

# MK9\_BartoszKloc\_59122

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[1]: import numpy as np
      import matplotlib.pyplot as plt

np.random.seed(0)

N_BITS = 32

def int_to_bits(x, n_bits=N_BITS):
    return np.array(list(np.binary_repr(x, width=n_bits))).astype(np.int8)

def bits_to_int(bits):
    return int("".join(bits.astype(str)), 2)

def sigmoid(z):
    z = np.clip(z, -50, 50)
    return 1.0 / (1.0 + np.exp(-z))

def dsigmoid(y):
    return y * (1 - y)

def tanh(z):
    return np.tanh(z)

def dtanh(h):
    return 1 - h**2

class RNNAdder:
    def __init__(self, hidden_size=128, lr=0.1, clip=1.0):
        self.H = hidden_size
        self.lr = lr
        self.clip = clip

        self.Wxh = np.random.randn(2, self.H) * 0.1
        self.Whh = np.random.randn(self.H, self.H) * 0.1
        self.Why = np.random.randn(self.H, 1) * 0.1

        self.bh = np.zeros((1, self.H))
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    self.by = np.zeros((1, 1))

def forward(self, X):
    """
    X: (B, T, 2) gdzie T=32
    Zwraca: hs (B, T+1, H), ys (B, T, 1)
    """
    B, T, _ = X.shape
    hs = np.zeros((B, T+1, self.H))
    ys = np.zeros((B, T, 1))

    for t in range(T):
        x_t = X[:, t, :] # (B,2)
        hs[:, t+1, :] = tanh(x_t @ self.Wxh + hs[:, t, :] @ self.Whh + self.
        ↵bh)
        ys[:, t, :] = sigmoid(hs[:, t+1, :] @ self.Why + self.by)

    return hs, ys

def loss_bce(self, Y_pred, Y_true):
    """
    BCE po wszystkich bitach i batchu
    Y_pred, Y_true: (B, T, 1)
    """
    eps = 1e-8
    Y_pred = np.clip(Y_pred, eps, 1-eps)
    return -np.mean(Y_true*np.log(Y_pred) + (1-Y_true)*np.log(1-Y_pred))

def backward(self, X, Y_true, hs, ys):
    """
    BPTT pełne po 32 krokach.
    """
    B, T, _ = X.shape

    dWxh = np.zeros_like(self.Wxh)
    dWhh = np.zeros_like(self.Whh)
    dWhy = np.zeros_like(self.Why)
    dbh = np.zeros_like(self.bh)
    dby = np.zeros_like(self.by)

    dY = (ys - Y_true) / (B * T) # (B, T, 1)

    dh_next = np.zeros((B, self.H))

    for t in reversed(range(T)):
        #

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dWhy += hs[:, t+1, :].T @ dY[:, t, :]
dby += np.sum(dY[:, t, :], axis=0, keepdims=True)

dh = dY[:, t, :] @ self.Why.T + dh_next
dz = dh * dtanh(hs[:, t+1, :])

dbh += np.sum(dz, axis=0, keepdims=True)
dWxh += X[:, t, :].T @ dz
dWhh += hs[:, t, :].T @ dz

dh_next = dz @ self.Whh.T

for D in [dWxh, dWhh, dWhy, dbh, dby]:
    np.clip(D, -self.clip, self.clip, out=D)

return dWxh, dWhh, dWhy, dbh, dby

def step(self, grads):
    dWxh, dWhh, dWhy, dbh, dby = grads
    self.Wxh -= self.lr * dWxh
    self.Whh -= self.lr * dWhh
    self.Why -= self.lr * dWhy
    self.bh -= self.lr * dbh
    self.by -= self.lr * dby

def make_batch(batch_size=32, n_bits=N_BITS):
    a = np.random.randint(0, 2**31, size=batch_size, dtype=np.int64)
    b = np.random.randint(0, 2**31, size=batch_size, dtype=np.int64)
    c = a + b

    A = np.stack([int_to_bits(x, n_bits) for x in a]) # (B,32)
    B = np.stack([int_to_bits(x, n_bits) for x in b])
    C = np.stack([int_to_bits(x, n_bits) for x in c])

    A = A[:, ::-1]
    B = B[:, ::-1]
    C = C[:, ::-1]

    X = np.stack([A, B], axis=2).astype(np.float64)
    Y = C[:, :, None].astype(np.float64)
    return X, Y, a, b, c

def predict_bits(model, X):
    hs, ys = model.forward(X)
    bits = (ys > 0.5).astype(np.int8)
    return bits, ys

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def batch_bit_accuracy(pred_bits, true_bits):
    return np.mean(pred_bits == true_bits)

model = RNNAdder(hidden_size=128, lr=0.2, clip=1.0)

loss_hist = []
acc_hist = []

