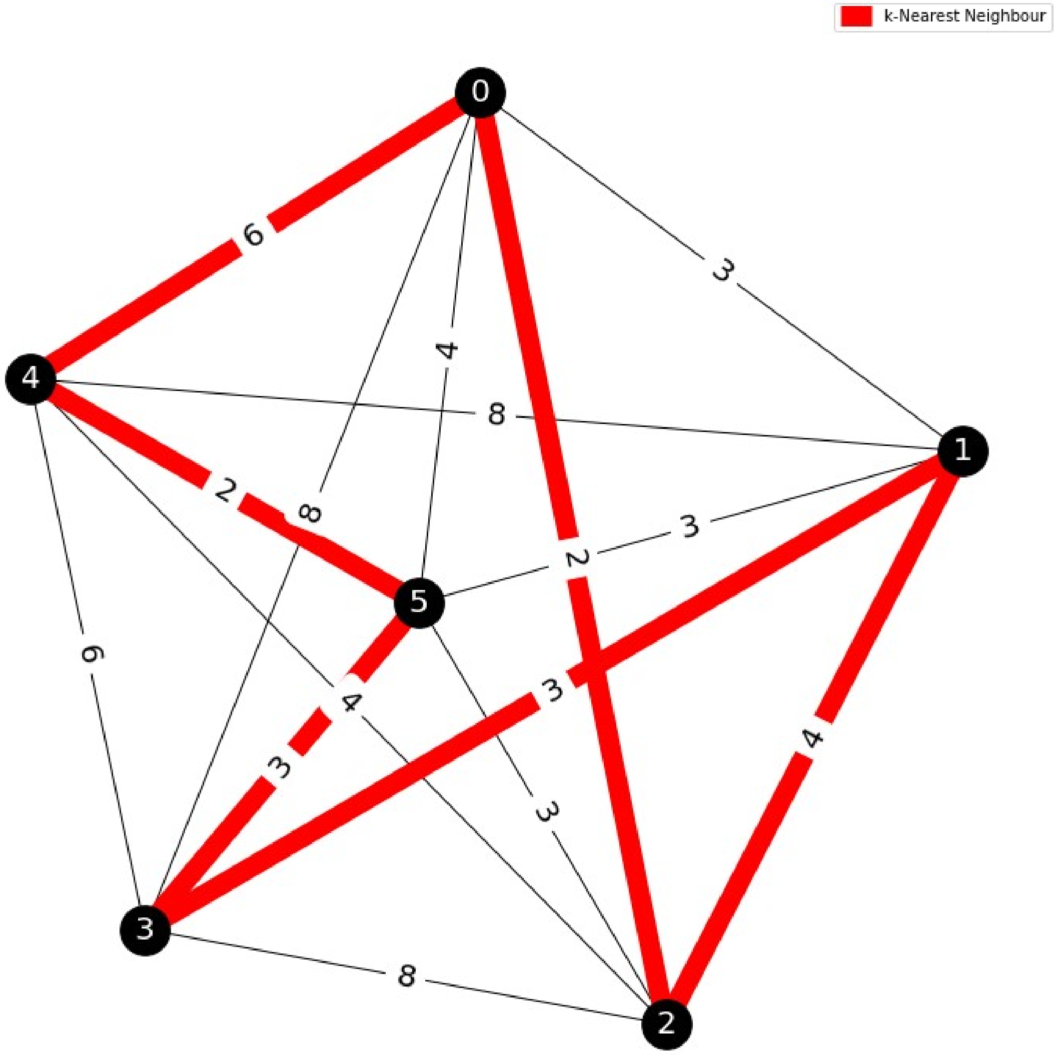
In [38]:
```
NN_mult = TSP_NN_mult(arr)
print("Multiple Nearest neighbour algorithm: The cost of the path {} is
DrawPath(arr, [NN_mult[1]], save=True, fname="nn_mult_network.pdf", lab
```

```
Multiple Nearest neighbour algorithm: The cost of the path [2 0 1 3
5 4 2] is 17
```



