University of Pisa

Computer Engineering Master's degree

Control Tower

Performance Evaluation of Computer Systems and Networks

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Contents

Introduction

A control tower manages the air traffic of an airport.
The airport has one runway for landing and taking off and one parking area for airplanes.
The runway can be occupied by one airplane at a time, whereas the parking area can contain one or more of them simultaneously. The parking area has infinite capacity.

Whenever an airplane reaches the airport area, it goes through the following steps:

1. The airplane queues for landing until it receives the ok from the control tower
2. The airplane performs the landing operation which takes a time t_l
3. The airplane remains in the parking area for a time t_p
4. The airplane queues for take off until it receives the ok from the control tower
5. The airplane takes off in a time t_o and leaves the system.

When the runway is unoccupied, the control tower serves one airplane according to the following policy:

1. It serves an airplane queued for take off
2. If the take-off queue is empty, it serves an airplane in the landing queue.

The aim of this project is to model the system described above and study the waiting time in both landing and take-off queues with a varying workload.
It has the purpose of studying how the number of airplanes in the parking area varies in relation with t_l and t_p.

In order to model the system the following assumptions have been made:

- Airplane's interarrival times are IID RVs
- t_l, t_o and t_p are IID RVs

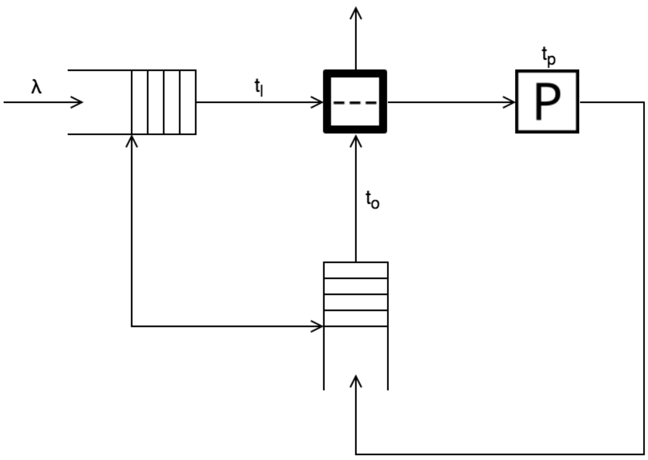
The analysis was focused on the following scenarios:

- Constant interarrival times and constant service time
- Exponential distribution of all the above RVs with the same means as the previous case

1 Simulation Analysis

1.1 Model

The system can be modeled as a queueing network as shown in the following figure.



The model consists of four modules: Sky, Runway, Parking and Takeoff. Sky represents the set of airplanes that has just entered the system and have to perform the landing.
Runway does not have any particular purpose but it makes the system more clear so we decided to insert a module for it. Airplanes coming from Sky proceed toward Parking, while those that come from Takeoff exit the system.
Parking module represents the set of airplanes that is waiting in the parking area in order to queue for the take-off.
Finally, Takeoff module represents the set of airplanes that has to take off.

The function of the control tower is modeled with the interaction between Sky and Takeoff modules. Takeoff asks Sky for the runway as soon as the first airplane arrives from Parking and releases it when no more airplanes have to take off.

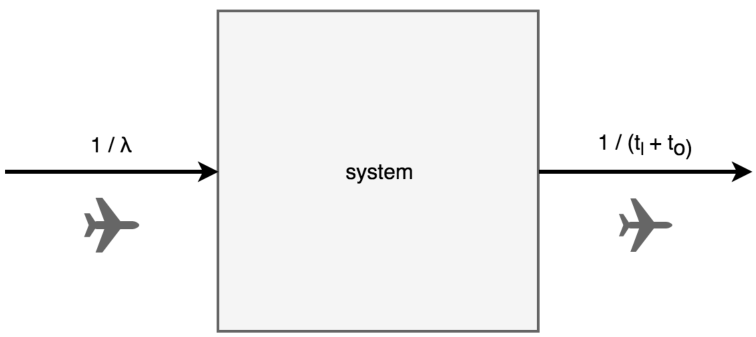
1.1.1 Assumptions

- There is no propagation delay for the runway request
- The airplanes always have enough fuel to remain in the air for an arbitrary time
- The parking area has infinite capacity
- The landing and take-off queues have infinite capacity

1.2 General System's Behavior and Stability

The system's behavior is heavily influenced by the workload and the service times of each module.
Every time that an airplane arrives in Sky's queue, it lands, then waits in the parking area and finally queues in Takeoff and leaves the system. Depending on the workload λ and on system's parameters t_l, t_p and t_o, landing and take-off operations may interfere and cause delays.

Before analyzing how the system is influenced by these parameters, a general stability condition has been set.
Let's consider the system as a single entity with interarrival time λ and service time t_l+t_o.



This assumption is possible since the runway cannot be used at the same time for both landings and take-offs. Furthermore, the Parking's service time t_p can be ignored since it does not influence stability.

Under these assumptions, the global utilization of the system can be written as

$$\frac{\frac{1}{\lambda}}{\frac{1}{t_{\mathrm{l}}+t_{\mathrm{o}}}}$$

If the utilization is greater than 1, then the system will not be able to reach the steady state. So the equation for the stability condition is

$$t_{\mathrm{l}} + t_{\mathrm{o}} < \lambda$$

The formula has been validated in all operating conditions with both constant and exponential parameters.

2 Simulator's verification

2.1 Instability condition

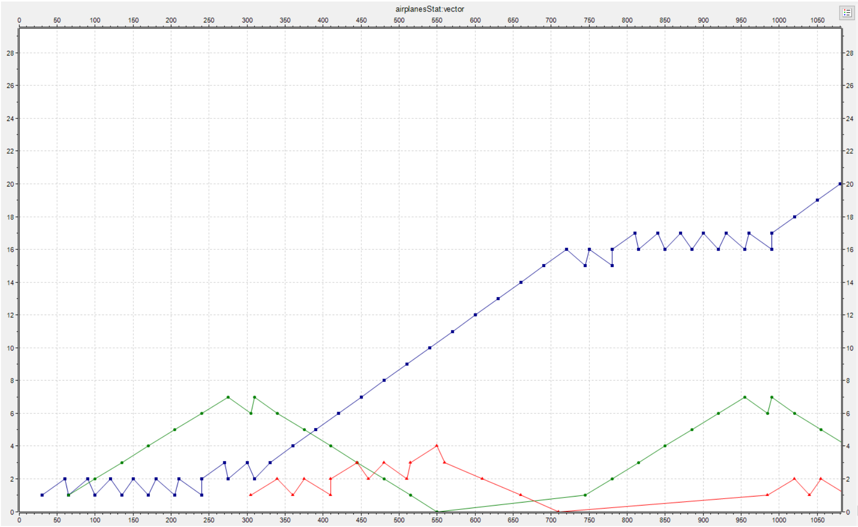


Figure 1: Instability condition: λ=30s, t_l=35s, t_p=240s, t_o=50s. Sky=blue, Takeoff=red, Parking=green.

In the simulation analysis we discarded the cases that do not respect $t_l + t_o < \lambda$ since they lead to unstable situations for the Sky's queue.

The above figure shows a case of instability condition. The blue line is the number of Airplanes in Sky's queue, which increases over and over.
Under the assumptions made, Sky is the only module that can diverge. In contrast, Takeoff and Parking's queues will always have a finite number of airplanes.
Takeoff periodically empties. It has the priority on the runway and during the take-offs landings are stopped, thus blocking the arrival of new airplanes in the module.
Parking is a delay on airplanes and being the input rate limited by $\frac{1}{t_l}$ it will never diverge.

2.2 Stability condition

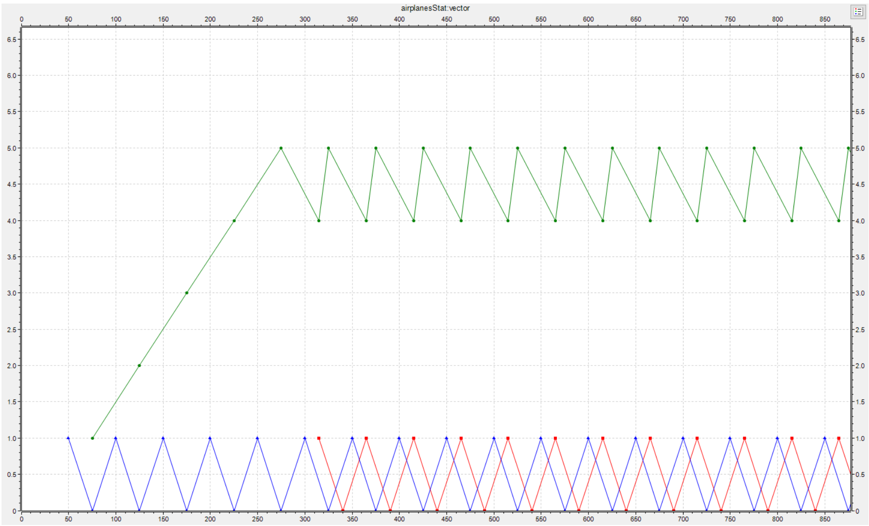


Figure 2: Stability condition: λ=50s, t_l=25s, t_p=240s, t_o=15s. Sky=blue, Takeoff=red, Parking=green.

In the stability condition, during the warm up time some airplanes land and take place in the parking. Spent the Parking's service time t_p, the first take-off is performed and, from this moment on, a landing and a take-off alternate maintaining more or less constant the airplanes' number in the parking area.

3 Examples

3.1 Example 1

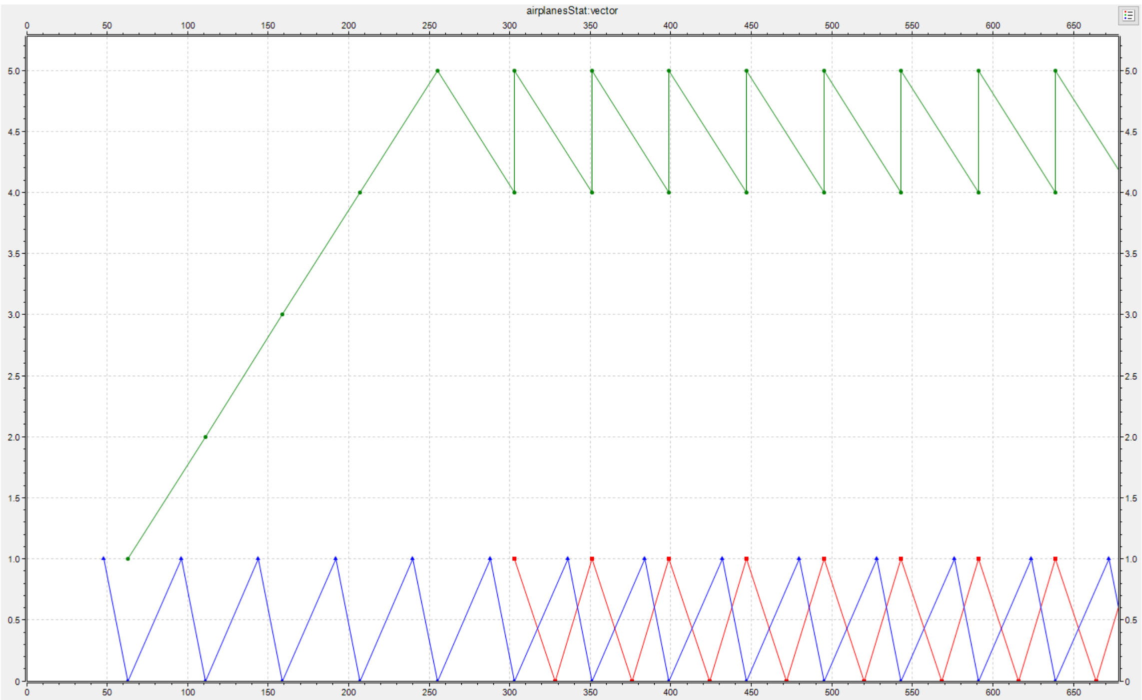


Figure 3: Airplanes' number without interference: λ=48s, t_l=15s, t_p=240s, t_o=25s. Sky=blue, Takeoff=red, Parking=green.

The above example is very similar to the one about the stability condition seen in the previous chapter. Following, the time table and the event table show a sample of the functioning.

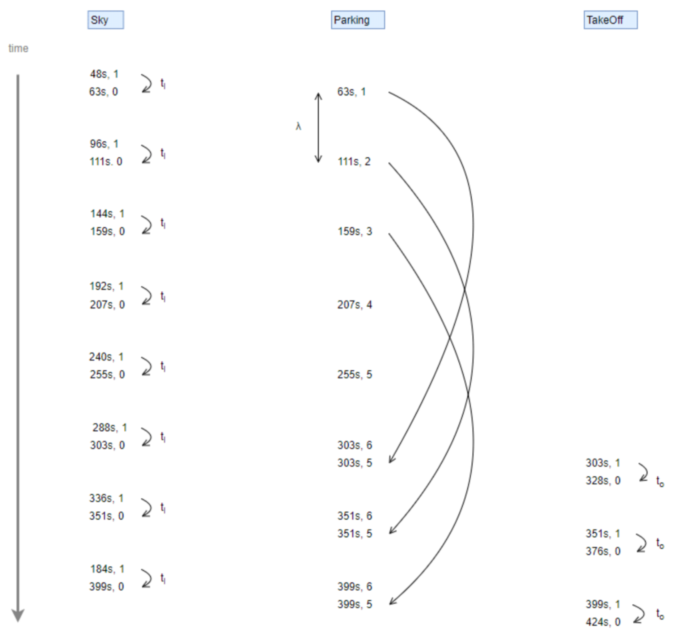


Figure 4: Time table

Item#	Event#	Time	Value
0	1	48	1.0
1	2	63	0.0
2	5	96	1.0
3	6	111	0.0
4	9	144	1.0
5	10	159	0.0
6	13	192	1.0
7	14	207	0.0
8	17	240	1.0
9	18	255	0.0
10	21	288	1.0
11	23	303	0.0
12	32	336	1.0
13	34	351	0.0
14	43	384	1.0
15	45	399	0.0

Item#	Event#	Time	Value
0	4	63	1.0
1	8	111	2.0
2	12	159	3.0
3	16	207	4.0
4	20	255	5.0
5	22	303	4.0
6	27	303	5.0
7	33	351	4.0
8	38	351	5.0
9	44	399	4.0
10	49	399	5.0

Item#	Event#	Time	Value
0	24	303	1.0
1	29	328	0.0
2	35	351	1.0
3	40	376	0.0
4	46	399	1.0
5	51	424	0.0
6	57	447	1.0
7	62	472	0.0
8	68	495	1.0

Figure 5: Sky, Parking, Takeoff's event table

Moving closer to the stability condition $t_o + t_l < \lambda$, landings and take-offs start to interfere with each other. The interference causes delays in Sky and Takeoff's queues and also makes the number of airplanes in Parking fluctuate. Following, an example with a higher λ to show how delays start to raise.

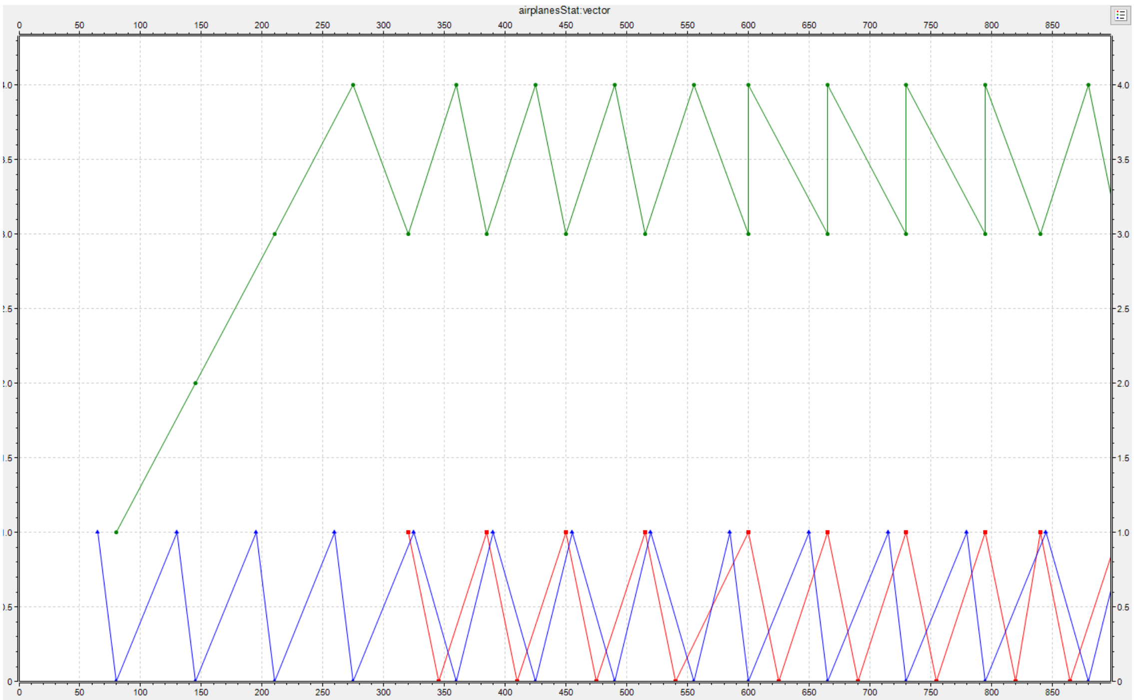


Figure 6: Airplanes' number: λ=65s, t_l=15s, t_p=240s, t_o=25s. Sky=blue, Takeoff=red, Parking=green.

