Ranking-based Algorithm for Facility Location with Constraints

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Abstract. The random search algorithm based on ranking of the search space elements is applied to solve the competitive facility location problem with constrain for the minimal market share for a new facility. The strategy for constraint handling is proposed and experimentally evaluated by solving complex instances of competitive facility location problem using real geographical data.

INTRODUCTION

The facility location (FL) deals with the determination of the optimal location for a facility (or a set of them) and is important for firms providing services to customers in a certain geographical region. There are a lot of models of facility location proposed in literature, e.g. [1, 2, 3, 4], which differ on their properties such as location space, describing possible locations for the facilities being located, attractiveness of facilities, or behavior of customers when choosing the most attractive facility to get a service.

In general a FL problem can be expressed as a mathematical optimization problem to find a decision variable, describing locations of the new facilities, which would be optimal with respect to a given objective function, describing the fitness of the new locations, e.g. obtained market share, costs of establishment, undesirable effect, etc.

Our research is focused on a discrete competitive FL problem (CFLP) for an entering firm, considering that a new firm wants to enter the market, where some firms already provide goods or service to the customers. All preexisting facilities are described by geographical coordinates and a quality indicator. All customers are aggregated to a set I of demand points each of which is described by geographical coordinates and a demand for a good or a service, which is expressed by the number of potential customers. The customers behavior is based on the partially binary rule [5], assuming that the full demand of a single demand point is served by all firms, but the customers patronize only one facility per firm – the one with the maximum attraction. Then the demand is split between those facilities in proportion with their attraction. The attraction that a customer feels to a facility is proportional to the quality indicator of the facility (which is known) and inversely proportional to the geographical distance between the customer and the demand point.

The goal of the entering firm is to locate a set of s new facilities $X = \{x_1, x_2, \ldots, x_s\}$ with respect to maximization of the total market share $M(X)$ of the new facilities. Here x_i stands for the index of a location in a given set of candidate locations $L = \{l_1, l_2, \ldots\}$.

In order to guarantee a profit for each new facility a constraint for minimal market share (α) is involved, which guarantees that each of the new facilities will attract at least α part of the market share. Solutions which do not satisfy the minimal market share constraint are considered as unfeasible and cannot be considered as possible solutions of the problem.

RANKING-BASED RANDOM SEARCH

The RDOA was proposed and adopted to solve discrete facility location problems in [6]. It starts with a randomly generated solution $X \subset L$, which is considered as the best solution found so far. A new solution $X' = \{x'_1, x'_2, \ldots, x'_s\}$ is generated by changing elements of the best known solution X to another ones, randomly selected from the set of all possible candidate locations. Each location $x_i \in X$ is changed with probability $1/s$ or is used to form X' without change with probability $1 - 1/s$. In the case of change, each candidate location $l_i \in L$ has a probability π_i to be selected to form X', which is evaluated by

$$\pi_i = \frac{r_i}{d(l_i, x_k) \sum_{j=1}^{|L|} \frac{r_j}{d(l_j, x_k)}}, \tag{1}$$

where r_i is a rank of candidate location l_i and $d(l_i, x_k)$ is a geographical distance between candidate location $l_i \in L$ and candidate location $x_k \in X$ which is being changed ($k = 1, 2, \ldots, s$).

The ranks of all candidate locations initially are equal to 1 and are automatically adjusted depending on success and failures when generating a new solution. If market share $M(X')$ captured by the new solution is grater than the market share $M(X)$ of the best known solution, then (1) the ranks of all locations which forms better solution X' are increased by one and (2) the ranks of all locations that forms outperformed solution X, but do not form X' are reduced by one. If $M(X')$ is not greater than $M(X)$, then the ranks of all candidate locations forming unsuccessfully generated solution X', but do not form the best known solution X, are reduced by one.

If the newly generated solution outperforms the best solution found so far, then X is changed by X' and iteration is assumed to be successful; otherwise, X remains unchanged and iteration is assumed to be unsuccessful.

For more details and justification of the strategy we refer to [6] where it was proposed and investigated.

CONSTRAINT HANDLING

The RDOA has been modified by involving constraint handling and has been applied to solve the above competitive facility location problem with the constraint for minimal market share.

Following the given constraint, a solution where at least one facility do not attract the minimal value α of market share is treated as not feasible and cannot be considered as candidate solution for the problem. In order do not stuck with an initial solution and move towards a feasible one, the non-feasible solutions are compared in sense of violation of constraints – a solution with smaller constraint violation is considered as better one comparing to a solution which has larger violation.

Lets denote by $m_1, m_2, \ldots, m_s$ the market share captured by facilities located in $x_i \in X$. Then violation of the minimal market share constraint of facility located in x_i is expressed by market share deficiency: $v_i = \alpha - m_i$, if $m_i < \alpha$ or $v_i = 0$, otherwise. Then total violation of solution X is

$$V(X) = \sum_{i=1}^{s} v_i. \tag{2}$$

Smaller $V(X)$ means slighter violation of the constrain and $V(X) = 0$ means that X fully satisfies the minimal market share constraint. Depending on whether a feasible solution is known or not, the violation or the market share criterion is used to compare two solutions: (1) if X is not feasible, then any newly generated solution X' such that $V(X') < V(X)$ is accepted as the best solution found so far; (2) if X is feasible, then any X' which is feasible and $M(X') > M(X)$ is accepted as the best solution found so far. If any of the above situations occur, then iteration is assumed as successful, and the iteration is assumed as unsuccessful otherwise. Depending on success of the iteration, candidate locations ranks are updated using the same strategy as in RDOA.