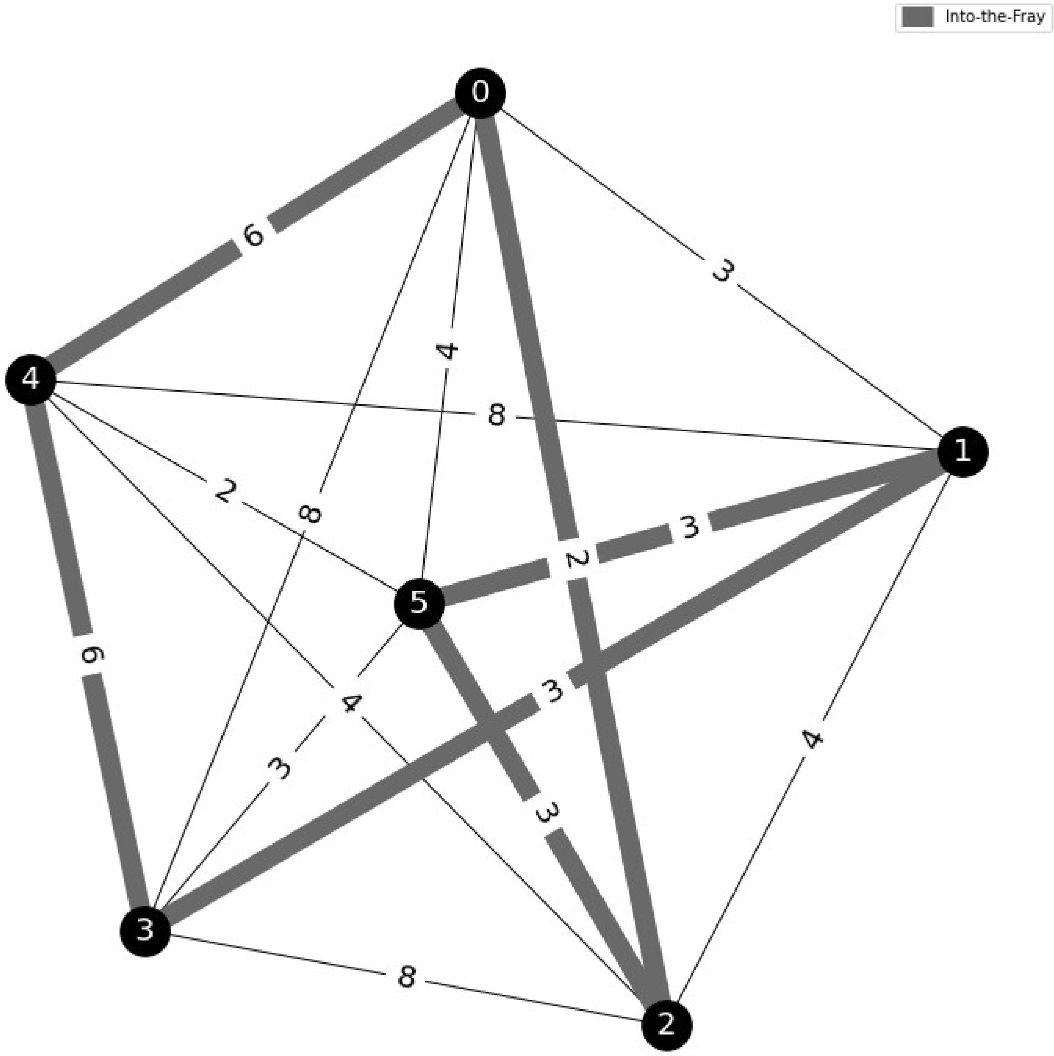
In [39]:
```
KNN = TSP_KNN(arr,k=3)
print("k-Nearest neighbour algorithm: The cost of the path {} is {}".fo
```

```
k-Nearest neighbour algorithm: The cost of the path [0 2 1 3 5 4 0]
is 20
```



