Predicting the number of mistakes in chess games based on the chosen opening, time control and ranking difference

Authors:

- Bartosz Więcek, 400421
- Dominik Czyżyk, 401858

1. Problem formulation.

a) Problem

The impact of the chess opening on the outcome of a game is a captivating subject that garners the attention of both professional players and enthusiasts of this noble game. Choosing the right opening can be crucial to the course of the entire game, affecting the number of errors made and, consequently, the chances of winning or even achieving an easy draw.

The chess opening refers to the initial moves made by each player on the board. There is an immense number of different openings, each with its unique characteristics and strategies. Some openings focus on rapid pawn development, while others emphasize control of central squares or the deployment of pieces. The choice of a specific opening may stem from a player's preferences but also from the analysis and knowledge of various variations.

The objective of this project is to examine the impact of a specific opening on the number of errors committed during a chess game. By analyzing the results of conducted matches, we will be able to assess whether there is a correlation between the chosen opening and subsequent mistakes. Do certain openings lead to a higher number of blunders, while others facilitate a more stable and controlled game?

Investigating such a relationship can contribute to the refinement of chess-playing strategies for both professional players and amateurs. Understanding which openings are riskier or safer can influence tactical choices during gameplay. Furthermore, analyzing the impact of the opening on the game's outcome can provide valuable insights for predicting errors and making appropriate decisions.

In the further stages of the project, we will delve into various chess openings and analyze game results, focusing on identifying patterns and trends. Through this study, we aim to answer the question of whether the choice of opening can indeed influence the course and outcome of a chess game.

b) Use Cases

This project aims to analyze the influence of different chess openings on the number of errors committed during a game. By studying the data and conducting statistical analyses, we can uncover patterns and trends regarding the impact of specific openings on gameplay accuracy. Such insights can have practical applications in several use cases, including:

- Chess Training and Coaching: Understanding the relationship between openings and errors can aid chess trainers and coaches in developing more effective training programs. By identifying openings that tend to lead to higher error rates, trainers can focus on addressing specific weaknesses in their students' gameplay and provide targeted guidance.
- Game Strategy Development: The analysis of opening choices and their impact on errors can assist chess players in refining their game strategies. Players can leverage this knowledge to select openings that align with their playing style and minimize the likelihood of making mistakes, thereby enhancing their overall performance.
- Opening Repertoire Selection: For both amateur and professional players, the choice of opening repertoire is a crucial decision. The analysis conducted in this project can provide valuable insights into the relative strengths and weaknesses of different openings, allowing players to construct a well-rounded repertoire tailored to their individual preferences and objectives.
- Opponent Analysis: Examining the relationship between openings and errors can also be useful for analyzing opponents' gameplay tendencies. By understanding the typical errors associated with certain openings, players can anticipate their opponents' moves and capitalize on potential weaknesses.

