Homework Assignment 4

Lecturer: Kyunghyun Cho

March 29, 2018

1. RBFN for XOR problem (20 points) The 2-dimensional XOR problem calls for a classifier that works on the following data distribution: for any $\mathbf{x} \in \mathbb{R}^2$, the class is positive if x_1 and x_2 share the same sign, and negative otherwise. It is a typical example that linear models are not applicable without any feature engineering.

For the 2-dimensional XOR problem, we select the following four basis vectors:

$$\mathbf{r}^1 = [-1, -1]^\top$$
$$\mathbf{r}^2 = [1, 1]^\top$$
$$\mathbf{r}^3 = [-1, 1]^\top$$
$$\mathbf{r}^4 = [1, -1]^\top.$$

Show that the XOR problem is solved by the radial basis function network with the following weight vector:

$$\mathbf{w} = [1, 1, -1, -1, 0]^\top.$$

2. Nearest-neighbour classifier by RBFN (20 points) A nearest-neighbour classifier can be constructed as a radial basis function network by selecting all the input vectors in a training set as basis vectors. In a multi-class classification setting (i.e., there are more than two categories), provide a description on how a weight matrix could be built.

3. Adaptive RBFN (20 points) Unlike a fixed basis function network, an adaptive basis function network adapts basis vectors so as to maximize the classification accuracy (i.e. to minimize the empirical cost.) In order to do so, we need to be able to compute the gradient of the (logistic regression) distance function with respect to each and every basis vector. Derive this gradient

$$\nabla_{\mathbf{r}^k} D(y^*, M, \phi(\mathbf{x})),$$

assuming that M is a logistic regression classifier and that

$$\phi(\mathbf{x}) = \begin{bmatrix} \exp\left(-(\mathbf{x} - \mathbf{r}^1)^2\right) \\ \vdots \\ \exp\left(-(\mathbf{x} - \mathbf{r}^K)^2\right) \end{bmatrix}.$$

4. Programming Assignment (40 points in total) Please open `https://github.com/nyu-dl/Intro_to_ML_Lecture_Note/blob/master/homeworks/hw4.ipynb` and follow the instructions there.