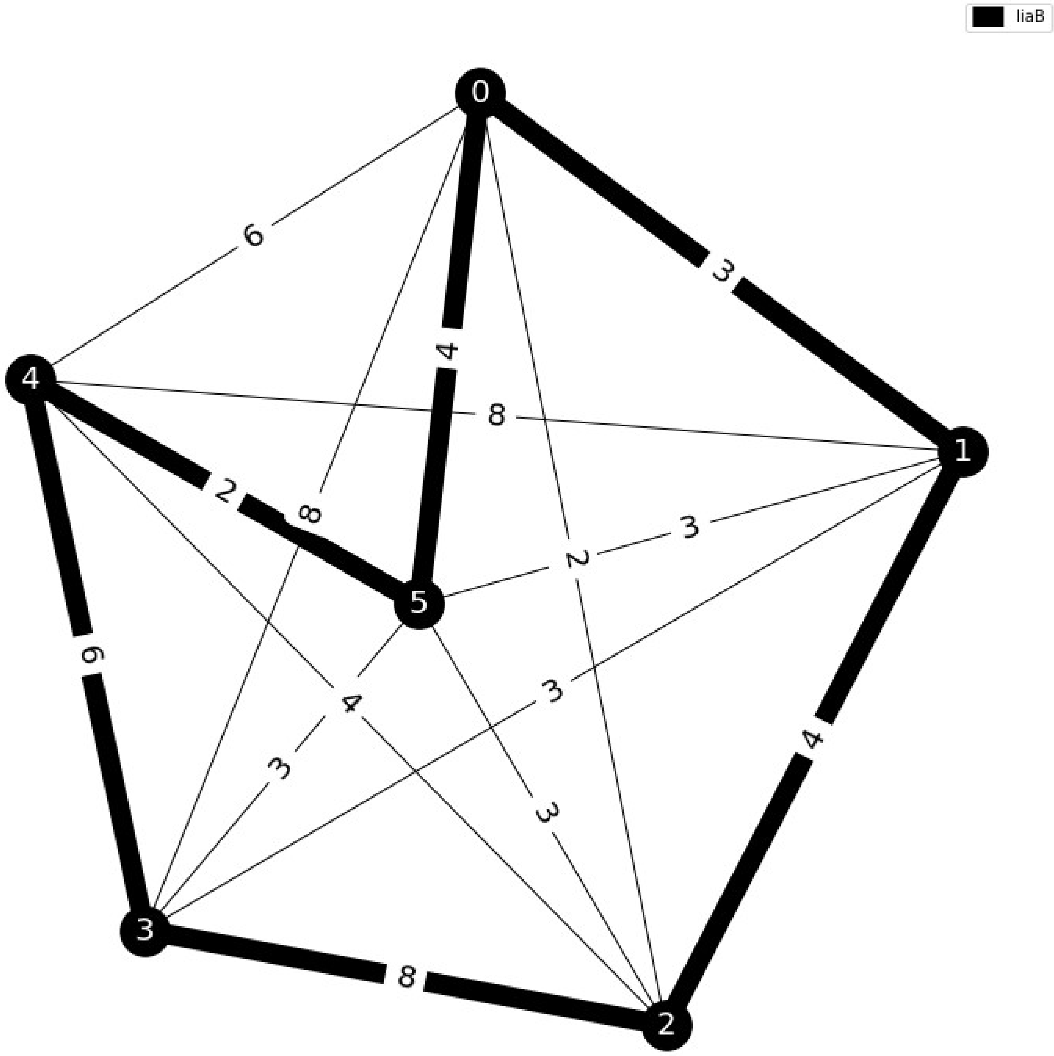
In [40]:
```
ItF = TSP_ItF(arr,k=2)
print("ItF algorithm: The cost of the path {} is {}".format(ItF[1], ItF
```

```
ItF algorithm: The cost of the path [0 2 5 1 3 4 0] is 23
```



