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June 2025

Introduction and Motivation

- Research Question: How does the inclusion of stochastic effects influence the economic evaluation in the context of epidemics?
- How do we perform these economic evaluations? Using epidemiological-economic (epi-econ) models, which:
 - 1 Incorporate individual behavioral responses to infection risk.
 - 2 Capture the **trade-off** between health outcomes and economic variables.

- Most epidemiological-economic models are developed by economists and include deterministic transmission rates.
- A small number of papers use stochastic transmission rates in epi-econ models, but few compare them directly to deterministic models.
- Hong et al. (2021): Focuses on how stochasticity affects asset pricing, not welfare analysis.
- Our Project: How do economic evaluations differ between deterministic and stochastic transmission models?
 - \Rightarrow We introduce stochasticity into an epi-econ model to explore this gap.

Why Do We Expect a Difference Between Deterministic and Stochastic Models

- Infection dynamics are non-linear; stochastic effects can lead to different epidemic paths (e.g., early fade-out, secondary waves).
- Stochastic models capture worst-case scenarios relevant for welfare under risk aversion.
- **Behavioral responses** (e.g., reducing activity) depend on observed infections, which fluctuate more under stochasticity.
- Economic cost distributions (health vs. activity) are wider under stochastic models.

Optimal Policy under stochastic dynamics

- In stochastic models, optimal policy interventions may only need to reduce infections to a small positive level, rather than zero, to allow fade-out of the epidemic.
- Deterministic models require driving infections all the way to **zero** to eliminate the epidemic.
- This can introduce a bias in economic evaluation:
 - Stochastic outcomes can reduce expected long-term costs,
- The same intervention may look more cost-effective in a stochastic model

Model Overview

We introduce **stochastic transitions** into an existing, deterministic epi-econ model framework. Our starting point is the model proposed by Farboodi et al. (2021).

- Individuals choose their level of social activity to reduce infection risk.
- Disease dynamics follow a standard SIRD model.
- Laissez-faire vs optimal policy
- This is a very **simple** framework, making it suitable for comparing deterministic vs stochastic economic evaluations.

Setup and Notation

We consider a population of homogeneous individuals who are unaware of their current health state, specifically whether they are susceptible or infected. Each individual's utility u(t), depending on social activity a(t), is described by:

$$u(a(t)) = \log(a(t)) - a(t) + 1$$

This specification ensures that the optimal activity is $a^*(t) = 1$, with a utility level of zero, so it can be interpreted as a **cost**.

Initial Welfare Function (Farboodi et al., 2021)

The individual maximizes the discounted difference between social activity costs and infection costs:

$$\max_{a(t)} \int_0^\infty e^{-(\rho+\delta)t} \left[\text{-social activity cost} - \text{infection cost} \right] dt$$

- $e^{-(\rho+\delta)t}$: discount factor capturing impatience and cure arrival
- Social activity cost: Utility lost by being less socially active than preferred
- Infection cost: Cost * probability dying

Implementation Challenges

- Attempted to implement this model several times in R.
- Model turned out te be very unstable- Only gets worse when adding stochasticity
- The original model operates in **continuous time**, which makes it badly suited for incorporating stochastic dynamics.
- Our solution: Simplify the welfare function to make it suitable for implementing stochasticity.

Welfare Function

We consider a myopic decision-maker maximizing instantaneous welfare:

$$W(a) = \log(a) - a + 1 - \beta a^2 N_s N_i \kappa$$

- First term: Social activity cost
- Second term: Expected infection cost
- No intertemporal maximization suitable for stochastic modeling

The disease transmission splits the population into:

- Susceptible $N_s(t)$
- Infected $N_i(t)$
- Recovered $N_r(t)$
- Deceased $N_d(t)$

The force of infection is:

Force of infection =
$$\beta a(t)^2 N_s(t) N_i(t)$$

Differential Equations

$$\frac{dN_s(t)}{dt} = -\beta a(t)^2 N_s(t) N_i(t)$$

$$\frac{dN_i(t)}{dt} = \beta a(t)^2 N_s(t) N_i(t) - \gamma N_i(t)$$

$$\frac{dN_r(t)}{dt} = (1 - \pi) \gamma N_i(t)$$

$$\frac{dN_d(t)}{dt} = \pi \gamma N_i(t)$$

Optimal Activity Level

Take the derivative of W(a):

$$\frac{1}{a} - 1 = 2\beta a N_s N_i \kappa$$

Define $M := \beta N_s N_i \kappa$. Solving this yields:

$$a^* = \frac{-1 + \sqrt{1 + 8M}}{4M} = \frac{-1 + \sqrt{1 + 8\beta N_s N_i \kappa}}{4\beta N_s N_i \kappa}$$

To better capture randomness in disease spread, we use binomial processes following Abrams et al. (2021) rather than stochastic β -shock only models.

Infection Transition Probability

- Rate of infection = $\beta a(t)^2 N_s(t) N_i(t)$
- $p_i = 1 \exp(-\beta a(t)^2 N_i(t))$
- $I_{\text{new},t+1} \sim \text{Binomial}(N_s(t), p_i)$

Incorporating Stochasticity in the Model

Recovery and Death Transitions

Recovery:

- Rate: $(1-\pi)\gamma N_i(t)$
- Probability: $p_r = 1 \exp(-(1 \pi)\gamma)$
- $Arr R_{\text{new},t+1} \sim \text{Binomial}(N_i(t), p_r)$

Death:

- Rate: $\pi \gamma N_i(t)$
- Probability: $p_d = 1 \exp(-\pi \gamma)$
- $D_{\text{new},t+1} \sim \text{Binomial}(N_i(t), p_d)$

Discrete-Time Population Dynamics

$$N_s(t+1) = N_s(t) - I_{\text{new},t+1}$$
 $N_i(t+1) = N_i(t) + I_{\text{new},t+1} - R_{\text{new},t+1} - D_{\text{new},t+1}$
 $N_r(t+1) = N_r(t) + R_{\text{new},t+1}$
 $N_d(t+1) = N_d(t) + D_{\text{new},t+1}$

Updated Simulation Results

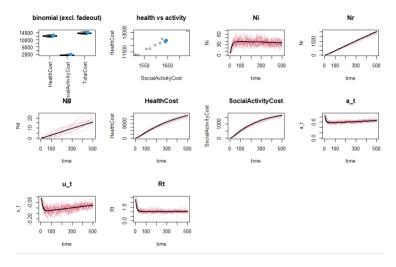
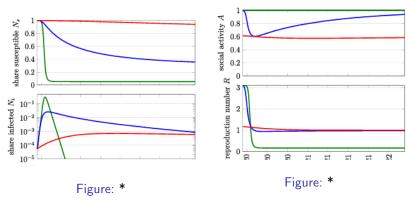


Figure: Simulation results using the revised welfare specification.

- Variation in social and health costs is limited.
- Average almost equal to median... Implying that we see no real fade-out effects.
- Optimal policy implementation doesn't work so far...
- Tried playing around with R0 and initial Ns or Ni, but no big differences so far...

Conclusion

- How does stochasticity affect economic evaluation in epidemic models?
- To do so, we simplify the welfare function used in **Farboodi** et al. (2021) to make it tractable in a stochastic, discrete-time setting.
- Results show a divergence in economic evaluation outcomes under stochastic dynamics.
- Pity that optimal policy isn't working out for stronger conclusions.



Susceptibles and infected shares

Social activity and reproduction number

Figure: Results from Farboodi et al. (2021).