Homework Assignment 1
Perceptron and Logistic Regression

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Submission Instruction You must typeset the answers to the theory questions using LaTeX or Microsoft Word and compile them into a single PDF file. For the programming assignment, complete the two Jupyter notebooks. Create a ZIP file containing both the PDF file and the two completed Jupyter notebooks, and name it "⟨Your-NetID⟩_hw1.zip". Submit the ZIP file on NYU Classes. The due date is **9:00 am on 02/12/2018.**

1. When we use empirical cost to approximate the expected cost,

$$\mathbb{E}_{\mathbf{x}\sim X}\left[D(M^*(\mathbf{x}),M,\mathbf{x})\right] \approx \frac{1}{N}\sum_{n=1}^{N} D(M^*(\mathbf{x}^n),M,\mathbf{x}^n)$$

is it okay to weigh each per-example cost equally? Given that we established that not every data $\mathbf{x}$ is equally likely, is taking the sum of all per-example costs and dividing by N reasonable? Should we weigh each per-example cost differently, depending on how likely each $\mathbf{x}$ is? Justify your answer.

2. A perceptron is defined as follows:

$$M(\mathbf{x}) = \text{sign}(\mathbf{w}^\top\mathbf{x}+\mathbf{b}),$$

where $\mathbf{w}\in\mathbb{R}^d, \mathbf{x}\in\mathbb{R}^d$ and $\mathbf{b}\in\mathbb{R}$. Why is the bias $\mathbf{b}$ necessary? Provide an example where it is necessary.

3. We used the following distance function for perceptron in the lecture:

$$D(M^*(\mathbf{x}),M,\mathbf{x}) = -\left(M^*(\mathbf{x})-M(\mathbf{x})\right)\left(\mathbf{w}^\top\mathbf{x}+\mathbf{b}\right).$$

This distance function has a problem of a trivial solution. What is the trivial solution? Propose a solution to this.

4. The distance function of logistic regression was defined as

$$D(y^*, \mathbf{w}, \mathbf{x}) = -(y^* \log M(\mathbf{x}) + (1 - y^*) \log(1 - M(\mathbf{x}))).$$

Derive its gradient with respect to the weight vector $\mathbf{w}$ step-by-step.

5. (Programming Assignment) Complete the implementation of perceptron and logistic regression using Python and scikit-learn. The completed notebooks must be submitted together with the answers to the questions above.

When submitting Jupyter notebooks, make sure to save printed outputs as well.

Perceptron `https://github.com/nyu-dl/Intro_to_ML_Lecture_Note/blob/master/notebook/Perceptron1.ipynb`

Logistic Regression `https://github.com/nyu-dl/Intro_to_ML_Lecture_Note/blob/master/notebook/Logistic%20Regression%201.ipynb`