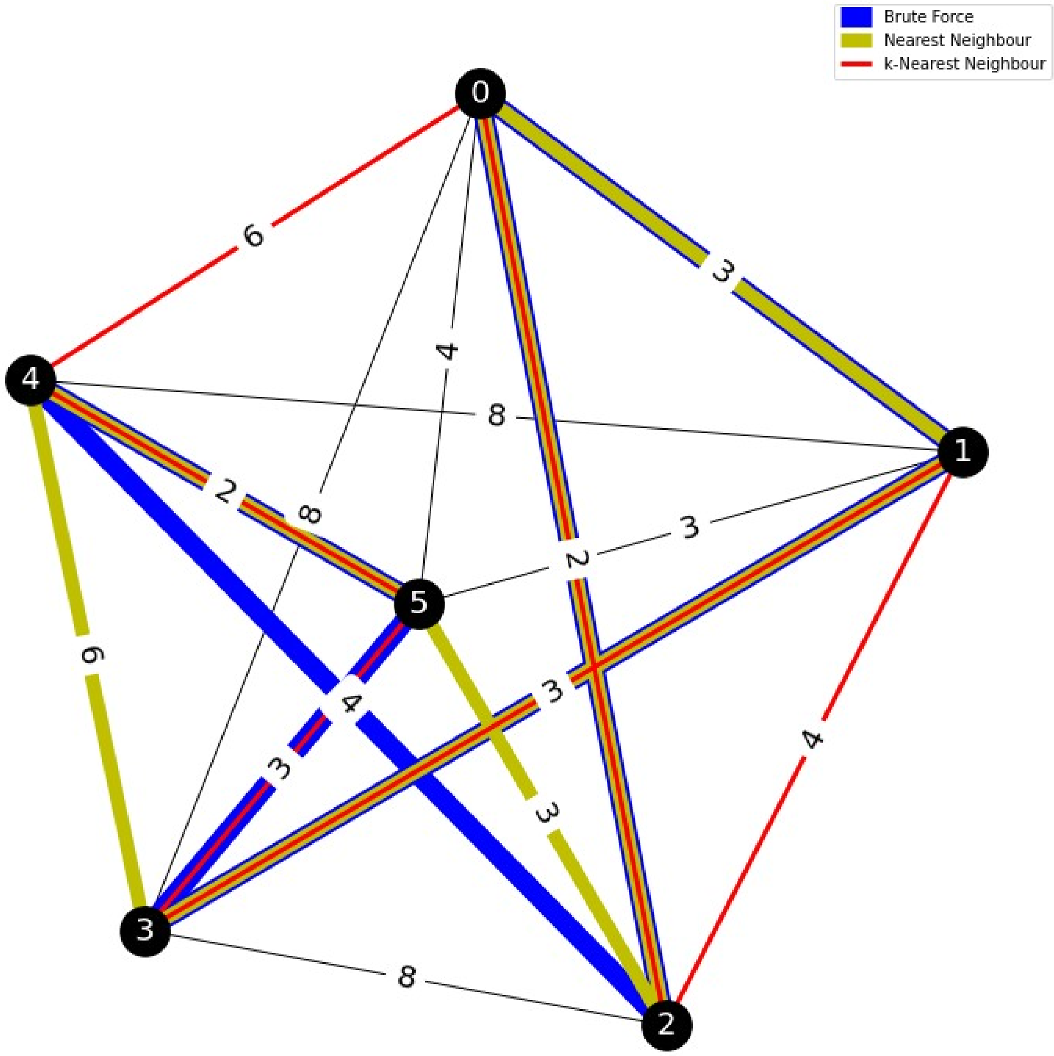
In [41]:
```
IiaB = TSP_IiaB(arr)
print("IiaB algorithm: The cost of the shortest path {} is {}".format(I
```

```
IiaB algorithm: The cost of the shortest path [0 1 2 3 4 5 0] is 27
```



In [42]:
```
print(arr)
k=3
BF = TSP_BF(arr)
NN = TSP_NN(arr)
KNN = TSP_KNN(arr,k)

## Shows user the cost of the path found by each network
print("Brute force algorithm: The cost of the shortest path {} is {}".f
print("Nearest neighbour algorithm: The cost of the path {} is {}".form
print("k-Nearest neighbour algorithm: The cost of the path {} is {}".fc
```

```
[[0 3 2 8 6 4]
 [3 0 4 3 8 3]
 [2 4 0 8 4 3]
 [8 3 8 0 6 3]
```

