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NONLINEAR CONTROL FOR A TANDEM ROTOR HELICOPTER

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Abstract: In this paper a Lyapunov control algorithm for a tandem rotor helicopter is obtained using backstepping techniques. A simple model dynamics for a behavior of a helicopter close to hover is obtained. Some simulation results are presented to illustrate the performance of such controller.

Keywords: Aerospace Control, Helicopter Control, Helicopter Dynamics, Lyapunov Methods, Multivariable Control Systems, Recursive Control Algorithms.

1. INTRODUCTION

Generally, the study of helicopters was mainly concerned with single-rotor helicopters. Thus, few studies have been conducted in order to develop controllers for tandem rotor helicopters. NASA has started the research with studies of the flying-qualities such as directional stability, lateral oscillations, turn characteristics (Kenneth and Tapscott, 1954) and speed stability (Tapscott and Kenneth, 1956), all this for a tandem helicopter with nonoverlapped-rotors and with the purpose of reducing the disadvantages of this type of instability; in the same way, but for overlapped-rotors, (Tapscott, 1958) and (Yeates, 1958) made studies on the longitudinal stability characteristics, vibrations in landing approach and yawed flight (these experiments was the base for the development of the VTOL aircraft). (Sridhar and Lindorff, 1973) presents a very brief control design using pole-placement theory where the feedback gains are obtained by a least squares solution, the dynamic model is based on a forward flight with linearized equations of motion. The research by (Stengel *et al.*, 1978), proposes two control laws for attitude-command and velocity-command control using digital control design and estimators. The paper of (Downing and Bryant, 1987) is based on the work of (Stengel *et al.*, 1978) and the authors add a trajectory generator and guidance algorithms in order to obtain an autoland system. A more recent study (Huang *et al.*, 1999) proposes two reconfigurable control laws that modify their control gains in presence of a actuator's failure in the system.

This article presents a simple dynamic model for a tandem rotor helicopter in hover conditions (Section 2), the section 3 focuses in the control law design that is based on backstepping techniques used in previous works for a single-rotor helicopter (Mahony *et al.*, 1999b) and (Dzul *et al.*, 2001). Section 4 shows the simulation results that illustrate the performance of such controller. Finally, we present the conclusion of this work.

2. DYNAMIC MODEL

A tandem rotor configuration (Figure 1) uses two contrarotating rotors of equal size and loading, so there is not net yaw moment on the helicopter because the torques of the rotors are equal and opposing. Typically, the two rotors are overlapped by around 20% to 50% of the radius (r) of the rotor disk, so the shaft separation is thus around $1.8r$ to $1.5r$. To minimize the aerodynamic interference created by the operation of the rear rotor in the wake of the front, the rear rotor is elevated on a pylon ($0.3r$ to $0.5r$ above the front rotor).

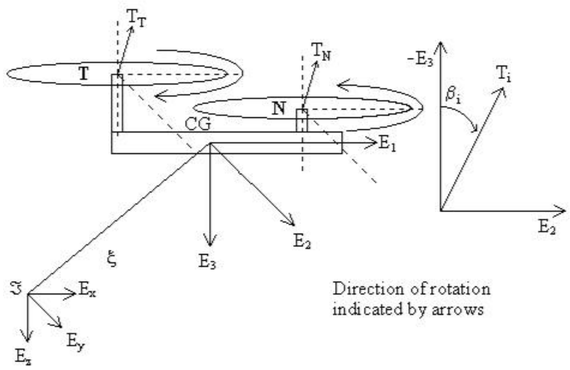

Figure 1. A tandem helicopter configuration.

In a tandem rotor helicopter, pitch moment is achieved by differential change of the main rotors thrust magnitude (by collective pitch), roll moment is controlled by lateral thrust tilt using cyclic pitch (Figure 2), yaw moment is obtained by differential lateral tilt of the thrust on the two main rotors with cyclic pitch (Figure 3), finally the vertical force is achieved by the change of the main rotor collective pitch.

