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Output Controller for Quadcopters with Wind Disturbance Cancellation

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***Abstract*— In the paper an output control approach for a class of nonlinear MIMO systems is presented. A multicopter with four symmetrical rotors, i.e. a quadcopter, is chosen to illustrate proposed adaptive control approach based on the high-gain principle, so-called, "consecutive compensator". Output controller is designed by decomposition of the mathematical model on two parts. The first one is a static MIMO transformation (more precisely, in the considering case a system of linear equations which relates control signals for actuators and virtual control inputs). The second one is a few SISO channels. Such trick allows to design a control law in two steps. At the first step we design virtual controls for each SISO channel. Here we apply the mentioned systematic approach "consecutive compensator", which also allows to cancel unknown wind disturbances changing their values and directions negligibly slowly. Then after the inverse MIMO transformation we get control signals for actuators.**

Key words: Output control, robust control, nonlinear systems, MIMO systems.

I. INTRODUCTION

From the practical point of view it is important not only to solve the control problem itself, but also to develop an easily implementable approach. Controllers of a simple structure with clear parameters' adjustment schemes are highly attractive for engineers in different areas including aerospace, robotics, and power systems [7], [11], [12], [15], [21], [22]. Particularly, output adaptive control methods in demand for a plenty of practical applications, where plant state measurement is hard or even impossible to realize.

This paper is focused on the recent advantages in the development of adaptive output control approach intensively using system passification principle named by the authors as "consecutive compensator", that was considered in the number of previous works [4]–[7].

In works [4], [6], [7] models consisting of linear part with unknown parameters and static nonlinearity block are considered. Pacification approach (see [9], [10]) is in the basis of proposed method. Controller has a simple structure and can be implemented as a proportional feedback providing strictly positive realness of the closed-loop system. In this sense developed algorithm is very close to results introduced in [2], [3], [12], but with weaker requirements for the plant. Specifically, it is known that almost strictly positive realness property is equal to hyper minimum phaseness [1]. In turn the Hurwitz numerator of the transfer function is proved to be sufficient condition for our algorithm application in SISO case. Thus we extended approach for the open-loop unstable systems with an arbitrary relative degree of the linear part. It became allowable owing to the development of the linear filter of special structure.

Exponential stability for the undisturbed closed-loop system and L_∞ stability for the bounded disturbance case correspondingly have been proved for "consecutive compensator" method in [8] and [19]. In [20] these results were extended for the time-varying delay. Proposed control algorithms demonstrate their effectiveness under conditions of an external disturbance and an unknown time-delay presence. Key point in this context is system stability.

In this paper the algorithm of control design for quadcopters is considered. This multicopter is controlled by varying rotors speed and thus by lift force changes. The quadcopter is the most maneuver vertically taking off air vehicle. It belongs to a class of autonomous robots which movement occurs without any contact with some supporting surface. A quadcopter as a plant is a multi-input/multi-output nonlinear dynamical system. Its mathematical model will be divided on two parts: static MIMO transformation and a few SISO channels that allows to design the control law in two steps. At the first step we design the virtual controls for each SISO channel and then after the inverse MIMO transformation we can get the control law for considered system. Also it is assumed that unknown wind disturbances act on the each channel of the quadcopter. Their values and directions change negligibly slowly. So, we can consider them as unknown constant signals which must be canceled.

II. CONTROL STRATEGY

Let us consider the class of MIMO systems as follows

$$y_i(t) = G_i(u_1, u_2, \ldots, u_r, f_i), \tag{1}$$

where $i = 1..l$, $y_i(t)$ are the output variables, $u_1, \ldots, u_r$ are the actuators inputs, G_i are the nonlinear differential operators in common case, and f_i are the unknown wind

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disturbances which change their values and directions negligibly slowly.

Controller design for the nonlinear MIMO systems can be divided on two steps. First of all it is necessary to decompose the nonlinear dynamic model on a static MIMO function and independent SISO dynamical channels. Further we will assume that the plant is such that mentioned decomposition is allowed. Each channel assigned with corresponding output variable $y_i, i = 1..l$. For every channel we introduce some virtual control input $U_i, i = 1..l$, that is superposition of all real actuators forces.

After decomposing the model (1) looks like

$$y_i(t) = \overline{G}_i(U_i, f_i), \quad U_i = \widetilde{G}_i(u_1, u_2 \dots, u_r), \tag{2}$$

where $\overline{G}_i, i = 1, ..l$ are the differential SISO channels and $\widetilde{G}_i, i = 1, ..l$ are the MIMO static functions.