In this example some delays happen. These are due to the overlaps of landings and take-offs.
Between an airplane's take-off and the landing of the next one there is a margin of time. This margin decreases as the Sky's and Takeoff's service times increase due to the approaching of the instability condition.

The margin can be quantified as

$$r = mod(t_p, \lambda)$$

Introducing this parameter, which will be called "rest" in the whole text, the absence of delays can be expressed by $t_l + t_o + r < \lambda$.

The rest is the time difference between the exit and the entry of an airplane in the parking area.

3.2 Example 2

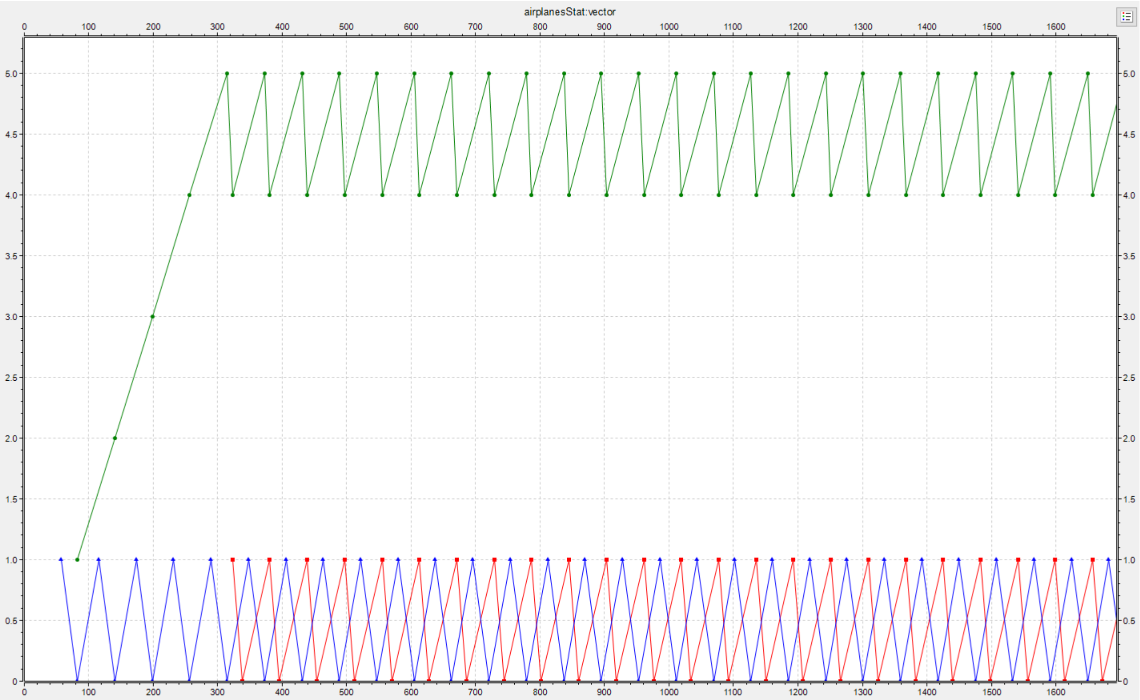


Figure 7: Airplanes' number with no interference: λ=58s, t_l=25s, t_p=240s, t_o=15s. Sky=blue, Takeoff=red, Parking=green.

Also this case represents a stability condition since it respects $t_l + t_o + r < \lambda$

$$r = mod(240, 58) = 8s$$

$$25s + 15s + 8s < 58s$$

As shown in the figure, initially some airplanes enter the system and accumulate in Parking. Once reached the steady state, landings and take-offs alternate keeping the airplanes in the parking area around the same values.

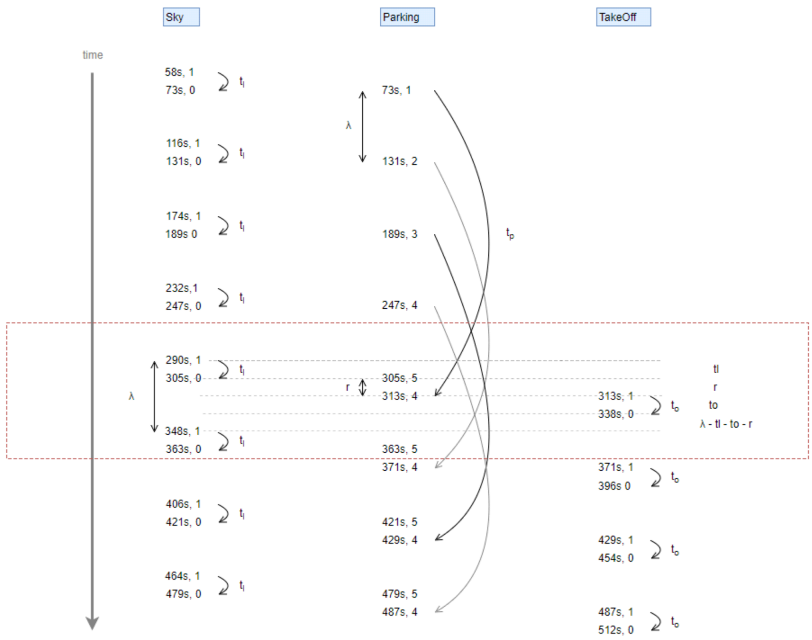


Figure 8: Time table

Item#	Event#	Time	Value
0	1	58	1.0
1	2	73	0.0
2	5	116	1.0
3	6	131	0.0
4	9	174	1.0
5	10	189	0.0
6	13	232	1.0
7	14	247	0.0
8	17	290	1.0
9	18	305	0.0
10	28	348	1.0
11	29	363	0.0
12	39	406	1.0
13	40	421	0.0
14	50	464	1.0
15	51	479	0.0

Item#	Event#	Time	Value
0	4	73	1.0
1	8	131	2.0
2	12	189	3.0
3	16	247	4.0
4	20	305	5.0
5	21	313	4.0
6	31	363	5.0
7	32	371	4.0
8	42	421	5.0
9	43	429	4.0
10	53	479	5.0

Item#	Event#	Time	Value
0	22	313	1.0
1	25	338	0.0
2	33	371	1.0
3	36	396	0.0
4	44	429	1.0
5	47	454	0.0
6	55	487	1.0
7	58	512	0.0

Figure 9: Sky, Parking, Takeoff's event table

3.3 Example 3: light interference

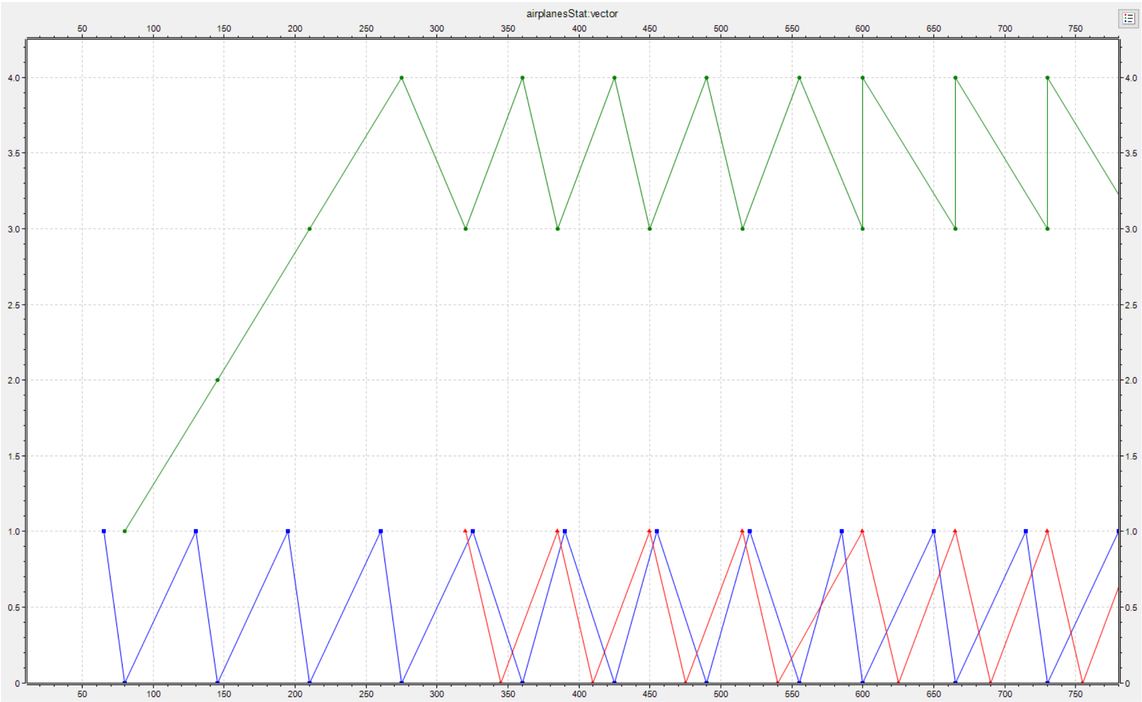


Figure 10: Airplanes' number with interference: λ=65s, t_l=15s, t_p=240s, t_o=25s. Sky=blue, Takeoff=red, Parking=green.

The Workload and service times in this example do not respect the stability condition.

$$r = mod(240, 65) = 55s$$

$$15s + 25s + 55s \nless 65s$$

Figure 10 shows how the number of Airplanes behaves for each module in presence of interference.

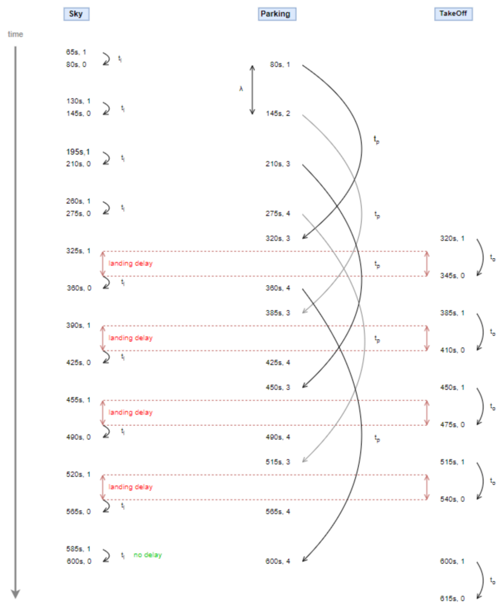


Figure 11: Time table

Item#	Event#	Time	Value
0	1	65	1.0
1	2	80	0.0
2	5	130	1.0
3	6	145	0.0
4	9	195	1.0
5	10	210	0.0
6	13	260	1.0
7	14	275	0.0
8	21	325	1.0
9	25	360	0.0
10	32	390	1.0
11	36	425	0.0
12	43	455	1.0
13	47	490	0.0
14	54	520	1.0
15	58	555	0.0
16	61	585	1.0
17	63	600	0.0

Item#	Event#	Time	Value
0	4	80	1.0
1	8	145	2.0
2	12	210	3.0
3	16	275	4.0
4	17	320	3.0
5	27	360	4.0
6	28	385	3.0
7	38	425	4.0
8	39	450	3.0
9	49	490	4.0
10	50	515	3.0
11	60	555	4.0
12	62	600	3.0
13	67	600	4.0

Item#	Event#	Time	Value
0	18	320	1.0
1	22	345	0.0
2	29	385	1.0
3	33	410	0.0
4	40	450	1.0
5	44	475	0.0
6	51	515	1.0
7	55	540	0.0
8	64	600	1.0
9	69	625	0.0

Figure 12: Sky, Parking, Takeoff's event table

Regular landings and delayed landings alternate. This happens because the delays in landings change the time distance between the airplanes in Parking and therefore the times in which they will be ready to take off.

3.4 Example 4: heavy interference

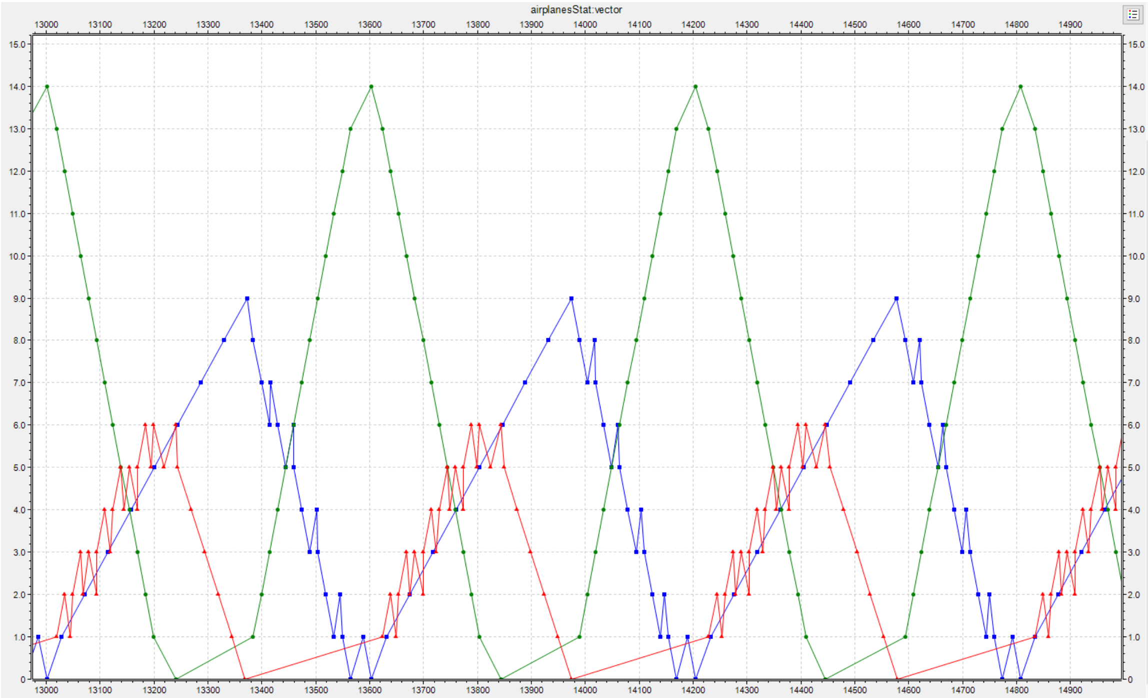


Figure 13: Airplanes with heavy interference: λ=43s, t_l=15s, t_p=240s, t_o=25s. Sky=blue, Parking=green, Takeoff=red.

In this case $t_l + t_o + r < \lambda$ is not respected, so we are in a situation with interference.

$$r = mod(240, 43) = 25s$$

$$15s + 25s + 25s \not< 43s$$

Differently from the previous case, here t_o is greater than t_l. Hence, when t_o+t_l+r $>$ λ and $t_o > t_l$ there could be heavy interference.