For simplicity we will present here the dynamic model of a tandem main rotor helicopter in hovering. We propose a dynamical tandem helicopter model based on the Newton's equations of motion (Goldstein, 1980) with the next hypothesis

2.1 The main rotor blades are assumed to hinge directly from the hub. That is, the flapping hinge offset is assumed to be zero. The coning angle is assumed to be zero. As a consequence each rotor will always lie in a disk termed the rotor disk.
2.2 The nose rotor blades are assumed to rotate in an anti-clockwise direction when viewed from above and the tail rotor blades rotate in an clockwise direction, see figure 1.
2.3 It is assumed that the cyclic lateral tilts are measurable and controllable. That is that the flapping angles are used directly as control inputs. Along with the tail rotor and nose rotor thrust these form the control inputs of the helicopter.
2.4 The only air resistance modeled are simple drag forces opposing the rotation of the two rotors.
2.5 The operation of two or more rotors in close proximity will modify the flow field at each, and hence the performance of the rotor system will not be the same as for the isolated rotors. We will not consider this phenomenon to simplify the dynamical model.

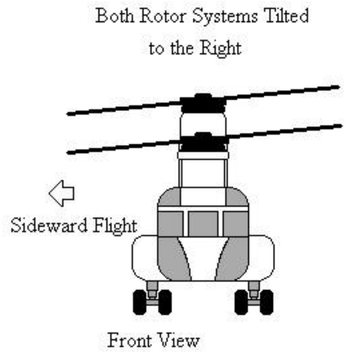

Figure 2. A tandem helicopter in sideward flight.

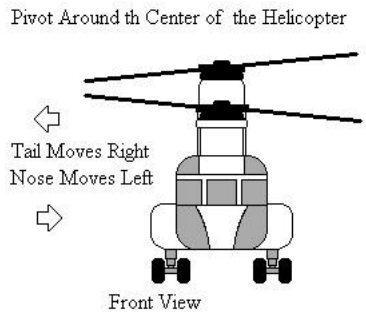

Figure 3. Control direction for a tandem helicopter.

In order to obtain the final dynamic equations, we have separated the aerodynamic forces in two groups. The first part is composed by translational forces and the second is related to the rotational forces of motion. More details on tandem rotor helicopter dynamics can be found in (Johnson, 1980).

2.1 *Translational Forces.*

Denote by T_N and T_T the thrust generated by the nose (N) and tail (T) rotors respectively (Figure 1). These forces are not E_1 component, then the thrust vectors are defined by :

$$T_N = T_N^2 E_2 - T_N^3 E_3, \qquad T_T = T_T^2 E_2 - T_T^3 E_3 \quad (1)$$

By simple geometric analysis we obtain expressions in terms of β, where β is the angle between the axis E_3 and the actual thrust vector.

$$T_N = |T_N| \sin\beta_N E_2 - |T_N| \cos\beta_N E_3 \qquad (2)$$

$$T_T = |T_T| \sin\beta_T E_2 - |T_T| \cos\beta_T E_3 \qquad (3)$$

The thrust vector can be represented by the following expression :

$$T_i = |T_i| \begin{bmatrix} 0 \\ \sin\beta_i \\ -\cos\beta_i \end{bmatrix} = |T_i| \begin{bmatrix} 0 \\ \beta_i \\ -1 \end{bmatrix} \qquad (4)$$

where $i = N$ or T, and considering values of β_i sufficiently small. Another force applied to the tandem rotor helicopter is the gravitational force given by $f_g = mgE_z$ where m is the complete mass of the helicopter and g is the gravitational constant. The above expression is defined in the inertial frame $\mathcal{I}$. In terms of the body fixed frame, it is necessary to multiply f_g by the inverse of the rotation matrix $R(\eta)$ that represents the orientation of the body fixed frame CG with respect to $\mathcal{I}$. The orientation vector η (yaw, pitch, roll) is defined by $\eta = [\phi, \theta, \psi]^T$. and

