Statistical Learning

Ridge Regression

1. Generate orthonormal ($X^TX = I$) matrix od dimension 1000×950. Consider the regression model

$$Y = X\beta + \epsilon,$$

with $\epsilon \sim N(0, I_{n\times n})$ and the vector of regression coefficients $\beta_1 = \ldots = \beta_k = 3.5$ and $\beta_{k+1} = \ldots = \beta_{950} = 0$ with

   a) k = 20,

   b) k = 100,

   c) k = 200.

   For each of these cases

   i) To be done by hand: Calculate the value of the tuning parameter λ for the ridge regression, so as to minimize the mean square error of the estimation of β.

   ii) To be done by hand: Calculate the bias, the variance and the mean squared error of this optimal estimator.

   iii) To be done by hand: Find the critical value and calculate the power of the statistical test based on the ridge estimator and controlling FWER at the level 0.1.

   iv) Generate 200 replicates of the above model and analyze the data using ridge regression and OLS. Compare empirical bias, variance, mse and the power of the test based on the ridge estimator with the theoretical values of these parameters, calculated above and with the corresponding parameters of OLS.

2. Generate the design matrix $X_{1000\times 950}$ such that its elements are iid random variables from $N(0, \sigma = 1/\sqrt{n})$. Then generate the vector of the response variable according to the models proposed in Task 1, above.

   Estimate the parameters of this model using the ridge regression with the tuning parameter selected by

   a) minimizing the SURE criterion

   b) 10 fold cross-validation and with

   c) OLS

   d) OLS within the model selected by mBIC2.

   Compare the estimation errors $||\hat{\beta} - \beta||^2$ and $||X(\hat{\beta} - \beta)||^2$ for these three approaches.

   Repeat the above experiment 100 times and compare the mean square errors of estimation of β and $\mu = X\beta$ for the above approaches.

3. Repeat 2 with $\beta_1 = \ldots = \beta_k = 5$.

4. Repeat 2 and 3 when rows of X are iid random vectors from $\frac{1}{n}N(0, \Sigma)$, where $\Sigma_{ii} = 1$ and for $i \neq j$ $\Sigma_{ij} = 0.5$.

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