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Output Controller for Quadcopters Based on Mathematical Model Decomposition

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***Abstract*— In the paper an output control approach for a class of nonlinear MIMO systems is presented. A multicopter with four symmetrical rotors, i.e. a quadcopter, is chosen to illustrate effectiveness of the proposed adaptive control approach based on the high-gain principle so-called "consecutive compensator". Output controller is designed by decomposition of the mathematical model on two parts. The first one is a static MIMO transformation (more precisely, in the considering case a system of linear equations which relates lift forces generated by the actuators and virtual control inputs). The second one is a few SISO channels. Such trick allows to design a control law in two steps. At the first step we design virtual controls for each SISO channel. Here we apply the mentioned systematic approach "consecutive compensator". And then after the inverse MIMO transformation we get a set of real lift forces.**

Key words: Output control, robust control, time-delay systems, nonlinear systems, MIMO systems.

I. Introduction

From a practical point of view it is important not only to solve the control problem but also to develop a simplicity of implementation. Controllers of a simple structure with clear parameters' adjustment schemes are highly attractive for engineers of different areas including aerospace, robotics, and power systems [7], [11], [12], [15], [21], [22]. Particularly, output adaptive control methods are demand for a plenty of practical applications, where plant state measurement is hard or even impossible to realize.

This paper is focused on the recent advantages in the development of adaptive output control approach intensively using system pacification principle named by the authors as "consecutive compensator", that was considered in a number of previous works [4]–[7].

In works [4], [6], [7] models consisting of linear part with unknown parameters and static nonlinearity block are considered. Pacification approach (see [9], [10]) underlies of a proposing method. Controller has a simple structure and can be implemented as a proportional feedback providing strictly positive realness of the closed-loop system. In this sense the developing algorithm is very close to results introduced in [2], [3], [12], but with weaker requirements for a plant. Specifically, it is known that almost strictly positive realness property is equal to hyper minimum phaseness [1]. In turn the Hurwitz numerator of the transfer function is proved to be sufficient condition for our algorithm application in SISO case. Thus we have extended the approach for open-loop unstable systems with an arbitrary relative degree of the linear part. It became valid through the development of the linear filter with a special structure.

Exponential stability for the undisturbed closed-loop system and L_∞ stability for the bounded disturbance case correspondingly have been proved for "consecutive compensator" method in [8] and [19]. In [20] these results were extended for a time-varying delay. The proposing control algorithms demonstrate their effectiveness under conditions of an external disturbance and unknown time-delay presence. Key point in this context is a system stability.

In this paper an algorithm of control design for quadcopters is considered. Multicopters can be by controlled through speed varying of rotors rotation. A quadcopter is the most maneuver vertically taking off air vehicle. It belongs to a class of autonomous robots which movement occurs without any contact with a supporting surface. A quadcopter as a plant is a multi-input/multi-output nonlinear dynamical system. Its mathematical model will be divided on two parts: static MIMO transformation and a few SISO channels that allows to design the control law in two steps. At the first step we need to design so-called virtual controls for each SISO channel and then after an inverse MIMO transformation we can get a control law for the considering system.

II. Control Strategy

Let us consider the class of MIMO systems as follows

$$\begin{aligned} y_1(t) &= F_1(u_1, u_2, ..., u_r), \\ y_2(t) &= F_2(u_1, u_2, ..., u_r), \\ &\vdots \\ y_l(t) &= F_l(u_1, u_2, ..., u_r), \end{aligned} \tag{1}$$

where $y_1, ..., y_l$ are the output variables, $u_1, ..., u_r$ are the actuators inputs, and $F_1, ..., F_l$ are the non-linear differential operators in common case.

Controller design for the nonlinear MIMO systems can be divided on two steps. First of all it is necessary to decompose the nonlinear dynamic model on a static MIMO function and independent SISO dynamical channels. Each channel

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assigned with corresponding output variable $y_i, i = 1..l$. For every channel we introduce some virtual control input $U_i, i = 1..l$, that is superposition of all real actuators forces.