If the nonlinear MIMO model can be divided on static MIMO transformation and several SISO channels like (2) then we will design control laws for virtual inputs $U_i, i = 1..l$ of each SISO channels. At the second step we design the control law for real actuators $u_j, j = 1..r$ basing on virtual controls for SISO channels $U_i, i = 1..l$ obtained at the first step making the inverse transformation of static nonlinearity.

We will consider the following SISO nonlinear system

$$a(p)y_i(t) = b(p)U_i(t) + f_i, \tag{3}$$

where $p = d/dt$ denotes the differential operator; output $y_i(t)$ are measured, but theirs derivatives are not measured; $b(p) = b_m p^m + \dots + b_1 p + b_0$, and $a(p) = p^n + \dots + a_1 p + a_0$ are monic coprime polynomials with unknown coefficients; the number $r \leq n - 1$; the transfer function $\frac{b(p)}{a(p)}$ has a relative degree $\rho = n - m$ that is known for controller designer; polynomial $b(p)$ is Hurwitz and parameter $b_m > 0$; the unknown constants f_i represent negligibly slow-varying wind disturbances for all the channels.

The first goal is to provide the exponential stability of nonlinear system (3). The second one is to extend first result for quadcopters in tracking mode with slowly varying wind disturbance cancellation. The input data is three linear coordinates and yaw angle. The purpose of control is to ensure stabilization of the quadcopter with some desired orientation in a specified point.

III. Control Design for SISO system

In this section the notation i which corresponds to particular SISO channel of the model (2) will be omitted for conciseness.

After differentiation of the model (3) we get

$$[pa(p)]y(t) = [pb(p)]u(t) + pf, \tag{4}$$

where $pf = 0$. Let $u(t) = \frac{1}{p}U(t)$, then (4) becomes

$$[pa(p)]y(t) = b(p)U(t). \tag{5}$$

Following results [8], [19], [20] choose the control $U(t)$

$$U(t) = -\mu\alpha(p)\hat{y}(t), \tag{6}$$

where the number μ and the polynomial $\alpha(p)$ are such that the transfer function $\frac{\mu b(p)\alpha(p)}{pa(p)+\mu\alpha(p)b(p)}$ is SPR and the function $\hat{y}(t)$ is the estimation of the output $y(t)$. It is calculated according to the following algorithm

$$\begin{cases} \dot{\xi}_1 = \sigma\xi_2, \\ \dot{\xi}_2 = \sigma\xi_3, \\ \dots \\ \dot{\xi}_{\rho-1} = \sigma\left(-k_1\xi_1 - \dots - k_{\rho-1}\xi_{\rho-1} + k_1(y - y^*)\right), \end{cases} \tag{7}$$

$$\hat{y} = \xi_1, \tag{8}$$

where the number $\sigma > \mu$ (see proof of the *theorem 1*, the inequality (41) in [20]) and parameters k_i are calculated for the system (7) to be exponentially stable. Such control law is known as "Consecutive compensator" approach [4], [6].

Substituting (6) into the equation (5), we obtain

$$pa(p)y(t) = -b(p)\mu\alpha(p)\hat{y}(t). \tag{9}$$

Introduce the error $\varepsilon(t) = y(t) - \hat{y}(t)$ and rewrite (9)

$$[pa(p) + \mu\alpha b(p)]y(t) = b(p)\mu\alpha(p)\varepsilon(t). \tag{10}$$

After simple transformations, for the model (10) we have

$$y(t) = \frac{\mu b(p)\alpha(p)}{pa(p) + \mu\alpha(p)b(p)}\varepsilon(t), \tag{11}$$

where the transfer function $W(p) = \frac{\mu b(p)\alpha(p)}{pa(p)+\mu\alpha(p)b(p)}$ is SPR.

Let us present the model (11) in the matrix form

$$\dot{x}(t) = Ax(t) + B\mu\varepsilon(t), \tag{12}$$

$$y(t) = C^T x(t), \tag{13}$$

where $x \in \mathbb{R}^n$ is a state vector of the system (12); A, B, and C are appropriate matrices of the transition from the model (11) to (12), (13).