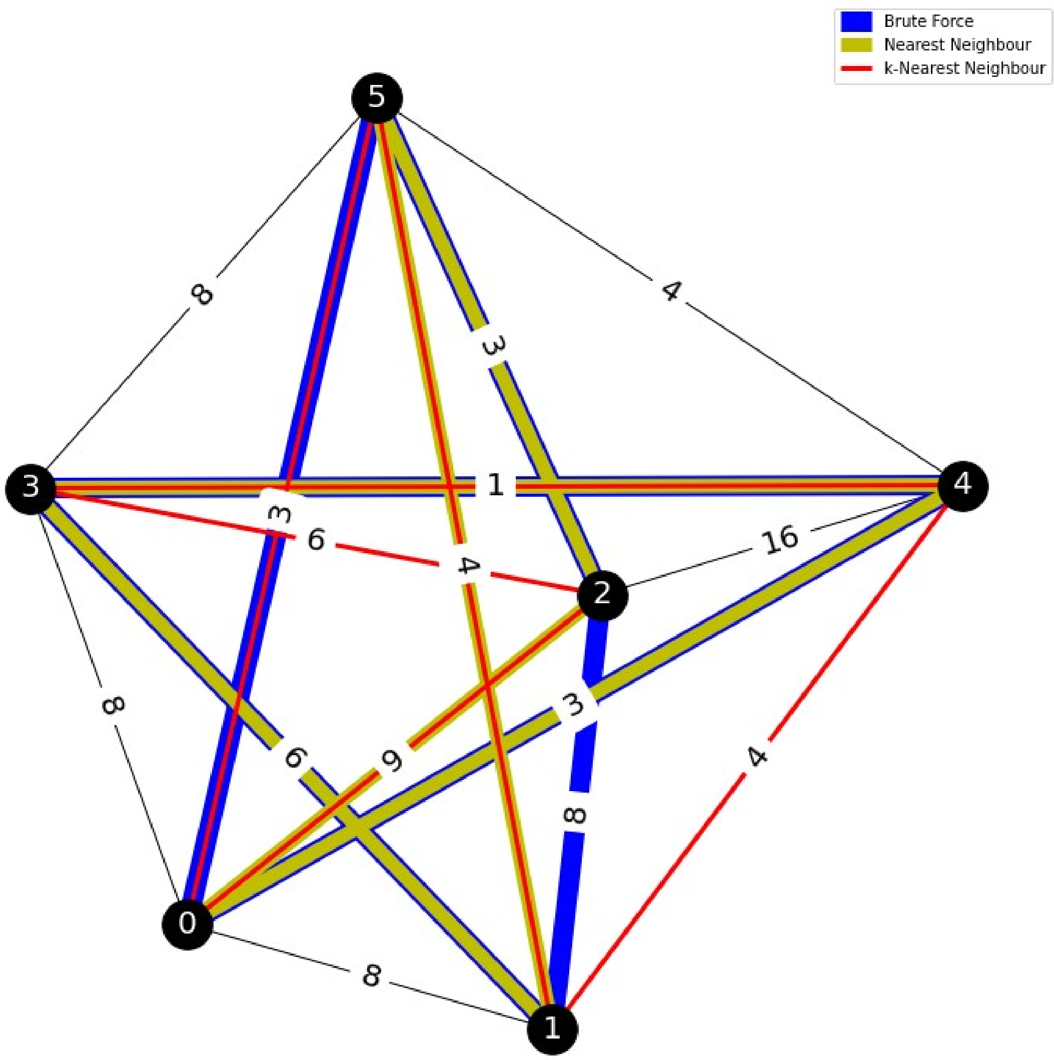


Firing Range

In [16]:
```
## Testing Random Arrays
n = 6 # np.random.randint(4, 8)
k = 3
arr = GetRandomArray(n)

## Prints weighted network array
print(arr)

## Applies algorithms to network
BF = TSP_BF(arr)
NN = TSP_NN(arr)
KNN = TSP_KNN(arr,k)

## Shows user the cost of the path found by each network
print("Brute force algorithm: The cost of the shortest path {} is {}".f
print("Nearest neighbour algorithm: The cost of the path {} is {}".form
print("k-Nearest neighbour algorithm: The cost of the path {} is {}".fc
```

```
if NN[0] == BF[0]:
    print("Nearest neighbour guessed the correct shortest path!")

if KNN[0] == BF[0]:
    print("k-Nearest neighbour guessed the correct shortest path!")
```

```
[[ 0  8  9  8  3  3]
 [ 8  0  8  6  4  4]
 [ 9  8  0  6 16  3]
 [ 8  6  6  0  1  8]
 [ 3  4 16  1  0  4]
 [ 3  4  3  8  4  0]]
Brute force algorithm: The cost of the shortest path [0 4 3 1 2 5 0]
is 24
Nearest neighbour algorithm: The cost of the path [0 4 3 1 5 2 0] is
26
k-Nearest neighbour algorithm: The cost of the path [0 5 1 4 3 2 0]
is 27
```



This cell is dedicated to abtaining error data for each algortihm

In this case we are going from a 4 node network to a 10 node network and running 100 trials for each 'n'.

This data is saved into a `.csv` for analysis and to stop the need to run the cell multiple times

In [19]:
```
## !!!WARNING!!! INCREASING n to > 12 may cause memory problems and/or
##  a computer crash. Use at ones own discretion

## Define range of 'n' values being observed
n = 10
nstart = 4
nrange = n - (nstart-1)

## Names of models in use. This is for the .csv file and will occupy th
model_names = ['KNN (k=n-1)', 'KNN (k=n-2)', 'KNN (k=n-3)', 'NN', 'NN_m

## finds how many models are being analysed
models = len(model_names)

## Attempt 100 trials
trials = 100

