

# Lab session 1: Signals and Systems

## Problem 1: Cartesian coordinates to Polar coordinates

Write a program that reads a Cartesian coordinate  $(x, y)$  from the input and converts it into the corresponding Polar coordinate  $(r, \theta)$ . The input of this problem consists of two integers:  $x$  and  $y$ . The output of your program should be two floating point numbers (use `doubles` in your programs):  $r$  (length, so a non-negative number) and  $\theta$  (in radians). The output values should be rounded and have two digits after the decimal dot. Note that, by convention,  $-\pi \leq \theta < \pi$ .

### Example 1:

input:  
2 0  
output:  
2.00 0.00

### Example 2:

input:  
0 2  
output:  
2.00 1.57

### Example 3:

input:  
-2 0  
output:  
2.00 -3.14

## Problem 2: Polar coordinates to Cartesian Coordinates

Write a program that reads a Polar coordinate  $(r, \theta)$  from the input and converts it into the corresponding Cartesian coordinate  $(x, y)$ . The input consists of two floating point numbers:  $r$  (a non-negative number) and  $\theta$  (in radians). The output of your program should be two floating point numbers (rounded to two digits after the decimal dot):  $x$  and  $y$ .

### Example 1:

input:  
1.00 1.57  
output:  
0.00 1.00

### Example 2:

input:  
1.00 0.00  
output:  
1.00 0.00

### Example 3:

input:  
1.00 -1.57  
output:  
0.00 -1.00

## Problem 3: Summing two sinusoids having the same frequency

The input of this problem consists of 5 floating point values:  $f$  (in Hz),  $A_0$ ,  $\varphi_0$  (in radians),  $A_1$ , and  $\varphi_1$  (in radians). Write a program that outputs the equation for the sum of the signals  $A_0 \cos(2\pi ft + \varphi_0) + A_1 \cos(2\pi ft + \varphi_1)$  in the form  $x(t) = A \cos(2\pi ft + \varphi)$ . Note that the equation should be  $x(t) = 0.00$  if  $A = 0$ . All numbers in the output must be rounded and have two digits after the decimal dot.

### Example 1:

input:  
42 1 1.047198 1 0.523599  
output:  
 $x(t) = 1.93 * \cos(2 * \pi * 42.00 * t + 0.79)$

### Example 2:

input:  
1 42 0 42 -3.1415927  
output:  
 $x(t) = 0.00$

### Example 3:

input:  
42 100 0.78540 100 2.35619  
output:  
 $x(t) = 141.42 * \cos(2 * \pi * 42.00 * t + 1.57)$

## Problem 4: Uniform sampling of a sinusoid

The input of this problem consists of 5 values:  $f_s$  (sampling rate, integer),  $n$  (number of samples, integer),  $A$  (amplitude),  $f$  (in Hz), and  $\varphi$  (in radians). Write a program that outputs  $n$  samples of the discrete time signal that is obtained after uniform sampling of  $x(t) = A \cos(2\pi ft + \varphi)$  using the sampling rate  $f_s$ . The samples are integers. Do not round the samples, but rely on C's conversion of floating point values to integers (i.e. truncation). Make sure that your output has the same format as the following examples.

### Example 1:

input:  
10 10 42 10 0  
output:  
[42, 42, 42, 42, 42, 42, 42, 42, 42, 42]

### Example 2:

input:  
20 10 42 10 0  
output:  
[42, -42, 42, -42, 42, -42, 42, -42, 42, -42]

### Example 3:

input:  
4 10 1 1 -3.14159265  
output:  
[-1, 0, 1, 0, -1, 0, 1, 0, -1, 0]

**Problem 5: Uniform sampling of a sum of sinusoids**

The input of this problem consists of several lines. The first line contains two integers:  $f_s$  (sampling rate), and  $n$  (number of samples). The second line contains a positive integer  $m$  (number of components). Next follow  $m$  lines, each containing three values describing a sinusoid:  $A$  (amplitude),  $f$  (in Hz), and  $\varphi$  (in radians). Write a program that outputs  $n$  samples of the discrete time signal that is obtained after uniform sampling the sum of the  $m$  sinusoids. As in the previous exercise, samples are integers and should not be rounded.