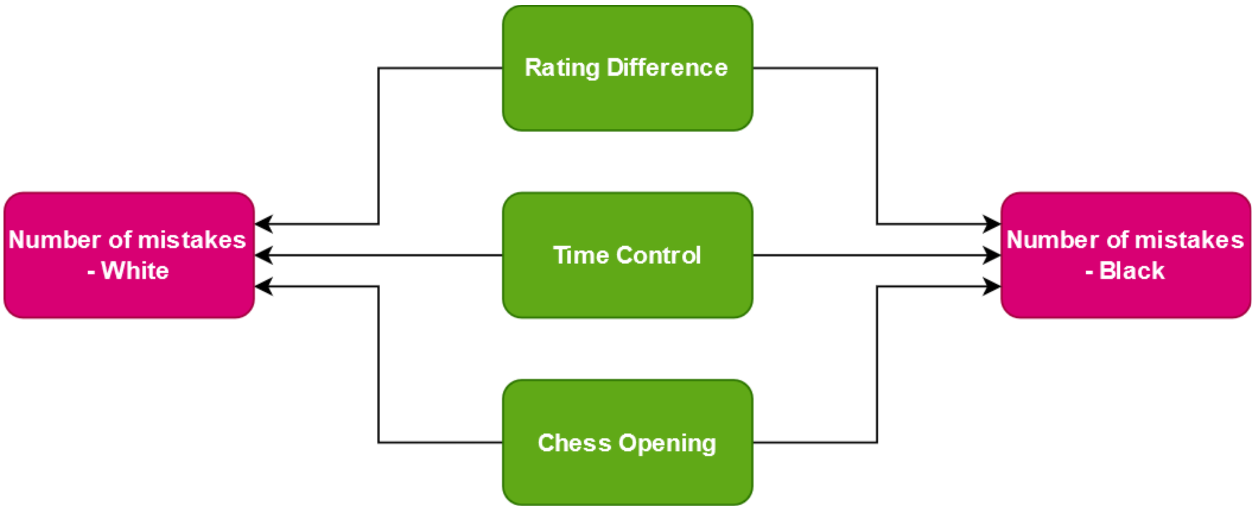
c) Data origin

The dataset used in the project comes from the Kaggle platform and is called "Lichess Chess Game Dataset". It can be found at the following link: https://www.kaggle.com/datasets/asrinyigit/lichess-chess-game-dataset or in a csv file: lichess_games_dataset.csv

This dataset was collected from the Lichess platform, which is a popular chess game website. It contains an extensive collection of recorded chess games that have been played on this platform.

The dataset contains a variety of information about the games played, such as the moves made by players, the time taken to make a move, the ranking of players, the result of the game and much more. The available data allows us to conduct detailed analyses on strategy, tactics, game results and other aspects related to the game of chess.

d) DAG



e) Confoundings

Confounding is a phenomenon in which the influence of one variable on the outcome variable is distorted by the presence of another variable, known as a confounding factor. In the considered model, there are two types of confounding:

- Pipe: "Time Conrol" can have an influence on the "Chess Opening", creating a relationship between these two factors. It is important to account for this relationship in the model to avoid potential confounding and to assess the associations between variables more accurately.
- Collider: The variables "Number of mistakes - White" and "Number of mistakes - Black" are colliders because they are directly dependent on different factors, such as "Time Conrol", "Chess Opening" and "Rating Difference." The presence of a collider can lead to erroneous conclusions about the relationship between mistakes made by white and black players.

Program

Importing neccessary libraries.

```
from cmdstanpy import CmdStanModel
import seaborn as sns

import arviz as az
import numpy as np
import scipy.stats as stats

import matplotlib.pyplot as plt
import pandas as pd

from copy import deepcopy
```

```
import warnings
warnings.filterwarnings("ignore")
```

```
/usr/local/lib/python3.9/site-packages/tqdm/auto.py:22: TqdmWarning: IProgress not found. Please update jupyter and ipywidgets. See https://ipywidgets.readthedocs.io/en/stable/user_install.html
  from .autonotebook import tqdm as notebook_tqdm
```

2. Data preprocessing

a) Data loading

There are two csv files needed:

- "lichess_games_dataset.csv" - file with chess games data
- "chess_openings.csv" - file with chess openings names used in ECO (opening code) to opening name decoding.

```
df_games = pd.read_csv('lichess_games_dataset.csv', usecols=["Time Control", "White Rating", "Black Rating", "Opening ECO", "White's Number of Mistakes", "White's Number of Blunders",
                                                             "Black's Number of Mistakes", "Black's Number of Blunders", "Winner"])
df_openings = pd.read_csv('chess_openings.csv')

df_games.head()
```

```
  Time Control  White Rating  Black Rating Opening ECO  \
0       10 + 0          1441          1559         C20
1      10 + 15          1258          1567         C46
2      60 + 10          1697          1712         C68
3       10 + 0          1978          1868         D31
4       10 + 0          2073          1816         A01

   White's Number of Mistakes  White's Number of Blunders  \
0                           2                           0
1                           0                           1
2                           0                           0
3                           0                           0
4                           2                           0

   Black's Number of Mistakes  Black's Number of Blunders Winner
0                           4                           1  White
1                           0                           0  Black
2                           0                           2  White
3                           0                           1  White
4                           3                           0  White
```

```
df_openings.head()
```

```
   ECO                                 name              moves
0  A00                  Anderssen's Opening                1.a3
1  A00  Polish Gambit, Anderssen's Opening        1.a3 a5 2.b4
2  A00              Creepy Crawly Formation  1.a3 e5 2.h3 d5
3  A00                          Andersspike        1.a3 g6 2.g4
4  A00               Ware; Meadow Hay; Crab                1.a4
```