$$R(\eta) = \begin{pmatrix} c_\theta c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ c_\theta s_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\psi s_\theta s_\phi - s_\psi c_\phi \\ -s_\theta & s_\psi c_\theta & c_\psi c_\theta \end{pmatrix} \tag{5}$$

where the following shorthand notation is used :

$$c_\alpha = \cos\alpha, \quad s_\alpha = \sin\alpha$$

Denote by f the total translational force applied to the helicopter expressed in the inertial frame $\mathcal{I}$

$$\begin{aligned} f = &(|T_N|\beta_N + |T_T|\beta_T)RE_y \\ &-(|T_N| + |T_T|)RE_z + mgE_z \end{aligned} \tag{6}$$

2.2 *Torques and anti-torques.*

The torques generated by the thrust vectors T_N and T_T are due to separation between the center of the mass CG and the rotor hubs (called τ_N and τ_T respectively). The gravitational force does not generate a torque since the helicopter is free to rotate around its center of mass. Before beginning, it is necessary to define the measured distances between the center of mass of the tandem rotor helicopter to the hubs of the two rotors (denoted l_N for the nose rotor and l_T for the tail rotor). If we express these vectors in terms of the body fixed frame, one has :

$$l_N = l_N^1 E_1 - l_N^3 E_3 \tag{7}$$

$$l_T = -l_T^1 E_1 - l_T^3 E_3 \tag{8}$$

The torques applied to the airframe by the thrust vectors are defined by

$$\tau_N = l_N \times T_N \textit{ and } \tau_T = l_T \times T_T \tag{9}$$

The total torque generated by the nose and tail rotors is given by

$$\tau_{NT} = \tau_N + \tau_T \tag{10}$$

$$\tau_{NT} = \begin{bmatrix} l_N^3|T_N|\beta_N + l_T^3|T_T|\beta_T \\ l_N^1|T_N| - l_T^1|T_T| \\ l_N^1|T_N|\beta_N - l_T^1|T_T|\beta_T \end{bmatrix} \tag{11}$$

Additionally, the aerodynamic drags on the rotors generate some pure torques acting through the rotor hubs. Evoking the hypothesis 2.2, the anti-torques are defined by

$$Q_N = |Q_N|E_3, \quad Q_T = -|Q_T|E_3 \tag{12}$$

Finally, the total torque applied to the tandem rotor helicopter (expressed in the body fixed frame) is given by

$$\tau = \tau_{NT} + |Q_N|E_3 - |Q_T|E_3. \tag{13}$$

2.3 *Complete Model*

For the translational motion of the helicopter let $v = \dot{\xi}$ denote the velocity of its center of mass expressed in the inertial frame $\mathcal{I}$. Let m denote the complete mass of the helicopter, then Newton's equations yields $m\dot{v} = f$, where f is the external translational force (6).

Newton's equations show that the rotational component of motion in a non-inertial frame is given by $\mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + \tau$, where Ω is the angular velocity expressed in the non-inertial frame; $\mathbf{I}$ denote the inertia of the helicopter around its center of mass with respect to the body fixed frame and τ is the applied external torque in the body fixed frame. Finally, recalling (6) and (13) the full dynamic model is given by

$$\begin{aligned} \dot{\xi} &= v && (14) \\ m\dot{v} &= (|T_N|\beta_N + |T_T|\beta_T)RE_y \\ &\quad -(|T_N| + |T_T|)RE_z + mgE_z && (15) \\ \dot{R} &= R\hat{\Omega} && (16) \\ \mathbf{I}\dot{\Omega} &= -\Omega \times \mathbf{I}\Omega + |Q_N|E_3 - |Q_T|E_3 + \tau_{NT} && (17) \end{aligned}$$

where $\Omega \in \Re^3$ and

$$\hat{\Omega} = \begin{pmatrix} 0 & -\Omega^3 & \Omega^2 \\ \Omega^3 & 0 & -\Omega^1 \\ -\Omega^2 & \Omega^1 & 0 \end{pmatrix}. \tag{18}$$

For the purposes of control design, we will rewrite (14 - 17) in an equivalent model. Define a nominal control for the translation dynamic of the system $u_1 > 0$ and $u_2 > 0$ defined by $u_1 = |T_N|$, $u_2 = |T_T|$.