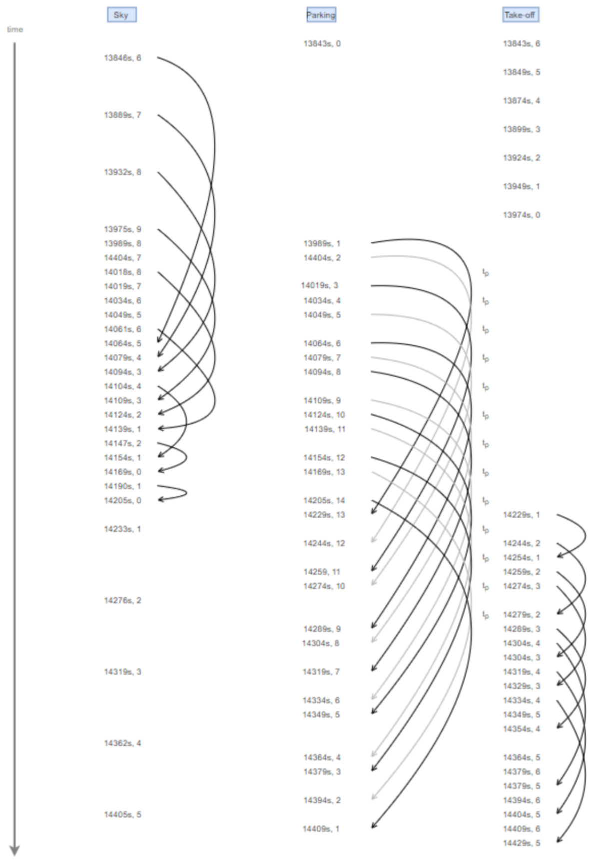


Figure 14: Time table

Long queues form and periodically empty in all the three main modules. Specifically, the difference between t_l and t_o assures that some airplanes initially accumulate in the take-off queue. These airplanes interrupt landings for a long period of time: they take off in sequence causing an accumulation of airplanes in Sky which will land afterward. This leads to have a subsequent even longer queue in Takeoff.

Hence, there is a burst operation of landings and take-offs rather than an alternation of them as we observed in the previous examples without interference.

Three conditions of stable functioning have been identified:

- without delays: $t_l+t_o+r < \lambda$
- with light delays: $t_l+t_o < \lambda < t_l+t_o+r$
- with probable significant delays: $t_l+t_o < \lambda < t_o+t_l+r$ and $t_o > t_l$

Under these conditions we found the maximum size of the queues:

1. No interference or light interference:
   - Sky: 1
   - Parking: $\lceil \frac{t_p}{\lambda} \rceil$
   - Takeoff: 1
2. Heavy interference:
   - Sky: $\lceil \frac{t_p}{t_l} \cdot \frac{t_o}{\lambda} \rceil$, that is the number of airplanes which arrives in Sky during the time needed to let take off the maximum possible number of airplanes in Parking.
   - Parking: $\lceil \frac{t_p}{t_l} \rceil$, that is the maximum number of airplanes that accumulate from Sky during Parking's waiting time.
   - Takeoff: $\lceil \frac{t_p}{t_l} \cdot t_o \cdot (\frac{1}{t_l} - \frac{1}{t_o}) \rceil$, that is the maximum number of airplanes that can accumulate in Takeoff considering arrivals from Parking with a rate $\frac{1}{t_l}$

With non constant time the system has delays with a frequency that depends on the parameters.
Showing examples is not significant in this case because the interference's levels that have been identified in constant time are no longer distinguishable.
However, the stability condition is still $t_l + t_o < \lambda$.
In general, the average length of the queues and the waiting times tend to stabilize on larger values when the sum of the service times approaches λ. Exceeding λ it does not stabilize.

3.5 Exponential Scenarios

Hereinafter, three scenarios with increasing workloads and t_l=20s, t_p=240s and t_o=20s.
The Sky, Parking and Takeoff's queues are respectively in blue, green and red colour.

3.5.1 λ=90s

With this workload the system reaches the stability and the queues of Sky and Takeoff tend to empty. Parking module contains a low number of airplanes.

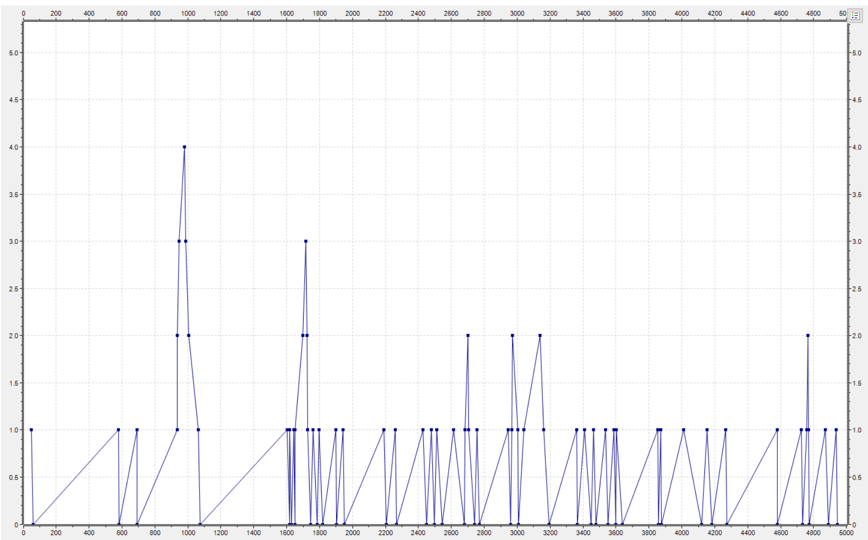


Figure 15: Airplanes in Sky

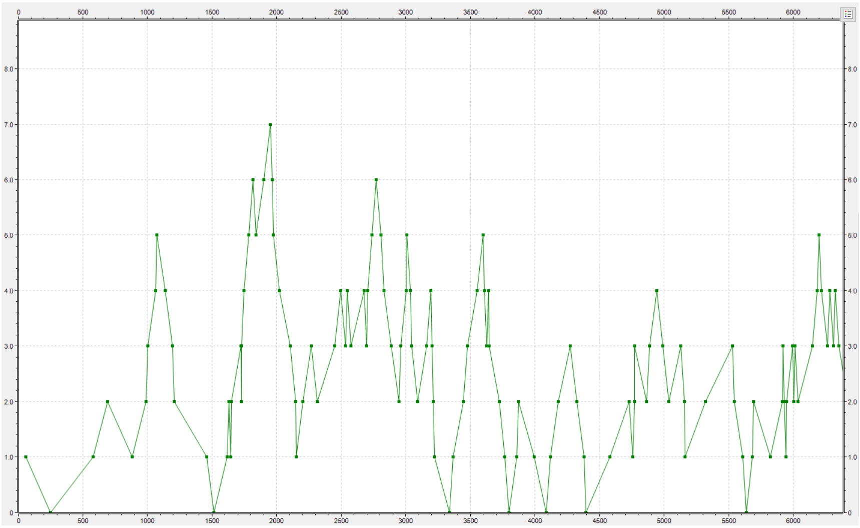


Figure 16: Airplanes in Parking

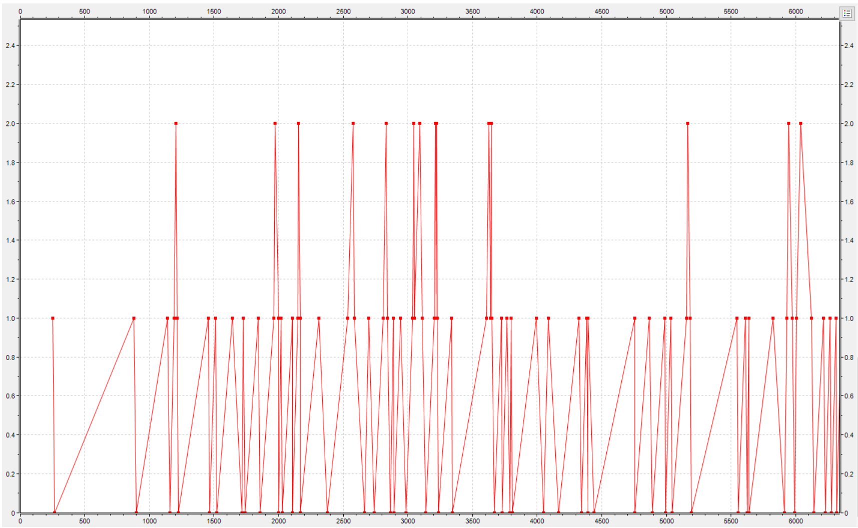


Figure 17: Airplanes in Takeoff

3.5.2 λ=60s

By increasing the workload the queues of Sky and Takeoff empty and have higher peaks than in the previous case. The Parking module contains on average a higher number of airplanes.

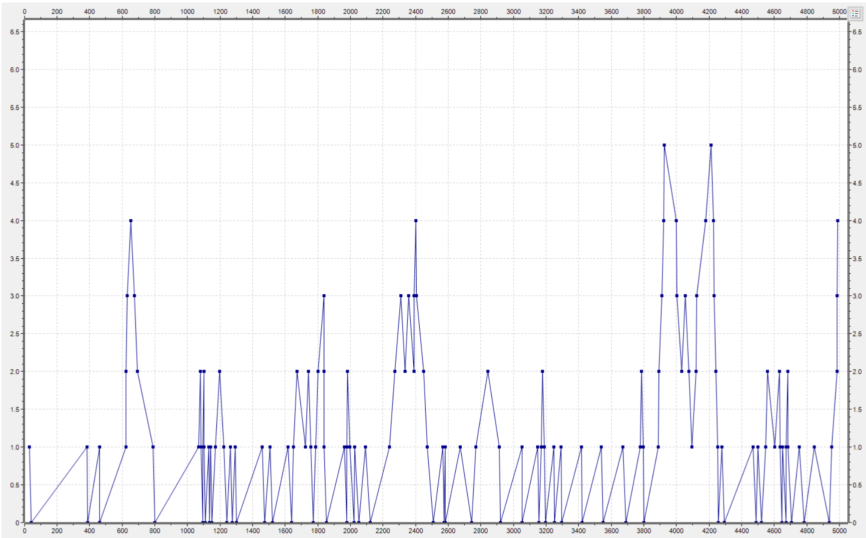


Figure 18: Airplanes in Sky

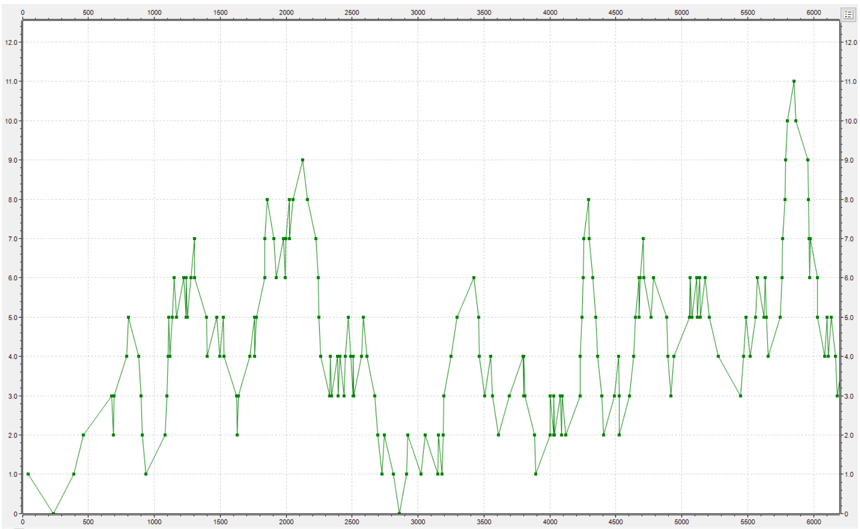


Figure 19: Airplanes in Parking

Figure 20: Airplanes in Takeoff

3.5.3 λ=41s

With this workload the system is still within the stability limit.
The Sky's queue contains a high number of airplanes for a long time in the final part of the test.
Also Takeoff and Parking contain on average a larger number of airplanes.

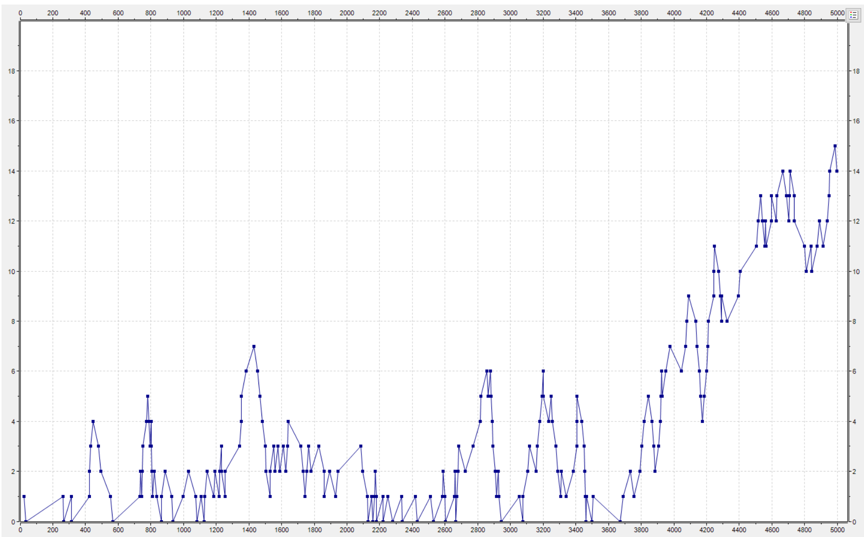


Figure 21: Airplanes in Sky

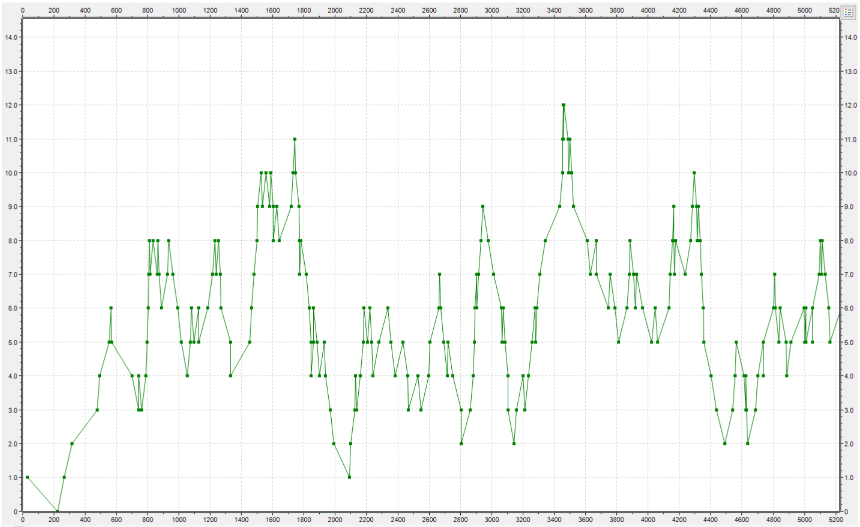


Figure 22: Airplanes in Parking

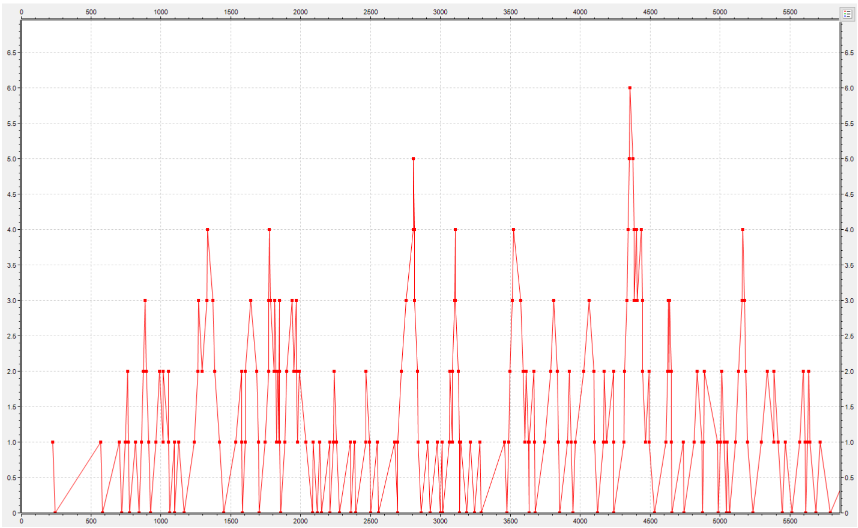


Figure 23: Airplanes in Takeoff

3.6 An unstable scenario

In the following, an unstable scenario with λ=35s, t_l=20s, t_p=240s and t_o=20s.
Going beyond the stability limit, Sky's queue diverges over time. Takeoff and Parking queues increase the average size compared to the previous examples but they do not diverge.