After decomposing the model (1) looks like

$$\begin{aligned} y_1(t) &= \bar{F}_1(U_1), \qquad U_1 = \widetilde{F}_1(u_1, u_2, ..., u_r), \\ y_2(t) &= \bar{F}_2(U_2), \qquad U_2 = \widetilde{F}_2(u_1, u_2, ..., u_r) \\ &\vdots \\ y_l(t) &= \bar{F}_l(U_l), \qquad U_l = \widetilde{F}_l(u_1, u_2, ..., u_r), \end{aligned} \tag{2}$$

where $\bar{F}_i, i = 1, ..l$ are the differential SISO channels and $\widetilde{F}_i, i = 1, ..l$ are the MIMO static functions.

If the nonlinear MIMO model can be divided on static MIMO transformation and several SISO channels like (2) then we will design control laws for virtual inputs $U_i, i = 1..l$ of each SISO channels. At the second step we design the control law for real actuators $u_j, j = 1..r$ basing on virtual controls for SISO channels $U_i, i = 1..l$ obtained at the first step making the inverse transformation of static nonlinearity.

We will consider the following SISO nonlinear system

$$y(t) = \frac{b(p)}{a(p)}U(t) + \frac{c(p)}{a(p)}\omega(t), \tag{3}$$

where $p = d/dt$ denotes differential operator; output $y(t)$ is measured, but its derivatives are not measured; $b(p) = b_m p^m + \cdots + b_1 p + b_0$, $c(p) = c_r p^r + \cdots + c_1 p + c_0$, and $a(p) = p^n + \cdots + a_1 p + a_0$ are monic coprime polynomials with unknown coefficients; number $r \le n - 1$; transfer function $\frac{b(p)}{a(p)}$ has relative degree $\rho = n - m$ that is known for controller designer; polynomial $b(p)$ is Hurwitz and parameter $b_m > 0$; unknown function $\omega(t) = \varphi(y)$ is such that:

$$|\varphi(y)| \le C_0 |y| \quad \text{for all } y, \tag{4}$$

where the number C_0 is unknown.

The first goal is to provide the exponential stability of nonlinear system (3). The second one is to extend first result for quadcopters in tracking mode. The input data is three linear coordinates and yaw angle. The purpose of control is to ensure stabilization of the quadcopter in some specified point with some specified orientation.

III. Control Design for SISO system

Following results [8], [19], [20] choose the control

$$U(t) = -\alpha(p)(\mu + \kappa)\hat{y}(t), \tag{5}$$

where number μ and polynomial $\alpha(p)$ are such that the transfer function $\frac{b(p)\alpha(p)}{a(p)+\mu b(p)\alpha(p)}$ is SPR, the positive parameter κ is used for compensation of the uncertainty $\varphi(y(t-\tau))$ (see proof of the *theorem 1*, inequality (43) in [20]) and the function $\hat{y}(t)$ is the estimation of output $y(t)$. The function $\hat{y}(t)$ is calculated according to the following algorithm

$$\begin{cases} \dot{\xi}_1 = \sigma\xi_2, \\ \dot{\xi}_2 = \sigma\xi_3, \\ \dots \\ \dot{\xi}_{\rho-1} = \sigma\left(-k_1\xi_1 - \ldots - k_{\rho-1}\xi_{\rho-1} + k_1(y - y^*)\right), \end{cases} \tag{6}$$

$$\hat{y} = \xi_1, \tag{7}$$

where number $\sigma > \mu + \kappa$ (see proof of the *theorem 1*, inequality (41) in [20]) and parameters k_i are calculated for the system (6) to be exponentially stable. Such control law is known as “Consecutive compensator” approach [4], [6].

Substituting (5) into equation (3), we obtain

$$\begin{aligned} y(t) &= \frac{b(p)}{a(p)}\left[-\alpha(p)(\mu+\kappa)\hat{y}(t)\right] + \frac{c(p)}{a(p)}\omega(t) \\ &= \frac{b(p)}{a(p)}\left[-\alpha(p)(\mu+\kappa)y(t) + \alpha(p)(\mu+\kappa)\varepsilon(t)\right] \\ &\quad + \frac{c(p)}{a(p)}\omega(t), \end{aligned} \tag{8}$$

where the error $\varepsilon(t) = y(t) - \hat{y}(t)$.