EXPERIMENTAL INVESTIGATION

The database with real geographical and population data with 1000 largest municipalities in Spain has been used to enable the facility location model. The set of 50 most populated demand points was used as the set of candidate

TABLE 1. Distribution of preexisting locations.

Chain	Indices of demand points	
	3 facilities per chain	5 facilities per chain
J_1	1, 4, 7	1, 4, 7, 10, 13
J_2	2, 5, 8	2, 5, 8, 11, 14
J_3	3, 6, 9	3, 6, 9, 12, 15

locations L. It was considered that there are 3 preexisting firms J_1, J_2, and J_3 with 3 and 5 facilities per firm, located in the most populated demand points. Distribution of candidate locations among the firms using 3 preexisting facilities per firm and 5 preexisting facilities per firm is presented in Table 1.

The minimal market share for a new facility is expressed as percent of estimated average market share per existing facility after location. The average market share per facility can be expressed by

$$\tilde{m} = \frac{W}{|J_1| + |J_2| + |J_3| + s}, \tag{3}$$

where W is the total demand of all customers in I. Then the minimal market share is considered as 10% to 80% of $\tilde{m}$, depending on the problem instance. The values of minimal market share α is presented in Table 2, assuming that the total demand W is 33208423. Here FPC stands for the number of preexisting facilities per chain.

The RDOA has been applied to solve the CFLP instances described above. Due to stochastic nature of the algorithm, computations have been run for 100 independent runs. The results are presented in Table 3, where average, minimal, and maximal objective values of the best solution are presented in percents of the total market share. Additionally, the number of trials when the maximal solution was found (#Max), the number of trials when a feasible solution was not found (#FNF), and average number of function evaluations required to find a feasible solution (#FFA) are presented in the table. The #FFA was recorded at every thousand of function evaluations.

One can see from Table 3 that RDOA is able to find a feasible solution for most of problem instances. No feasible solutions were found for instances with the largest constraint for the minimal market share $\alpha = 80\%$. The algorithm is able to find a feasible solution within 1000 function evaluations for all instances with $\alpha \leq 50\%$ and for some instances with α equal to 50 and 60%; e.g. the instances with $s = 5$. The instances with large α-value required more than 1000 function evaluations to determine a feasible solution; e.g. instances with $s = 10$ and $\alpha = 70\%$ required around 4000 function evaluations in average.

The RDOA was unable to find a feasible solution with the largest α value for both instances. This happens because the alpha-value is so high and it is impossible to find optimal locations for the new facilities that follows the constraint for the minimal market share. The instances with 10 variables and larger α-value appeared to be more complicated since a feasible solution were not found in all trials; e.g. a feasible solution was not found in 68 trials when solving CFLP with $\alpha = 70\%$.

The best known solution was determined at least 87 times out of 100 for CFLP instances with $s = 5$, and at least 17 times for the instances with $s = 10$, depending on the α-value. In all cases the algorithm always converges to a close region of the best known solution as the difference between solutions with the minimal and the maximal objective values is less than 3% except the instance with 5 facilities per chain, $s = 10$, $\alpha = 70\%$ and $\alpha = 80\%$, where the differences between minimal and the maximal solutions is around 14%. Since the differences between average and the maximal solutions is several percents, this larger difference can be explained by occurrence of several solutions with exceptionally small objective values.

TABLE 2. Minimal market share per new facility (α) for different CFLP instances.

FPC	s	10%	20%	30%	40%	50%	60%	70%	80%
3	5	237203	474406	711609	948812	1186015	1423218	1660421	1897624
3	10	174781	349562	524344	699125	873906	1048687	1223468	1398249
5	5	166042	332084	498126	664168	830211	996253	1162295	1328337
5	10	132834	265667	398501	531335	664168	797002	929836	1062670

TABLE 3. Results obtained using RDOA for solving CFLP with 1000 demand points.

FPC	s	$\alpha(\%)$	$M(X)$					
			Average	Min	Max	#Max	#FNF	FFA
3	5	10	36.35	36.00	36.38	91	0	1000
3	5	20	36.37	36.00	36.38	97	0	1000
3	5	30	36.35	36.00	36.38	93	0	1000
3	5	40	36.37	36.00	36.38	97	0	1000
3	5	50	36.34	36.00	36.38	90	0	1000
3	5	60	36.34	36.00	36.38	90	0	1000
3	5	70	36.33	36.00	36.38	87	0	1130
3	5	80	FNF	FNF	FNF	FNF	100	FNF
5	10	10	39.96	39.57	40.06	21	0	1000
5	10	20	39.96	39.56	40.06	28	0	1000
5	10	30	39.95	39.56	40.06	32	0	1000
5	10	40	39.92	39.26	40.06	24	0	1000
5	10	50	39.39	38.82	39.56	39	0	1000
5	10	60	38.77	30.33	39.00	20	3	1155
5	10	70	37.40	33.63	38.91	17	68	4219
5	10	80	FNF	FNF	FNF	FNF	100	FNF

CONCLUSIONS

The strategy for constraint handling in the Ranking-based Discrete Optimization Algorithm were proposed and experimentally investigated. Results of the experimental investigation show that the proposed strategy makes the algorithm suitable to solve optimization problems with constraints. The modified algorithm finds a feasible solution (if any) in early stage of the computations – usually within 1000 function evaluations. After 10.000 function evaluation the algorithm provides the best known solution with less than 4% discrepancy in objective function value.

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