A Flexible Generator of Constrained Global Optimization Test Problems

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Abstract. In the present work, further development of an approach to constructing test global optimization problems with nonlinear constraints is considered. In the generated problems, the location of the global minimum is known. The considered generator is featured by an option of specifying desired number of constraints and the fraction of feasible domain relative to the whole global search domain. In the updated version of the generator, the options of forming the problems with a conditional minimum located at the boundary of the feasible domain and of controlling the number of constraints active at the minimizer have been added. The demonstration of the developed approach is given on the example of a known index method for solving complex multiextremal optimization problems with non-convex constraints.

INTRODUCTION

In the present paper, the methods for generating global optimization test problems with non-convex constraints

$$\varphi(y^*) = \min\{\varphi(y) : y \in D,\ g_i(y) \le 0,\ 1 \le i \le m\}, \tag{1}$$

$$D = \left\{ y \in R^N : a_i \le y_i \le b_i, 1 \le i \le N \right\} \tag{2}$$

are considered. The objective function $\varphi(y)$ (henceforth denoted by $g_{m+1}(y)$) and the left-hand sides $g_i(y),\ 1 \le i \le m$, of the constraints are supposed to satisfy the Lipschitz condition

$$\left| g_i(y') - g_i(y'') \right| \le L_i \left\| y' - y'' \right\|,\ y', y'' \in D,\ 1 \le i \le m+1,$$

with the Lipschitz constants unknown a priori. The analytical formulae of the problem functions may be unknown, i.e. they may be defined by an algorithm for computing the function values in the search domain (so called "black-box"-functions).

The problem of developing the generators of the multiextremal test problems with non-convex constraints is relevant since classes of test problems are used usually for investigating and comparing global optimization algorithms. Well known generators construct the problems without constraints as a rule. For example, a method for constructing two-dimensional test functions has been proposed by Grishagin (its description can be found in [1, 2]). Another generator (*GKLS generator*) for the functions of arbitrary dimensionality has been proposed in [3]. GKLS generator has been successfully applied in the investigation of several global optimization methods [4, 5, 6]. Based on the GKLS generator, an approach to constructing the constrained problems has been proposed (Emmental-type GKLS-based test problems [7]). In this generator, the modified GKLS-function was used as the objective function, the constraints were the exterior of the balls with given centers and radii. The use of other objective functions and/or constraints was not allowed that was a disadvantage of this generator.

In the present work, further development of the approach to generating the global optimization problems with non-convex constraints proposed originally in [8] has been conducted. When generating the test problems, the necessary number of constraints and the desired fraction of the feasible domain relative to the whole global search domain can be specified. At that, the location of the global minimum is known for the generated problems that essentially simplifies evaluating the results of the numerical experiments. In addition to the specified options, the following functions have been added in the current version of the generator:

- The possibility to generate the problems with the constrained minimizer located at the boundary of the feasible domain.
- The possibility to control the number of constraints active at the optimum point.

The above capabilities allow emulating the properties of the applied constrained global optimization problems more adequately.

TEST PROBLEM CLASSES

The generator GCGen (Global Constrained optimization problem Generator) which allows to generate the test global optimization problems with m constraints has been proposed in [8]. In the mentioned paper the rules which allow formulating the constrained global optimization problems have been proposed so that:

- one could control the size of feasible domain with respect to the whole domain of the parameters' variation;
- the global minimizer of the objective function would be known a priori taking into account the constraints;
- the global minimizer of the objective function without accounting for the constraints would be out of the feasible domain;
- the value of the gradient at the global solution of the constrained problems is zero.

The location of the minimizer at the boundary of the feasible domain is an important property featuring the applied constrained optimization problems. When generating the problems according to the scheme described in [8], the constrained minimum point, in general, will be located inside the feasible domain. Therefore, the options of shifting the minimizer to the boundary of the feasible domain as well of controlling the number of the active constraints at the global solution have been added in the new version of the problem generator.

The shifting of the minimizer to the boundary of the feasible domain is performed in the following way.