Now we can rewrite (11) as follows :

$$\tau_{NT} = \begin{bmatrix} l_N^3 u_1\beta_N + l_T^3 u_2\beta_T \\ l_N^1 u_1 - l_T^1 u_2 \\ l_N^1 u_1\beta_N - l_T^1 u_2\beta_T \end{bmatrix} \tag{19}$$

Define σ as the small body forces of the helicopter

$$\sigma = (u_1\beta_N + u_2\beta_T) \tag{20}$$

Taking these considerations and rearranging terms the full dynamic model becomes :

$$\dot{\xi} = \upsilon \quad (21)$$

$$m\dot{\upsilon} = -(u_1 + u_2)RE_z + mgE_z + R\sigma E_y \quad (22)$$

$$\dot{R} = R\hat{\Omega} \quad (23)$$

$$\mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + |Q_N|E_3 - |Q_T|E_3 + \tau_{NT} \quad (24)$$

3. CONTROLLER DESIGN

In this section a backstepping control design is proposed for the approximate model ($\sigma = 0$) in the case of tracking a desired smooth trajectory $\xi^d = (x^d, y^d, z^d$ and $\phi^d)$.

The control problem considered is to find a control law $(u, \gamma^1, \gamma^2, \gamma^3)$ depending only on the measurable states $(\xi, \dot{\xi}, \eta, \dot{\eta})$ and arbitrary many derivatives of the smooth trajectory (ξ^d, ϕ^d) such that the tracking error

$$\mathcal{E} := (\xi(t) - \xi^d(t), \phi(t) - \phi^d(t)) \in \Re^4 \quad (25)$$

is asymptotically stable for all initial conditions. u is defined as the total thrust $(u_1 + u_2)$ and $\gamma = [\gamma^1, \gamma^2, \gamma^3]^T$ is the control input for roll, pitch and yaw respectively.

Define a partial error $\delta_1 := \xi - \xi^d$ and the first storage function as :

$$S_1 = \frac{1}{2}\delta_1^T\delta_1 = \frac{1}{2}\left|\delta_1\right|^2. \quad (26)$$

The time derivative of S_1 is given by

$$\dot{S}_1 = \delta_1^T\dot{\delta}_1 = \delta_1^T(\upsilon - \upsilon^d), \quad (27)$$

where υ^d represents the velocity of tracked trajectory. Let υ^v be the virtual control for this first stage of this procedure and be chosen as

$$\upsilon^v = \upsilon^d - k_1\delta_1. \quad (28)$$

Introducing the above into (27) we get

$$\dot{S}_1 = -k_1|\delta_1|^2 + \frac{1}{m}\delta_1^T\delta_2. \quad (29)$$

Where δ_2 is defined as

$$\delta_2 = m\upsilon - m\upsilon^v. \quad (30)$$

Differentiating δ_2 and using equation (22) without the small body forces, it follows

$$\dot{\delta}_2 = -uRE_z + mgE_z - m\dot{\upsilon}^v. \quad (31)$$

Define a second storage function associated with the second error term δ_2

$$S_2 = \frac{1}{2}\left|\delta_2\right|^2. \quad (32)$$

Therefore, from (31)

$$\dot{S}_2 = \delta_2^T(-uRE_z + mgE_z - m\dot{\upsilon}^v).$$

Consider the new virtual control :

$$X = (uRE_z)^v = mgE_z - m\dot{\upsilon}^v + \frac{1}{m}\delta_1 + k_2\delta_2. \quad (33)$$