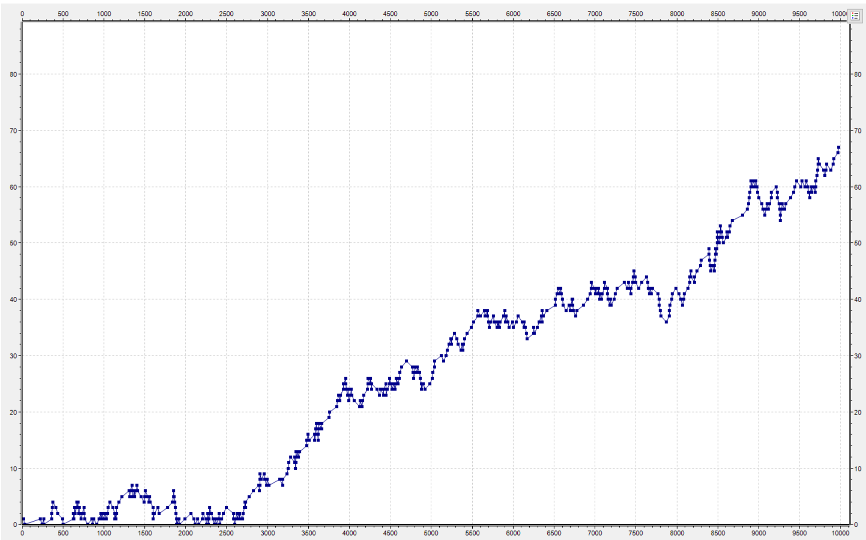


Figure 24: Airplanes in Sky

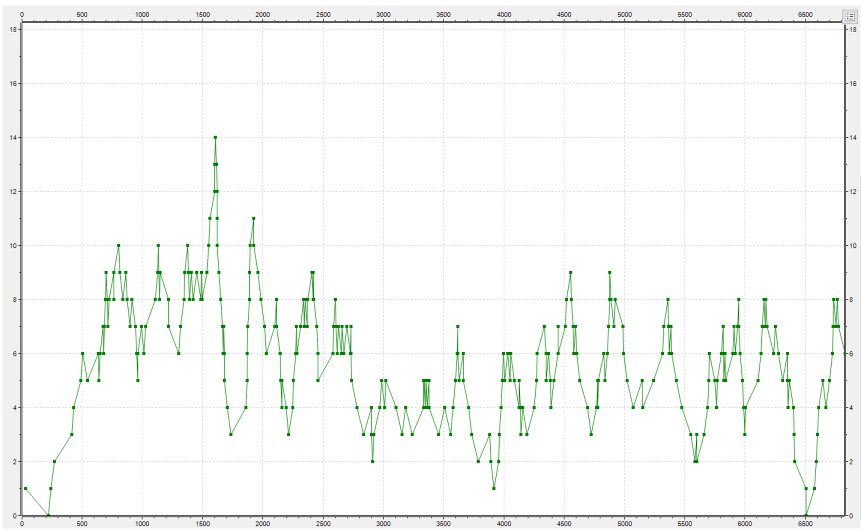


Figure 25: Airplanes in Parking

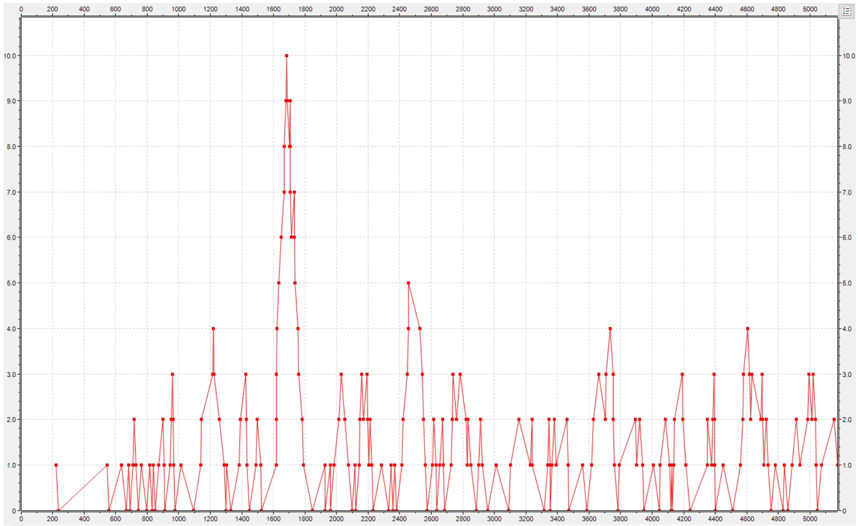


Figure 26: Airplanes in Takeoff

4 Analysis

In order to perform the study of the system, a range for the parameters has been set. Parking's service time is usually an order of magnitude greater than the other service times to maintain realistic relationships between parameters.
t_l and t_o usually have similar values and λ is often used as a fixed parameter of workload.

4.1 Waiting times constant analysis

The waiting time is the time period that an airplane has to wait in Sky or Takeoff's queues before landing or taking off. It is the sum of the service time, of the waiting in the queue and of any delay due to interference.
The landing waiting time and the take-off waiting time are lower limited respectively by t_l and t_o.
Hence, under interference conditions the waiting time will become larger than its respective service time.
In the simulator they are obtained by recording the time in which an airplane arrives in the module and the time in which the landing or take-off operation completes.

The workload appeared to be relevant for the system because according to its value it stabilizes with or without delays.

As already mentioned, $t_l+t_o+r < \lambda$ represents the set of safe cases where no interference occurs. In these cases waiting times are equal to service times since airplanes never land or take off at a time when the runway is busy and never find airplanes' queues reaching the module.
Under light interference conditions, that is $t_l + t_o < \lambda < t_l + t_o + r$, waiting times are limited by the sum of the service times of Sky and Takeoff. No queues and at most one airplane waiting for landing or taking off.
When also $t_o > t_l$, heavy interference may occur. Parking will contain more airplanes and queues will raise in Takeoff. These queues interrupt landings for a considerable time further increasing waiting times.

4.1.1 Example without interference

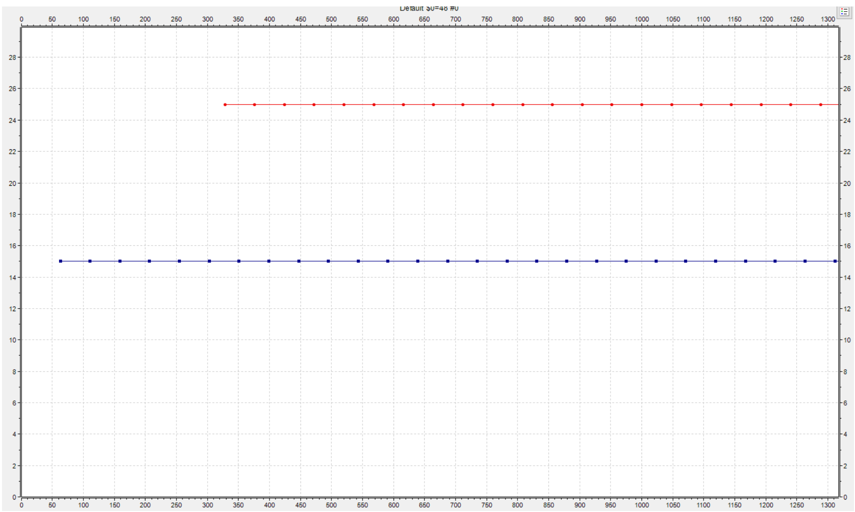


Figure 27: Waiting time without interference: λ=48s, t_l=15s, t_p=240s, t_o=25s. Sky=blue, Takeoff=red.

The waiting times are equal to the Sky and the Takeoff's service time since there are no delays due to interference. There are no intereferences since $t_l + t_o + r < \lambda$.

4.1.2 Example with light interference

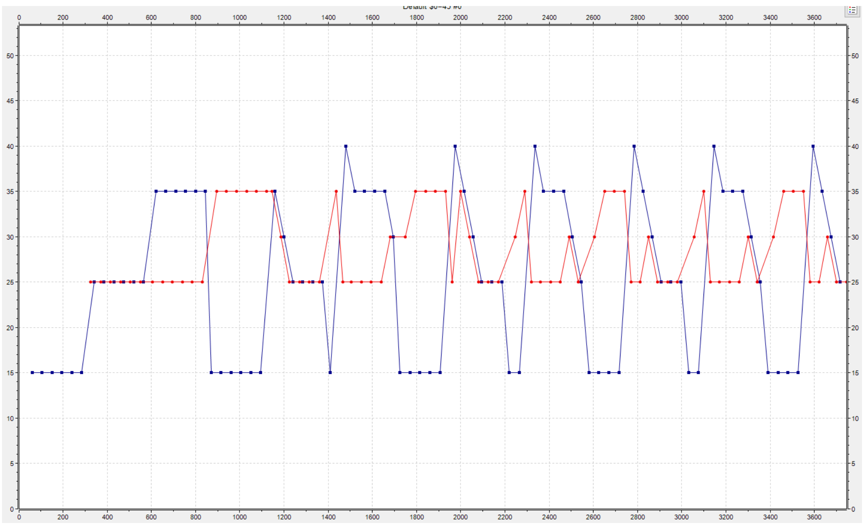


Figure 28: Waiting time with light interference: λ=45s, t_l=15s, t_p=240s, t_o=25s. Sky=blue, Takeoff=red.

In this case $t_l + t_o + r \not< \lambda$, so landings and take-offs with and without delays alternate.
Queues never raise and the maximum waiting time is equal to the sum of service time of module itself and that of the concurrent module.

4.1.3 Example with heavy interference

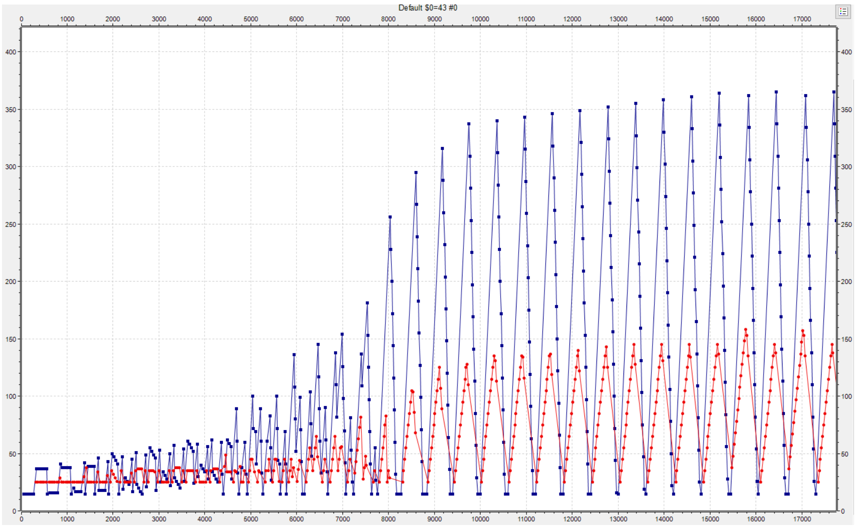


Figure 29: Waiting time with heavy interference: λ=43s, t_l=15s, t_p=240s, t_o=20s. Sky=blue, Takeoff=red

This case achieves stability but queues raise in Sky and Takeoff.
$t_l + t_o + r < \lambda$ and $t_o > t_l$, with these parameters heavy interferences occur.

The total understanding of the constant case would require a rather complex arithmetic analysis.
As already mentioned, particular cases occur when $t_o > t_l$ and the equation for interference-free functioning (i.e. t_l+t_o+r $<$ λ) is not respected.
The warm-up time is very long. The waiting times range between very high values and service times. Since Sky queue is the largest, the landing waiting time reaches higher values.

4.1.4 Sensitivity on parameters

Following, a study of how delays occur for certain values of λ, t_l, t_o and t_p.

All simulations have been performed with a simulation time of $10^6 s$ and warm-up time of $2 \cdot 10^4 s$. The high warm-up time is due to the fact that cases with heavy interference take a long time to reach stability.

- **Workload**
  The waiting times are equal to the service times for any value of λ except for those in which the equation $t_l + t_o + r < \lambda$ is not verified. No more delays occur beyond $\lambda > t_l + t_o + t_p$ because when $\lambda > t_p$ the rest (i.e. $mod(t_p, \lambda)$) is equal to t_p, so for greater values of λ the free-interference condition is always respected.
  In absence of interference and in stable conditions no queues raise, so the workload does not increase the waiting time.

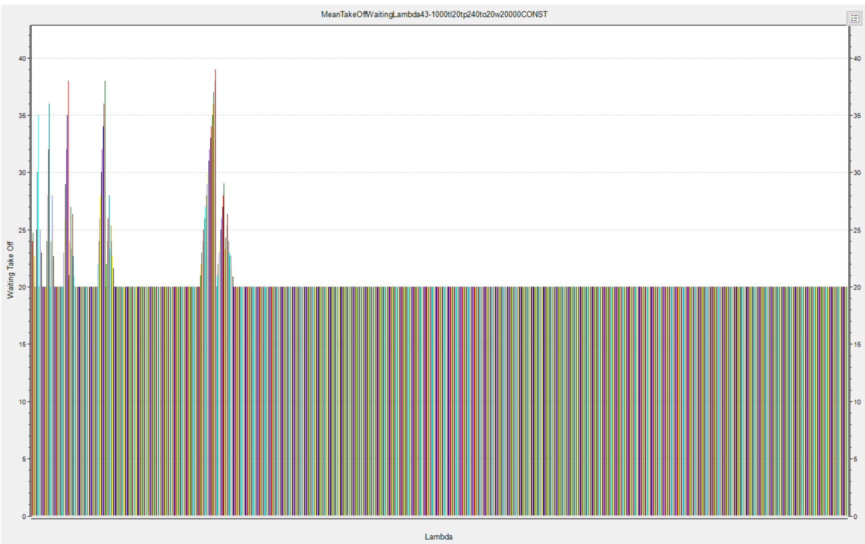


Figure 30: Takeoff's average waiting time: λ=[43,1000]s, t_l=20s, t_p=240s, t_o=20s, warm up=$2 \cdot 10^4$s, time=10^6s

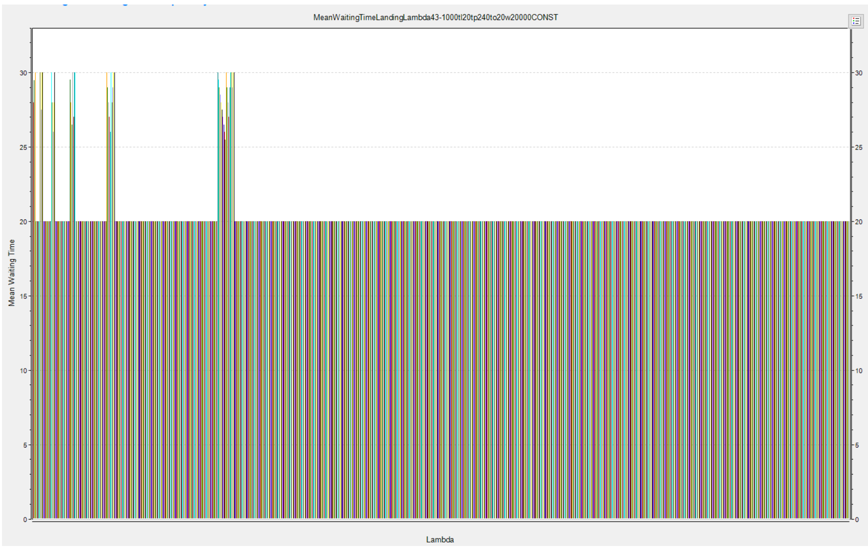


Figure 31: Sky's average waiting time: λ=[43,1000]s, t_l=20s, t_p=240s, t_o=20s, warm up=$2 \cdot 10^4$s, time=10^6s

- **Service time t_l**
  Sky's waiting time is equal to the service time t_l (i.e. no waiting) until $t_l+t_o+r < \lambda$ is not exceeded.
  Takeoff's waiting time stays at the fixed value until the threshold is reached.