## Initialises errors array to hold each result from the trial
##  for example, errors[2, 4, 87] will be the result from the 86th tria
errors = np.zeros((nrange, models, trials))


## Iterates through 'n' values
for i in range(nstart, n+1):

    ## Begins trials
    for j in range(trials):
        ## Generates random ixi array
        arr = GetRandomArray(i)

        ## Obtains shortest using BF
        BF = TSP_BF(arr)

        ## Obtains path lengths of NN and IiaB
        NN = TSP_NN(arr)
        NN_mult = TSP_NN_mult(arr)
        IiaB = TSP_IiaB(arr)

        ## Calculates relative error of each model
        ##  and puts it into it's place in the error array
        errors[i-nstart,3,j] = (BF[0] - NN[0]) / BF[0]
        errors[i-nstart,4,j] = (BF[0] - NN_mult[0]) / BF[0]
        errors[i-nstart,8,j] = (BF[0] - IiaB[0]) / BF[0]

        ## Begins looking at KNN and ItF for k = n-1, n-2, n-3
        ##  need indexer
        ind = 0
        for k in [i-1, i-2, i-3]:

            ## Obtains path lengths from KNN and ItF
            KNN = TSP_KNN(arr, k)
            ItF = TSP_ItF(arr, k)

            ## Calculates relative error of each model
```

```
            ##  and puts it into it's place in the error array
            ##  ItF is 4 places ahead in the model names list
            errors[i-nstart,ind,j] = (BF[0] - KNN[0]) / BF[0]
            errors[i-nstart,ind+5,j] = (BF[0] - ItF[0]) / BF[0]

            ind += 1


## After analysis done, saves result in seperate .csv for each value of
for i in range(nrange):
    with open('errors_' + str(i+nstart) +'.csv', 'w', newline='') as f:
        writer = csv.writer(f)


        ## Saves model names to header
        writer.writerow(model_names)

        ## Saves errors of each model to each row in the .csv
        for j in range(trials):
            writer.writerow(errors[i,:,j])
```