b) Openning ECO mapping to name

Mapping is done using chess_openings dataset ("chess_openings.csv").

```
def map_ECO_to_name(ECO):
    mapped_ECO = list(df_openings.loc[df_openings["ECO"] == ECO]
["name"])[0]
    try:
        i = 1
        while mapped_ECO == ECO:
            mapped_ECO = list(df_openings.loc[df_openings["ECO"] ==
ECO]["name"])[i]
            i += 1
    except IndexError:
        return None
    if ";" in mapped_ECO:
        mapped_ECO = mapped_ECO.split(";")[0] if ECO not in
mapped_ECO.split(";")[0] else mapped_ECO.split(";")[1]
    if "," in mapped_ECO:
        mapped_ECO = mapped_ECO.split(",")[0] if ECO not in
mapped_ECO.split(",")[0] else mapped_ECO.split(",")[1]
    return mapped_ECO

ECOs = list(df_games["Opening ECO"])
ECOs_names = [map_ECO_to_name(ECO) for ECO in ECOs]
df_games.insert(4, "Oppening Name", ECOs_names)
```

c) Selecting four openings and coding them.

```
number_of_records = {}

for n, record in df_games.iterrows():
    opening = record['Oppening Name']

    try:
        number_of_records[opening]
    except KeyError:
        number_of_records[opening] = 1
        continue
    number_of_records[opening] += 1

selected_openings  = {}
```

```
for op, num in number_of_records.items():
    if num <= 300:
        continue
    print(f'{op}:\t{num}')
    selected_openings[op] = num
```

```
King Pawn Game: 306
Italian Game:   555
Franco-Sicilian Defense:    637
Center Counter: 630
Queen Pawn Game:678
Indian Defense: 381
Petroff Defense:308
Ware Defense:   563
Caro-Kann Defense:     324
Philidor Defense:      496
Hartlaub Gambit:511
Anderssen's Opening:  635
Sicilian Defense:     486
Zukertort Variation:  348
```

Chosen openings are Petroff Defense, Sicilian Defense, Queen Pawn Game, Italian Game. Those openings were chosen because of two main reasons:

- for each of them there is at least 300 records (it is enough number to get reasonable results),
- One of them (Petroff Defense) is considered debut of a more tranquil kind. We therefore expect these games to be characterized by fewer errors. Another one (Sicilian Defense), on the other hand, is considered aggressive debut, leading to more dynamic and therefore less accurate play. The last two openings (Queen Pawn Game, Italian Game) can (depending on the variant) lead to both calmer and more complicated play, and thus the number of errors can vary quite a bit.

```
df_games = df_games.loc[(df_games["Oppening Name"] == 'Sicilian Defense') | (df_games["Oppening Name"] == 'Petroff Defense') |
                        (df_games["Oppening Name"] == 'Queen Pawn Game') | (df_games["Oppening Name"] == 'Italian Game')]

df_games.insert(loc=5, column="Opening Name Mapped", value='')

df_games["Opening Name Mapped"].mask(df_games["Oppening Name"] == 'Sicilian Defense', 1, inplace=True)
df_games["Opening Name Mapped"].mask(df_games["Oppening Name"] == 'Petroff Defense', 2, inplace=True)
df_games["Opening Name Mapped"].mask(df_games["Oppening Name"] == 'Queen Pawn Game', 3, inplace=True)
df_games["Opening Name Mapped"].mask(df_games["Oppening Name"] == 'Italian Game', 4, inplace=True)
```

Coding opening names to integers is needed needed for use them in the model.