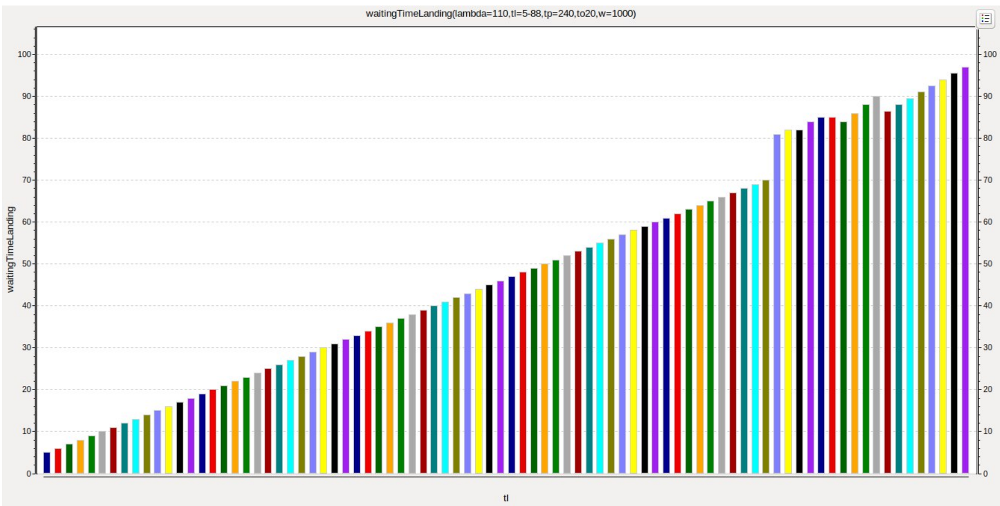


Figure 32: Sky's average waiting time: λ=110s, t_l=[5,88]s, t_p=240s, t_o=20s, warm up=1000s, time=10^6s

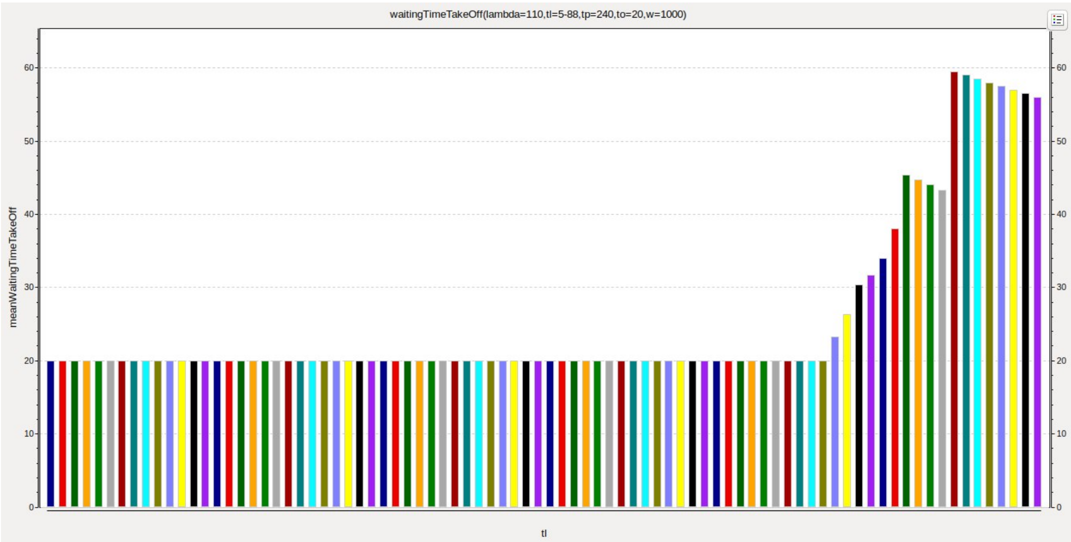


Figure 33: Takeoff's average waiting time: λ=110s, t_l=[5,88]s, t_p=240s, t_o=20s, warm up=1000s, time=10^6s

- **Service time t_o**
  Takeoff's waiting time remains equal to the respective service time t_o and Sky's waiting time stays at the fixed value until $t_l+t_o+r < \lambda$ is not exceeded. When the threshold is exceeded the values start to raise rapidly. In this case the delays are higher because $t_o > t_l$.

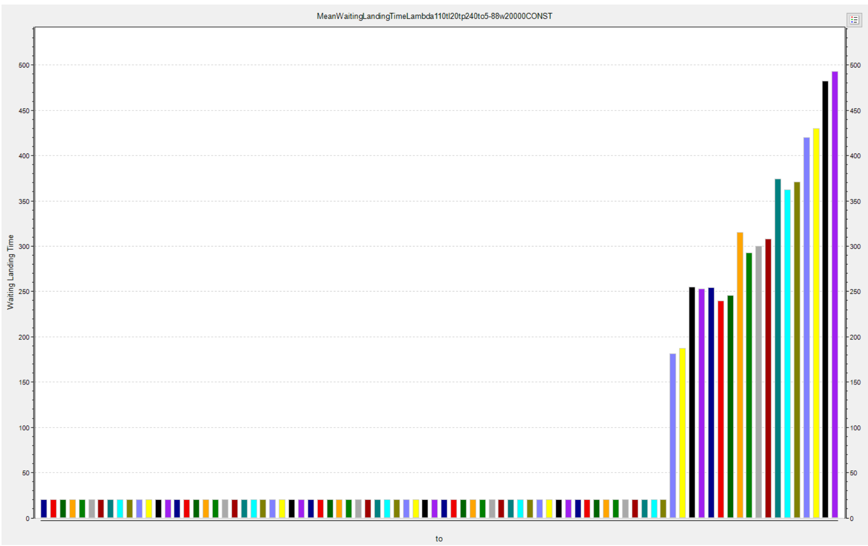


Figure 34: Sky's average waiting time: λ=110s, t_l=20s, t_p=240s, t_o=[5,88]s, warm up=$2 \cdot 10^4$s, time=10^6s

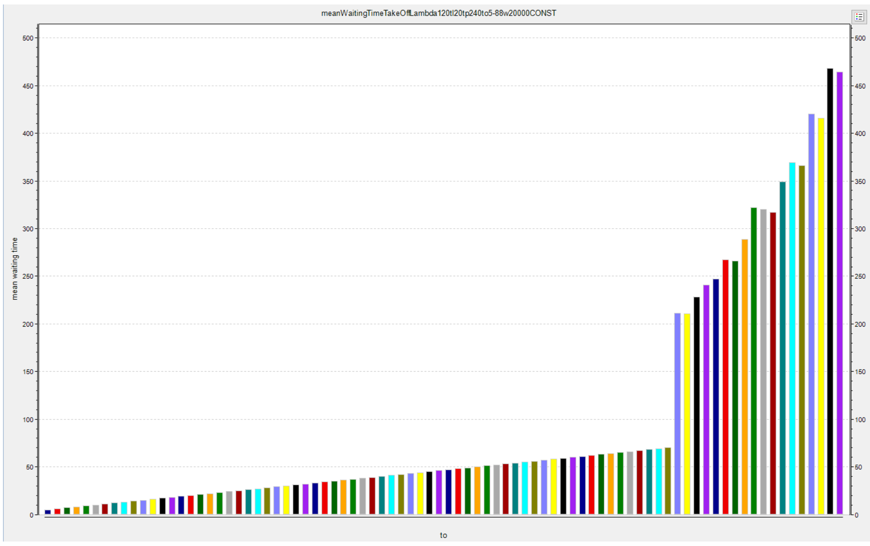


Figure 35: Takeoff's average waiting time: λ=110s, t_l=20s, t_p=240s, t_o=[5,88]s, warm up=$2 \cdot 10^4$s, time=10^6s

- **Service time t_p**
  The waiting times are equal to the service times when $t_l+t_o+r<\lambda$.
  The periodic interference is due to the variation of the rest (i.e. r=mod(t_p,λ)) that continuously changes.

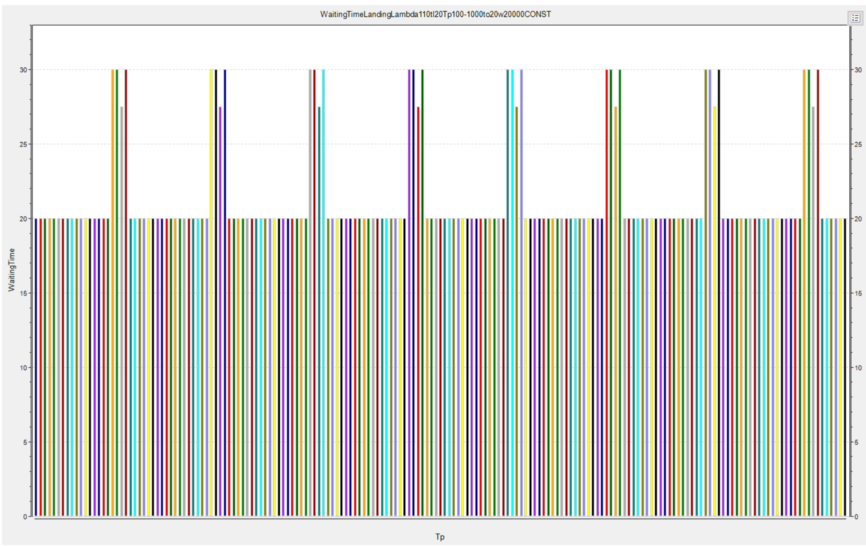


Figure 36: Sky's average waiting time: λ=110s, t_l=20s, t_p=[100,1000]s, t_o=20s, warm up=$2 \cdot 10^4$s, time=10^6s

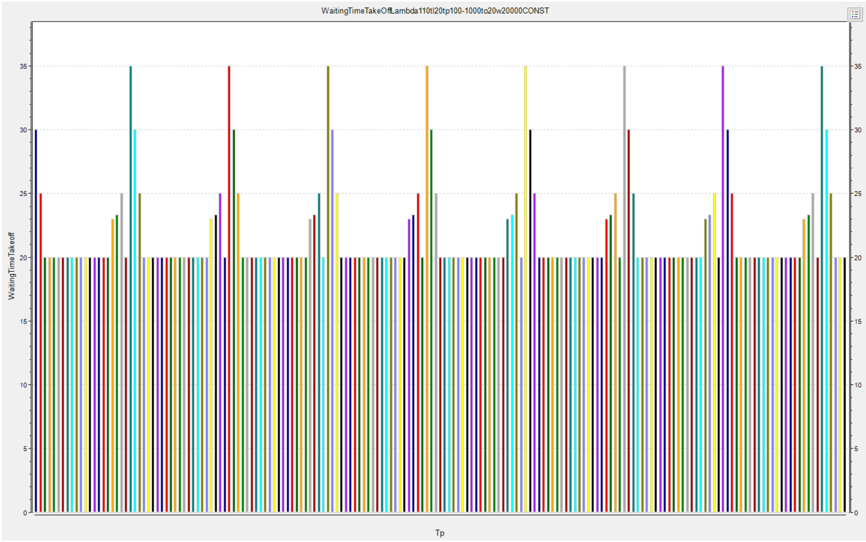


Figure 37: Takeoff's average waiting time: λ=110s, t_l=20s, t_p=[100,1000]s, t_o=20s, warm up=$2 \cdot 10^4$s, time=10^6s

4.1.5 Conclusions

In the constant case waiting times are equal to service times once the stability is achieved and there is no interference.
In case of light interference, i.e. $t_l + t_o < \lambda < t_l + t_o + r$, waiting times are limited by the sum of the two service times t_l and t_o. No queues are formed.
In case of heavy interference, a possible option is trying to set a maximum limit on waiting times. Not being able to precisely understand when heavy interference occurs, this maximum limit will be valid in all cases in which t_l+t_o+r>λ and t_o>t_l.

- Maximum landing waiting time:

$$MaxSkyQueueLength \cdot t_l + MaxParkingQueueLength \cdot t_o$$

$$\lceil \frac{t_p}{t_l} \cdot \frac{t_o}{\lambda} \rceil \cdot t_l + \lceil \frac{t_p}{t_l} \rceil \cdot t_o$$

  Airplanes in Sky have to wait for the take-off of airplanes in Parking and for the landing of those that come first in Sky's queue.

- Maximum takeoff waiting time:

$$MaxTakeOffQueueLength \cdot t_o$$

$$\lceil \frac{t_p}{t_l} \cdot t_o \cdot (\frac{1}{t_l} - \frac{1}{t_o}) \rceil \cdot t_o$$

Takeoff queue has the priority on Sky queue so Airplanes in Takeoff will only have to wait for the emptying of the take-off queue.

4.2 Waiting times exponential analysis

With exponential parameters different functionings are no longer identifiable as they were in the constant case. Interferences always occur and parameters will affect less sharply but incrementally.

4.2.1 2^k *Analysis*

A 2^k Analysis has been performed by fixing the workload and assigning t_l [20,30]s, t_o [20,30]s and t_p [250,300]s.

- λ=120s:

	t_l	t_p	t_o
landingWaitingTime	$\sim$ 75%	$\sim$ 0%	$\sim$ 25%
takeoffWaitingTime	$\sim$ 10%	$\sim$ 0%	$\sim$ 90%

- λ=200s:

	t_l	t_p	t_o
landingWaitingTime	$\sim$ 90%	$\sim$ 0%	$\sim$ 10%
takeoffWaitingTime	$\sim$ 5%	$\sim$ 0%	$\sim$ 95%

- λ=300s:

	t_l	t_p	t_o
landingWaitingTime	$\sim$ 95%	$\sim$ 0%	$\sim$ 5%
takeoffWaitingTime	$\sim$ 2.5%	$\sim$ 0%	$\sim$ 97.5%

The results of this analysis deal only with a limited range of parameters and their precision is not very reliable. However, the results show that the Parking time does not affect waiting times in the queues.
Furthermore, the more the workload decreases the more the waiting times of the two queues are dependent on the respective service time and not on that of the competing module.
This is due to the lower frequency of interferences.

Taking into account that the Parking time is irrelevant, a 2^k analysis has been performed by fixing the parking time t_p at 300s and assigning λ [120,150]s, t_l [20,30]s and t_o [20,30]s.

- λ=[120,150]s:

	λ	t_l	t_o
landingWaitingTime	$\sim$ 10%	$\sim$ 65%	$\sim$ 20%
takeoffWaitingTime	$\sim$ 4%	$\sim$ 7%	$\sim$ 88%

- λ=[180,210]s:

	λ	t_l	t_o
landingWaitingTime	$\sim$ 3%	$\sim$ 85%	$\sim$ 10%
takeoffWaitingTime	$\sim$ 1%	$\sim$ 5%	$\sim$ 93%

- λ=[270,300]s:

	λ	t_l	t_o
landingWaitingTime	$\sim$ 0.5%	$\sim$ 94%	$\sim$ 5%
takeoffWaitingTime	$\sim$ 0.25%	$\sim$ 2%	$\sim$ 96%

This second analysis confirms how the influence of the workload on waiting times increases with the workload itself. As it increases, the mutual influence between the two queues also increases.
In perfect functioning conditions with very low workloads, waiting times would exactly correspond to the service times and would be independent to small variations of the workload.

4.2.2 Examples with different workloads

1. $\lambda = 90s$:
   With a low workload most of the waiting times are close to the value of the service times and queues empty. Moreover, Sky's waiting time tends to be higher than Takeoff's.

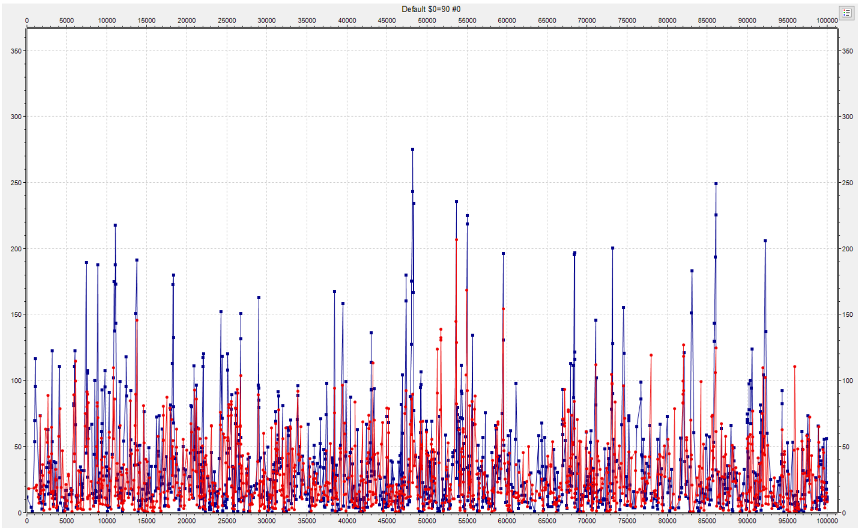


Figure 38: Waiting time:λ=90s, t_l=20s, t_p=240s, t_o=20s. Sky=blue, Takeoff=red.

2. $\lambda = 60s$:
   Increasing the workload, waiting times increase too. The difference between landing waiting time and take-off waiting time grows. The queues frequently empty.

