Report: Teoria i Metody Optymalizacji, Project A

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1. Introduction

The aim of this project was to apply the golden ratio method and gradient method to find the minimum of 4 functions (three of them come from the site: http://infinity77.net/global_optimization/test_functions_1d.html) which depend on only one variable (unimodal or multimodal functions). The search for the minimum of selected functions was supposed to take place in two cases: without constraints and with constraints, including where the minimum is on a constraint. For without constraints, it is assumed that starting condition is from -infinity to infinity. In order to get the final calculations, I assumed that the search area is in range <-1000, 1000>, which is much bigger than in the second case (with constraints) which is around the minimum point.

2. Objectives

- first function: $\mathrm{f_1(x)} = \mathrm{x}^2 + 1$

First selected function is a quadratic function, which is a unimodal function, because it has only one extreme (minimum) in point x = 0. When the argument of a function goes to infinity, the value of the function tends to plus infinity.

- second function: $\mathrm{f_2(x)} = \frac{\mathrm{x^2 - 5 \cdot x + 6}}{\mathrm{x^2 + 1}}$

The value of second function goes to one, when the argument goes to infinity. This function has two extremes (one maximum and one minimum). The wanted minimum is in point around: x= 2.41.

- third function: $\mathrm{f_3(x)} = \sin(\mathrm{x}) + \sin\left(\frac{10}{3} \cdot \mathrm{x}\right)$

The third function consists of the sum of two periodic function and because of this, function has the infinity number of extremes in the range minus infinity to plus infinity. E.g., in the range $x \in < 2.5, 7.5 >$ the global optimum is in point around x = 5.15.

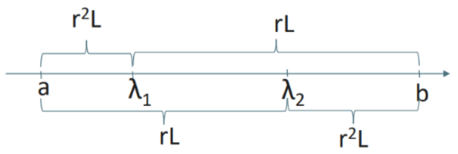
- fourth function: $\mathrm{f_4(x)} = -\mathrm{e}^{-\mathrm{x}} \cdot \sin(2 \cdot \pi \cdot \mathrm{x})$

The value of fourth function goes to zero, when the argument goes to plus infinity. E.g. in the range $x \in < 0,4 >$ the global optimum is in point around x = 0.22.

3. Methods

3.1. Golden ratio method

Golden ratio method consists in the preliminary determination of the interval [a, b], in this interval, we determine two points λ_1 and λ_2, which are calculated according to the convection of the golden ratio - by solving the following equation: $rL + r^2\mathrm{L} = \mathrm{L}$.



If $f(\lambda_2)> f(\lambda_1)$ then we change the boundaries of the interval to $[a, \lambda_2]$ otherwise we change the boundaries of the interval to $[\lambda_1, b]$. We lead calculation till we fulfill the condition: $|a - b| < \varepsilon$. We choose epsilon as the initial parameter. The found minimum is calculated by: $\frac{b+a}{2}$.

3.2. Simple gradient descent method

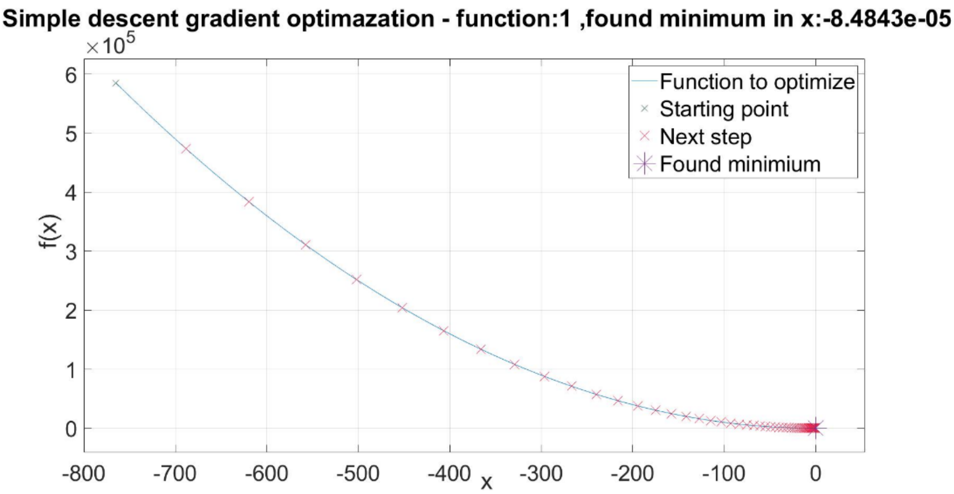
Simple gradient descent consists in selecting the starting point x_s and calculating the anti-gradient: $-\nabla f(x_s)$ at this point, which indicates the decrease in the function value. The point in the next step is calculated according to the formula: $x_{s+1} = x_s + \alpha \cdot (-\nabla f(x_s))$, where the coefficient α is a constant parameter of the method. The calculations are performed until the following condition is met: $\left|\left|x_{s+1} - x_s\right|\right| < \varepsilon$ or $\left|\left|\nabla f(x_s)\right|\right| < \varepsilon$, where epsilon is a function parameter. The last point determines the minimum of the function.