**Example 1:****input:**

20 10

1

42 10 0

**output:**

[42, -42, 42, -42, 42, -42, 42, -42, 42, -42]

**Example 2:****input:**

20 10

2

42 10 0

42 10 0

**output:**

[84, -84, 84, -84, 84, -84, 84, -84, 84, -84]

**Example 3:****input:**

20 10

2

42 10 0

42 10 -3.14159265

**output:**

[0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

**Problem 6: Aliases?**

The input of this problem consists of three lines. The first line contains a positive integer  $f_s$  (the sampling rate). The remaining two lines each contain the parameters that define a sinusoid:  $A$  (amplitude),  $f$  (in Hz), and  $\varphi$  (in radians). Write a program that outputs YES if uniform sampling of the two sinusoids using the given sampling rate yields the same discrete output. Otherwise, it should output NO.

**Example 1:****input:**

100

1 10 0

1 20 0

**output:**

NO

**Example 2:****input:**

100

1 100 0

1 200 0

**output:**

YES

**Example 3:****input:**

100

50 120 0

50 80 0

**output:**

YES

**Problem 7: Microphones**

Two sound sources are placed on the x-axis, i.e. at locations  $(x_0, 0)$ , and  $(x_1, 0)$ . Both sources produce the same sinusoid. Next, a microphone is also placed on the x-axis at location  $(x_2, 0)$ . The objective is to compute the amplitude of the sinusoid that is observed by the microphone. Take the speed of sound to be 343 m/s, and ignore sound degradation.

The input of this problem consist of two lines. The first line defines the sinusoid:  $A$  (amplitude),  $f$  (in Hz), and  $\varphi$  (in radians). The second lines contains the three locations  $x_0$ ,  $x_1$  and  $x_2$  (in meters). The output of the program must be the amplitude of the observed signal, rounded to two digits after the decimal dot.

**Example 1:****input:**

5 100 0

-100 100 0

**output:**

10.00

**Example 2:****input:**

5 100 0

-10 10 1

**output:**

2.58

**Example 3:****input:**

5 100 0

-10 10 -1

**output:**

2.58

**Problem 8: Periodic signal**

The input of this problem consists of several lines. The first line contains a positive integer  $n$ . Next follow  $n$  lines, each containing the parameters that define a sinusoid:  $A$  (amplitude),  $T$  (period, integer, in microseconds), and  $\varphi$  (in radians). The sum of the  $n$  sinusoids is also periodic. The output of your program must be the fundamental period  $T_0$  (integer, in microseconds) of this sum.

**Example 1:****input:**

4  
4 3 0.79  
9 6 1.38  
8 9 2.62  
6 4 0.38

**output:**

36

**Example 2:****input:**

10  
5 6 2.51  
1 2 1.19  
9 6 2.09  
1 9 1.40  
4 8 0.42  
3 5 1.19  
8 4 1.13  
8 3 0.80  
8 6 0.51  
6 10 0.26

**output:**

360

**Example 3:****input:**

10  
4 1 1.12  
8 4 1.64  
5 10 2.06  
1 7 0.35  
5 4 2.69  
4 5 0.84  
3 4 1.76  
3 7 0.63  
6 7 2.62  
8 1 2.25

**output:**

140

**Problem 9: Sum to Product**

The input of this problem consists of two frequencies:  $f_0$  and  $f_1$  (both in Hz, integer). These input values define the signal  $x(t) = \cos(2\pi f_0 t) + \cos(2\pi f_1 t)$ . This signal is a beat note (assuming  $f_0 \neq f_1$ ), which can be rewritten as  $x(t) = A \cos(2\pi f_2 t) \cos(2\pi f_3 t)$ . The output of your program must be the frequencies  $f_2$  and  $f_3$  (where  $f_2 < f_3$ ). The output must be rounded to two digits after the decimal dot.

**Example 1:****input:**

672 648

**output:**

12.00 660.00

**Example 2:****input:**

362 134

**output:**

114.00 248.00

**Example 3:****input:**

919 732

**output:**

93.50 825.50