After simple transformations, for model (8) we have

$$\begin{aligned} y(t) &= \frac{b(p)\alpha(p)}{a(p) + \mu\alpha(p)b(p)}\left[-\kappa y(t) + (\mu+\kappa)\varepsilon(t)\right] \\ &\quad + \frac{c(p)}{a(p) + \mu\alpha(p)b(p)}\omega(t), \end{aligned} \tag{9}$$

where transfer function $W(p) = \frac{b(p)\alpha(p)}{a(p)+\mu\alpha(p)b(p)}$ is SPR.

Let us present model (9) in the form

$$\dot{x}(t) = Ax(t) + B(-\kappa y(t) + (\mu+\kappa)\varepsilon(t)) + q\omega(t), \tag{10}$$

$$y(t) = C^T x(t), \tag{11}$$

where $x \in \mathbb{R}^n$ is a state vector of system (10); A, B, q and C are appropriate matrix and vectors of transition from model (9) to model (10), (11).

Since transfer function $W(p)$ is SPR then

$$A^T P + PA = -R, \qquad PB = C, \tag{12}$$

where $R = R^T$ and parameters of matrix R depend on μ and do not depend on κ.

Let us rewrite model (6), (7) in the form

$$\dot{\xi}(t) = \sigma(\Gamma\xi(t) + dy(t)), \tag{13}$$

$$\hat{y}(t) = h^T\xi(t), \tag{14}$$

where $\Gamma = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_1 & -k_2 & -k_3 & \dots & -k_{\rho-1} \end{bmatrix}$, $d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ k_1 \end{bmatrix}$, and $h^T = \begin{bmatrix} 1 & 0 & 0 & \dots 0 \end{bmatrix}$.

Consider vector

$$\eta(t) = hy(t) - \xi(t), \tag{15}$$

then by force of vector h structure the error $\varepsilon(t)$ will become

$$\begin{aligned}\varepsilon(t) = y(t) - \hat{y}(t) &= h^T h y(t) - h^T \xi(t) \\ &= h^T (hy(t) - \xi(t)) = h^T \eta(t).\end{aligned} \tag{16}$$

For derivative of $\eta(t)$ we obtain

$$\begin{aligned}\dot{\eta}(t) &= h\dot{y}(t) - \sigma(\Gamma(hy(t) - \eta(t)) + dy(t)) \\ &= h\dot{y}(t) + \sigma\Gamma\eta(t) - \sigma(d + \Gamma h)y(t).\end{aligned} \tag{17}$$

Since $d = -\Gamma h$ (can be checked by substitution), then

$$\dot{\eta}(t) = h\dot{y}(t) + \sigma\Gamma\eta(t), \varepsilon(t) = h^T\eta(t), \tag{18}$$

where matrix Γ is Hurwitz by force of calculated parameters k_i of system (6) and

$$\Gamma^T N + N\Gamma = -M, \tag{19}$$

where $N = N^T > 0$, $M = M^T > 0$.

Theorem 1: Consider the nonlinear system (10), (11), (18). Let number $\rho = n - m \geq 1$ and unknown function $\omega(t) = \varphi(y(t))$ be such that:

$$|\varphi(y)| \leq C_0 |y|, \tag{20}$$

where C_0 is unknown.

There exists such constants κ_0 and σ_0 depended on plant parameters that for all $\kappa \geq \kappa_0 > 0$ and $\sigma \geq \sigma_0 > 0$ the nonlinear system (10), (11), (18) is exponentially stable at the origin in the sense of the norm

$$\left(\|x(t)\|^2 + \|\eta(t)\|^2\right)^{1/2}. \tag{21}$$

The corresponding proof of this theorem one can find in [19], [20] as a particular case when the state delay τ is absent.