d) Merging mistakes with blunders

Blunders in chess are just "more serious mistakes". That's why they can be added to make the total number of mistakes (of all weights) done by players.

```
df_games['White\'s Number of Mistakes'] = df_games['White\'s Number of Mistakes'] + df_games['White\'s Number of Blunders']
df_games['Black\'s Number of Mistakes'] = df_games['Black\'s Number of Mistakes'] + df_games['Black\'s Number of Blunders']
df_games.drop(columns=['White\'s Number of Blunders', 'Black\'s Number of Blunders'], inplace=True)
df_games.head()
```

```
   Time Control  White Rating  Black Rating Opening ECO    Oppening Name  \
6        15 + 0          1500          2058         C50     Italian Game
26        3 + 0          1883          1899         C50     Italian Game
28      10 + 15          1595          1483         D00  Queen Pawn Game
36      60 + 10          2005          1986         C42  Petroff Defense
37        5 + 5          2038          1996         D00  Queen Pawn Game

   Opening Name Mapped  White's Number of Mistakes  \
6                    4                           2
26                   4                           4
28                   3                           2
36                   2                           0
37                   3                           4

    Black's Number of Mistakes Winner
6                            4  White
26                           8  White
28                           1  Black
36                           4  White
37                           6  White
```

e) Time Control mapping to name and coding

Mapping helps to create three main groups used in chess (Classic, Rapid and Blitz) from many time cotrol values. Coding time control to integers is needed needed for use them in the model.

```
df_games["Time Control"].unique()
```

```
array(['15 + 0', '3 + 0', '10 + 15', '60 + 10', '5 + 5', '3 + 2',
       '30 + 25', '60 + 0', '10 + 0', '5 + 0', '30 + 0'],
dtype=object)

df_games.insert(loc=1, column="Time Control Name", value='')
df_games["Time Control Name"].mask((df_games["Time Control"] == '3 +
2') | (df_games["Time Control"] == '5 + 5') | (df_games["Time
Control"] == '5 + 0') | (df_games["Time Control"] == '3 + 0'),
'Blitz', inplace=True)
df_games["Time Control Name"].mask((df_games["Time Control"] == '10 +
0') | (df_games["Time Control"] == '10 + 15') | (df_games["Time
Control"] == '15 + 0'), 'Rapid', inplace=True)
df_games["Time Control Name"].mask((df_games["Time Control"] == '60 +
10') | (df_games["Time Control"] == '60 + 0') | (df_games["Time
Control"] == '30 + 0') | (df_games["Time Control"] == '30 + 25'),
'Classic', inplace=True)

df_games.insert(loc=2, column="Time Control Mapped", value='')
df_games["Time Control Mapped"].mask((df_games["Time Control Name"] ==
'Blitz'), 1, inplace=True)
df_games["Time Control Mapped"].mask((df_games["Time Control Name"] ==
'Rapid'), 2, inplace=True)
df_games["Time Control Mapped"].mask((df_games["Time Control Name"] ==
'Classic'), 3, inplace=True)
```

f) Calculating rating difference

```
df_games.insert(loc=3, column="Rating Difference", value='')
df_games['Rating Difference'] = df_games['White Rating'] -
df_games['Black Rating']
df_games.drop(columns=['White Rating', 'Black Rating'], inplace=True)
df_games.head()

   Time Control Time Control Name Time Control Mapped  Rating
Difference  \
6        15 + 0             Rapid                   2               -
558
26        3 + 0             Blitz                   1                -
16
28       10 + 15            Rapid                   2
112
36       60 + 10          Classic                   3
19
37        5 + 5             Blitz                   1
42

   Opening ECO    Oppening Name Opening Name Mapped  \
6          C50     Italian Game                   4
26         C50     Italian Game                   4
28         D00  Queen Pawn Game                   3
```

```
36         C42  Petroff Defense                    2
37         D00  Queen Pawn Game                    3

    White's Number of Mistakes  Black's Number of Mistakes Winner
6                            2                           4  White
26                           4                           8  White
28                           2                           1  Black
36                           0                           4  White
37                           4                           6  White
```

