UNIVERSITÀ DI PISA

Performance Evaluation of Computer Systems and Networks

Control Tower System Analysis

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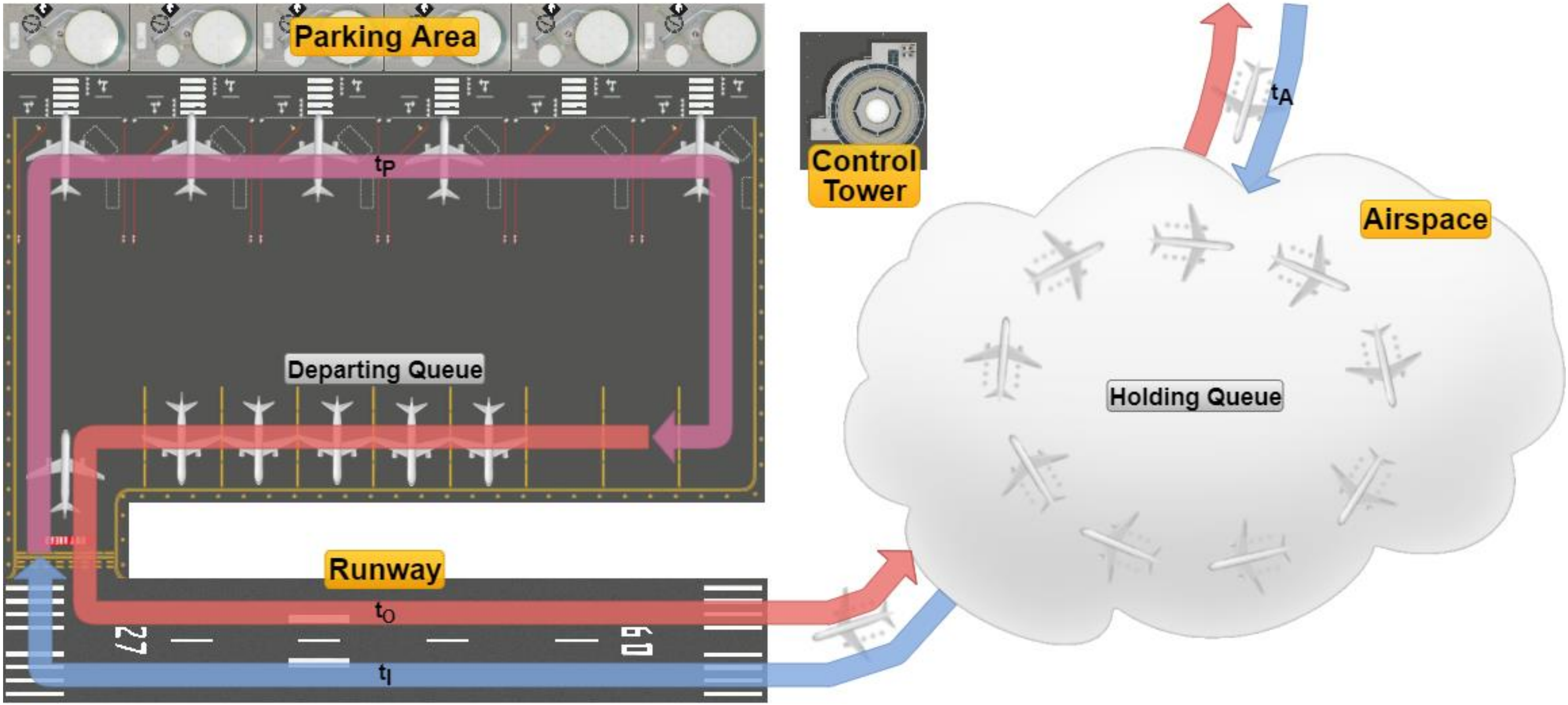
System Model



Working Hypotheses

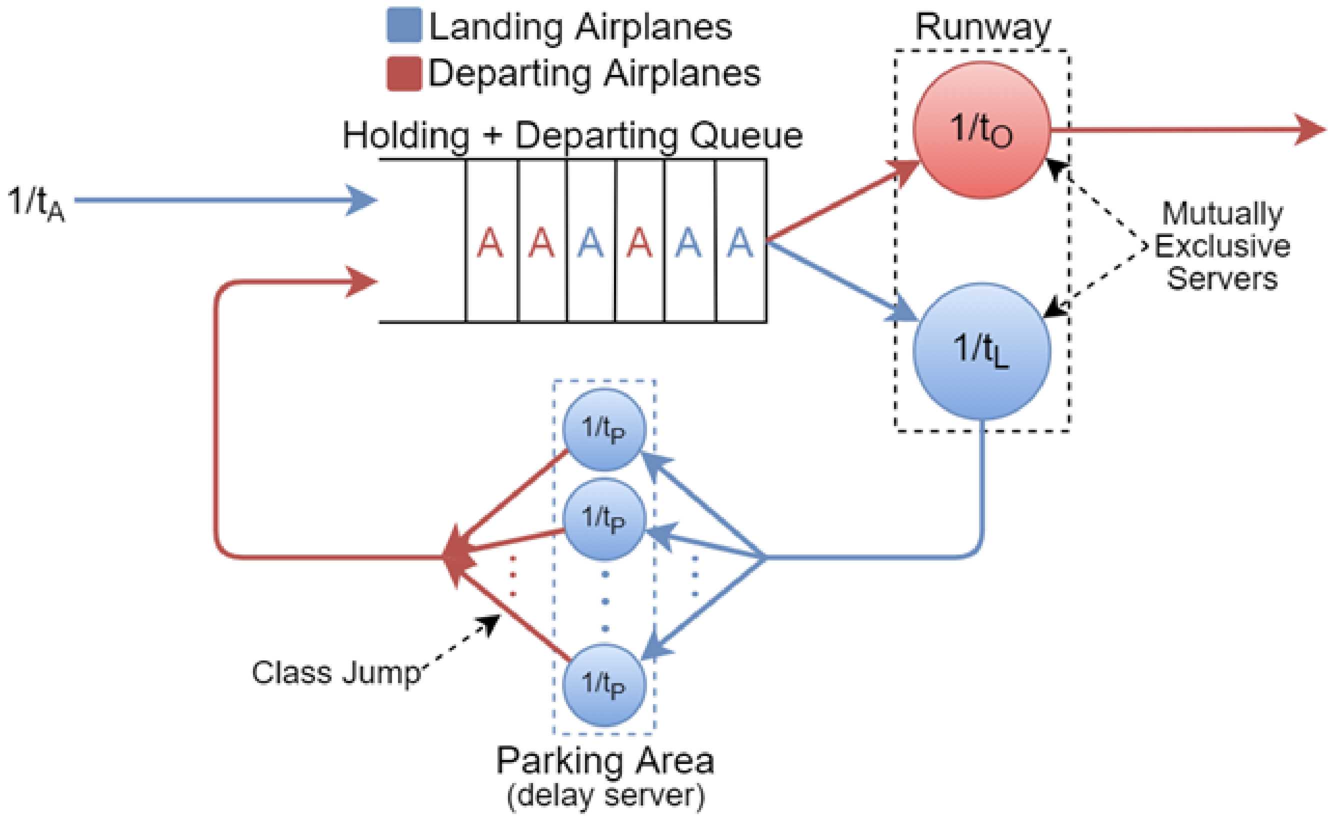
1) Analyse in both deterministic and exponential regimes.
2) Airplanes waiting for landing have an infinite fuel supply.
3) The parking area has an infinite airplane capacity.