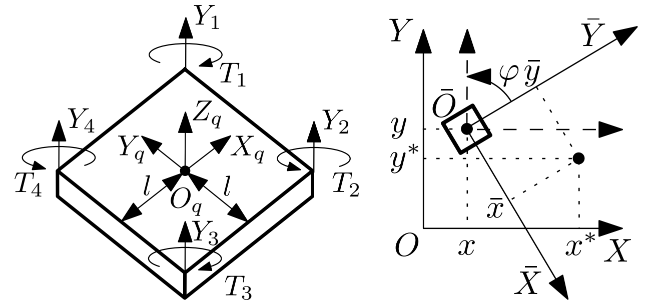
IV. Control Design for Quadrocopter

Now let us apply the presented approach to a real MIMO system, namely a quadcopter. Its position with respect to the fixed coordinate system can be specified by a translation vector with components (x, y, z) and pitch, roll, and yaw angles (θ, ψ, φ). A rotation of a rigid body around Cartesian axes in three-dimensional space by these three angles can be presented by a rotation matrix R in the following form

$$R = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\varphi - s_\psi c_\varphi & c_\psi s_\theta c_\varphi + s_\psi s_\varphi \\ s_\psi c_\theta & s_\psi s_\theta s_\varphi + c_\psi c_\varphi & s_\psi s_\theta c_\varphi - c_\psi s_\varphi \\ -s_\theta & c_\theta s_\varphi & c_\theta c_\varphi \end{bmatrix}, \tag{22}$$

where $s_\alpha \equiv \sin\alpha$, $c_\alpha \equiv \cos\alpha$, $\alpha = \{\theta, \psi, \varphi\}$.

Consider a three-dimensional scheme of the quadcopter shown in Fig. 1a, where Y_i, $i = \{1, 2, 3, 4\}$ is a Z-component of a lift force vector of the i-th rotor (other components are assumed to be neglected), T_i is a rotor speed. Assume that the first and third rotors rotate in clockwise direction and the second and fourth ones rotate in counterclockwise direction. Also admit that all the rotors rotate in the same plane, the rotation axes are perpendicular to the plane X_q, Y_q and intersect it in the points $(l, -l, 0)$, $(l, l, 0)$, $(-l, l, 0)$, $(-l, -l, 0)$.



(a) 3D scheme

(b) Coordinate systems and coordinates

Fig. 1: Schemes of the quadcopter

According to the information mentioned earlier, output variables of this MIMO system are three linear coordinates in Cartesian space (x, y, z) and three angles (θ, ψ, φ). Therefore, the corresponding model can be represented as a system of six differential equations of the form (1). Use the following dynamical model of the quadcopter

$$\begin{cases} m\ddot{x} &= \left(\sum Y_i\right)(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi), \\ m\ddot{y} &= \left(\sum Y_i\right)(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi), \\ m\ddot{z} &= \left(\sum Y_i\right)(\cos\theta\cos\psi) - mg, \\ J_1\ddot{\theta} &= l(-Y_1 - Y_2 + Y_3 + Y_4), \\ J_2\ddot{\psi} &= l(-Y_1 + Y_2 + Y_3 - Y_4), \\ J_3\ddot{\varphi} &= C(Y_1 - Y_2 + Y_3 - Y_4), \end{cases} \tag{23}$$

where m is a mass, J_1, J_2, J_3 are main mass moments of inertia.

Perform the decomposition of a complex dynamic model on independent linear models and corresponding static nonlinear relations. Introduce following virtual control inputs which are superpositions of Y_i

$$\begin{cases} U_1 &= Y_1 + Y_2 + Y_3 + Y_4, \\ U_2 &= -Y_1 - Y_2 + Y_3 + Y_4, \\ U_3 &= -Y_1 + Y_2 + Y_3 - Y_4, \\ U_4 &= Y_1 - Y_2 + Y_3 - Y_4. \end{cases} \tag{24}$$

These virtual control inputs will be generated according to the specified coordinates and orientation of the quadcopter. Values of Y_i will be calculated by the introduced system of the linear equations (24).

After substitution (24) in (23) we obtain the first simplified model

$$\begin{cases} m\ddot{x} &= U_1(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi), \\ m\ddot{y} &= U_1(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi), \\ m\ddot{z} &= U_1(\cos\theta\cos\psi) - mg, \\ J_1\ddot{\theta} &= lU_2, \\ J_2\ddot{\psi} &= lU_3, \\ J_3\ddot{\varphi} &= CU_4. \end{cases} \tag{25}$$

Consider the first two equations from the system (25). Write them in the matrix form

$$\begin{bmatrix} m\ddot{x} \\ m\ddot{y} \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} U_5 \\ U_6 \end{bmatrix}, \tag{26}$$

where U_5 and U_6 are auxiliary virtual control inputs defined by the equations

$$\begin{array}{rcl} U_5 & = & U_1 \sin\theta \cos\psi, \\ U_6 & = & -U_1 \sin\psi. \end{array} \quad (27)$$