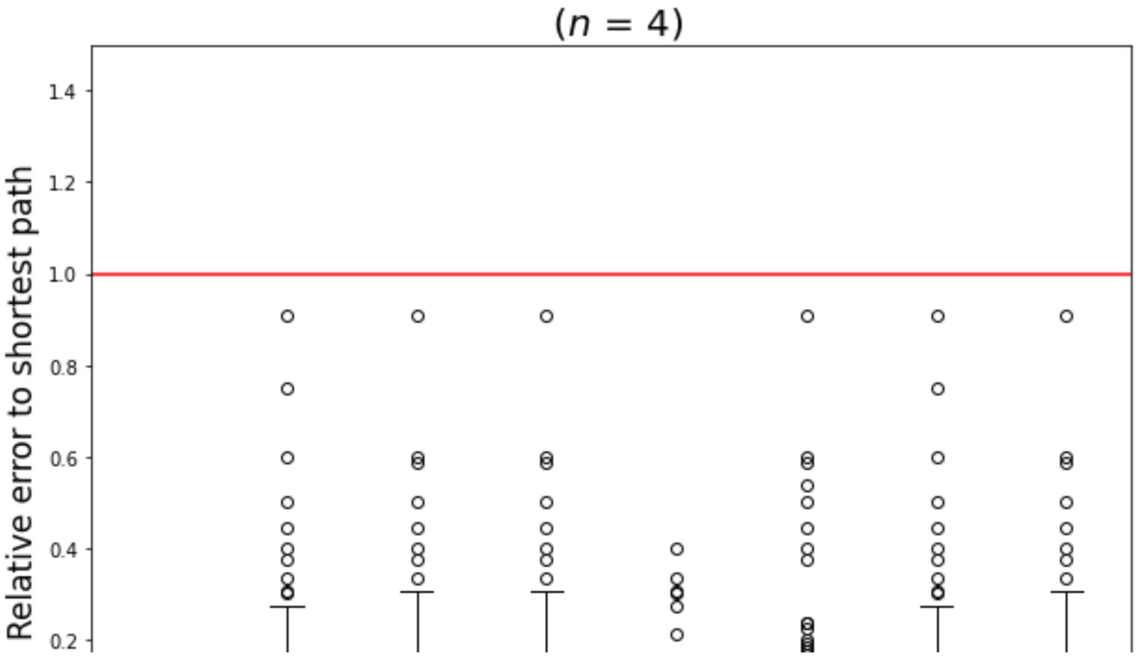
Now we produce boxplots for each `.csv`.

This is done to observe how each model performs relative to the ground truth. We would expect the error to be quite low for $k = n - 1$ `k-NN` model as it can make one step, and then essentially jump to the end of a network for its next step.

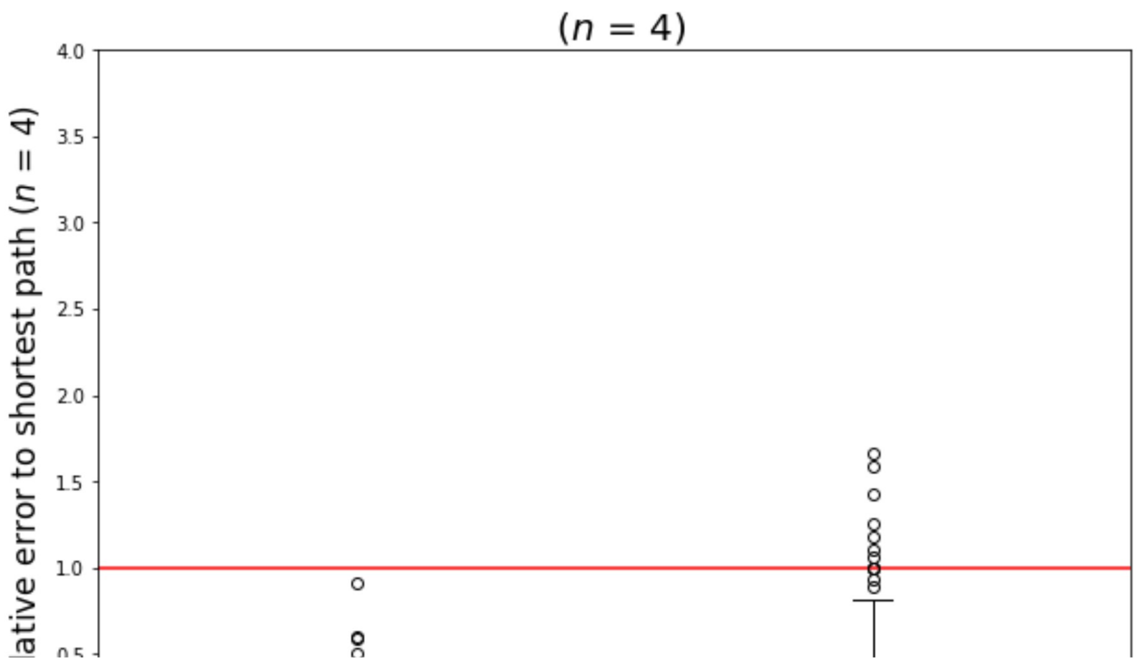
We would also expect that `IiaB` to perform extremely poorly. For the sake of clarity, it is omitted from this analysis. It would be better to compare it to `NN`, it's slighlty smarter counterpart.

