

Module - 2

Random Variables

outcomes!

$$X: \mathcal{S} \rightarrow \mathbb{R}, n \in \mathbb{R}$$

One-Dimensional Random Variable

$$X(\mathcal{S}) = \mathbb{R},$$

One-Dimensional Random Variables

Discrete Random Variable

A R.V. 'X' is said to be discrete R.V. if it fulfills the following

Conditions:

- (i) $P_i \geq 0, \forall i$
- (ii) $\sum_i P_i = 1, \forall i$
- probability mass function (pmf)

Continuous Random Variable

A con. R.V. X is said to be
a probability density function (pdf)
if it satisfies,

$$(i) \quad f(x) \geq 0 \quad \forall x \in \mathbb{R}$$

$$(ii) \quad \int_{-\infty}^{\infty} f(x) dx = 1.$$

Cumulative Distribution function (or) Distribution function (CDF)

A discrete or continuous R.V. has
a Cumulative D.F. denoted by

$F(x)$ and defined by

$$F(x) = P[X \leq x] = \begin{cases} \sum_x p_i, & \text{Discrete case} \\ \int_{-\infty}^x f(x) dx, & \text{Cont..} \end{cases}$$

Relationship between PDF and CDF

$$f(x) = \frac{d[F(x)]}{dx}$$

Note :-

$$\text{Mean} = \int_{-a}^a x f(x) dx$$

$$= \sum_i x p(x)$$

$$\text{Variance} = \int_{-\infty}^{\infty} (x - \text{mean})^2 f(x) dx$$

(k₂)

$$= \sum_i (x_i - \text{mean})^2 p(x_i)$$

Sh. Janssens

$$\mu_2 = \int_{-\infty}^{\infty} (x - \text{mean})^2 f(x) dx$$

$$\underline{\mu_0, \mu_1, \mu_2, \dots}$$

$$\left. \begin{array}{l} \mu_0 = 1 \\ \mu_1 = 0 \end{array} \right\}$$

$$\mu_2 = \text{var}$$

$$\mu_3 = \text{Skew}$$

$$\mu_4 = \text{Kurtosis}$$

⋮

Ex

1. Given is the prob. distribution of X ,

$X :$	-2	-1	0	1	2	3
$P(X) :$	0.1	k	0.2	$2k$	0.3	$3k$

find the value of k , $P(X > -1)$

$P(-1 < X < 2)$ and also

cdf

Soln :-

$$\sum_i P_i = 1$$

$$0.1 + K + 0.2 + 2K + 0.3 + 3K = 1$$

$$0.6 + 6K = 1$$

$$6K = 1 - 0.6 = 0.4$$

$$K = \frac{0.4}{6} = \frac{1}{15}$$

$x:$	-2	-1	0	1	2	3
$P(x):$	0.1	$\frac{1}{15}$	0.2	$\frac{2}{15}$	0.3	$\frac{3}{15}$

$$\begin{aligned}
 \text{c.ii)} \quad P[X > -1] &= 1 - P[X \leq -1] \\
 &= 1 - \{P(X = -1) + P(X = -2)\} \\
 &= 1 - \left\{ \frac{1}{15} + \frac{1}{10} \right\} \\
 &= 1 - \left\{ \frac{25}{150} \right\} = 0.83
 \end{aligned}$$

$$\begin{aligned}
 \text{cii) } P(-1 < X < 2) &= P[X=0] + P[X=1] \\
 &= 0.2 + 0.13 \\
 &= 0.33
 \end{aligned}$$

C d f :-

$X:$	-2	-1	0	1	2	3
$P(X):$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{2}{10}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{3}{15}$
$F(X):$	$\frac{1}{10}$	$\frac{1}{6}$	$\frac{11}{30}$	$\frac{15}{30}$	$\frac{24}{30}$	1 //

2) A R.V has the following prob-
distribution:

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

find K , $P(X < 6)$, $P(X \geq 6)$, $F(x)$.