1. A problem with an arbitrary location of the constrained minimum point $\bar{y}$ is generated;
2. From the point $\bar{y}$, a coordinate search of a feasible point y^*, for which at least one constraint is active (i.e. there is such index j, $1 \le j \le m$ that $\left|g_j(y^*)\right| \le \delta$) is performed with given step h.
3. A rough estimate of y^* obtained is refined by a local method.
4. The coordinate transformation transferring the point $\bar{y}$ to the point y^* is performed. This way, the objective function reaches its minimum at the boundary of feasible domain.

Note that since the search of the point y^* is performed in each coordinate separately, the computation costs would increase linearly with increasing the dimensionality of the problem. At that, no more than $2N\,\|b-a\|\,/h$ computations of the objective function and constraints values will be required.

In the case, when it is necessary to generate a problem with a number of active constraints at the minimizer not less than S, the rules are modified as follows.

1. A problem with an arbitrary location of a constrained minimizer $\bar{y}$ is generated;
2. From the point $\bar{y}$, a coordinate search of a feasible point y^*, for which at least one constraint is active (i.e. there is such index j, $1 \le j \le m$, that $\left|g_j(y^*)\right| \le \delta$) is performed with a predefined step h.
3. A rough estimate y^* obtained is refined by a local method.
4. The number K of the constraints active at the point y^* is determined.
5. If $K < S$, $S - K$ constraints are selected from the set of the inactive ones, and these ones are set to be the active by adding a positive parameter q to the right hand side.
6. The coordinate transformation transferring the point $\bar{y}$ to the point y^* is performed. This way, the objective function reaches its minimum at the boundary of the feasible domain with the number of active constraints not less than S.

NUMERICAL RESULTS

As an illustration, the level lines of the objective functions and the zero-level lines of three constraints for the problem constructed on the base of functions from [2] are shown in Fig. 1. The feasible domains are highlighted by green. Figures 1 (a,b,c,d) correspond to different methods of generating the test problems.

(a) The problem has been generated with the constrained optimizer located inside the feasible domain.
(b) A modification of the problem from item (a): unconstrained optimizer is located outside the feasible domain.
(c) A modification of the problem from item (b): constrained optimizer is located at the boundary of the feasible domain, the number of active constraints is not predefined.
(d) A modification of the problem from item (c): the number of active constraints is defined to be two.

Note that the volume fraction of the feasible domain for the first three problems will be equal to $\Delta = 0.5$, and will be no more than $\Delta = 0.5$ for the last problem (because the volume of feasible domain can decrease when generating the problems with controlled number of active constraints).