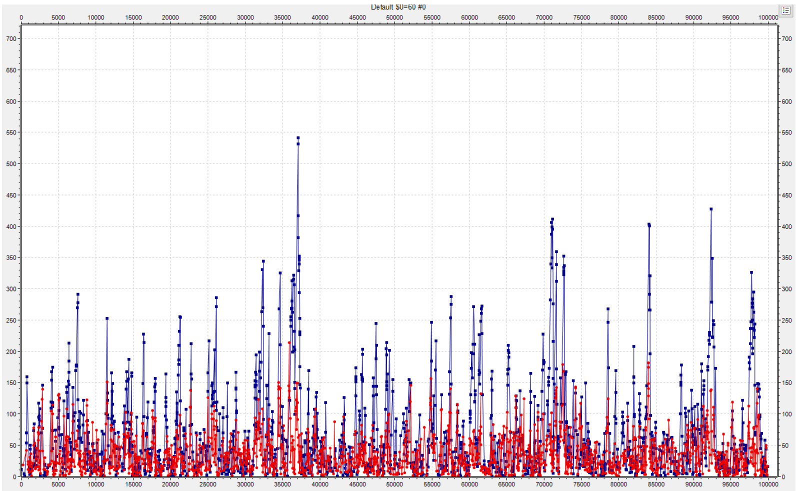


Figure 39: Waiting time: λ=60s, t_l=20s, t_p=240s, t_o=20s. Sky=blue, Takeoff=red.

3. $\lambda = 50s$:
   With this workload, queues stay long periods of time without emptying. The difference between Sky and Takeoff's waiting times has further increased.

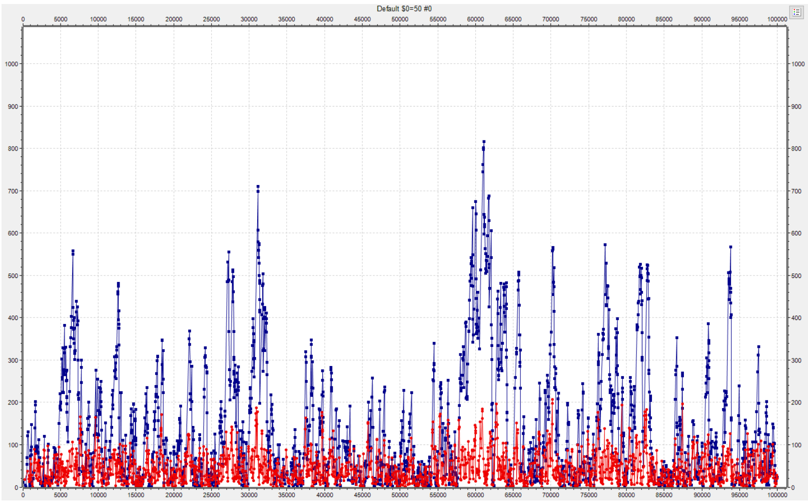


Figure 40: Waiting time: λ=50s, t_l=20s, t_p=240s, t_o=20s. Sky=blue, Takeoff=red.

4. $\lambda = 41s$:
   At the stability limit Takeoff's waiting time stays almost the same as in the previous case while Sky's one increases a lot. Sky's queue tends to empty rarely and to have a high average length.

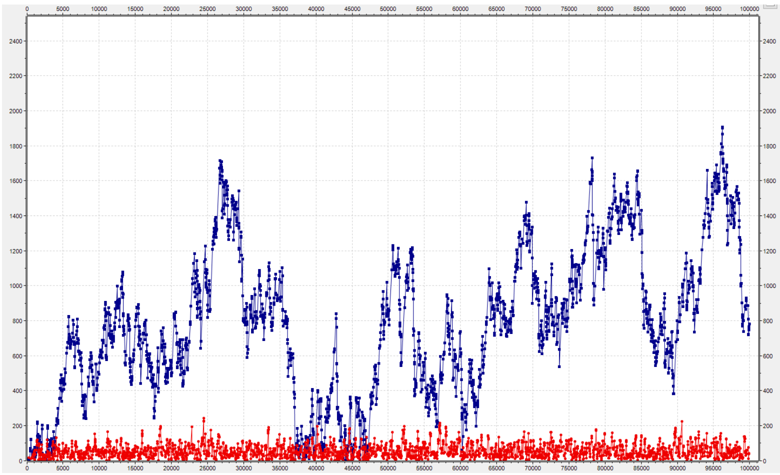


Figure 41: Waiting time: λ=41s, t_l=20s, t_p=240s, t_o=20s. Sky=blue, Takeoff=red.

5. $\lambda = 35s$, unstable case:
   Exceeding the stability threshold the Sky's queue diverges and so its waiting time.

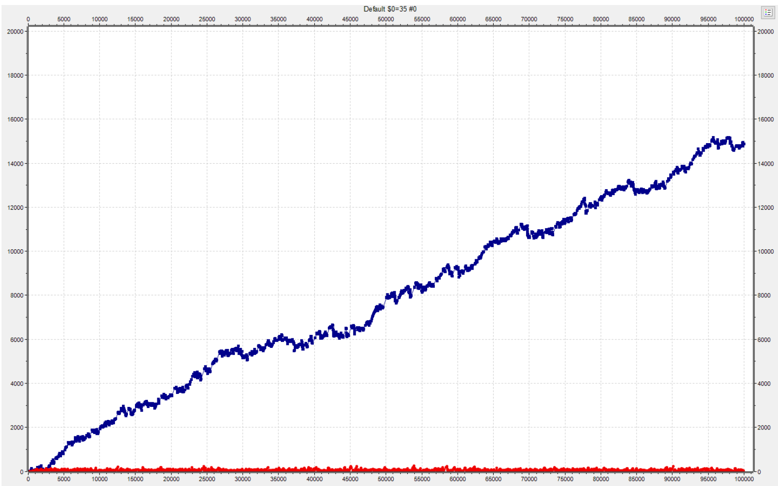


Figure 42: Waiting time: λ=35s, t_l=20s, t_p=240s, t_o=20s. Sky=blue, Takeoff=red.

4.2.3 Waiting times distribution

The average waiting time has been studied on 500 experiments with different seeds over a time of 10^6s and warm up time of $5 \cdot 10^4$s.

- Stability condition: λ=80s, t_l=20s, t_p=240s, t_o=20s
  - Landing waiting time:
    Normal distribution - R^2=0.997
  - Takeoff waiting time:
    Normal distribution - $R^2 = 1$
- Almost unstable condition: λ=41s, t_l=20s, t_p=240s, t_o=20s
  - Landing waiting time:
    Exponential distribution - R^2=0.990
  - Takeoff waiting time
    Normal distribution - R^2=0.997

Landing waiting time reacts differently to the increase in workload from take-off waiting time. In particular, the variance of landing waiting time increases significantly as the workload increases.

4.2.4 Sensitivity on parameters

All the experiments were performed with a simulation time of 10^6s and warm up time of $5 \cdot 10^4$s.

- **Workload**
  Landing waiting time increases slowly when workload's values are far away from the instability condition and faster when instability condition is reached.
  The model that comes closest is the Lognormal one, even if the fitting is not particularly satisfying. The coefficient of determination with a Lognormal model fitting is 0.91.

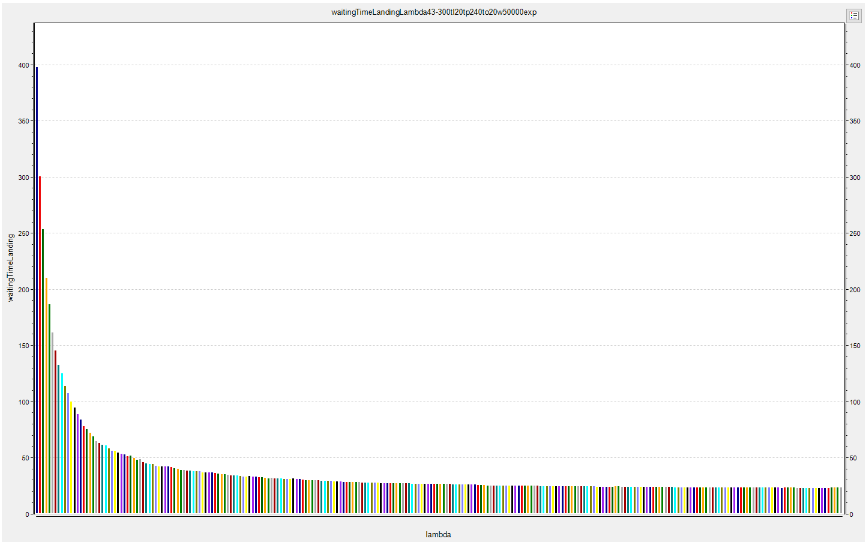


Figure 43: Landing average waiting time: λ=[43,300]s, t_l=20s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

Take-off waiting time is less influenced by the increase of the workload than the landing one.
The model that comes closest is the Exponential one, with a coefficient of 0.98.

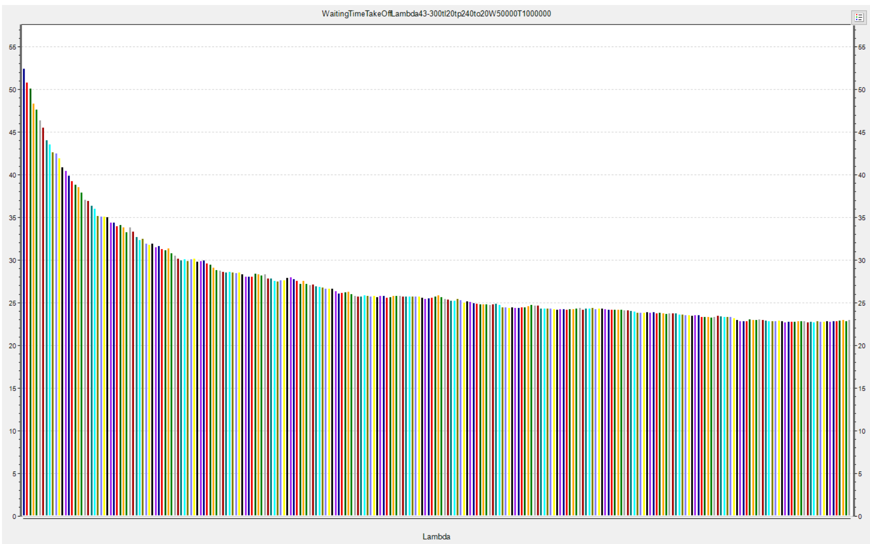


Figure 44: Takeoff average waiting time: λ=[43,300]s, t_l=20s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

- **Service time t_l**
  As the service time t_l increases, the landing waiting time increases. The closest model is the Lognormal one, with a coefficient of 0.94.

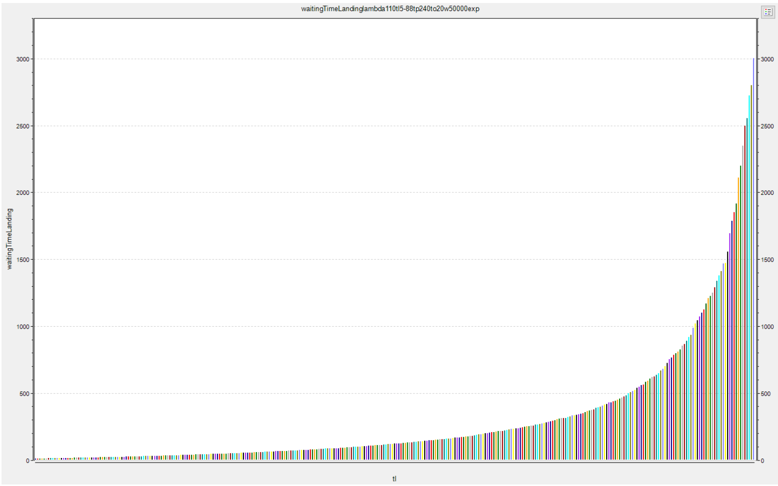


Figure 45: Landing average waiting time: λ=110s, t_l=[5,88]s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

Takeoff's waiting time grows gradually as t_l increases.
The model that better describes growth is $\beta(0.5, 1)$, with a coefficient of determination that is almost 1.

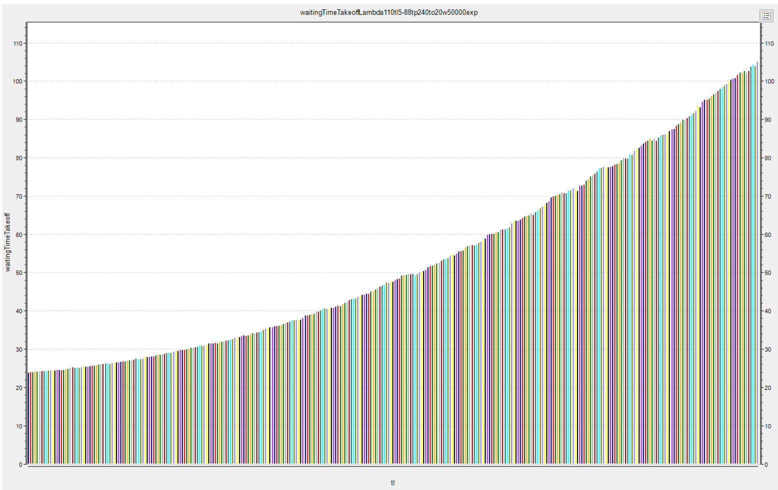


Figure 46: Takeoff average waiting time: λ=110s, t_l=[5,88]s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

- **Service time t_o**
  As t_o increases the waiting times grow.
  The landing waiting time shows an exponential growth with a coefficient of determination of 0.94.
  The jagged pattern to the right of the graph is not easily explicable. It could be due to the more frequent occurrence of interferences between the modules.

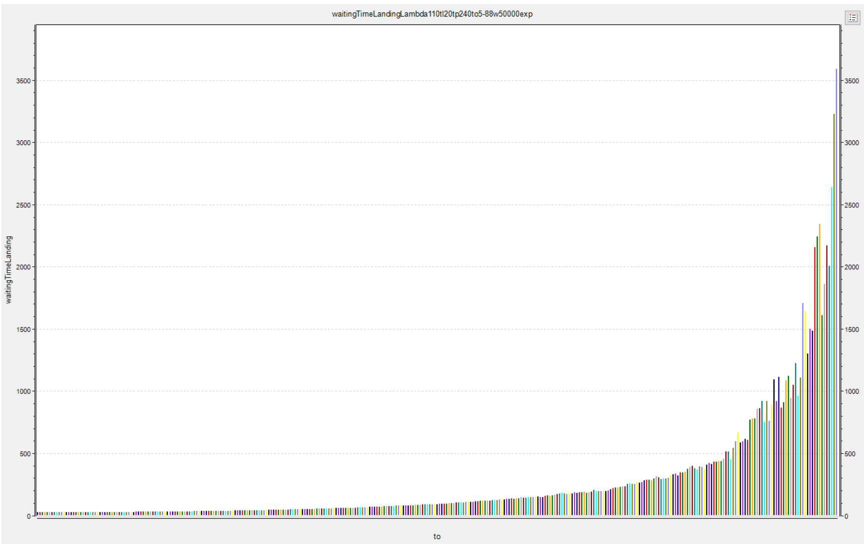


Figure 47: Landing average waiting time: λ=110s, t_l=20s, t_p=240s, t_o=[5,88]s, warm up=$5 \cdot 10^4$s, time=10^6s

The take-off waiting time increases slowly. The closest model is $\beta(0.7, 1.4)$ with a determination coefficient of about 1.

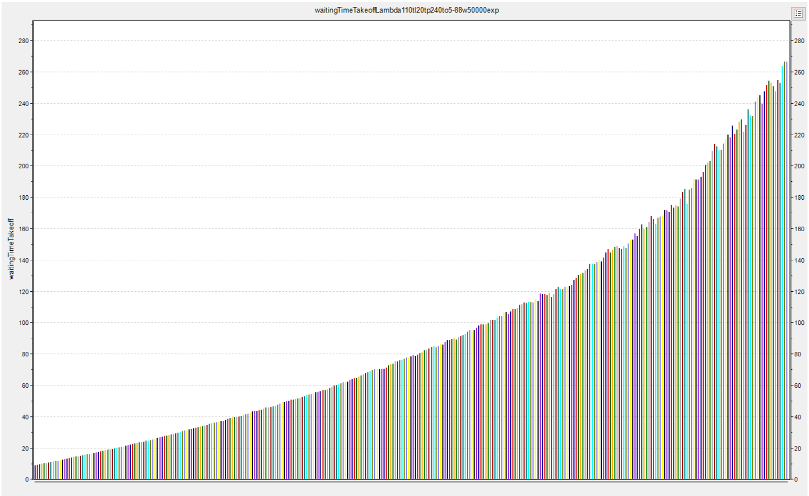


Figure 48: Takeoff average waiting time: λ=110s, t_l=20s, t_p=240s, t_o=[5,88]s, warm up=$5 \cdot 10^4$s, time=10^6s

- **Service time t_p**
  When t_p increases there is a constant trend on waiting times. Being simply a delay, t_p has no influence on the length of Sky and Takeoff's queues.

