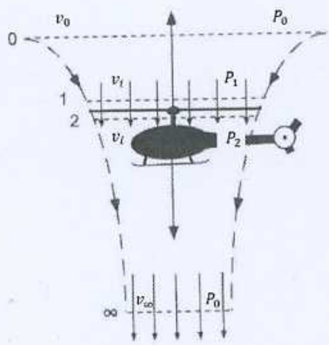

Fig. 2. Momentum Theory in Hover (Adapted From [14])

rotation of the blades must be considered along side the overall speed of the vehicle. As the craft manoeuvres in space, the air flow through the rotor has significant complexities which complicates the analysis. Since the rotorcraft is expected to perform in a variety of flight styles it is important to understand these models, and their flaws.

To simplify, initially consider a helicopter in a hovering state (Weight(W) = Thrust(T)). The rotor 'smooths' out the air as it forces it through the disk area. This more uniform air creates an edge known as the slipstream or wake boundary, with the surrounding air remaining dormant [14]. Inside the wake boundary, the average velocity of the air is tangible and effective, where outside the slipstream edge, the average air velocity is negligible and obsolete. The force required to push that mass of air through the disk space is, by Newton's third law, returned by the air unto the rotor, Thus giving the rotor blades a thrust component.

Rankine-Froude's Momentum Theory looks at this induced velocity as well as the displacement of air through the propeller, and attempts to quantify the induced thrust. The variable naming convention for the equations is shown in figure 2 below and is common to Leishman et al [14] in their naming of the components. Subscripts 0, 1, 2 and ∞ refer to the locations of quiescent flow, inflow directly before the rotor, airflow immediately after the disk and the slipstream[1] or far wake condition respectively. The velocities are shown as the induced velocity in and out the rotor (v_i), the far wake velocity (v_∞) and finally v_0 represents the zone with zero flow rate. There is no velocity jump across the rotor, the energy being fed into the system by the rotor is represented by a pressure change between P2 and P1.

As described above, it is by forcing the air through the disk that lift is generated. The mass flow rate of this air can then be described by (2), where ρ is the density of air and A is the area of one full blade rotation. The rate at which this mass of air is displaced becomes a crucial variable in rotor dynamics and is directly proportional to thrust (T). This relationship can be quantified as shown in (3). Thrust can also be calculated by finding the difference in pressures over the rotor disk as in (4)

$$\dot{m} = \rho A v_i \quad (2)$$
$$T = \dot{m} a \quad (3)$$
$$T = A(P_2 - P_1) \quad (4)$$

Since v_0 is zero during hover and acceleration is the difference in v_∞ and v_0, (5) can be obtained.

$$T = \rho A v_i v_\infty \quad (5)$$

Thrust can now be quantified ~~if~~ once the slipstream and induced velocities are known. Then by applying Bernoulli's equation of conservation to both sides of the rotor disk, the change in pressure across the disk can be quantified as shown in (6). That change in pressure fits into one of the initial definitions of thrust (4). Equating both of those definitions yields an important relationship between the three velocities, as shown in (7). The relationship simply states that the induced velocity at the rotor is the average of the quiescent flow above and the far wake velocities. This definition proves useful at a later stage in the rotor theory definitions.

$$P_2 - P_1 = \frac{1}{2}\rho(v_\infty^2 - v_0^2) \quad (6)$$

and describes

$$v_i = \frac{1}{2}(v_\infty + v_0) \quad (7)$$

C. Disk and Power Loading

Disk loading (DL) is a term seen often in the world of rotor crafts, it is a simple but important ratio between thrust and the area a rotating disk makes. It is represented in it's simplest form in the beginning of (8). Since the pressure drop across each rotor is considered uniform, the disk loading for each rotor will equate to the pressure drop across that disk. Equation (6) first shows the difference in pressure and by taking v_0 as zero (state of hover), the second half of equation (8) can be formed.

$$DL(\frac{N}{m^2}) = \frac{T}{A} = \frac{1}{2}\rho v_\infty^2 \quad (8)$$

For multi-rotor crafts, the disk loading is assumed uniform across all rotors [14]. The overall disk loading of a single rotor craft such as a traditional helicopter will be lower than