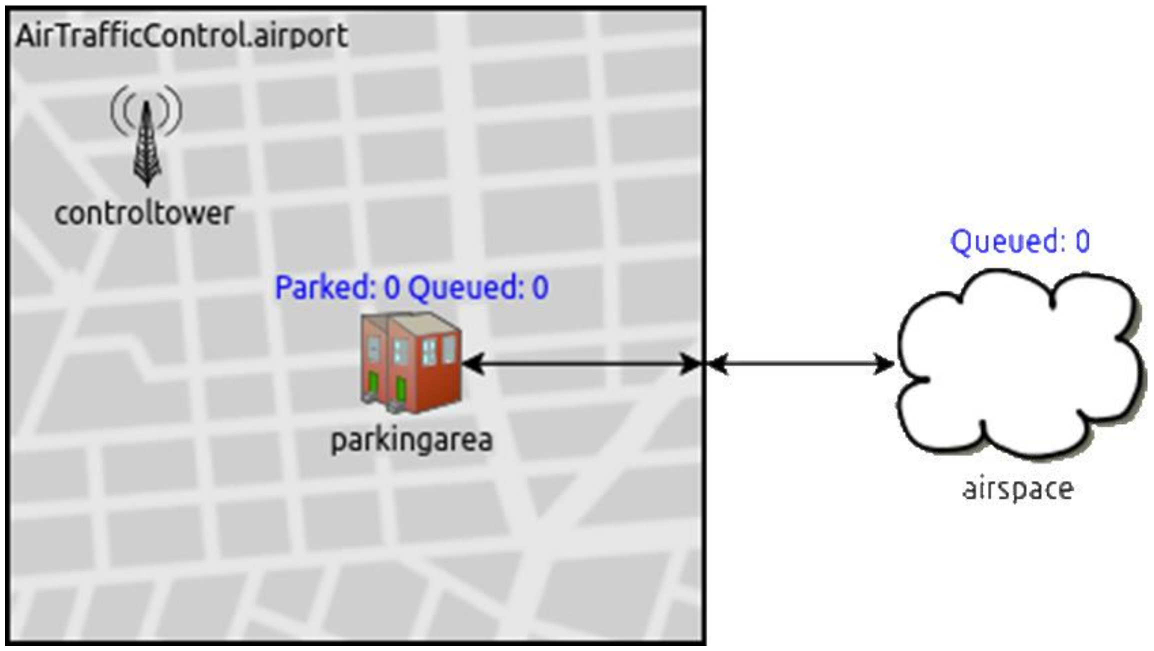
Additional Hypotheses

4) System analysis starts from its empty state.
5) t_P starts when leaving the runway, ends when enqueuing for take-off.
6) In case of conflicts, the airplane waiting for landing has the priority.
7) No real-world delays or other factors are taken into account.

QT Equivalent Model

Simulation Model





- Queues merged in a single logical queue
- Control Tower has no correspondence
- Steady-state equations can’t be derived

- Three simple modules
- The runway is a connection
- Control tower is a logical module

System Statistics

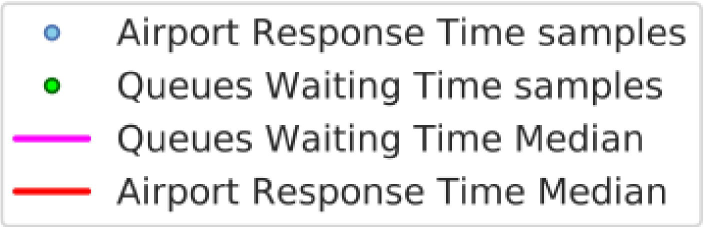
- The number of airplanes waiting in both queues (*Holding Queue Size* and *Depart Queue Size*)
- The airplanes’ waiting times in both queues (*Holding Queue Waiting Time* and *Depart Queue Waiting Time*)
- The number of parked airplanes (*Parked Planes*)
- The system total response time (*Airport Response Time*)

System Stability Analysis

Stability Condition: $$t_A > t_L + t_O \Rightarrow \lambda_A < \frac{\mu_L \mu_O}{\mu_L + \mu_O}$$

Equivalent Service Rate: $$\mu_E = \frac{\mu_L \mu_O}{\mu_L + \mu_O} < \mu_L, \mu_O \quad \mu_L, \mu_O > 0$$

System Utilization Factor: $$\rho = \frac{\lambda_A}{\mu_E} = \frac{\lambda_A(\mu_L + \mu_O)}{\mu_L \mu_O} = \frac{t_L + t_O}{t_A}$$