Since the transfer function $W(p)$ is SPR then there exist some matrices $P = P^T > 0$ and $Q = Q^T > 0$ such that

$$A^T P + PA = -Q, \qquad PB = C. \tag{14}$$

Let us rewrite the model (7), (8) in the form

$$\dot{\xi}(t) = \sigma(\Gamma\xi(t) + dy(t)), \tag{15}$$

$$\hat{y}(t) = h^T\xi(t), \tag{16}$$

where $\Gamma = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_1 & -k_2 & -k_3 & \dots & -k_{\rho-1} \end{bmatrix}$, $d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ k_1 \end{bmatrix}$,
and $h^T = \begin{bmatrix} 1 & 0 & 0 & \dots 0 \end{bmatrix}$.

Consider the vector

$$\eta(t) = hy(t) - \xi(t), \tag{17}$$

then, by force of the vector h structure, the error $\varepsilon(t)$ becomes

$$\begin{aligned} \varepsilon(t) = y(t) - \hat{y}(t) &= h^T hy(t) - h^T\xi(t) \\ &= h^T(hy(t) - \xi(t)) = h^T\eta(t). \end{aligned} \tag{18}$$

For the derivative of $\eta(t)$ we obtain

$$\begin{aligned}\dot{\eta}(t) &= h\dot{y}(t) - \sigma(\Gamma(hy(t) - \eta(t)) + dy(t)) \\ &= h\dot{y}(t) + \sigma\Gamma\eta(t) - \sigma(d + \Gamma h)y(t).\end{aligned} \tag{19}$$

Since $d = -\Gamma h$ (can be checked by substitution), then

$$\dot{\eta}(t) = h\dot{y}(t) + \sigma\Gamma\eta(t), \varepsilon(t) = h^T\eta(t), \tag{20}$$

where the matrix Γ is Hurwitz by force of calculated parameters k_i of the system (7) and

$$\Gamma^T N + N\Gamma = -M, \tag{21}$$

where $N = N^T > 0$, $M = M^T > 0$.

Theorem 1: Consider the nonlinear system (12), (13), (20). Let number $\rho = n - m \geq 1$. There exists constant σ_0 depending on plant parameters such that for all $\sigma \geq \sigma_0 > 0$ the nonlinear system (12), (13), (20) is exponentially stable at the origin in the sense of the norm

$$\left(\|x(t)\|^2 + \|\eta(t)\|^2\right)^{1/2}. \tag{22}$$

The corresponding proof of this theorem one can find in [19], [20] as a particular case.

IV. Control Design for Quadrocopter

Now let us apply the presented approach to a real MIMO system, namely a quadcopter. Its position with respect to the fixed coordinate system can be specified by a translation vector with components (x, y, z) and pitch, roll, and yaw angles (θ, ψ, φ). A rotation of a rigid body around Cartesian axes in three-dimensional space by these three angles can be presented by a rotation matrix R in the following form

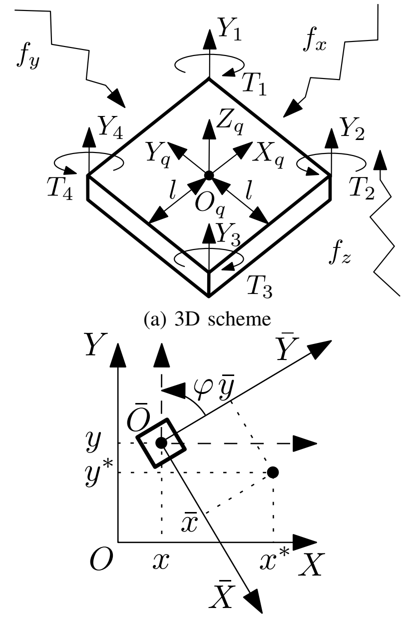
$$R = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\varphi - s_\psi c_\varphi & c_\psi s_\theta c_\varphi + s_\psi s_\varphi \\ s_\psi c_\theta & s_\psi s_\theta s_\varphi + c_\psi c_\varphi & s_\psi s_\theta c_\varphi - c_\psi s_\varphi \\ -s_\theta & c_\theta s_\varphi & c_\theta c_\varphi \end{bmatrix}, \tag{23}$$

where $s_\gamma \equiv \sin\gamma$, $c_\gamma \equiv \cos\gamma$, $\gamma = \{\theta, \psi, \varphi\}$.

Consider a three-dimensional scheme of the quadcopter shown in Fig. 1a, where Y_i, $i = \{1, 2, 3, 4\}$ is a Z-component of a lift force vector of the i-th rotor (other components are assumed to be neglected), T_i is a rotor speed. Assume that the first and third rotors rotate in clockwise direction and the second and fourth ones rotate in counterclockwise direction. Also admit that all the rotors rotate in the same plane, the rotation axes are perpendicular to the plane X_q, Y_q and intersect it in the points $(l, -l, 0)$, $(l, l, 0)$, $(-l, l, 0)$, $(-l, -l, 0)$.

