

Deficit Scheduler

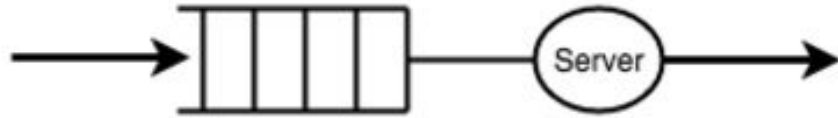
Project 13

Gabriele Lagani
Luca Maddau
Indrit Kertusha

Model

FIFO Queue with some periods of inactivity called *vacations*.

Stability Condition



- Like an M/M/1 system

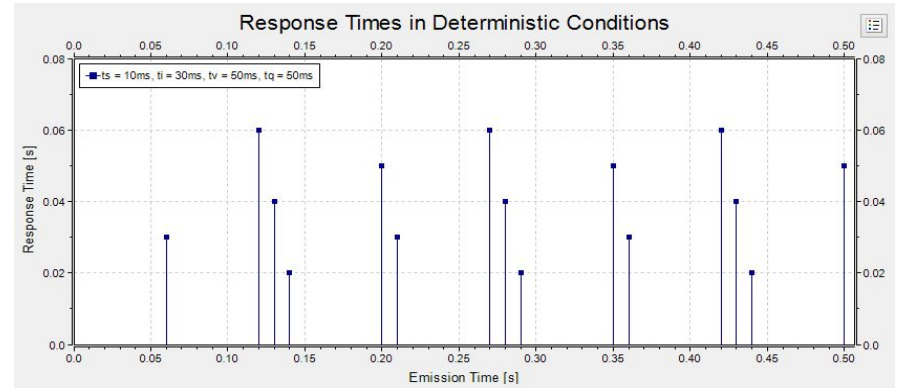
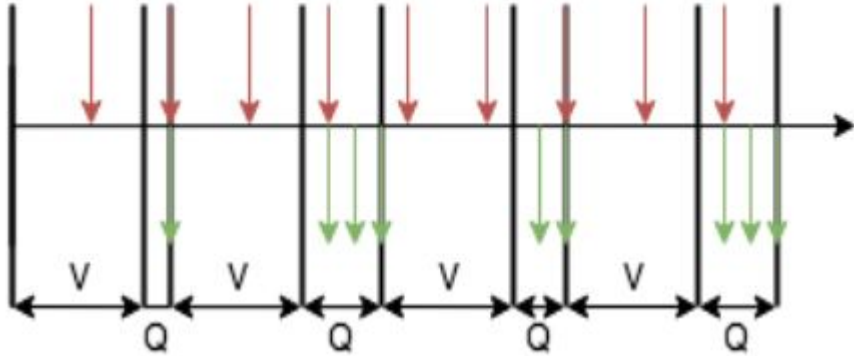
$$t_i > t_s$$

- The num. of jobs that the system can serve within a turn has to be larger than the number of jobs arrived during the previous vacation and the turn itself