g) Removing rows with NaN values

```
df_games.dropna()
```

```
      Time Control Time Control Name Time Control Mapped  Rating
Difference  \
6           15 + 0             Rapid                   2
-558
26           3 + 0             Blitz                   1
-16
28         10 + 15             Rapid                   2
112
36         60 + 10           Classic                   3
19
37           5 + 5             Blitz                   1
42
...            ...               ...                 ...
...
18374        5 + 0             Blitz                   1
-125
18380       10 + 0             Rapid                   2
24
18404        5 + 5             Blitz                   1
13
18424       15 + 0             Rapid                   2
-4
18454       30 + 0           Classic                   3
342

      Opening ECO     Oppening Name Opening Name Mapped  \
6             C50      Italian Game                   4
26            C50      Italian Game                   4
28            D00   Queen Pawn Game                   3
36            C42   Petroff Defense                   2
37            D00   Queen Pawn Game                   3
...           ...               ...                 ...
18374         B20  Sicilian Defense                   1
18380         C50      Italian Game                   4
18404         C42   Petroff Defense                   2
18424         D00   Queen Pawn Game                   3
```

```
18454         B20  Sicilian Defense                   1

       White's Number of Mistakes  Black's Number of Mistakes Winner
6                               2                           4  White
26                              4                           8  White
28                              2                           1  Black
36                              0                           4  White
37                              4                           6  White
...                           ...                         ...    ...
18374                           4                           6  White
18380                           2                           3  White
18404                           3                           3  Black
18424                           2                           4  White
18454                           1                           0  Black

[2027 rows x 10 columns]
```

h) Saving data to csv

```
df_games.to_csv('Chess_data_after_preprocessing.csv')
```

3. Model

a) Models specification

- Model 1 - it takes into account time control and ranking difference. This model is designed to analyze the effect of time control and ranking difference on the number of errors of each party. The model takes into account two independent variables: "time_control" (representing the type of time controlled) and "rating_difference" (representing the ranking difference between players). By appropriately assigning coefficients (e.g. gamma_white), the model estimates the effect of these variables on the number of player errors. Analysis of this model will allow us to assess how time control and ranking difference affect the quality of the game.
- Model 2 - This model extends the "Model 1" by also considering the "opening" variable, which represents the choice of opening in the game. We assume that "opening" can affect game performance, regardless of time control and ranking difference. In the model, we include factors such as "time_control," "rating_difference" and "opening" that affect the outcome variable "mistakes." By estimating the relevant coefficients, the model will allow us to assess the impact of these variables on the number of player mistakes.

b) Difference between models

In "Model 1", we analyze the effect of time control and ranking difference on game performance. We assume that the number of player errors depends only on these two variables. In "Model 2", we extend the analysis by additionally considering the "opening" variable, which represents the choice of opening in the game. Both models allow us to evaluate the impact of these variables on game performance, but the second model additionally includes the choice of opening as a factor affecting player errors.

c) Models difference justification

Such different models will be the best for studying the effect of chess opening on the number of errors made by players (which is the goal of this project), since the only difference between them is precisely the fact of taking into account or not taking into account the chosen debit.

d) Models description

Common elements of the models (formulas and parameters that fulfill the same roles, differing at most in values between models):

- Inputs include information about the number of samples (N), the difference in ranking (rating_difference), time control (time_control), and the number of mistakes of the white player (mistakes_white_model) and the black player (mistakes_black_model).
- Model parameters include "gamma_white" and "gamma_black" (influence of ranking) and "time_controll_coeff_blitz", "time_controll_coeff_rapid" and "time_controll_coeff_classic" (influence of type of time controlled).

Elements included in "Model 2" only:

- The variables "opening_coeff_sicilian", "opening_coeff_petroff", "opening_coeff_queen_pawn" and "opening_coeff_italian" (influence of opening choice) were added to the model parameters.

In the model section, calculations are made for the number of player errors based on the parameters and input data. The poisson_log function is used, which operates on the logarithms of the expected number of events to account for the positivity condition of the lambda parameter in the Poisson distribution. The logarithmic transformation allows modeling the value of λ in terms of the whole real number, which is important because of the positivity requirement for λ in a Poisson distribution. The poisson_log function generates a Poisson distribution based on the logarithm of the expected number of events, and the poisson_log_rng function generates random values from this distribution. Using the poisson_log function allows you to take into account the fact that the lambda parameter must be add.