Deterministic

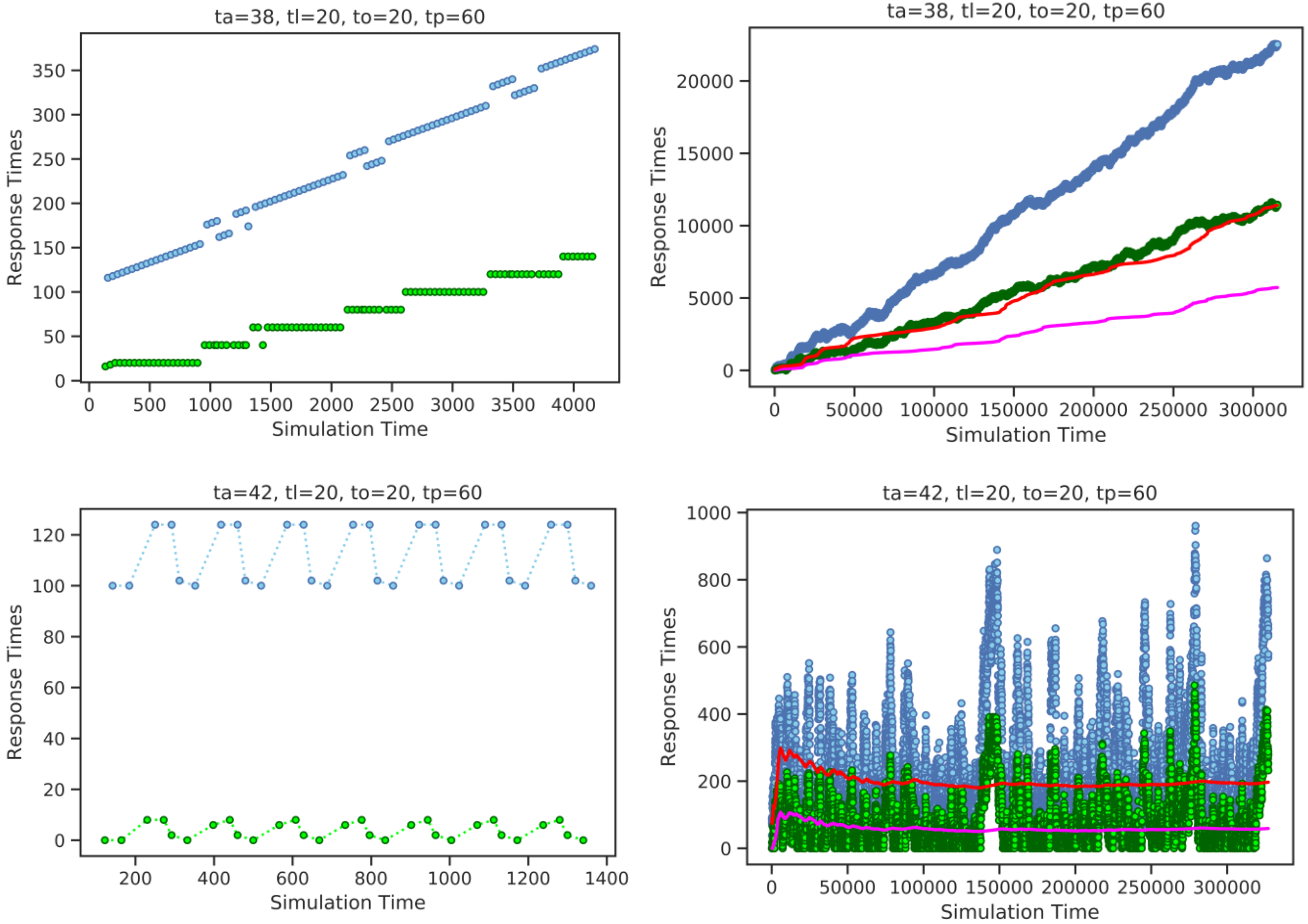
Exponential

System Instability

$(t_A < t_L + t_O)$

System Stability

$(t_A > t_L + t_O)$

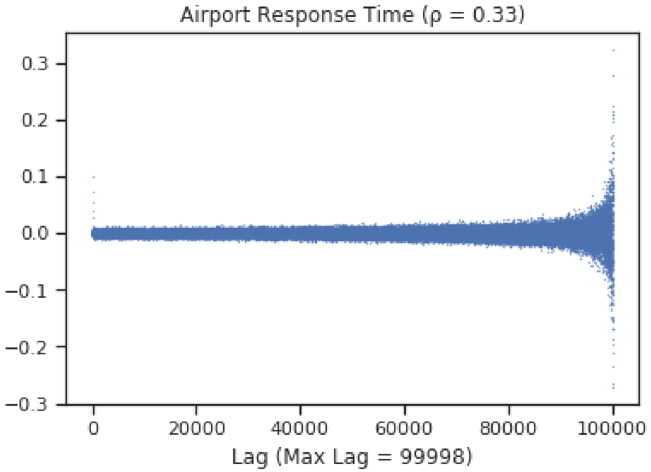


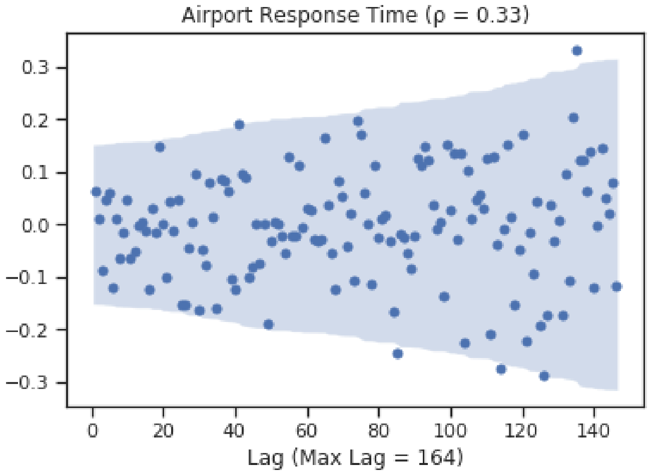
Subsampling and Confidence Levels

Statistics autocorrelation dependent on the utilization factor ρ

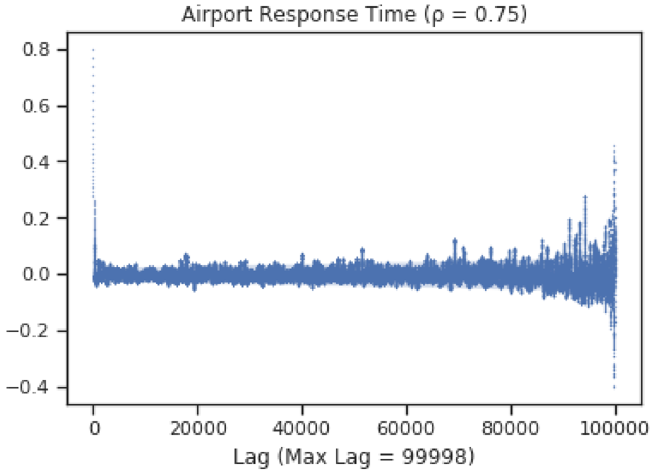
Before Subsampling / After Subsampling

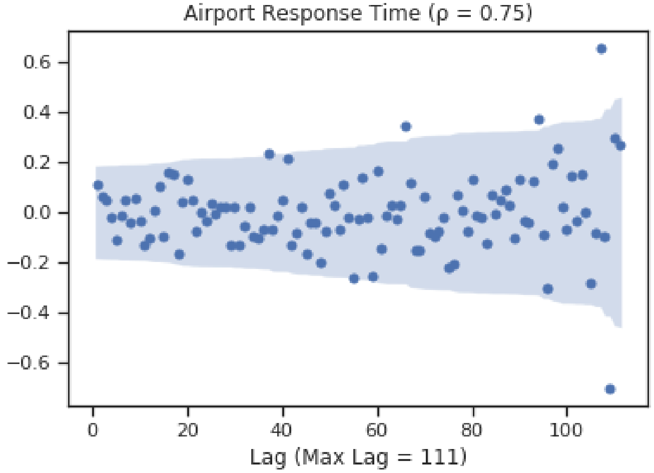
Low Utilization