$$\frac{t_q}{t_s} > \frac{t_v}{t_i} + \frac{t_q}{t_i}$$

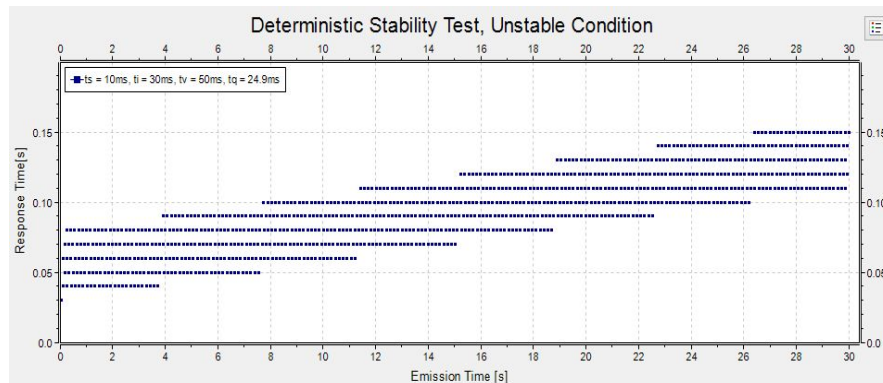
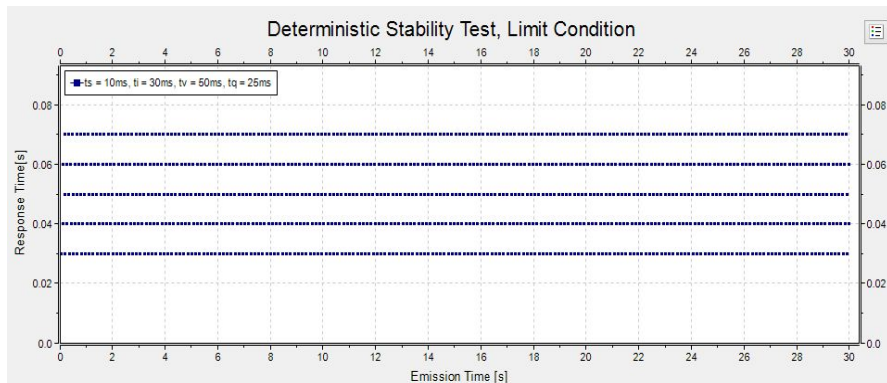
Deterministic Regime Tests

Output of the system tested against handmade computations



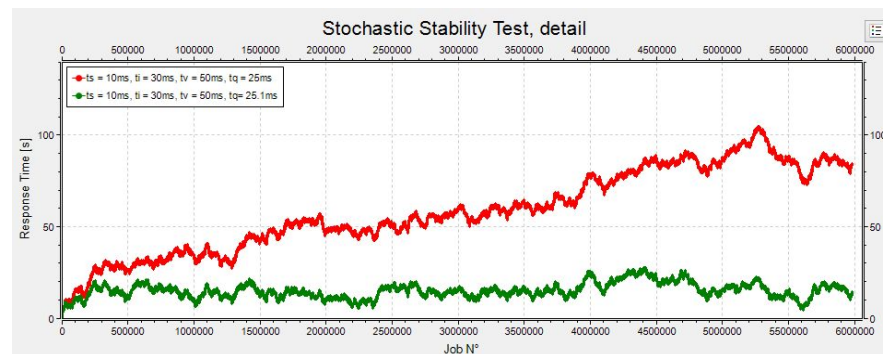
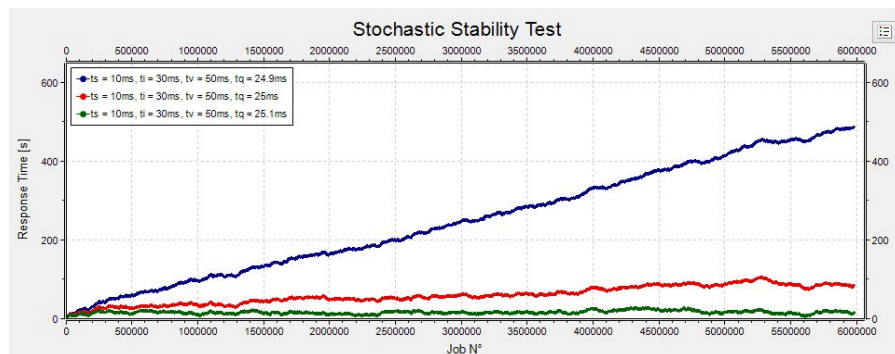
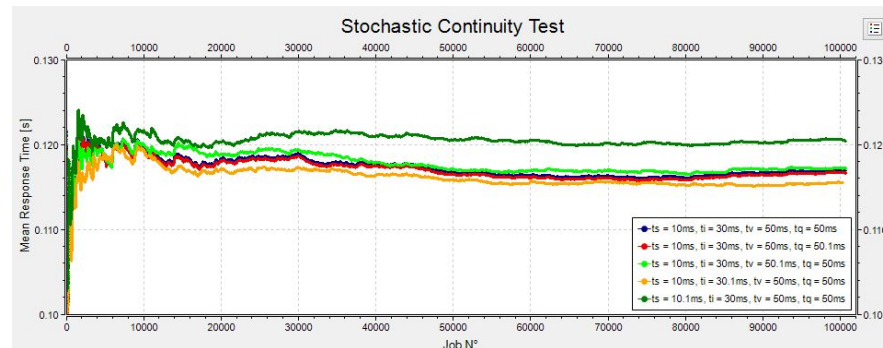
Deterministic Stability Test

