CSE 527 HW 1

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1. (a). D: disease ; $\bar{D}$: non-disease ; P: positive (testing) ; $\bar{P}$: negative (testing)

Given: $P(P|D) = 0.99$, $P(\bar{P}|\bar{D}) = 0.99$; $P(D) = 0.0001$; $P(\bar{D}) = 0.9999$

Calculate. $P(D|P) = \frac{P(P|D)\cdot P(D)}{P(P)}$

$$= \frac{P(P|D)\cdot P(D)}{P(P|D)\cdot P(D) + (1-P(P|D))\cdot P(\bar{D})} \quad (1).$$

$$= \frac{0.99 \times 0.0001}{0.99 \times 0.0001 + (1-0.99) \times 0.9999} \approx 0.0098$$

Why good news?

from Equation (1): $P(D|P) = \frac{1}{1 + \underbrace{\left(\frac{1}{P(P|D)}-1\right)}_{\text{let it } = C > 0} \cdot \left(\frac{1}{P(D)} - 1\right)}$

$$\frac{\partial P(D|P)}{\partial P(D)} = \left[\frac{C}{(1-C)P(D)+C}\right]^2 > 0 \quad (\text{for } \forall P(D)).$$

~~The smaller~~ The ~~rear~~ rarer the disease is, the smaller the chances will be.

(b) to prove $P(A,B|E) = P(A|B,E)\cdot P(B|E)$

$\text{left} = \frac{P(A,B,E)}{P(E)}$ $\quad \text{right} = \frac{P(A,B,E)\cdot P(B|E)}{P(B,E)}$

$$= \frac{P(A,B,E)\cdot P(B|E)}{P(B|E)\cdot P(E)} = \frac{P(A,B,E)}{P(E)}$$

left = right

proven!

to prove $p(A|B,E) = \frac{P(B|A,E)\cdot p(A|E)}{p(B|E)}$

$$\text{right} = \frac{\frac{P(A,B,E)}{P(A,E)} \cdot \frac{P(A,E)}{P(E)}}{\frac{P(B,E)}{P(E)}} = \frac{P(A,B,E)}{P(B,E)} = P(A|B,E) = \text{left}.$$

proven!

2. a). $P(x_1 | x_2, x_3)$

$\theta_{x_1|x_2=up,x_3=up}$, $\theta_{x_1|x_2=up,x_3=down}$, $\theta_{x_1|x_2=down,x_3=up}$, $\theta_{x_1|x_2=down,x_3=down}$

b) $$L(\theta:D) = \prod l(D:\theta)$$
$$= L(x[1],\dots,x[M];\ \theta_{x_1|pa_1},\dots\theta_{x_i|pa_i})$$
$$= \prod_{t=1}^{M} P(\theta x_1,\dots x_N)$$
$$= \prod_{t=1}^{M(pa_i)} \prod_{i=1}^{N} \cancel{\theta^t_{x_i|pa_i}}\,(P(x_i^t|pa_i).$$

c) $$L(\theta:D) = \prod_{i=1}^{N} \left(\theta_{x_i|pa_i}^{M[x_i,pa_i]} \cdot (1-\theta_{x_i|pa_i})^{M[pa_i]-M[x_i,pa_i]}\right)$$

$$\log L(\theta:D) = \sum_{t=1}^{N} \left(M[x_i,pa_i]\cdot \log\theta_{x_i|pa_i} + (M[pa_i]-M[x_i,pa_i])\log(1-\theta_{x_i|pa_i}\right).$$

$$\frac{\partial(\log L(\theta:D))}{\partial\theta_{x_i|pa_i}} = \frac{M[x_i,pa_i]}{\theta_{x_i|pa_i}} - \frac{M[pa_i]-M[x_i,pa_j]}{1-\theta_{x_i|pa_i}} = 0$$

$$\hat{\theta}_{x_i|pa_i} = \frac{M[x_i,pa_i]}{M[pa_i]}$$

3 a) Model 1: θ_{Gal80}; $\theta_{Gal4|Gal80=high}$; $\theta_{Gal4|Gal80=low}$;
$\theta_{Gal2|Gal4=high}$; $\theta_{Gal2|Gal4=low}$.