($\rho = 0.33$)





High Utilization
($\rho = 0.75$)





Confidence intervals taken with a confidence level of 95% ($\alpha = 0.05$)

Warm-up Time Study

System warm-up time corresponds to the time required by the mean throughput of airplanes on the feedback loop (*Parking Area*) to stabilize.

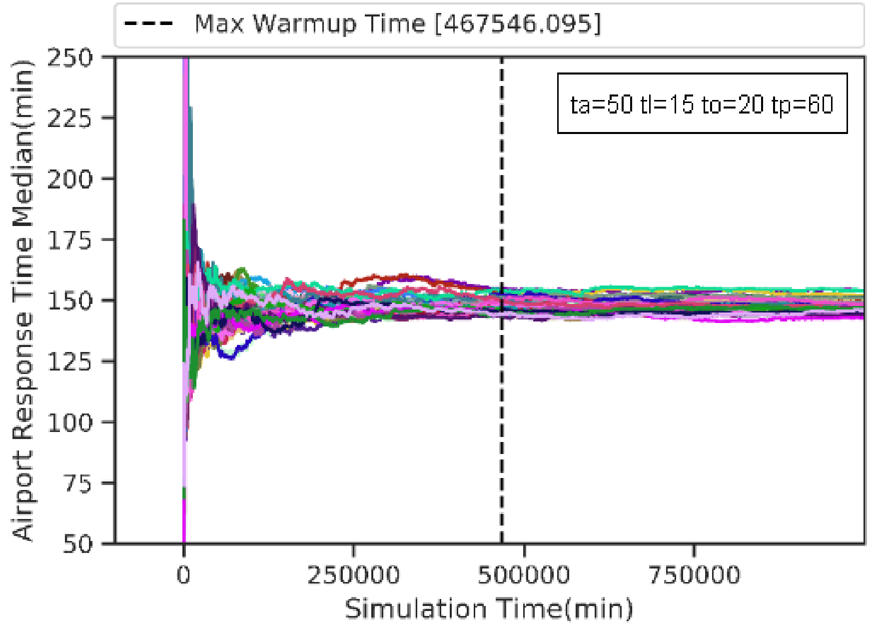
Deterministic Warm-up Time:

$$t_{WA} = t_A + t_L + t_P$$

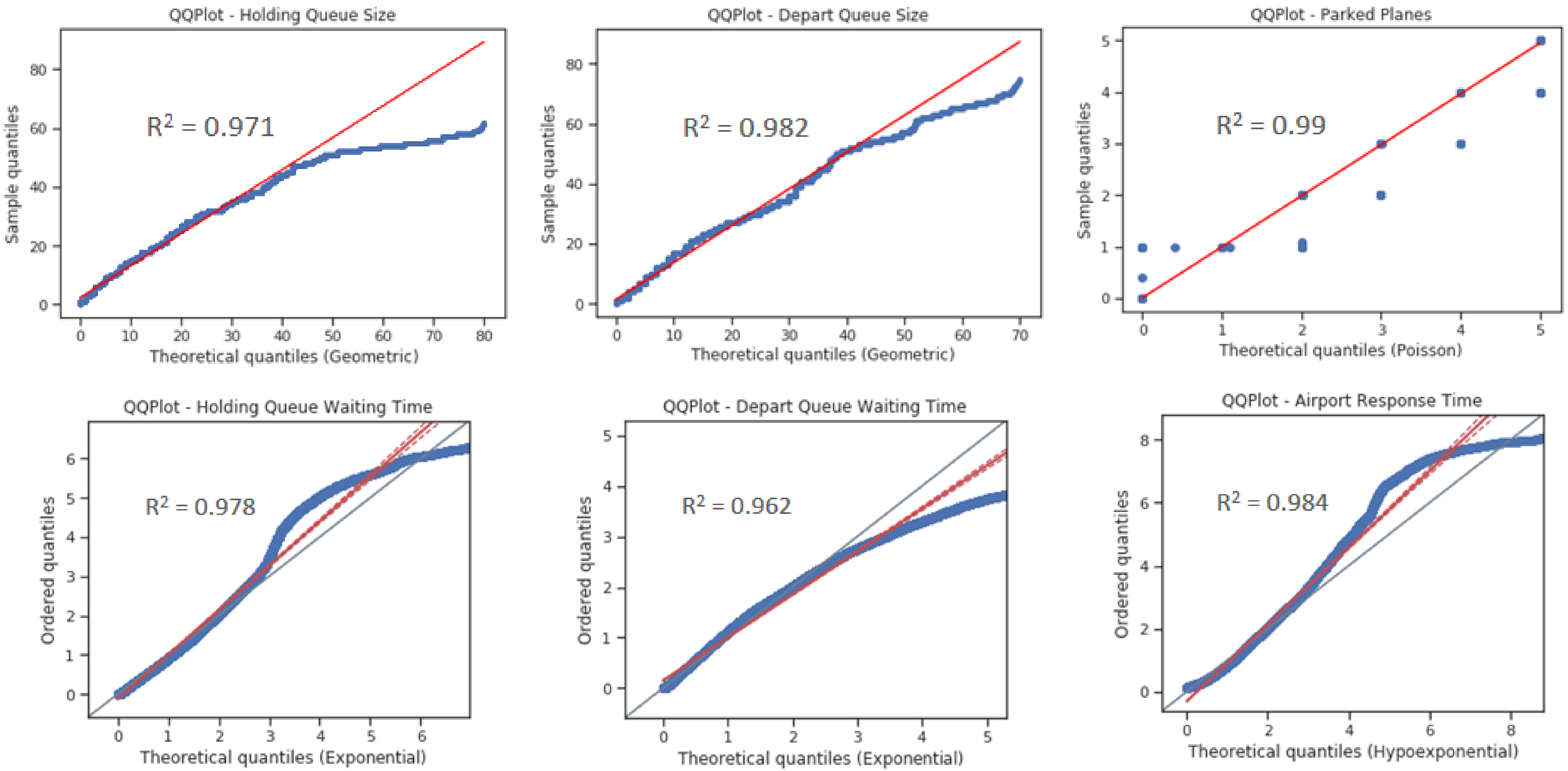
Exponential Warm-up Time:
(attempt)