According to the information mentioned earlier, output variables of this MIMO system are three linear coordinates in Cartesian space (x, y, z) and three angles (θ, ψ, φ). Therefore, the corresponding model can be represented as a system of six differential equations of the form (1). Use the following dynamical model of the quadcopter

$$\left\{\begin{array}{rcl} m\ddot{x} &=& \left(\sum Y_i\right)(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi) + f_x, \\ m\ddot{y} &=& \left(\sum Y_i\right)(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi) + f_y, \\ m\ddot{z} &=& \left(\sum Y_i\right)(\cos\theta\cos\psi) - mg + f_z, \\ J_1\ddot{\theta} &=& l(-Y_1 - Y_2 + Y_3 + Y_4) + f_\theta, \\ J_2\ddot{\psi} &=& l(-Y_1 + Y_2 + Y_3 - Y_4) + f_\psi, \\ J_3\ddot{\varphi} &=& C(Y_1 - Y_2 + Y_3 - Y_4) + f_\varphi, \end{array}\right. \tag{24}$$



(a) 3D scheme

(b) Coordinate systems

Fig. 1: Schemes of the quadcopter

where m is a mass, J_1, J_2, J_3 are main mass moments of inertia, f_x, f_y, f_z, f_θ, f_ψ, and f_φ are slow-varying wind disturbances of the each channel. The constant C means some efficient value of the displacement vector that makes the torque from rotational forces made by propellers.

Perform the decomposition of a complex dynamic model on independent linear models and corresponding static relations. Introduce following virtual control inputs which are superpositions of Y_i

$$\left\{\begin{array}{rcl} U_1 &=& Y_1 + Y_2 + Y_3 + Y_4, \\ U_2 &=& -Y_1 - Y_2 + Y_3 + Y_4, \\ U_3 &=& -Y_1 + Y_2 + Y_3 - Y_4, \\ U_4 &=& Y_1 - Y_2 + Y_3 - Y_4. \end{array}\right. \tag{25}$$

These virtual control inputs will be generated according to the specified coordinates and orientation of the quadcopter. Values of Y_i are calculated by the introduced system of the linear equations (25).

After substitution (25) in (24) we obtain the first simplified model

$$\left\{\begin{array}{rcl} m\ddot{x} &=& U_1(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi) + f_x, \\ m\ddot{y} &=& U_1(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi) + f_y, \\ m\ddot{z} &=& U_1(\cos\theta\cos\psi) - mg + f_z, \\ J_1\ddot{\theta} &=& lU_2 + f_\theta, \\ J_2\ddot{\psi} &=& lU_3 + f_\psi, \\ J_3\ddot{\varphi} &=& CU_4 + f_\varphi. \end{array}\right. \tag{26}$$

Consider the first two equations from the system (26). Write them in the matrix form

$$\begin{bmatrix} m\ddot{x} \\ m\ddot{y} \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} U_5 \\ U_6 \end{bmatrix} + \begin{bmatrix} f_x \\ f_y \end{bmatrix}, \tag{27}$$

where U_5 and U_6 are auxiliary virtual control inputs defined by the equations

$$\begin{array}{rcl} U_5 & = & U_1 \sin\theta \cos\psi, \\ U_6 & = & -U_1 \sin\psi. \end{array} \quad (28)$$

In accordance to the problem formulation control of the quadcopter must be performed in the earth-fixed coordinate system (O, X, Y). However, for convenience consider an auxiliary coordinate system $(\bar{O}, \bar{X}, \bar{Y})$ rigidly attached to the quadcopter (see Fig. 1b).

Introduce coordinates of the displacement in the longitudinal $\bar{x}$ and transversal $\bar{y}$ directions. Indeed, any transition of the quadcopter can be represented both in the absolute coordinate system and in the local one. Write the relation between these coordinates

$$\left[\begin{array}{c} x^* \\ y^* \end{array}\right] = \left[\begin{array}{c} x \\ y \end{array}\right] + \left[\begin{array}{cc} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{array}\right] \left[\begin{array}{c} \bar{x} \\ \bar{y} \end{array}\right], \quad (29)$$

where (x^*, y^*) and (x, y) are desired and current coordinates of the quadcopter respectively in the absolute coordinate system. Matrix in this equation is a rotation matrix for planar coordinate systems.