- Parameters set to the limit of the stability condition
- Slight change
- The system becomes unstable



Stochastic Regime Tests

- Continuity test
- Stability test



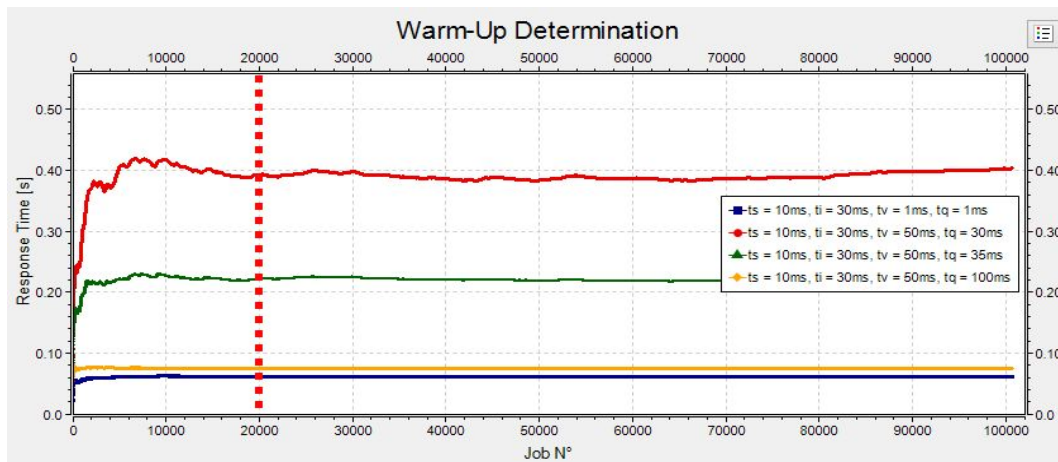
Warm-Up and Simulation Duration Determination

- Simulation duration was set in order to gather 10^5 observations
- Warm-up time was set in order to remove the first 20000 observations
- Times computed as nt_i

$$n = \left(\frac{z_{\alpha/2} S}{rX} \right)^2$$

$\sim 10^4$ (pointing to n)
 $\sim 10^0$ (pointing to $z_{\alpha/2}$)
 $\sim 10^{-1}$ (pointing to S)
 $\sim 10^{-3}$ (pointing to rX)

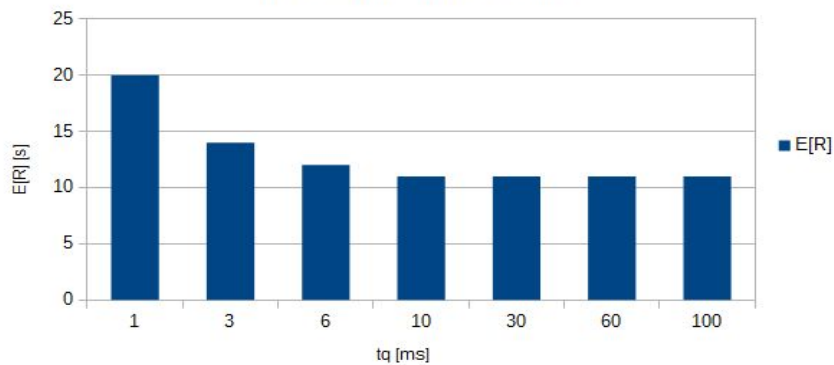
Considering further subsampling $\rightarrow 10^5$