$$t_{WA} = k(t_A + t_L + t_P + t_O)$$

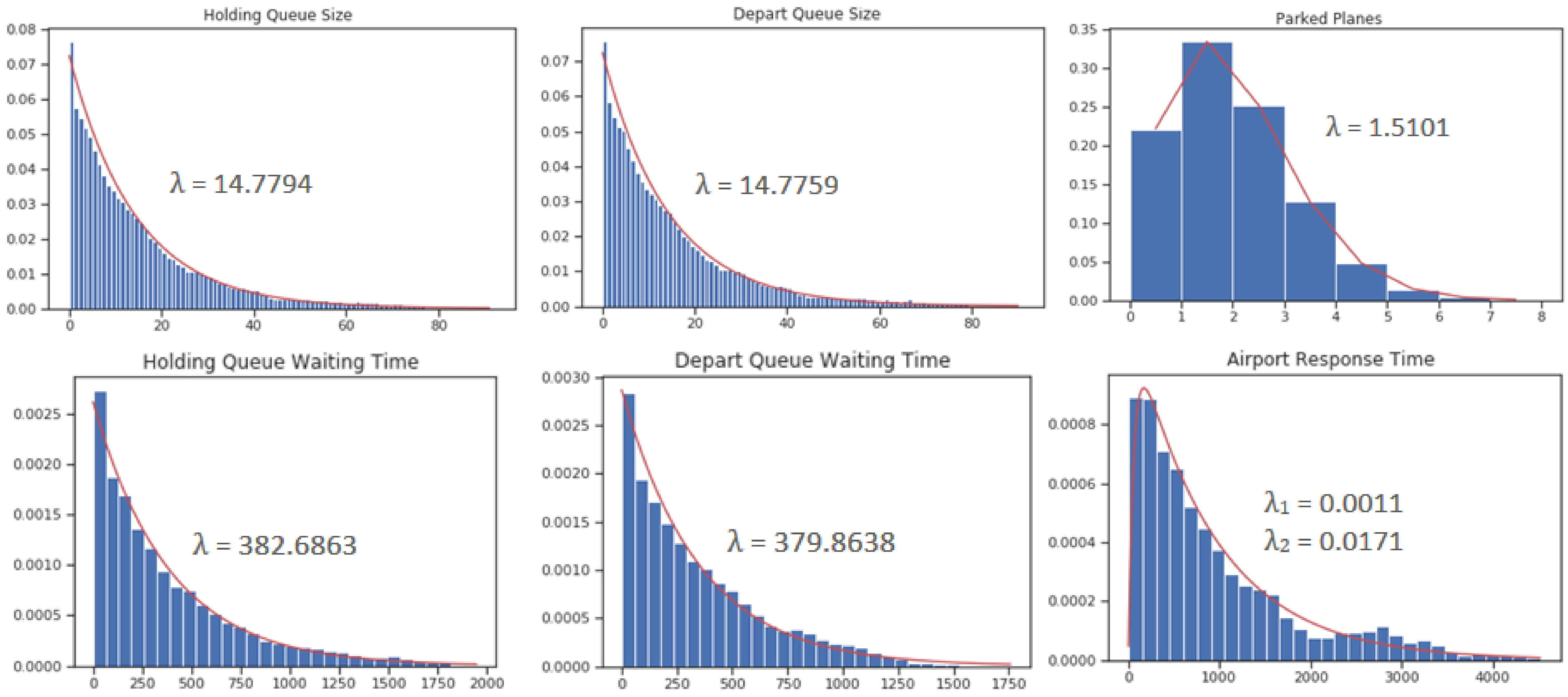
Exponential Warm-up Time:
(rigorous method)