Model 2: θ_{Gal80}; θ_{Gal4}; $\theta_{Gal2|Gal80=high,Gal4=high}$; $\theta_{Gal2|Gal80=high,Gal4=low}$.
$\theta_{Gal2|Gal80=low,Gal4=high}$, $\theta_{Gal2|Gal80=low,Gal4=low}$.

(b) Model 1

$$L(\theta:D) = \prod_m P[x_{80}[m], x_4[m], x_2[m]; \theta].$$

$$= \prod_m \{ P(x_{80}[m]) \cdot P(x_4[m] \mid x_{80}[m]=1) \cdot P(x_4[m] \mid x_{80}[m]=0] \cdot$$

$$P(x_2[m] \mid x_4[m]=1) \cdot P(x_2[m] \mid x_4[m]=0) \}.$$

$$= \theta_{80}^{M[1]} \cdot (1-\theta_{80})^{M[0]} \cdot \theta_{4 \mid Gal80=1}^{M[1]} \cdot (1-\theta_{4 \mid Gal80=1})^{M[0]} \cdot \theta_{2 \mid Gal4=1}^{M[1]} \cdot (1-\theta_{2 \mid Gal4=1})^{M[0]}$$

$$\theta_{4 \mid Gal80=0}^{M[1]} \cdot (1-\theta_{4 \mid Gal80=0})^{M[0]} \cdot \theta_{2 \mid Gal4=0}^{M[1]} \cdot (1-\theta_{2 \mid Gal4=0})^{M[0]}$$

Model 2.

$$L(\theta:D) = \prod_m \cdot P(x_{80}[m], x_4[m], x_2[m]; \theta)$$

$$= \prod_m \{ P(x_{80}[m]) \cdot P(x_4[m]) \cdot P(x_2[m] \mid x_{80}[m]=0, x_4[m]=0) \cdot$$

$$P(x_2[m] \mid x_{80}[m]=0, x_4[m]=1) \cdot P(x_2[m] \mid x_{80}[m]=1, x_4[m]=1)$$

$$P(x_2[m] \mid x_{80}[m]=1, x_4[m]=0) \}.$$

$$= \theta_{80}^{M[1]} \cdot (1-\theta_{80})^{M[0]} \cdot \theta_4^{M[1]} \cdot (1-\theta_4)^{M[0]} \cdot \theta_{2 \mid Gal80=0, Gal4=0}^{M[1]} (1-\theta_{2 \mid Gal80=0, Gal4=0})^{M[0]}$$

$$\theta_{2 \mid Gal80=0, Gal4=1}^{M[1]} \cdot (1-\theta_{2 \mid Gal80=0, Gal4=1})^{M[0]} \cdot \theta_{2 \mid Gal80=1, Gal4=0}^{M[1]} \cdot (1-\theta_{2 \mid Gal80=1, Gal4=0})^{M[0]}$$

$$\theta_{2 \mid Gal80=1, Gal4=1}^{M[1]} (1-\theta_{2 \mid Gal80=1, Gal4=1})^{M[0]}$$

(c) For both model1 and mode 2

According to 2 (c) $\hat{\theta}_{x_i \mid pa_i} = \dfrac{M[x_i, pa_i]}{M[pa_i]}$

Eg. Model 2. $\hat{\theta}_{2 \mid Gal80=1, Gal4=1} = \dfrac{M[x_2=1, Gal80=1, Gal4=1]}{M[Gal80=1, Gal4=1]}$

in this set, numbers of samples that Gal2 = 1

numbers of ~~all the~~ samples ~~is~~ statify the Condition that Gal80=1, Gal4=1

Model 1

M1_theta_80
0.482

	M1_theta_4
M1_80 = 0	0.69
M1_80 = 1	0.333

	M1_theta_2
M1_4 = 0	0.37
M1_4 = 1	0.707

Model 2

M2_theta_80
0.482

M2_theta_4
0.518

	M2_theta_2
M2_80 = 0, M2_4 = 0	0.333
M2_80 = 0, M2_4 = 1	0.8
M2_80 = 1, M2_4 = 0	0.389
M2_80 = 1, M2_4 = 1	0.5

	Likelihood(Log)
Model 1	-218.54
Model 2	-223.13

(e) Likelihood: Model 1 > Model 2; Model 1 is better than Model 2 according to MLE.