In this case the derivative of the second storage function becomes

$$\dot{S}_2 = -\frac{1}{m}\delta_2^T\delta_1 - k_2|\delta_2|^2 + \delta_2^T\delta_3, \quad (34)$$

where δ_3 is the third error used in this procedure. It is defined as :

$$\delta_3 = X - uRE_z \quad (35)$$

Consider the new storage function

$$S_3 = \frac{1}{2}|\delta_3|^2 + \frac{1}{2}|\epsilon_3|^2. \quad (36)$$

where

$$\epsilon_3 = \phi - \phi^d \quad (37)$$

penalizes the yaw. The yaw component of the error term is introduced at this stage of the backstepping procedure in order that the relative degree of δ_3 and ϵ_3 with respect to the controls u and γ match. Indeed, the relative degree of each control with respect either error is two. Differentiating the above expression and recalling (23), one has

$$\dot{S}_3 = \delta_3^T\dot{\delta}_3 + \epsilon_3(\dot{\phi} - \dot{\phi}^d) \quad (38)$$

or using (18), (23) and (35)

$$\begin{aligned}\dot{S}_3 &= \delta_3^T(\dot{X} - \dot{u}RE_z - uR\hat{\Omega}E_z) \\ &\quad + \epsilon_3(\dot{\phi} - \dot{\phi}^d) \quad (39)\\ &= \delta_3^T\left(\dot{X} - R\begin{bmatrix} u\Omega_2 \\ -u\Omega_1 \\ \dot{u}\end{bmatrix}\right) \\ &\quad + \epsilon_3(\dot{\phi} - \dot{\phi}^d) \quad (40)\end{aligned}$$

The value of $\dot{u}$ can be assigned directly via the following control law :

$$\dot{u} = E_z^T R^T(\dot{X} + \delta_2 + k_3\delta_3), \quad (41)$$

Note that if we assume that the measurements of $(\xi, \dot{\xi}, \eta, u)$ are available, then one can estimate the value of the derivative of the thrust $\dot{u}$. When the measurement of u is not available, but the measurement of $\ddot{\xi}$ is available, then one can estimate the value of u, using the following relation

$$|u| = m\left|\ddot{\xi} - gE_z\right|$$

Now, define the following virtual input :

$$\begin{pmatrix} u\Omega_2 \\ -u\Omega_1 \\ 0 \end{pmatrix}^v = [I - E_zE_z^T]R^T(\dot{X} + \delta_2 + k_3\delta_3) = Y.$$

To proceed we introduce the error variable :

$$\delta_4 = Y - \begin{pmatrix} u\Omega_2 \\ -u\Omega_1 \\ 0 \end{pmatrix},$$

Thus, equation (40) becomes

$$\dot{S}_3 = -\delta_3^T\delta_2 - k_3|\delta_3|^2 + \delta_3^T R\delta_4 + \epsilon_3(\dot{\phi} - \dot{\phi}^d). \quad (42)$$

Now consider the term associated with ϵ_3 and let $\dot{\phi}^v$ denote the virtual yaw velocity and choose

$$\dot{\phi}^v = \dot{\phi}^d - k_{31}\epsilon_3 \tag{43}$$

We can rewrite (42) as

$$\dot{S}_3 = -\delta_3^T\delta_2 - k_3|\delta_3|^2 + \delta_3^T R\delta_4 - k_{31}\epsilon_3^2 + \epsilon_3(\dot{\phi} - \dot{\phi}^v). \tag{44}$$

Define $\epsilon_4 = \dot{\phi} - \dot{\phi}^v$. With this choice the derivative of S_3 is

$$\dot{S}_3 = -\delta_3^T\delta_2 - k_3|\delta_3|^2 + \delta_3^T R\delta_4 - k_{31}\epsilon_3^2 + \epsilon_3\epsilon_4. \tag{45}$$