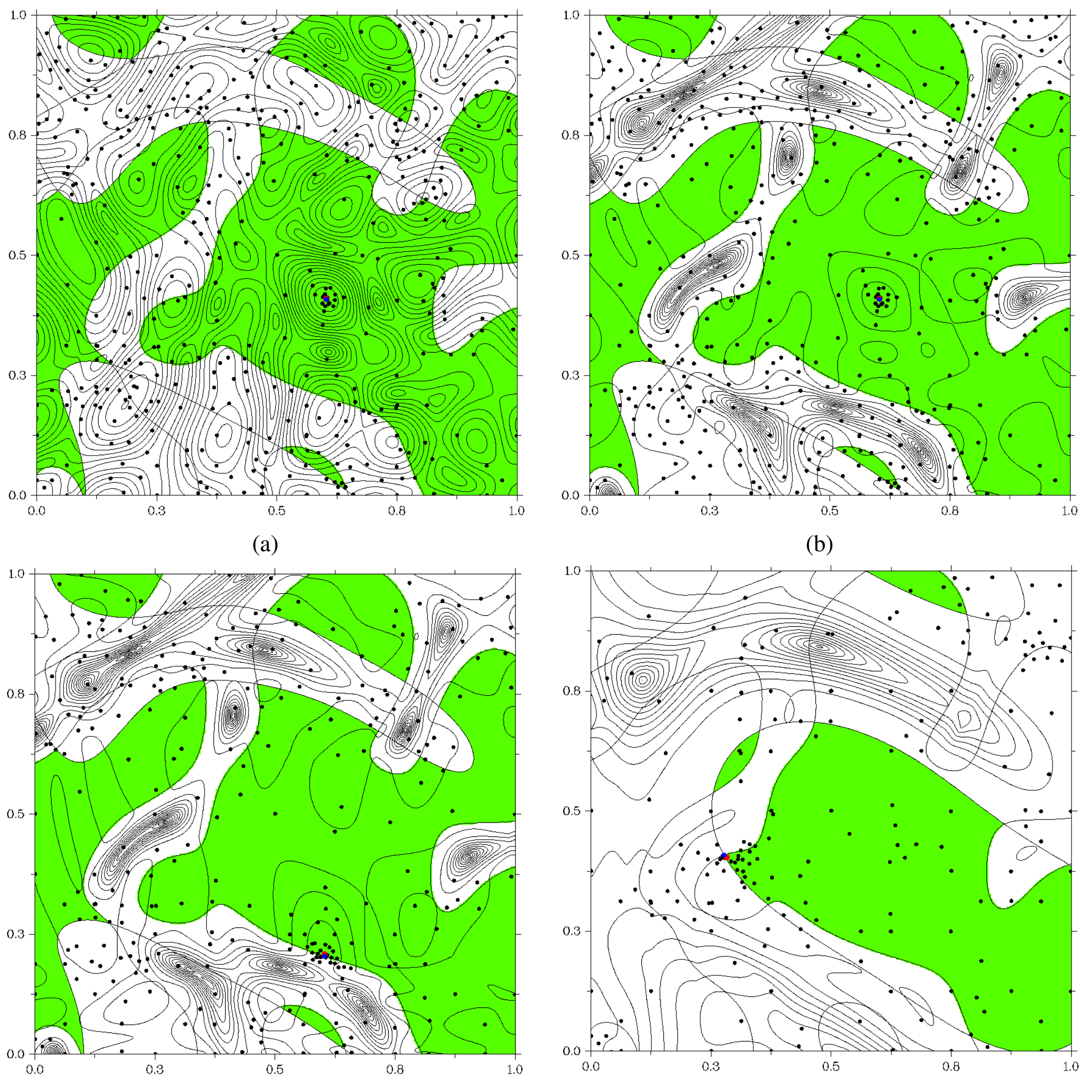

FIGURE 1. The test problems generated by GCGen

Figures 1 (a,b,c,d) also shows the points of 425, 381, 289 and 188 trials, correspondingly, performed by the *index method* for solving constrained global optimization problems until the required accuracy $\epsilon = 10^{-2}$ was achieved. The conditional optimizer is shown as a red point and the best estimation of the optimizer is shown as a blue point. The index method has been developed in [9, 10, 11]. The approach is based on a separate accounting for each constraint of the problem and is not related to the use of the penalty functions.

The index objective function utilized in the method essentially differs from the classical penalty one since the computing of the initial objective function value outside the feasible domain is not required in the index method. Moreover, it is not required to compute the values of all constraints outside the feasible domain as well but until the occurrence of the first violated one only whereas in the classical penalty problems all constraints along with the objective function are computed at every point. This allows the index method: (i) accounting for the information on each constraint separately and (ii) solving the problems, in which the function values may be undefined out of the feasible domain.

Conclusion

This paper considers the method for generating the global optimization test problems with non-convex constraints, which allows:

- controlling the size of the feasible domain with respect to the whole domain of the parameters' variation;
- knowing a priori the conditional global minimizer of the objective function;
- generating the unconditional global minimizer of the objective function out of the feasible domain;
- generating the problems with a conditional minimizer located at the boundary of the feasible domain;
- controlling the number of constraints active at the optimum point.

The developed approach allows generating any number of test global optimization problems with non-convex constraints for performing computational experiments in order to obtain a reliable evaluation of the efficiency of the developed optimization algorithms.

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