Statistics Distributions Fitting



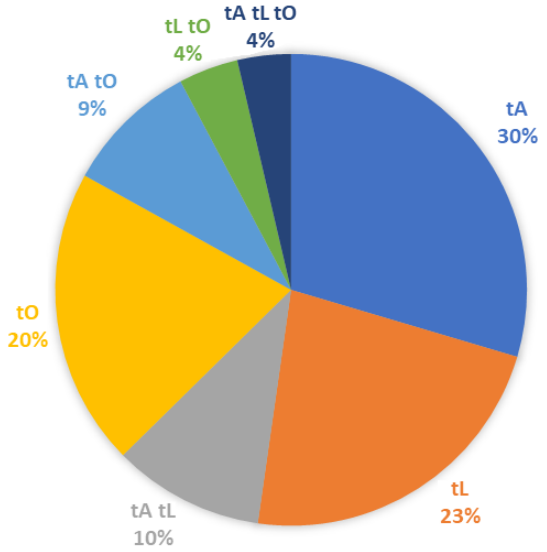
Maximum Likelihood Estimators



Both queues share the same distributions with the same parameters

$2^k r$ Factorial Analysis

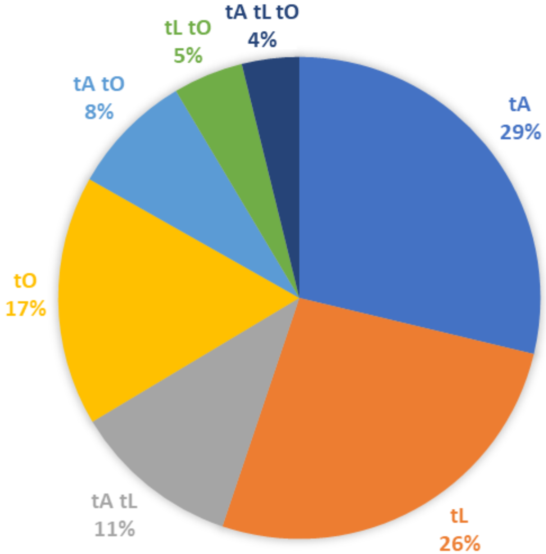
Holding Queue Size



SST = $4.59*10^3$

SSE = 0.901 (0.02%)

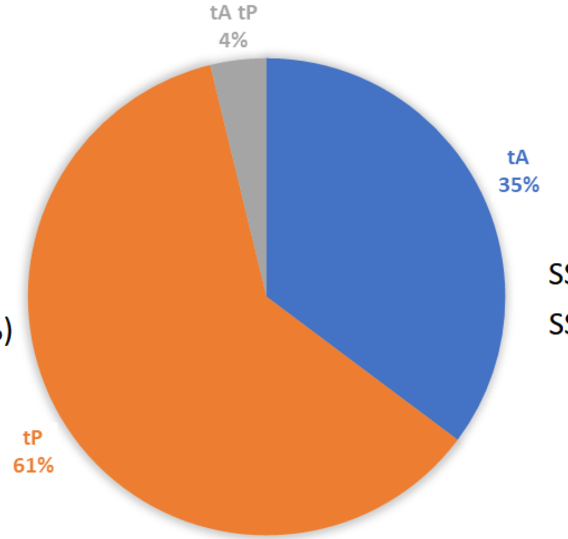
Depart Queue Size



SST = $4.63*10^3$

SSE = 0.905 (0.02%)

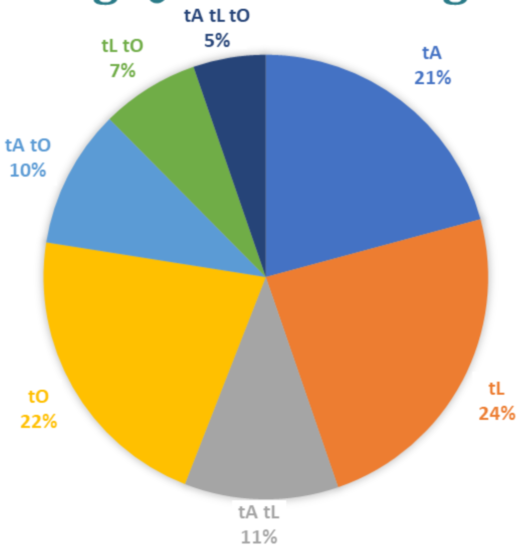
Parked Planes



SST = $2.1*10^3$

SSE = 0.0373 (0.00%)

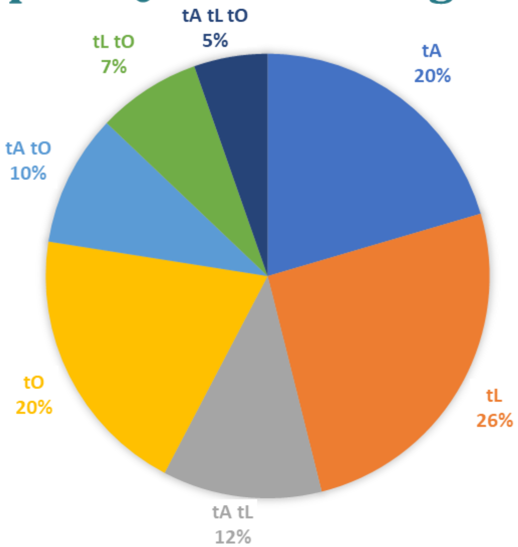
Holding Queue Waiting Time



SST = $1.12*10^7$

SSE = $2.25*10^3$ (0.02%)

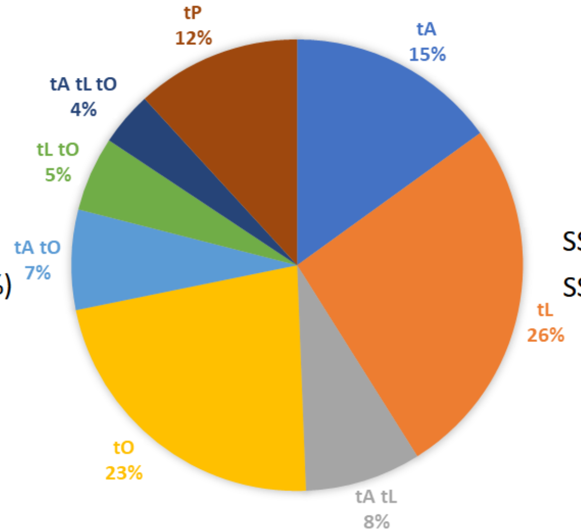
Depart Queue Waiting Time



SST = $1.12*10^7$

SSE = $2.25*10^3$ (0.02%)