4. Results

4.1. Without constraints

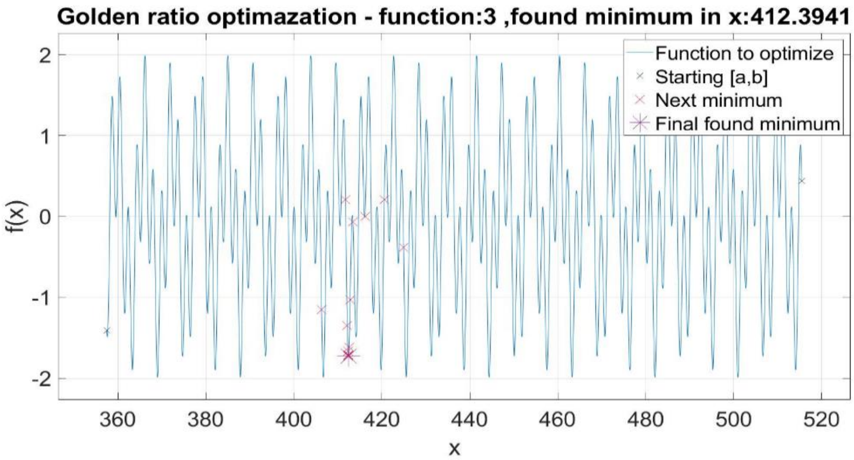
- first function

In this case the gradient method always find the point near the global minimum, which is the point x = 0. But the golden ratio method will find the point near the global minimum when the randomize a and b will be opposite signs.



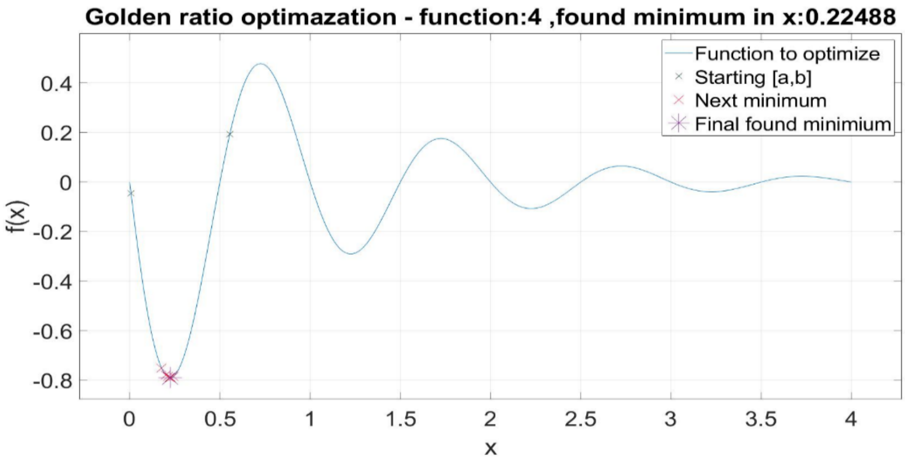
- second function, third function, fourth function

In these cases, it depends on a lot from the initial starting point – simple descent gradient method or starting intervals – golden ratio method. We are not able to predict what the minimum value will be and whether it will be the global minimum. Below figure for golden ratio method for third function in range around <357,515>. In this example I didn't find the global optimum.



4.2. With constraints, including where the minimum is on constraint

In this case we are able to randomize the initial starting point – simple descent gradient near the global optimum and the initial intervals – golden ratio method close to opposite sides of the global optimum in ranges describe in second paragraph. The figure on the right shows golden ratio optimization for function 4 in range around <0,4>.



5. Conclusion

The advantage of the gradient method for a unimodal function is that it will always result the global minimum (or maximum) with the accuracy of the method parameters values. The use of the golden ratio method will not always result in obtaining the global minimum for the unimodal function, unless the randomly drawn initial ranges are on the opposite sides of the global extreme. The advantage of the golden ratio method over the gradient method is that the initial values are more insensitive to the flatness of the starting point surroundings, which can cause getting stuck around at the starting point. Increasing the coefficient in the gradient method may speed up the finding of the minimum, but unfortunately too high its value may cause oscillation around the minimum point. On the other hand, a small value of the coefficient significantly extends the number of steps after which we find the extreme. In the case of the golden ratio method, reducing the distance between the extreme intervals results in a more precise approximation of the minimum, but unfortunately it lengthens the calculations.

Applying the constraints, including where the minimum is on a constraint for each of the four functions, causes, that it is always possible to find a global minimum in given range (second paragraph), which proves the correctness of the written algorithm for both these methods.