Express the coordinates $\bar{x}$ and $\bar{y}$ from the equation (29)

$$\left[\begin{array}{c} \bar{x} \\ \bar{y} \end{array}\right] = \left[\begin{array}{cc} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{array}\right] \left[\begin{array}{c} x^* - x \\ y^* - y \end{array}\right]. \quad (30)$$

Multiply (30) by m. Then assuming that φ changes slowly and $\dot{\varphi}$ is negligibly small after double differentiation we have

$$\left[\begin{array}{c} m\ddot{\bar{x}} \\ m\ddot{\bar{y}} \end{array}\right] = \left[\begin{array}{cc} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{array}\right] \left[\begin{array}{c} -m\ddot{x} \\ -m\ddot{y} \end{array}\right]. \quad (31)$$

Substitute (27) in (31) and obtain the model of a transition in the coordinates $\bar{x}$ and $\bar{y}$

$$\left[\begin{array}{c} m\ddot{\bar{x}} \\ m\ddot{\bar{y}} \end{array}\right] = \left[\begin{array}{c} -U_5 + f_x \\ -U_6 + f_y \end{array}\right]. \quad (32)$$

It is clear that changing of roll or pitch angles is a supporting process to ensure a transition in the coordinates $\bar{x}$ and $\bar{y}$. Changing of these values must be allowed in a small range, for instance, $[-\frac{\pi}{6}; \frac{\pi}{6}]$. In this case it is possible to use the approximate equations $\sin\gamma \approx \gamma$, $\cos\gamma \approx 1$ if $|\gamma| \leq \frac{\pi}{6}$. Then rewrite the third expression from (26) and the equalities (28) in the following form

$$m\ddot{z} = U_1 - mg + f_z, \quad (33)$$

$$U_5 = U_1\theta, \quad (34)$$

$$U_6 = -U_1\psi. \quad (35)$$

Therefore, we can write the final simplified model of the quadcopter

$$\left\{ \begin{array}{rcl} m\ddot{\bar{x}} & = & -U_5 + f_x, \\ m\ddot{\bar{y}} & = & -U_6 + f_y, \\ m\ddot{z} & = & U_1 - mg + f_z, \\ J_1\ddot{\theta} & = & lU_2 + f_\theta, \\ J_2\ddot{\psi} & = & lU_3 + f_\psi, \\ J_3\ddot{\varphi} & = & CU_4 + f_\varphi, \end{array} \right. \quad (36)$$

where the local coordinates $\bar{x}$ and $\bar{y}$ can be calculated according to the desired coordinates with respect to the absolute coordinate system by the equation (30), while the desired values of θ and ψ can be derived from (34) and (35)

$$\theta^* = \frac{U_5}{U_1}, \qquad \psi^* = -\frac{U_6}{U_1}. \quad (37)$$

Remark 1: Equations (37) have potential division by zero. To avoid this it is reasonable to modify the scheme of calculation the desired values of θ^* and ψ^*. Firstly, if U_1 is close to zero the values θ^* and ψ^* should be equal to zero. It means that the motion along the coordinate z driven by U_1 is more preferable. When the absolute value of U_1 is greater or equal then some preset constant one can use (37). Secondly, if the result of (37) is too big, i.e. the desired pitch or roll are bigger than some critical value like $\frac{\pi}{6}$, it is reasonable to bound the desired values of θ^* or ψ^* by some preset constant.

The MIMO model (36) contains the six dynamical SISO channels. It is easy to apply the output control law (6)–(8) to design the corresponding virtual controllers for U_i, $i = 1..6$ with $\rho = 2$ and $\alpha(p) = 2p^2 + p + 1$

$$U_i(t) \;=\; \mu_i \left(2\dot{\xi}_i + \xi_i + \int_0^t \xi_i \right), \quad (38)$$

$$\dot{\xi}_i \;=\; \sigma_i(-\xi_i + e_i), \quad (39)$$

where $e_i = \{z^* - z, \theta^* - \theta, \psi^* - \psi, \varphi^* - \varphi, \bar{x}, \bar{y}\}$ is the error of the corresponding channel, $\bar{x}$ and $\bar{y}$ can be obtained by (30), $\mu_i, \sigma_i, i = 1..6$ are tuning coefficients that can be chosen without knowledge of the plant parameters.