ITERS = 8000
BATCH = 32

for it in range(1, ITERS+1):
    X, Y, a, b, c = make_batch(BATCH)

    hs, ys = model.forward(X)
    L = model.loss_bce(ys, Y)
    grads = model.backward(X, Y, hs, ys)
    model.step(grads)

    pred_bits = (ys > 0.5).astype(np.int8)
    acc = batch_bit_accuracy(pred_bits, Y.astype(np.int8))

    loss_hist.append(L)
    acc_hist.append(acc)

    if it % 500 == 0:
        i = 0
        pred_int = bits_to_int(pred_bits[i, ::-1, 0])
        true_int = bits_to_int(Y[i, ::-1, 0].astype(np.int8))
        print(f"iter {it} | loss={L:.4f} | bit-acc={acc:.3f}")
        print(f"a={a[i]} b={b[i]} true={c[i]} pred={pred_int} ↵ok={pred_int==c[i]}")
        print("-"*60)

plt.figure()
plt.plot(loss_hist)
plt.title("RNN 32-bit: loss (BCE)")
plt.xlabel("iteracja")
plt.ylabel("loss")
plt.grid(True)
plt.show()

plt.figure()
plt.plot(acc_hist)
plt.title("RNN 32-bit: dokładność bitowa")
plt.xlabel("iteracja")

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plt.ylabel("accuracy")
plt.grid(True)
plt.show()

X_test, Y_test, a_t, b_t, c_t = make_batch(10)
pred_bits, pred_prob = predict_bits(model, X_test)
for i in range(5):
    pred_int = bits_to_int(pred_bits[i, ::-1, 0])
    print(f"TEST {i}: {a_t[i]} + {b_t[i]} = {c_t[i]} | pred={pred_int} | ok={pred_int==c_t[i]}")