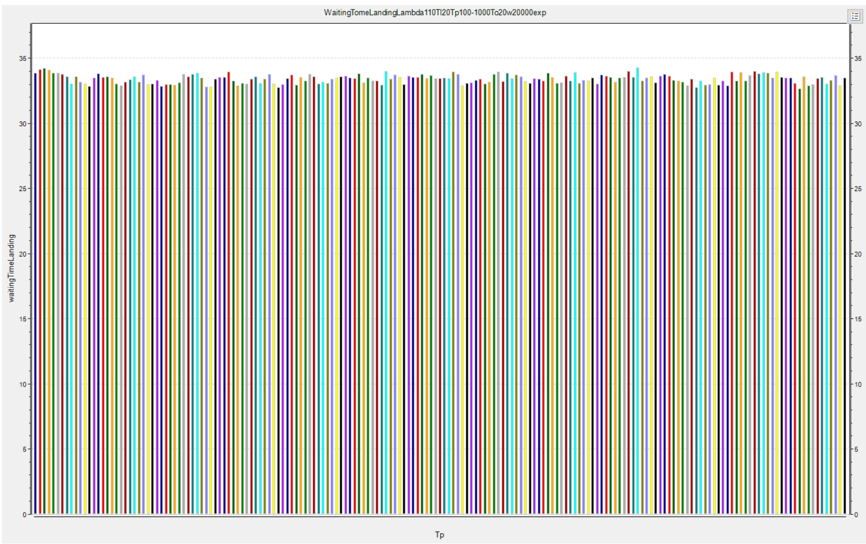


Figure 49: Landing waiting time: λ=110s, t_l=20s, t_p=[100,1000]s, t_o=20s, warm up=$2 \cdot 10^4$s, time=10^6s

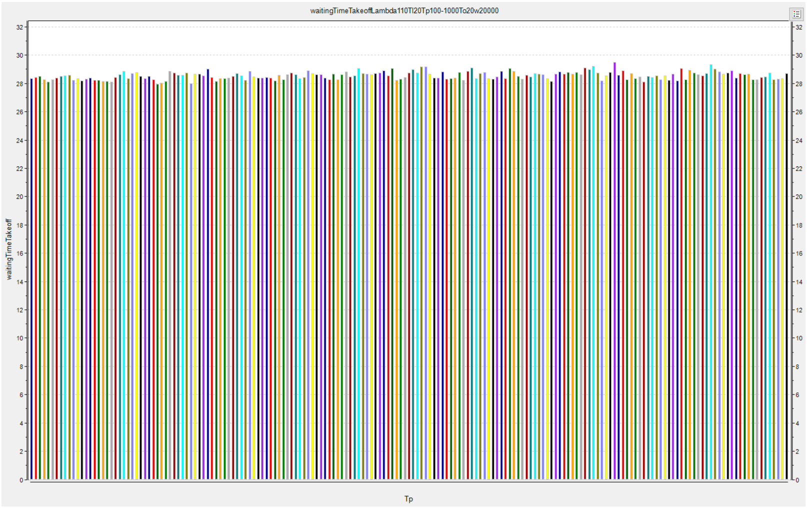


Figure 50: Takeoff waiting time: λ=110s, t_l=20s, t_p=[100,1000]s, t_o=20s, warm up=$2 \cdot 10^4$s, time=10^6s

In general, the Takeoff module shows lower delays than the Sky module. Takeoff cannot become unstable since take-offs have the priority over landings. Furthermore, a long take-off queue will prevent landings for a considerable amount of time and therefore it will slow down future arrivals in the queue.

	LandingWaitingTime	TakoffWaitingTime
Workload	Lognormal	Exponential
t_l	Lognormal	β(0.5,1)
t_o	Exponential	β(0.7,1.4)
t_p	Constant	Constant

The experiments have been made approaching the instability condition progressively, except for the case of t_p.

The functions have a Lognormal, Exponential, β(0.7,1.4) and β(0.5,1) slope.

The workload negatively affects the waiting times.

The increase in the service time of a module negatively affects the waiting time of the same module and, in a slightly reduced way, that of the other module.

Sky generally tends to be more subjected to delays.

4.3 Airplanes' number in Parking with constant parameters

When the stability condition is reached the number of airplanes in the parking area depends on whether the system is susceptible to interference or not. In absence of interferences, it is equal to $\frac{t_p}{\lambda}$ if t_p is a multiple of λ, otherwise it oscillates from $\lfloor \frac{t_p}{\lambda} \rfloor$ to $\lceil \frac{t_p}{\lambda} \rceil$ if there is a rest.
With light interference Parking will continue to contain this number of airplanes while with heavy interference the number of airplanes varies from 0 up to a maximum of $\lceil \frac{t_p}{t_l} \rceil$.

However, the average number of airplanes in the parking area remains the same in all conditions while the maximum number of airplanes changes and increases with the increase of interference.

With heavy interference the warm up time is very long and this will affect the warm up time of all the tests since we could not exactly established when heavy interference's cases occur.

4.3.1 Examples with increasing interference

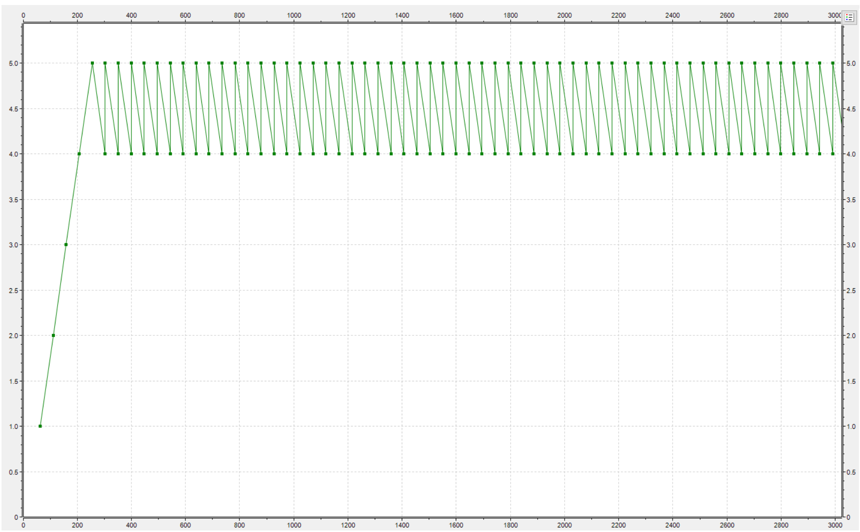


Figure 51: Airplanes in Parking without interference: λ=48s, t_l=15s, t_p=240s, t_o=25s

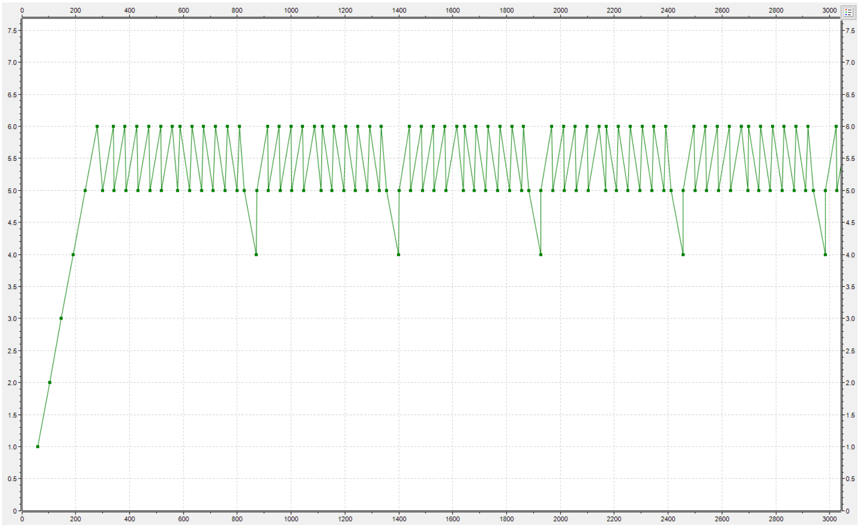


Figure 52: Airplanes in Parking with light interference: λ=44s, t_l=15s, t_p=240s, t_o=25s

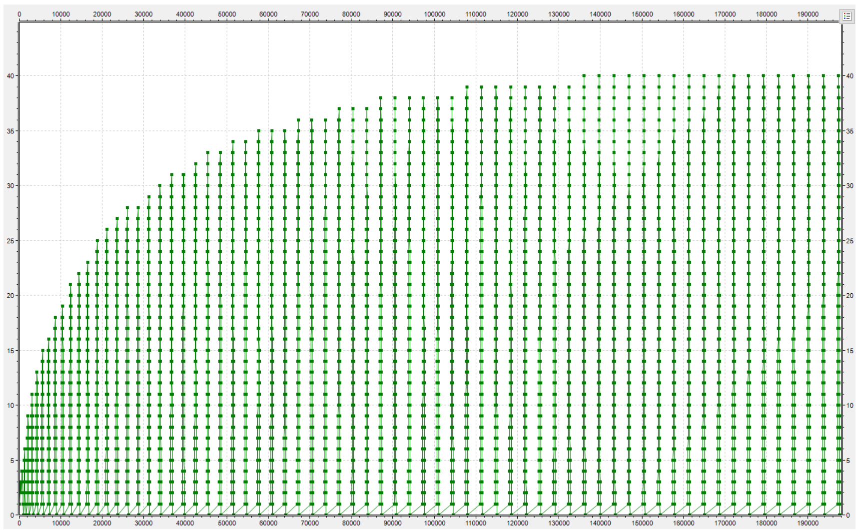


Figure 53: Airplanes in Parking with heavy interference: $\lambda = 90s$, t_l=5s, t_p=240s, t_o=84s

4.3.2 Sensitivity on parameters

- **Workload**
  As the workload increases, the average number of airplanes in the parking area increases exponentially with a coefficient of determination 0.97. The maximum number of airplanes changes exponentially as the workload increases, with a coefficient of determination 0.99.

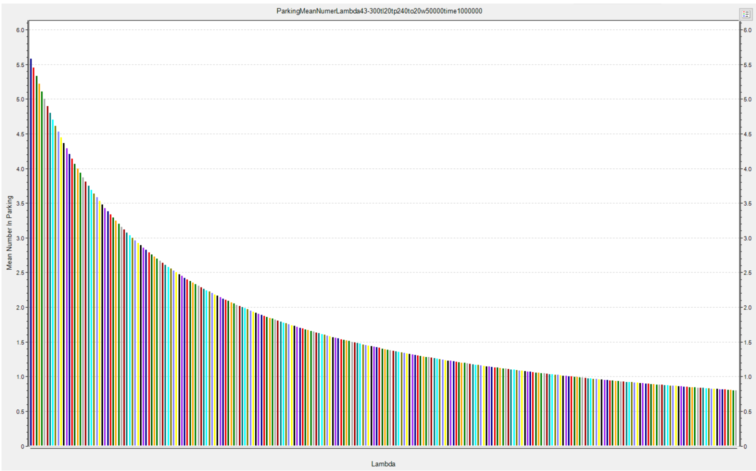


Figure 54: Average number of airplanes: λ=[43,300]s, t_l=20s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

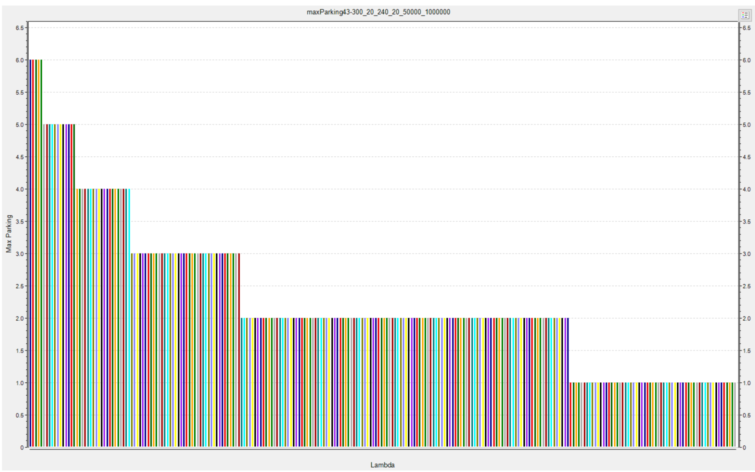


Figure 55: Max number of airplanes: λ=[43,300]s, t_l=20s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

- $\mathbf{t_l}$
  The average and the maximum numbers of airplanes in the parking area do not vary by changing t_l. t_l has not influence on them unless the system is in heavy interference.

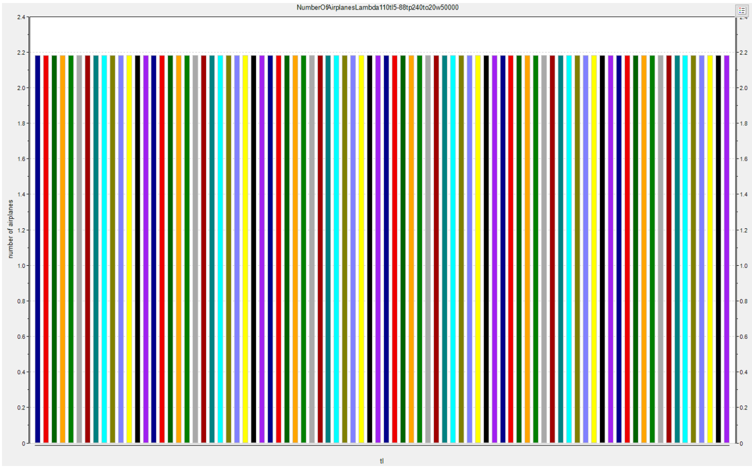


Figure 56: Average number of airplanes: $\lambda = 110s$, t_l=[5,88]s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

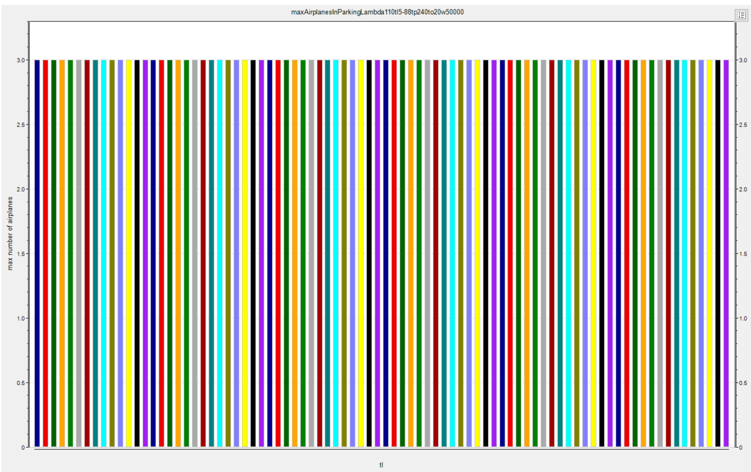


Figure 57: Max number of airplanes: $\lambda = 110s$, t_l=[5,88]s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

- $\mathbf{t_o}$
  The average number of airplanes in the parking area stays constant changing t_o.
  In functioning with light interference either the maximum nor the average number of airplanes in the parking area change.

Figure 58: Average number of airplanes: $\lambda = 110s$, t_l=20s, t_p=240s, t_o=[5,88]s, warm up=$5 \cdot 10^4$s, time=10^6s

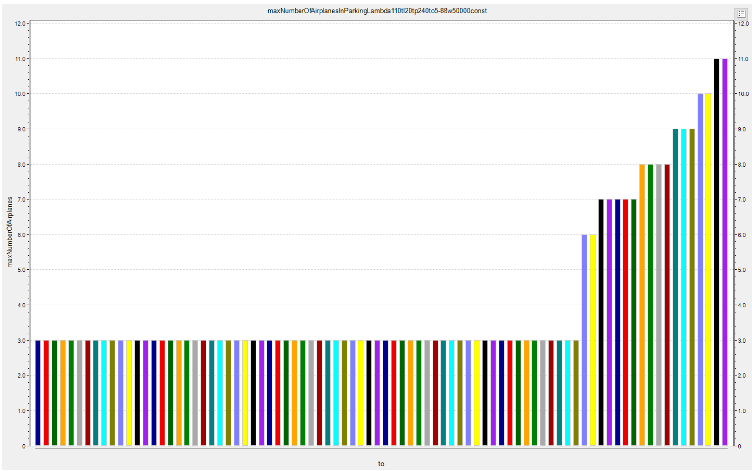


Figure 59: Max number of airplanes: $\lambda = 110s$, t_l=20s, t_p=240s, t_o=[5,88]s, warm up=$5 \cdot 10^4$s, time=10^6s

- $\mathbf{t_p}$

  As t_p changes, the number of airplanes in the parking area raises linearly (i.e. $\frac{t_p}{\lambda}$). Also the maximum number increases linearly, taking the ceiling of the average number.