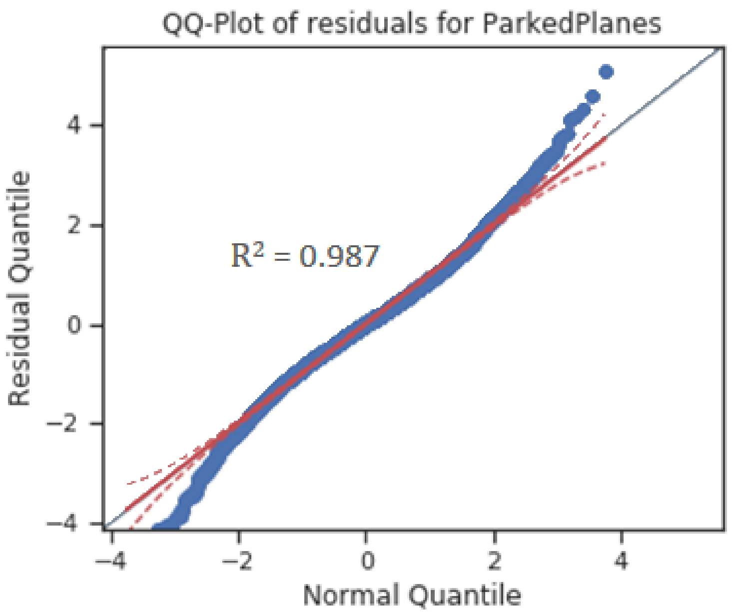
Airport Response Time

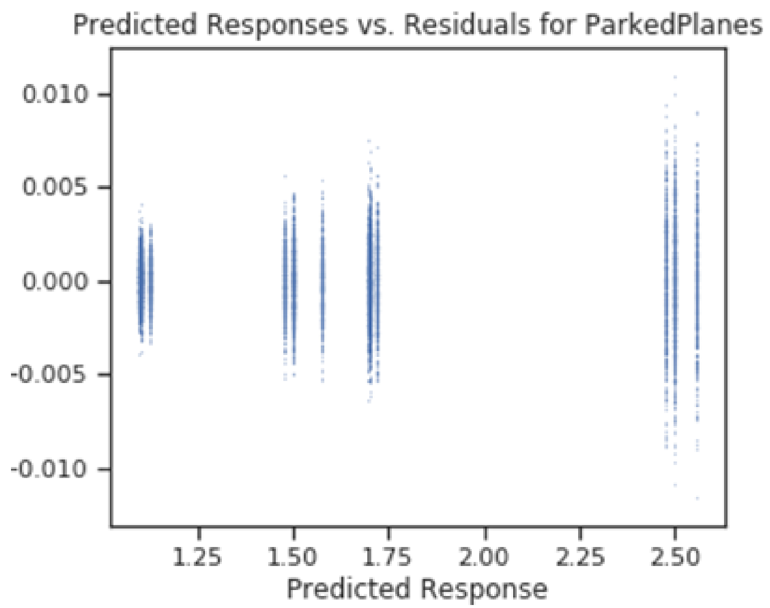


SST = $6.5*10^7$

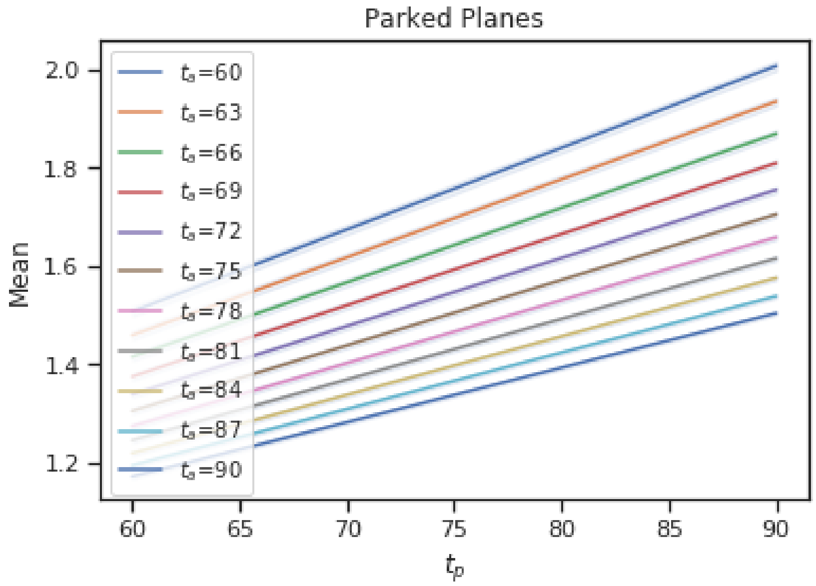
SSE = $9.2*10^3$ (0.01%)

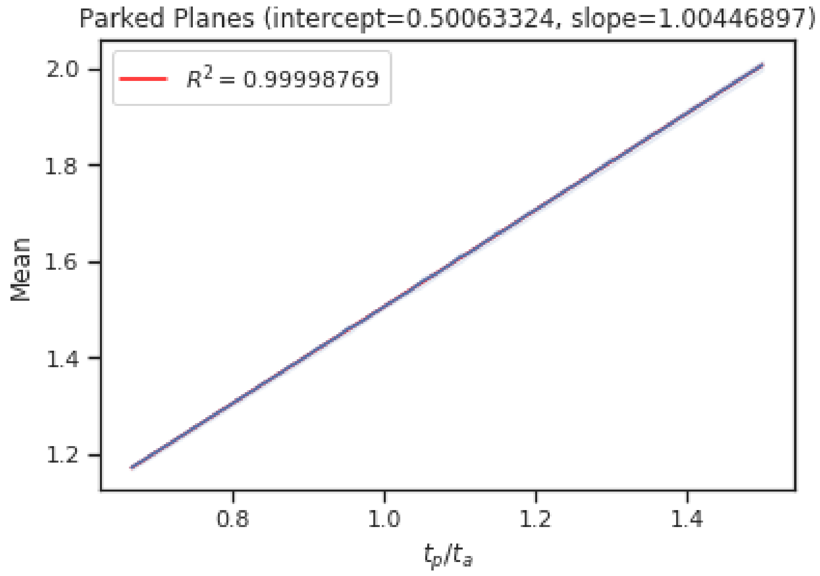
CI on the results can be computed for the *Parked Planes* only