In accordance to the problem formulation control of the quadcopter must be performed in the earth-fixed coordinate system (O, X, Y). However, for convenience consider an auxiliary coordinate system $(\bar{O}, \bar{X}, \bar{Y})$ rigidly attached to the quadcopter (see Fig. 1b). In accordance to the problem formulation control of the quadcopter must be performed in the earth-fixed coordinate system (O, X, Y). However, for convenience consider an auxiliary coordinate system $(\bar{O}, \bar{X}, \bar{Y})$ rigidly attached to the quadcopter (see Fig. 1b).

Introduce coordinates of the displacement in the longitudinal $\bar{x}$ and transverse $\bar{y}$ directions. Indeed, any transition of the quadcopter can be represented both in the absolute coordinate system and in the local one. Write the relation between these coordinates

$$\left[\begin{array}{c} x^* \\ y^* \end{array}\right] = \left[\begin{array}{c} x \\ y \end{array}\right] + \left[\begin{array}{cc} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{array}\right] \left[\begin{array}{c} \bar{x} \\ \bar{y} \end{array}\right], \quad (28)$$

where (x^*, y^*) and (x, y) are desired and current coordinates of the quadcopter respectively in the absolute coordinate system. Matrix in this equation is a rotation matrix for planar coordinate systems.

Express the coordinates $\bar{x}$ and $\bar{y}$ from the equation (28)

$$\left[\begin{array}{c} \bar{x} \\ \bar{y} \end{array}\right] = \left[\begin{array}{cc} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{array}\right] \left[\begin{array}{c} x^* - x \\ y^* - y \end{array}\right]. \quad (29)$$

Multiply (29) by m. Then after double differentiation we have

$$\left[\begin{array}{c} m\ddot{\bar{x}} \\ m\ddot{\bar{y}} \end{array}\right] = \left[\begin{array}{cc} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{array}\right] \left[\begin{array}{c} -m\ddot{x} \\ -m\ddot{y} \end{array}\right]. \quad (30)$$

Substitute (26) in (30) and obtain the model of a transition in the coordinates $\bar{x}$ and $\bar{y}$

$$\left[\begin{array}{c} m\ddot{\bar{x}} \\ m\ddot{\bar{y}} \end{array}\right] = \left[\begin{array}{c} -U_5 \\ -U_6 \end{array}\right]. \quad (31)$$

It is clear that changing of roll or pitch angles is a supporting process to ensure a transition in the coordinates $\bar{x}$ and $\bar{y}$. Changing of these values must be allowed in a small range, for instance, $[-\frac{\pi}{6}; \frac{\pi}{6}]$. In this case it is possible to use the approximate equations $\sin\alpha \approx \alpha$, $\cos\alpha \approx 1$ if $|\alpha| \leq \frac{\pi}{6}$. Then rewrite the third expression from (25) and the equalities (27) in the following form

$$m\ddot{z} = U_1 - mg, \quad (32)$$

$$U_5 = U_1\theta, \quad (33)$$

$$U_6 = -U_1\psi. \quad (34)$$

Therefore, we can write the final simplified model of the quadcopter

$$\left\{\begin{array}{rcl} m\ddot{\bar{x}} & = & -U_5, \\ m\ddot{\bar{y}} & = & -U_6, \\ m\ddot{z} & = & U_1 - mg, \\ J_1\ddot{\theta} & = & lU_2, \\ J_2\ddot{\psi} & = & lU_3, \\ J_3\ddot{\varphi} & = & CU_4, \end{array}\right. \quad (35)$$

where the local coordinates $\bar{x}$ and $\bar{y}$ can be calculated according to the desired coordinates with respect to the absolute coordinate system by the equation (29), while the desired values of θ and ψ can be derived from (33) and (34)

$$\theta^* = \frac{U_5}{U_1}, \qquad \psi^* = -\frac{U_6}{U_1}. \quad (36)$$

Remark 1: Equations (36) have potential division by zero. To avoid this it is reasonable to modify the scheme of calculation the desired values of θ^* and ψ^*. Firstly, if U_1 is close to zero the values θ^* and ψ^* should be equal to zero. It means that the motion along the coordinate z driven by U_1 is more preferable. When the absolute value of U_1 is greater or equal then some preset constant one can use (36). Secondly, if the result of (36) is too big, i.e. the desired pitch or roll are bigger than some critical value like $\frac{\pi}{6}$, it is reasonable to bound the desired values of θ^* or ψ^* by some preset constant.