The fourth storage function associated with the backstepping procedure is given by

$$S_4 = \frac{1}{2}|\delta_4|^2 + \frac{1}{2}|\epsilon_4|^2, \tag{46}$$

Taking the derivative of S_4, it yields

$$\dot{S}_4 = \delta_4^T\left(\dot{Y} - \frac{d}{dt}\begin{bmatrix} u\Omega_2 \\ -u\Omega_1 \\ 0 \end{bmatrix}\right) + \epsilon_4(\ddot{\phi} - \ddot{\phi}^v) \tag{47}$$

and the next equations describe the control inputs that achieve the desired dynamics of the closed-loop system, note that

$$\frac{d}{dt}\left(u\begin{bmatrix} \Omega_2 \\ -\Omega_1 \\ 0 \end{bmatrix}\right) = \dot{u}\begin{bmatrix} \Omega_2 \\ -\Omega_1 \\ 0 \end{bmatrix} + u\hat{E}_z\dot{\Omega} \tag{48}$$

where $\hat{e}_z = S(e_z)$ which is the skew-symetric matrix of e_z (see (18)). Consider the following control input transformation $\gamma = \dot{\Omega}$, introducing this equation into (48) we get

$$\frac{d}{dt}\left(u\begin{bmatrix} \Omega_2 \\ -\Omega_1 \\ 0 \end{bmatrix}\right) = \dot{u}\hat{\Omega}E_z + u\hat{E}_z\gamma. \tag{49}$$

Now to achieve the desired control, we choose :

$$u\hat{E}_z\gamma = u\begin{pmatrix} \gamma^2 \\ -\gamma^1 \\ 0 \end{pmatrix} = \dot{Y} + R^T\delta_3 + k_4\delta_4 - \dot{u}\hat{\Omega}e_3 \tag{50}$$

and

$$\ddot{\phi} = \ddot{\phi}^v - k_{41}\epsilon_4 - \epsilon_3. \tag{51}$$

Introducing the above equations into (47) we get

$$\dot{S}_4 = -\delta_4^T R^T\delta_3 - k_4|\delta_4|^2 - k_{41}|\epsilon_4|^2 - \epsilon_4\epsilon_3. \tag{52}$$

It remains to define the control signals equations as

$$\dot{u} = E_z^T R^T(\dot{X} + \delta_2 + k_3\delta_3), \tag{53}$$

$$\gamma^1 = -E_y^T(\dot{Y} + R^T\delta_3 + k_4\delta_4 - \dot{u}\Omega_1)/u, \tag{54}$$

$$\gamma^2 = E_x^T(\dot{Y} + R^T\delta_3 + k_4\delta_4 + \dot{u}\Omega_2)/u, \tag{55}$$

$$\gamma^3 = \frac{\cos\theta}{\cos\psi}(\ddot{\phi}^v - k_{41}\epsilon_4 - \epsilon_3 + E_x^T W_\eta^{-1}\dot{W}_\eta W_\eta^{-1}\Omega - \frac{\sin\psi}{\cos\theta}\gamma^2). \tag{56}$$

The equation for γ^3 is found in (Mahony and Hamel, 1999a) and is obtained by the second derivative of η. The value of W_η is defined as follows

$$W_\eta = \begin{pmatrix} -s_\theta & 0 & 1 \\ c_\theta s_\psi & c_\psi & 0 \\ c_\theta c_\psi & -s_\psi & 0 \end{pmatrix} \tag{57}$$

The Backstepping process showed before, accomplish the monotonic decrease of the following Lyapunov function

$$V = S_1 + S_2 + S_3 + S_4.$$
$$= \frac{1}{2}\sum_{i=1}^{4}|\delta_i|^2 + \frac{1}{2}|\epsilon_3|^2 + \frac{1}{2}|\epsilon_4|^2$$

One can directly verify that

$$\dot{V} = -k_1|\delta_1|^2 - k_2|\delta_2|^2 - k_3|\delta_3|^2$$
$$-k_4|\delta_4|^2 - k_{31}|\epsilon_3|^2 - k_{41}|\epsilon_4|^2$$

Note that δ_1 and ϵ_3 together form the original tracking error that we wish to minimize. Then the Lyapunov function V is monotonically decreasing and thus the control objective is achieved. Using simple algebra, we can recover the control inputs u_1 and u_2 using the equations $u = (u_1 + u_2)$ and $\tau_{NT}^2 = \gamma^2$. In the same way, control inputs β_N and β_T are recovered from the equations $\tau_{NT}^1 = \gamma^1$ and $\tau_{NT}^3 = \gamma^3$.