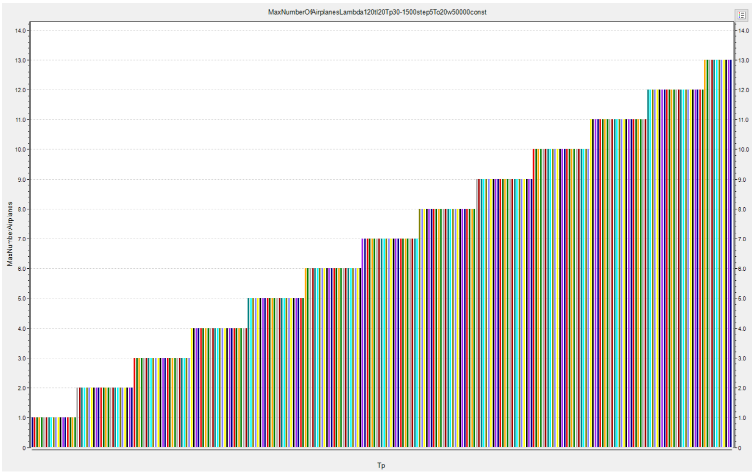


Figure 60: Max number of airplanes: $\lambda = 120s$, t_l=20s, t_p=[5,500]s step=5s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

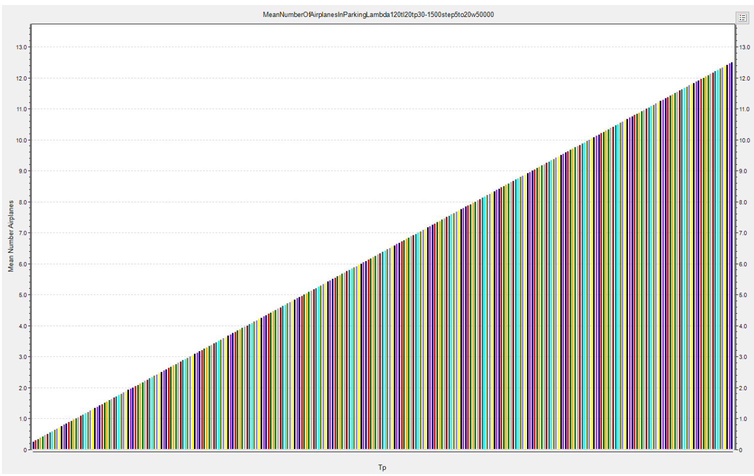


Figure 61: Average number of airplanes: $\lambda = 120s$, t_l=20s, t_p=[5,500]s step=5s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

4.3.3 Conclusions

Sensitivity:

	MeanNumberParking	MaxNumberParking
Workload	Exponential	Exponential
t_l	Constant	Constant
t_p	Linear	Linear
t_o	Constant	Constant (exp for high values)

Far from the instability condition the number of airplanes in Parking is influenced only by the workload and t_p. The workload has in particular a heavier influence than t_p. When close to the instability condition the increase of t_o is also relevant. It can cause heavy interference.

4.4 Airplanes' number in Parking with exponential parameters

A 2^k analysis has been performed for the evaluation of the parameters and workload's influence on the number of airplanes in Parking.

4.4.1 2^k *Analysis*

- Fixed workload: this analysis has been performed with fixed workloads and assigning t_l and t_o [20,30]s and t_p [250,300]s.
  λ equals to 120s, 200s and 300s give the same results.

	t_l	t_p	t_o
ParkingNumber	$\sim 0\%$	$\sim 100\%$	$\sim 0\%$

  Service times have no influence on the number of airplanes in the parking area.

- Assigning both t_l and t_o [20,30]s, t_p [250,300]s and the workload [120,150]s we obtained the following results.

	λ	t_l&t_o	t_p
ParkingNumber	$\sim$ 59%	$\sim$ 0%	$\sim$ 39%

Assigning λ [180,210]s we obtained the following results

	λ	t_l&t_o	t_p
ParkingNumber	$\sim$ 41%	$\sim$ 0%	$\sim$ 55%

Finally, assigning the workload [270,300]s we obtained the following results

	λ	t_l&t_o	t_p
ParkingNumber	$\sim$ 23%	$\sim$ 0%	$\sim$ 72%

The influence of the workload on the number of airplanes in the parking area decreases as the workload decreases.

4.4.2 Distribution

- The distribution of the average number of airplanes in the parking area has been studied on different repetitions of the same experiment.

- Stable condition
  - λ=80s, t_l=20s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^5$s, time=10^6s
  - 500 experiments
  - Normal distribution, with a coefficient of determination about 1
- Almost unstable
  - λ=41s, t_l=20s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^5$s, time=10^6s
  - 500 experiments
  - Normal distribution, with a coefficient of determination about 1

• Distribution on the single experiment

- Stable condition
  - λ=80s, t_l=20s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^5$s, time=10^6s
  - 500 IID samples
  - Poisson distribution

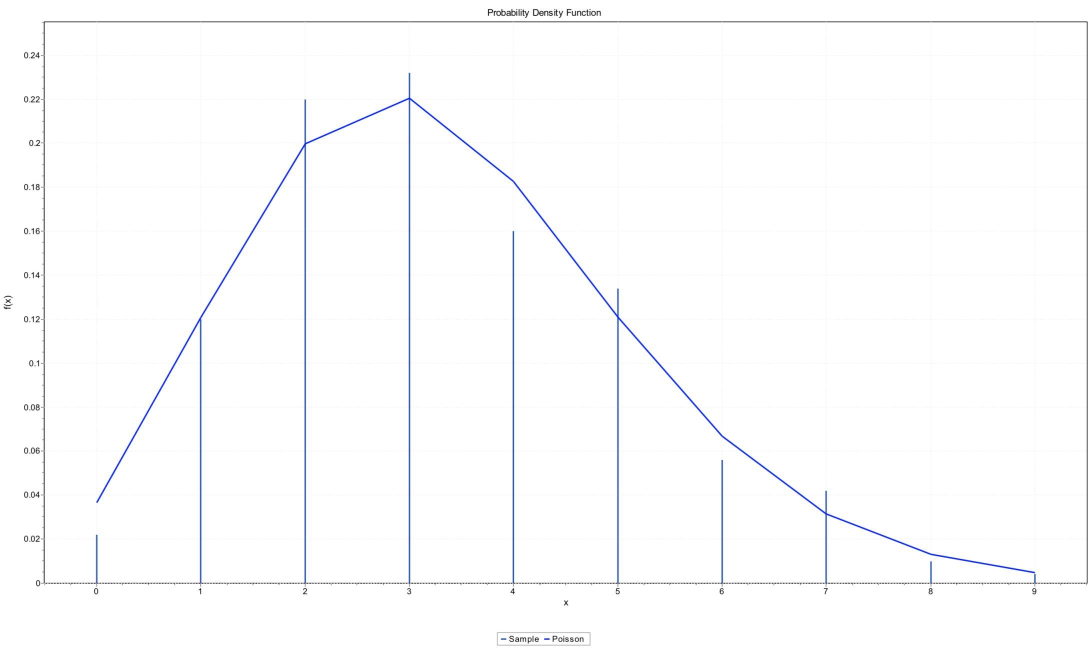


Figure 62: Probability density function of the airplanes' number in stable condition

The above graph shows the probability to find a certain number of airplanes within the parking area.

- Almost unstable condition
  - λ=41s, t_l=20s, t_p=240s, t_o=20s, warm up=$5 \cdot 10^5$s, time=10^6s
  - 500 IID
  - Poisson distribution

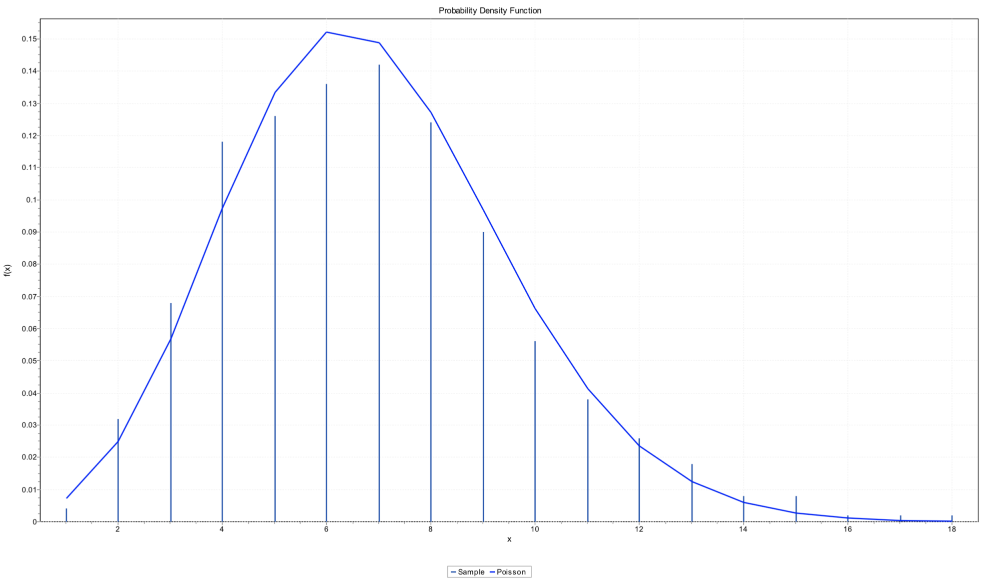


Figure 63: Probability density function of the airplanes' number in almost unstable condition

The graph represents the probability density function of the airplanes' number within the Parking, that is the probability to find a given number of airplanes.

4.4.3 Sensitivity on parameters

- **Workload**

Increasing the workload the average number of airplanes in the parking area increases exponentially with a determination coefficient of 0.97.

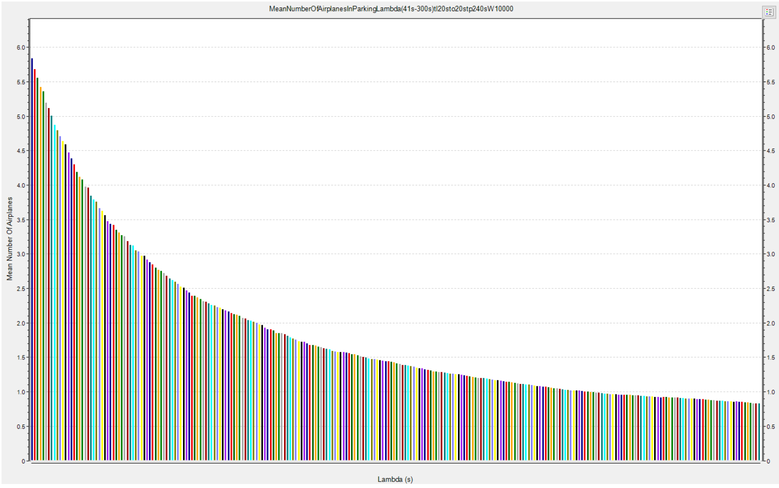


Figure 64: Average number of airplanes: λ=[41,300]s, t_l=20s, t_p=240s, t_o=20s, warm up=10^4s, time=10^6s

- **t_l**
  The average number of airplanes in the parking area remains constant varying t_l.

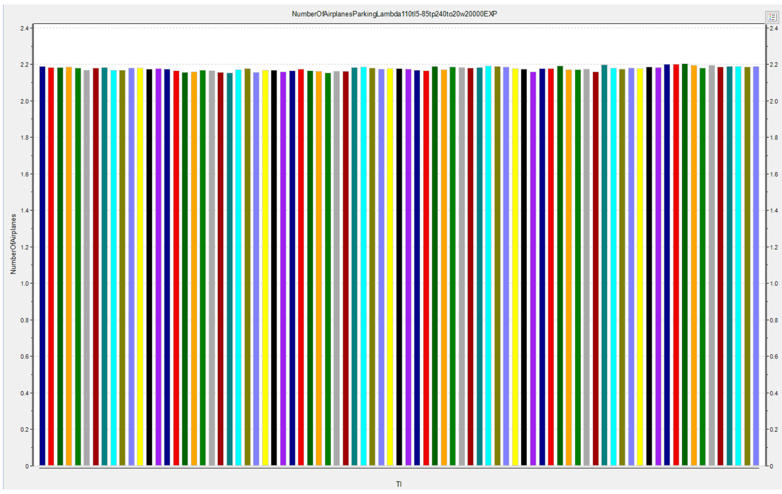


Figure 65: Average number of airplanes: λ=110s, t_l=[5,85]s, t_p=240s, t_o=20s warm up=$2 \cdot 10^4$s, time=10^6s

- $\mathbf{t_o}$
  The average number of airplanes in the parking area remains constant varying t_o.

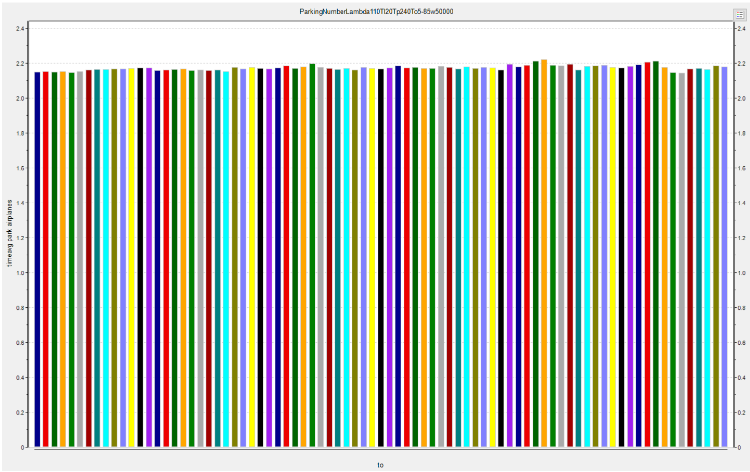


Figure 66: Average number of airplanes: λ=110s, t_l=20s, t_p=240s, t_o=[5,85]s, warm up=$5 \cdot 10^4$s, time=10^6s

- $\mathbf{t_p}$
  Increasing t_p the average number of airplanes in the parking area increases linearly with a determination coefficient of approximately 1.

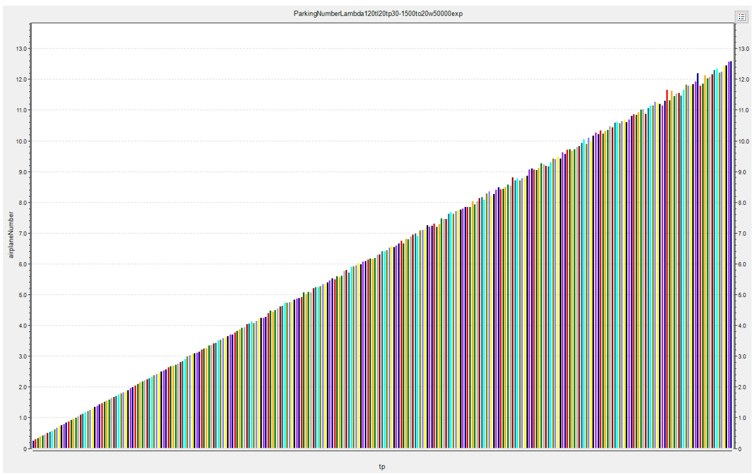


Figure 67: Average number of airplanes: λ=120s, t_l=20s, t_p=[30,1500]s step=5s, t_o=20s, warm up=$5 \cdot 10^4$s, time=10^6s

4.4.4 Conclusions

Sensitivity:

	MeanNumberParking
Workload	Exponential
t_l	Constant
t_p	Linear
t_o	Constant

The workload and t_p influence the number of airplanes in the parking area. Specifically, as the workload increases the average number of airplanes in the parking area increases exponentially, while as t_p increases the average number of airplanes in the parking area increases linearly.