The MIMO model (35) contains the six dynamical SISO channels. It is easy to apply the output control approach "consecutive compensator" [6], [8], [19], [20] to design the corresponding virtual controllers for U_i, $i = 1..6$ with $\rho = 2$

$$U_1 = mg + \mu_1(\xi_1 + \dot{\xi}_1), \quad \dot{\xi}_1 = \sigma_1(-\xi_1 + z^* - z), \quad (37)$$

$$U_2 = \mu_2(\xi_2 + \dot{\xi}_2), \qquad \dot{\xi}_2 = \sigma_2(-\xi_2 + \theta^* - \theta), \quad (38)$$

$$U_3 = \mu_3(\xi_3 + \dot{\xi}_3), \qquad \dot{\xi}_3 = \sigma_3(-\xi_3 + \psi^* - \psi), \quad (39)$$

$$U_4 = \mu_4(\xi_4 + \dot{\xi}_4), \qquad \dot{\xi}_4 = \sigma_4(-\xi_4 + \varphi^* - \varphi), \quad (40)$$

$$U_5 = \mu_5(\xi_5 + \dot{\xi}_5), \qquad \dot{\xi}_5 = \sigma_5(-\xi_5 + \bar{x}), \quad (41)$$

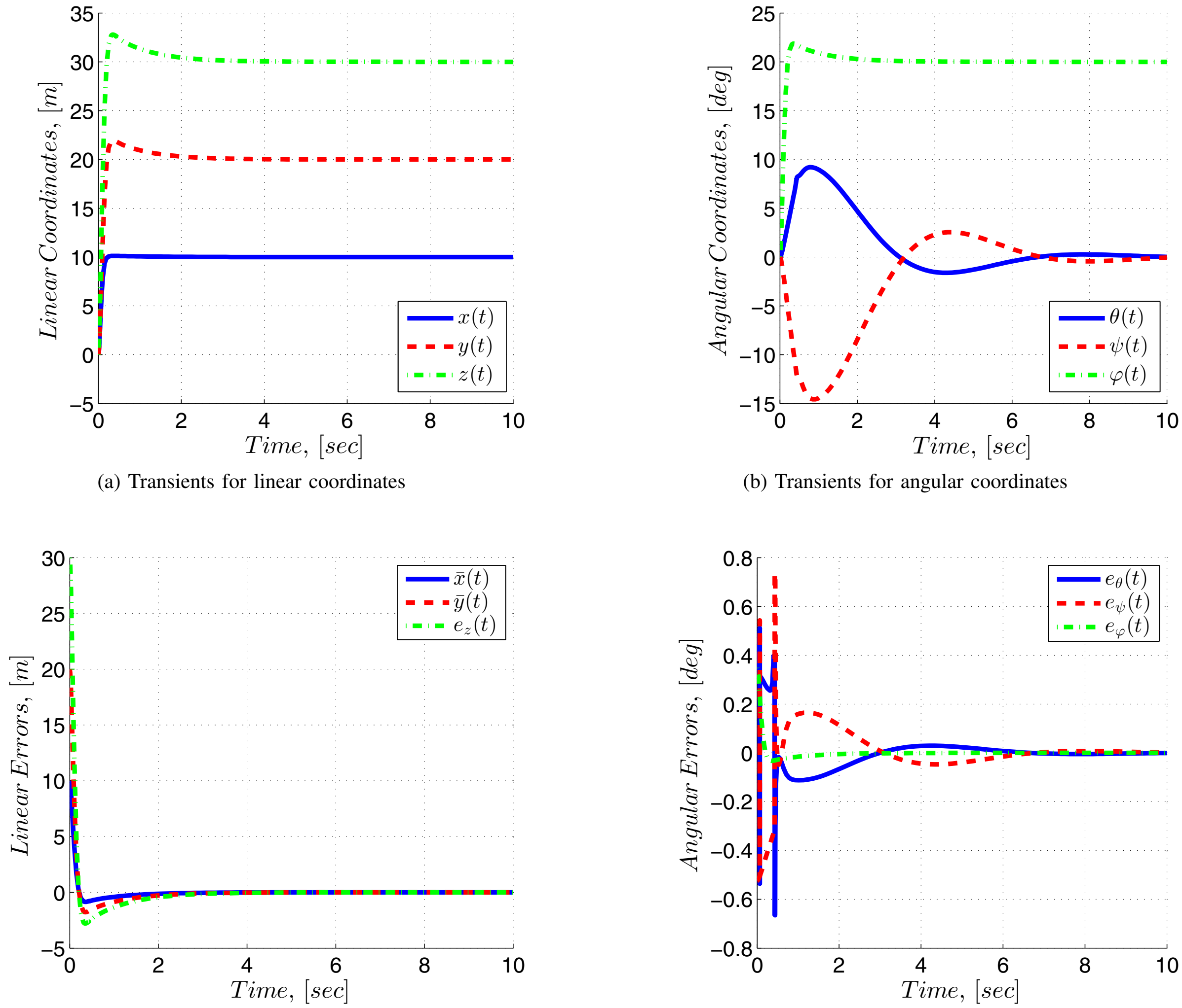
$$U_6 = \mu_6(\xi_6 + \dot{\xi}_6), \qquad \dot{\xi}_6 = \sigma_6(-\xi_6 + \bar{y}), \quad (42)$$

where $\bar{x}$ and $\bar{y}$ can be obtained by (29), $\mu_i, \sigma_i, i = 1..6$ are tuning coefficients that can be chosen without knowledge of the plant parameters.

As a result we will get all the virtual control inputs U_i, $i = 1..6$. Remind that the last two inputs, i.e. U_5 and U_6, were required to calculate the desired values of roll and pitch angles. To obtain expressions for lift forces generated by actuators of the plant in common case we need make the inverse transformation of static nonlinearity. But in our case it is sufficient to substitute the first four control inputs, i.e. U_1, U_2, U_3, and U_4, in the system of the linear equations (24). And then we will obtain the values of the lift forces Y_1, Y_2, Y_3, and Y_4.

V. Simulation

On Figs. 2a and 2b you can see simulation results of the presented approach. Here we consider the quadcopter with parameters $m = 1$, $l = 0.5$, $C = 1$, $J_1 = 1$, $J_2 = 1$, $J_3 = 1$, $g = 9.8$. The initial point has coordinates $x_0 = 0$, $y_0 = 0$, $z_0 = 0$, and the initial orientation is $\varphi_0 = 0$. The reference point has coordinates $x^* = 10$, $y^* = 20$, $z^* = 30$, and the desired orientation is $\varphi^* = 20°$. The quadcopter flies under the presented control law with coefficients $\mu_1 = 10$, $\mu_2 = 2$, $\mu_3 = 2$, $\mu_4 = 10$, $\mu_5 = 10$, $\mu_6 = 10$, $\sigma_1 = 30$, $\sigma_2 = 30$, $\sigma_3 = 30$, $\sigma_4 = 30$, $\sigma_5 = 30$, $\sigma_6 = 30$. Figs. 2a and



(a) Transients for linear coordinates

(b) Transients for angular coordinates

(c) Transients for linear errors

(d) Transients for angular errors

Fig. 2: Transients in the closed-loop control system

2b illustrate the transients for the coordinates of the closed-loop system, while Figs. 2c and 2d show the transients for the errors.

VI. Conclusions and Future Work

This paper is focused on the recent advantages in the development of the output control approach "consecutive compensator". Proposed algorithm has extended the theory of output feedback control of nonlinear MIMO systems.

The most interesting problem is the output control of the nonlinear system with parametric and functional uncertainties and the input delay. In [17], [18] the control problem is considered for the linear plants with the input delay. The feedback controller based on approaches presented in [13], [14] and [16] allows to reject an unknown biased sinusoidal disturbance for an internally unstable plant with the input delay.

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