Parked Planes Analysis:





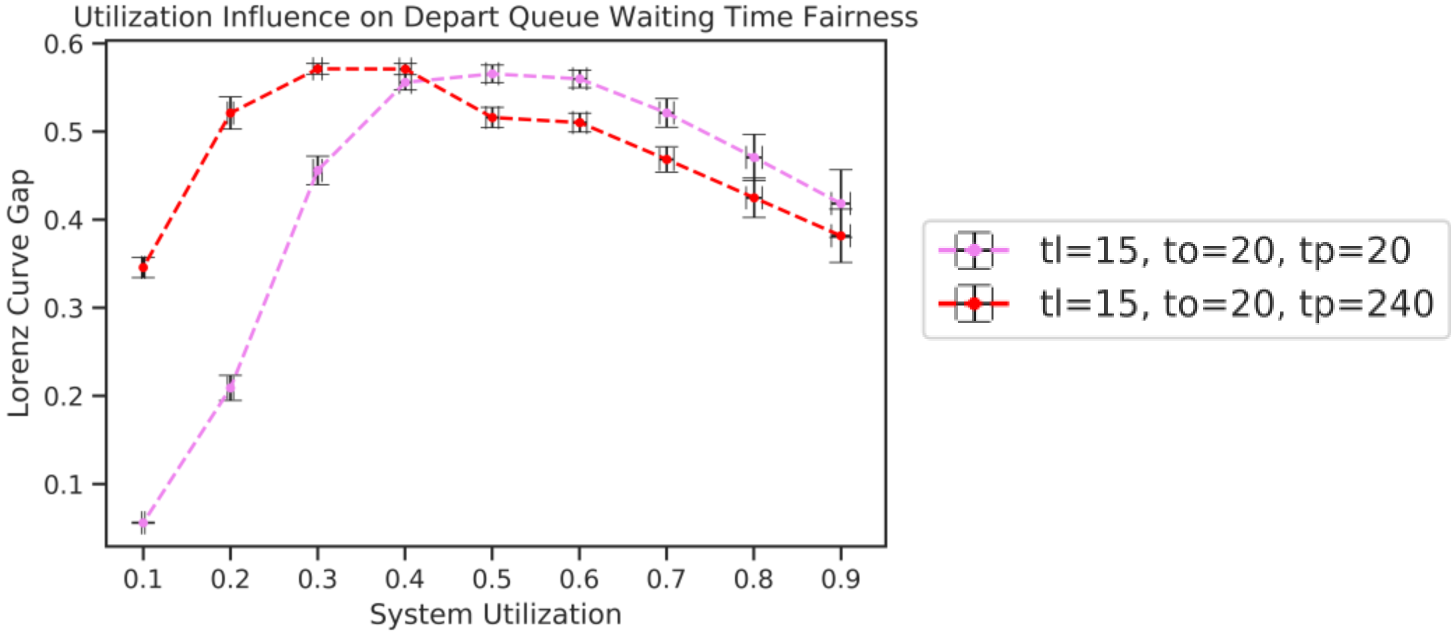
$$\mathrm{E[PP]} = \frac{t_P}{t_A} + 0.5$$

System Fairness Analysis

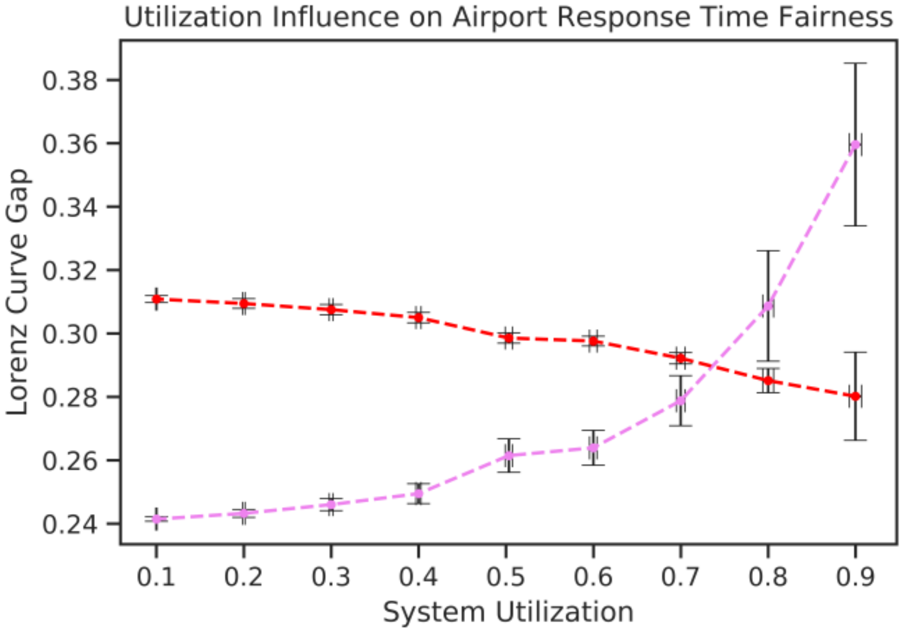
Fairness mainly influenced by three factors:

1) System Utilization Factor ρ.
2) Parking Time t_P, which decouples the two queues.
3) Number, weights and *likeliness* of the influencing parameters (CLT).

Queue Waiting Time Fairness



Airport Response Time Fairness



The two fairnesses are not directly related

Concluding Remarks

The analysed system is *unfair by nature*, with trends alternating in time spikes of high and dips of low values, a behaviour that can be mainly attributed to the mutually exclusive utilization of a single shared resource (the *runway*) by the two queues.

Performance-increasing Proposals

1) Introduce additional runways, which allows for a linear scaling of the system's equivalent service rate:

$$\mu_E = N \frac{\mu_L \mu_O}{\mu_L + \mu_O} \quad \Rightarrow \quad \lambda_A < N \frac{\mu_L \mu_O}{\mu_L + \mu_O}$$

N = Number of Runways

2) In a real-world scenario, all necessary measures should be taken in order to limit as much as possible the *randomness* of the system (e.g. by scheduling flights and enforcing strict time protocols within the airport).