Results

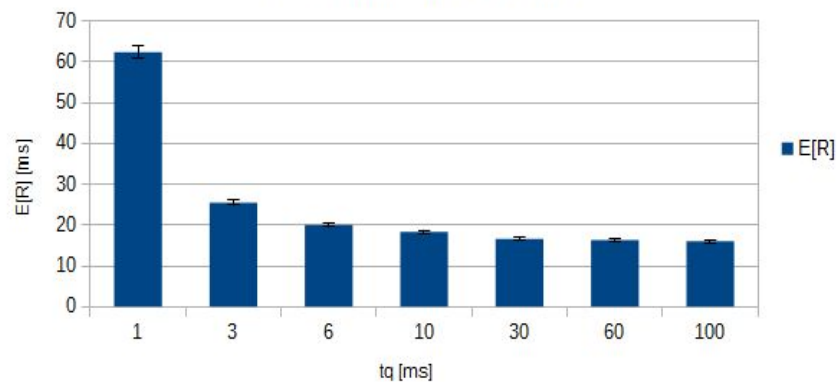
Response Times, Deterministic Regime

$t_s = 10\text{ms}$, $t_i = 30\text{ms}$, $t_v = 1\text{ms}$

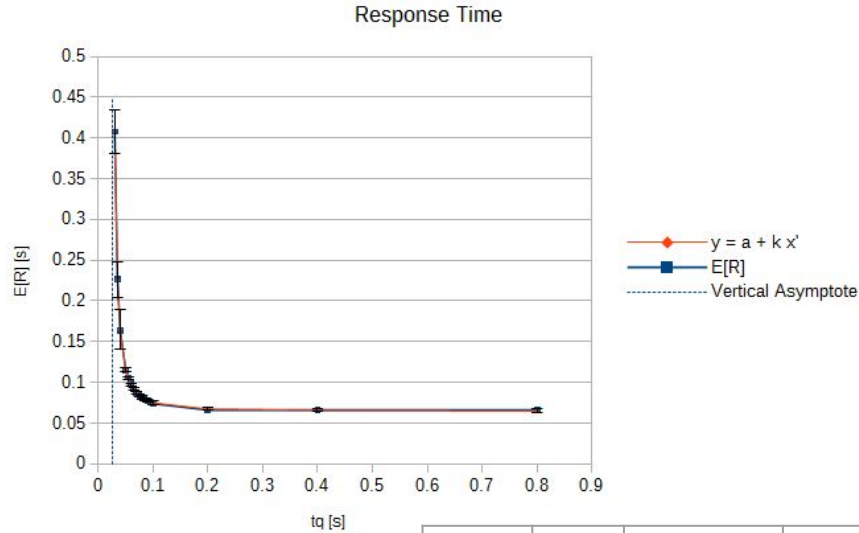


Response Times, Stochastic Regime

$t_s = 10\text{ms}$, $t_i = 30\text{ms}$, $t_v = 1\text{ms}$



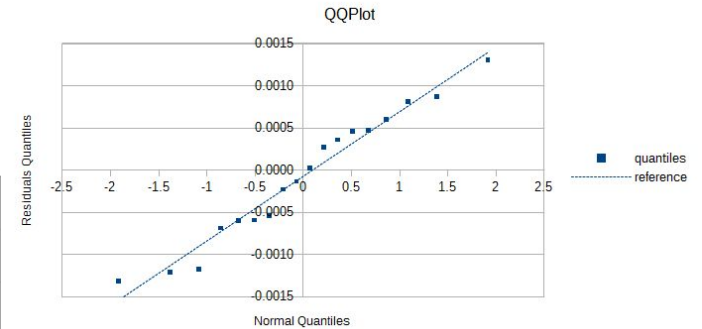
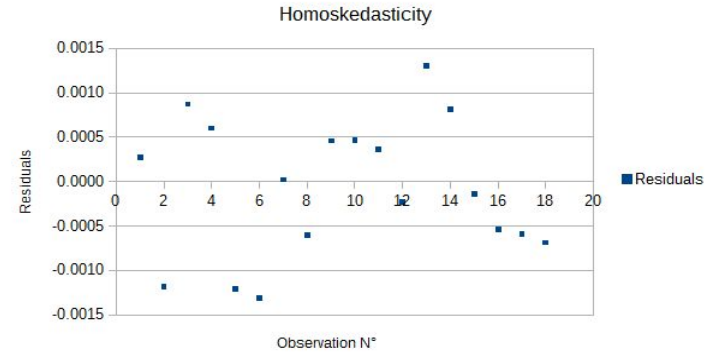
Response Time in Function of t_q