In [26]:
```
## Have model names here just to easily index which models you want to
#model_names = ['KNN (k=n-1)', 'KNN (k=n-2)', 'KNN (k=n-3)', 'NN', 'NN_

## Opens and obtains data for varying values of 'n'
for j in range(nstart, n+1):
    labels = np.genfromtxt('errors_' + str(j) + '.csv', delimiter=',',
    errors = np.genfromtxt('errors_' + str(j) + '.csv', delimiter=',',

    ## Obtains the absolute of the errors (Since the BF path will alway
    ##  As stated above, exclude IiaB from analysis
    errors = abs(errors[:, (0,1,2,3,4,5,6,7)])
    labels = labels[[0,1,2,3,4,5,6,7]]

    fig = plt.figure(j, figsize=(10, 7))

    ## Add horizontal line showing where the relative would be 1, or wh
    ##  would be twice the true shortest path
    plt.axhline(1, color='r')

    ## Plot definitions
    plt.boxplot(errors, patch_artist=True, boxprops=dict(facecolor='gra

    plt.xticks(ticks = [1,2,3,4,5,6,7,8], labels=labels, rotation=45, f

    plt.title(" ($n$ = " + str(j) + ")", fontsize=20)
    plt.ylabel("Relative error to shortest path", fontsize=17)
    plt.ylim(top=1.5)
```

```
## Saves boxplots for user
plt.savefig("images\errors_box_" + str(j) + ".pdf", dpi=300, bbox_i

plt.show()
```



In [27]:
```
## Opens and obtains data for varying values of 'n'
for j in range(nstart, n+1):
    labels = np.genfromtxt('errors_' + str(j) + '.csv', delimiter=',',
    errors = np.genfromtxt('errors_' + str(j) + '.csv', delimiter=',',

    ## Obtains the absolute of the errors (Since the BF path will alway
    ##  As stated above, exclude IiaB from analysis
    errors = abs(errors[:, (3,-1)])
    labels = labels[[3,-1]]

    fig = plt.figure(figsize =(10, 7))
    ## Add horizontal line showing where the relative would be 1, or wh
    ##  would be twice the true shortest path
    plt.axhline(1, color='r')

    ## Plot definitions
    plt.boxplot(errors, patch_artist=True, boxprops=dict(facecolor='gra

    plt.xticks(ticks = [1,2], labels=labels, rotation=45, fontsize=17)

    plt.title(" ($n$ = " + str(j) + ")", fontsize=20)

    plt.ylabel("Relative error to shortest path ($n$ = " + str(j) + ")"
    plt.ylim(top=4)

    ## Saves boxplots for user
    plt.savefig("images\errors_box_" + str(j) + "_w_iiab.pdf", dpi=300,
```



In [ ]:

In [ ]: