FROG LEAP ALGORITHM

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WHAT IS LEAP FROG ALGORITHM?

- It's a method used in numerical integration
- It's relatively easy to implement
- It works great for simulation of phenomena over long period of time:
  - It does not require to many data point per oscillation
  - It does not require so much computing power

HOW DOES IT WORK?

Steps and half steps

We divide time in our simulation to pieces of length Δt

We know:

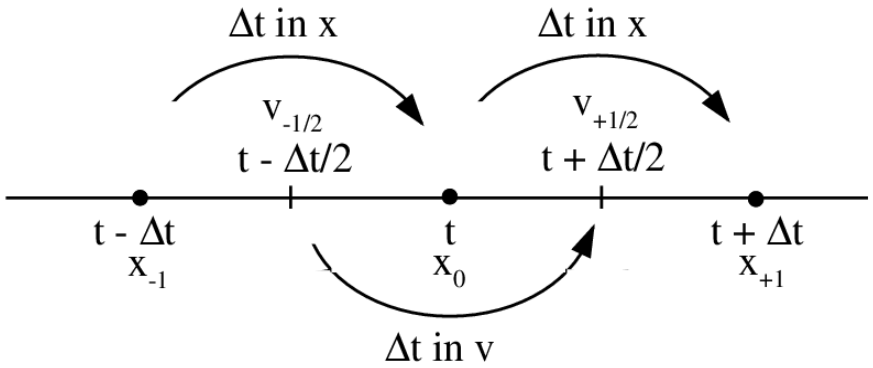
- Position in t=0
- Acceleration in t=0
- Velocity in t=-Δt/2

First we calculate velocity after one step using this formula:

- $v(t+\Delta t/2) = v(t-\Delta t/2)+a(t)\Delta t$

Then we can calculate position

- $x(t+\Delta t) = x(t)+v(t+\Delta t/2)$



Two-Dimensional Modelling of the Hall Thruster Discharge: Final Report - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Leapfrog-integration-scheme-for-macroparticle-motion_fig12_235043233 [accessed 17 Mar 2025]

PROJECTS FOR THIS METHOD:

- Simulating movement of planet around the sun
- Simulating movement of particle in magnetic field

USEFUL PHYSICS AND MATH FOR FIRST PROJECT

- Vectors can be expressed as arrays in python language
- Vectors have components along the axis (2D, 3D etc.)
- Gravity is given by formula:

$$\vec{\mathbf{F}}_{12} = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$$

- This formula contain vectors, versors and scalars

USEFUL PHYSICS AND MATH FOR FIRST PROJECT

- To simulate particle moving in magnetic field we need to use Lorentz formula

$$\overrightarrow{F} = q \cdot \vec{v} x \vec{B}$$

- Notice that x in this equation means vector product