$$x' = \frac{1}{x - b} \quad x \leq 40ms$$

$$x' = \frac{1}{x^2 - b^2} \quad x > 40ms$$

x	a	95% CI for a	k	95% CI for k
< 40ms	0.043	$a \pm 0.014$	0.0018	$k \pm 0.0002$
> 40ms	0.065	$a \pm 0.0001$	0.000095	$k \pm 3E-06$



Fitting (1)

- It is interesting to find a distribution that fits the data
 - Response time of job j made up by two components:
 - Sum of service times of jobs before j and j itself, like in M/M/1 -> exponential
 - Delay due to vacations -> exponential
- $E[R] = \frac{1}{\lambda} \frac{\rho}{1-\rho} + V$
- Sum of two independent exponentials with different rate -> hypoexponential distribution

$$\text{PDF: } f(x) = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} (e^{-x\lambda_2} - e^{-x\lambda_1})$$

MLE:

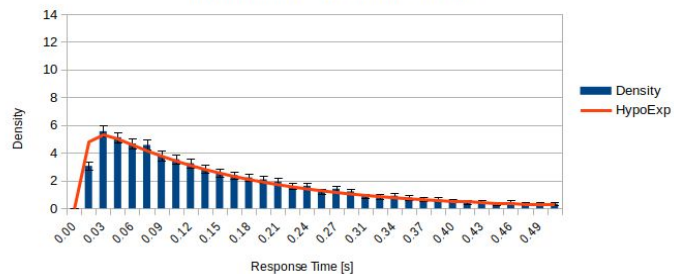
$$\lambda_1 = \frac{2}{\bar{x}} \left[1 + \sqrt{1 + 2(c^2 - 1)} \right]^{-1}$$

$$\lambda_2 = \frac{2}{\bar{x}} \left[1 - \sqrt{1 + 2(c^2 - 1)} \right]^{-1}$$

Fitting (2)

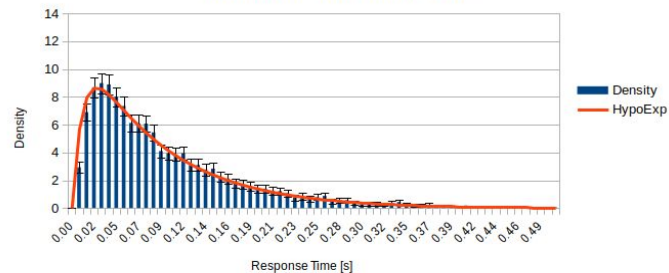
Histogram

ts = 10ms, ti = 30ms, tv = 50ms, tq = 30ms
hypo: lambda1 = 6.39, lambda2 = 105.99



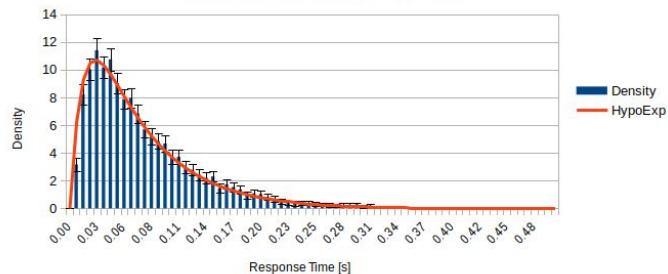
Histogram

ts = 10ms, ti = 30ms, tv = 50ms, tq = 60ms
hypo: lambda1 = 11.72, lambda2 = 95.59



Histogram

ts = 10ms, ti = 30ms, tv = 50ms, tq = 100ms
hypo : lambda1 = 16.65, lambda2 = 72.52



QQPlot

ts = 10ms, ti = 30ms, tv = 50ms, tq = 100ms
hypo : lambda1 = 16.65, lambda2 = 72.52