4. SIMULATIONS

In this section, we present the simulation of the behavior of the complete helicopter dynamics and the approximative dynamics used to obtain the control law. The experiment considers the case of stabilization of the tandem helicopter dynamics to a stationary configuration. We have used the following parameters for the tandem helicopter model (approximations based in the technical specifications of the CH-47SD Chinnok) :

$$Mass = 11549\,Kg$$
$$g = 9.8\,ms^{-2}$$
$$l_N = [5.95, 0, -3.75]\,m$$
$$l_T = [-5.95, 0, -4.55]\,m$$
$$\mathbf{I} = \begin{pmatrix} 18229.44 & 0 & 1633.36 \\ 0 & 250646.21 & 0 \\ 1633.36 & 0 & 257144.25 \end{pmatrix} kgm^2$$
$$|Q_N| = 0.02|T_N|$$
$$|Q_T| = 0.02|T_T|$$

For all simulations, the values of the gains are $k_1 = 0.5$, $k_2 = 0.5$, $k_3 = 1.5$, $k_{31} = 1$, $k_4 = 1.5$ and $k_{41} = 1$. The initial condition adopted for the control is $u = gm \approx 113180$, initial force input should be exactly equal to the force required for sustaining the tandem helicopter in stationary flight. The initial position chosen is $\xi_0 = [0, 4, -5]^T$ m., $\phi_0 = 0$ rd. And the desired position is given by $\xi^d = [2, 6, -15]^T$ m., $\phi^d = 0.5$ rd.

Figure 4 shows the helicopter system that does not take into consideration the small body forces. The results obtained when small body forces are present in the helicopter model is shown in Figure 5. In both the ideal case and when the small body

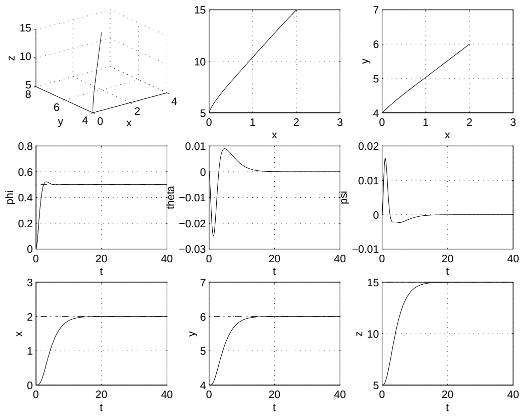

Figure 4. Position regulation of the helicopter dynamics in absence of small body forces.

forces are present the simulation indicate that the position regulations is achieved. We can see that the response when the small body forces are present, shows several effects of disturbances with respect to the axis y (ψ) since the small body forces act in this axis (see equation (22)).

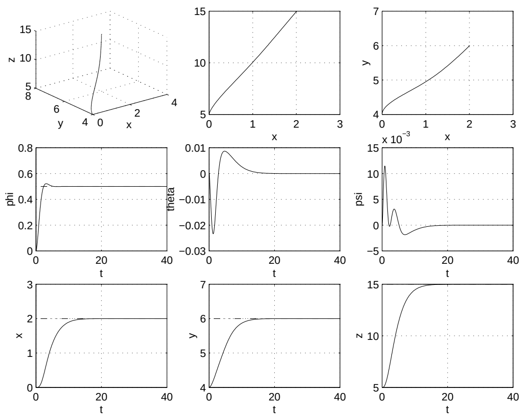

Figure 5. Position regulation of the helicopter dynamics in the presence of small body forces.

5. CONCLUSIONS

In this paper a simple model for the dynamics of a tandem rotor helicopter close to hover conditions is presented. A backstepping control was implemented based on a approximation of the system dynamics obtained by neglecting the small body forces associated with the torque control. Simulations showed that the performance of the controller is acceptable.

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