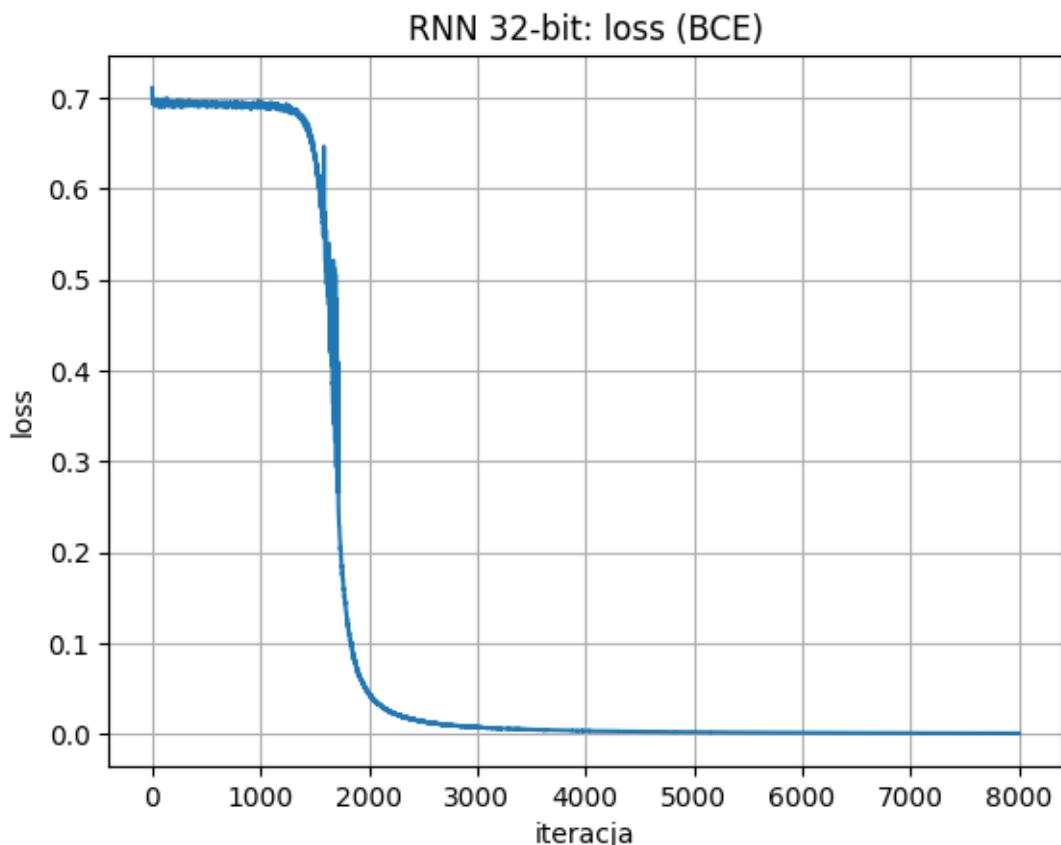
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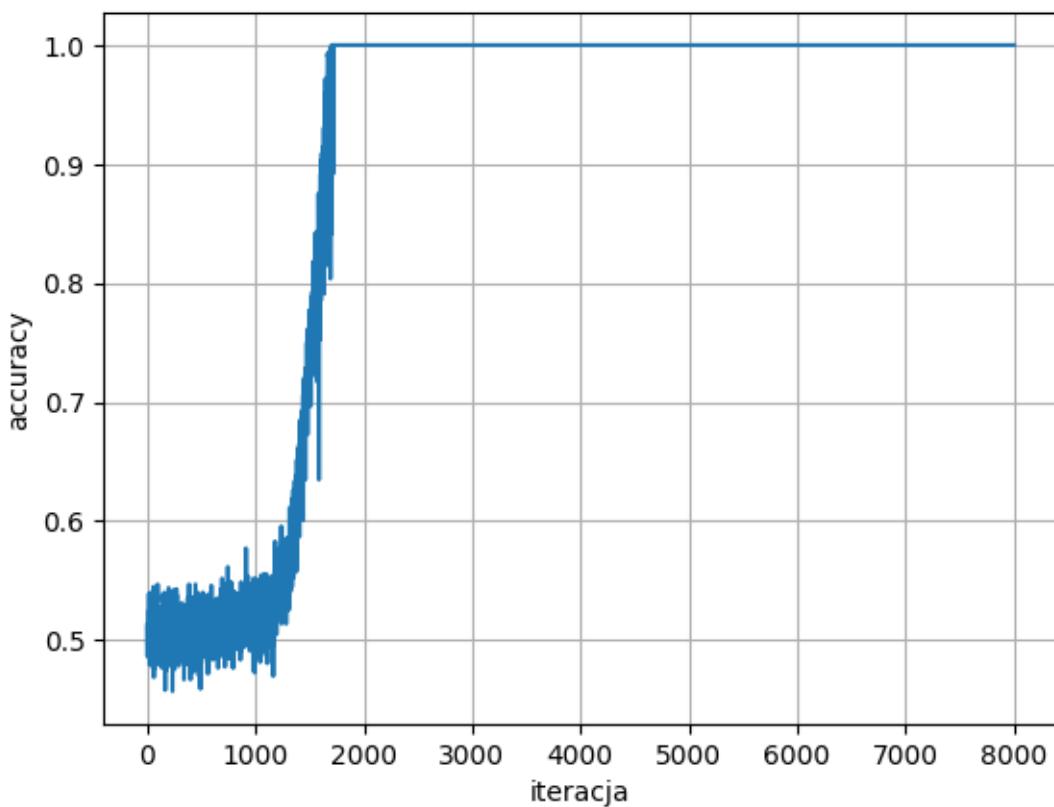
iter 500 | loss=0.6921 | bit-acc=0.521
a=236133877  b=976600362  true=1212734239  pred=1955004414  ok=False
-----
iter 1000 | loss=0.6897 | bit-acc=0.542
a=558908130  b=82446307  true=641354437  pred=112263118  ok=False
-----
iter 1500 | loss=0.6379 | bit-acc=0.745
a=94504340  b=1646290193  true=1740794533  pred=25321605  ok=False
-----
iter 2000 | loss=0.0448 | bit-acc=1.000
a=1070137103  b=193319915  true=1263457018  pred=1263457018  ok=True
-----
iter 2500 | loss=0.0142 | bit-acc=1.000
a=183903945  b=2090862107  true=2274766052  pred=2274766052  ok=True
-----
iter 3000 | loss=0.0077 | bit-acc=1.000
a=340159974  b=1144492818  true=1484652792  pred=1484652792  ok=True
-----
iter 3500 | loss=0.0051 | bit-acc=1.000
a=620518172  b=2087456420  true=2707974592  pred=2707974592  ok=True
-----
iter 4000 | loss=0.0038 | bit-acc=1.000
a=915740041  b=1727033284  true=2642773325  pred=2642773325  ok=True
-----
iter 4500 | loss=0.0031 | bit-acc=1.000
a=1448050105  b=1499093431  true=2947143536  pred=2947143536  ok=True
-----
iter 5000 | loss=0.0025 | bit-acc=1.000
a=1670102261  b=1690061818  true=3360164079  pred=3360164079  ok=True
-----
iter 5500 | loss=0.0021 | bit-acc=1.000
a=1206420596  b=1168385349  true=2374805945  pred=2374805945  ok=True
-----
iter 6000 | loss=0.0018 | bit-acc=1.000
a=1372690286  b=721467706  true=2094157992  pred=2094157992  ok=True
-----
iter 6500 | loss=0.0016 | bit-acc=1.000

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a=1104050634  b=1088997577  true=2193048211  pred=2193048211  ok=True
-----
iter 7000 | loss=0.0014 | bit-acc=1.000
a=574672248  b=1352216076  true=1926888324  pred=1926888324  ok=True
-----
iter 7500 | loss=0.0013 | bit-acc=1.000
a=844172986  b=297747325  true=1141920311  pred=1141920311  ok=True
-----
iter 8000 | loss=0.0011 | bit-acc=1.000
a=1082565209  b=1160428272  true=2242993481  pred=2242993481  ok=True
```



RNN 32-bit: dokładność bitowa



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TEST 0: 1222529569 + 1814931942 = 3037461511 | pred=3037461511 | ok=True
TEST 1: 1397937206 + 1576366678 = 2974303884 | pred=2974303884 | ok=True
TEST 2: 870291629 + 2009140019 = 2879431648 | pred=2879431648 | ok=True
TEST 3: 245717356 + 154953448 = 400670804 | pred=400670804 | ok=True
TEST 4: 1915241754 + 203588170 = 2118829924 | pred=2118829924 | ok=True
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