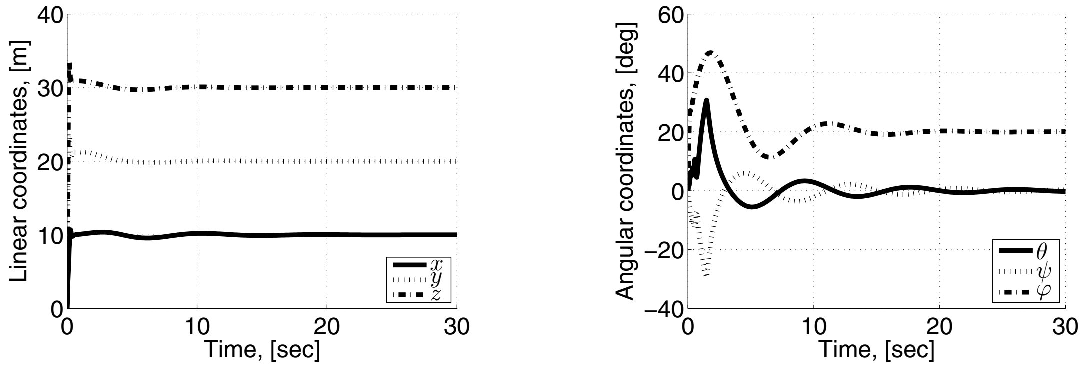
Remark 2: The polynomial $\alpha(p)$ may be arbitrary but Hurwitz. The choice of coefficients of the polynomial is important to guarantee desired quality of transients. So we decide to increase the impact of $\dot{\xi}_i$ in the control input (38) to decrease overshoots.

As a result we will get all the virtual control inputs U_i, $i = 1..6$. Remind that the last two inputs, i.e. U_5 and U_6, were required to calculate the desired values of roll and pitch angles. To obtain the control law for actuators of the plant in common case we need make the inverse transformation of static nonlinearity. But in our case it is sufficient to substitute the first four control inputs, i.e. U_1, U_2, U_3, and U_4, in the system of the linear equations (25). And then we will obtain the values of the real control signals Y_1, Y_2, Y_3, and Y_4:

$$\left\{ \begin{array}{rcl} Y_1 & = & (U_1 - U_2 - U_3 + U_4)/4, \\ Y_2 & = & (U_1 - U_2 + U_3 - U_4)/4, \\ Y_3 & = & (U_1 + U_2 + U_3 + U_4)/4, \\ Y_4 & = & (U_1 + U_2 - U_3 - U_4)/4. \end{array} \right. \quad (40)$$

V. Simulation

On Figs. 2a and 2b you can see simulation results of the presented approach under an assumption that output variables x, y, z, θ, ψ, φ are measured without noises. Here we consider the quadcopter with parameters $m = 1$, $l = 0.5$, $C = 1$, $J_1 = 1$, $J_2 = 1$, $J_3 = 1$, $g = 9.8$. The initial point has coordinates $x_0 = 0$, $y_0 = 0$, $z_0 = 0$, and the initial orientation is $\varphi_0 = 0$. The reference point has coordinates $x^* = 10\,[m]$, $y^* = 20\,[m]$, $z^* = 30\,[m]$, and the desired orientation is $\varphi^* = 20\,[deg]$. The values



(a) Transients for linear coordinates

(b) Transients for angular coordinates

Fig. 2: Transients in the closed-loop control system

of the wind disturbances for the each channel are chosen as $f_x = 10$, $f_y = 10$, $f_z = 10$, $f_\theta = 0.1$, $f_\psi = 0.1$, $f_\phi = 10$. The quadcopter flies under the presented control law with coefficients $\mu_1 = 10$, $\mu_2 = 2$, $\mu_3 = 2$, $\mu_4 = 10$, $\mu_5 = 10$, $\mu_6 = 10$, $\sigma_1 = 30$, $\sigma_2 = 30$, $\sigma_3 = 30$, $\sigma_4 = 30$, $\sigma_5 = 30$, $\sigma_6 = 30$. Fig. 2a illustrates the transients for the linear coordinates, while Fig. 2b shows the transients for the angular coordinates.

VI. Conclusions and Future Work

This paper is focused on the recent advantages in the development of the output control approach "consecutive compensator". Proposed algorithm has extended the theory of output feedback control of nonlinear MIMO systems.

The most interesting problem is the output control of the nonlinear system with parametric and functional uncertainties and the input delay. In [17], [18] the control problem is considered for the linear plants with the input delay. The feedback controller based on approaches presented in [13], [14] and [16] allows to reject an unknown biased sinusoidal disturbance for an internally unstable plant with the input delay.

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