$$\log \lambda_{color} = \gamma_{color} * rating_d ifference + time_c ontroll_c oeff + \left(oppening_c oeff f_{¿} \right)$$

$$mistakes_{color} \sim Poisson\left(\lambda_{color}\right)$$

4. Priors

Data loading

```
df = pd.read_csv('Chess_data_after_preprocessing.csv')
df.drop(columns=['Unnamed: 0'], inplace=True)
df

     Time Control Time Control Name  Time Control Mapped  Rating 
Difference  \
0           15 + 0             Rapid                    2   
-558
```

```
1            3 + 0             Blitz                     1
-16
2           10 + 15            Rapid                     2
112
3           60 + 10          Classic                     3
19
4            5 + 5             Blitz                     1
42
...            ...               ...                   ...
...
2022         5 + 0             Blitz                     1
-125
2023        10 + 0             Rapid                     2
24
2024         5 + 5             Blitz                     1
13
2025        15 + 0             Rapid                     2
-4
2026        30 + 0           Classic                     3
342

     Opening ECO     Oppening Name  Opening Name Mapped  \
0            C50      Italian Game                    4
1            C50      Italian Game                    4
2            D00   Queen Pawn Game                    3
3            C42   Petroff Defense                    2
4            D00   Queen Pawn Game                    3
...          ...               ...                  ...
2022         B20  Sicilian Defense                    1
2023         C50      Italian Game                    4
2024         C42   Petroff Defense                    2
2025         D00   Queen Pawn Game                    3
2026         B20  Sicilian Defense                    1

      White's Number of Mistakes  Black's Number of Mistakes Winner
0                              2                           4  White
1                              4                           8  White
2                              2                           1  Black
3                              0                           4  White
4                              4                           6  White
...                          ...                         ...    ...
2022                           4                           6  White
2023                           2                           3  White
2024                           3                           3  Black
2025                           2                           4  White
2026                           1                           0  Black

[2027 rows x 10 columns]
```

Setting values for constants

```
N = df.shape[0]
time_control_list = list(df['Time Control Mapped'])
opening_list = list(df['Opening Name Mapped'])
rating_difference_list = list(df['Rating Difference'])
mistakes_white_list = list(df['White\'s Number of Mistakes'])
mistakes_black_list = list(df['Black\'s Number of Mistakes'])

R = 100
```

Model 1 - prior

a) Prior parameters values with explanation:

- gamma_white = normal_rng(-0.0015, 0.0001) - gamma_white is a coefficient that determines the effect of the ranking difference (rating_difference) on the number of mistakes made by a player playing with the white color (mistakes_white). The value of μ is negative, because a positive rating difference means an advantage for the white color (we assume that less mistakes are made when playing with a weaker player).
- gamma_black = normal_rng(0.0015, 0.0001) - gamma_black is a coefficient that determines the effect of the ranking difference (rating_difference) on the number of mistakes made by a player playing with the white black (mistakes_black). The value of μ is positive, because a positive rating difference means an advantage for the white color (we assume that more mistakes are made when playing with a stronger player).
- time_controll_coeff - time_controll_coeff is a vector of coefficients that determine the effect of the time controll (time_controll) on the number of mistakes made by both players (mistakes_white & mistakes_black). The assumption is that the faster game is the more mistakes are done.
  - time_controll_coeff[1] = normal_rng(1.3, 0.1) - coefficient for blitz games (μ is the greatest among time_controll_coefficients)
  - time_controll_coeff[2] = normal_rng(0.8, 0.1) - coefficient for rapid games (μ is greater than for classics but lower than for blitz)
  - time_controll_coeff[3] = normal_rng(0.4, 0.1) - coefficient for classic games (μ is the lowest among time_controll_coefficients)

```
model_1_ppc = CmdStanModel(stan_file='stan_files/model_1_ppc.stan')

model_1_prior = model_1_ppc.sample(
    data={
        'rating_difference': rating_difference_list,
        'N': len(time_control_list),
        'time_control': time_control_list,
        },
    seed=19042023,
    fixed_param=True,
    iter_sampling=R,
    iter_warmup=0,
    chains=1
)
```

```
model_1_prior_df = model_1_prior.draws_pd()
model_1_prior_df.head()
```

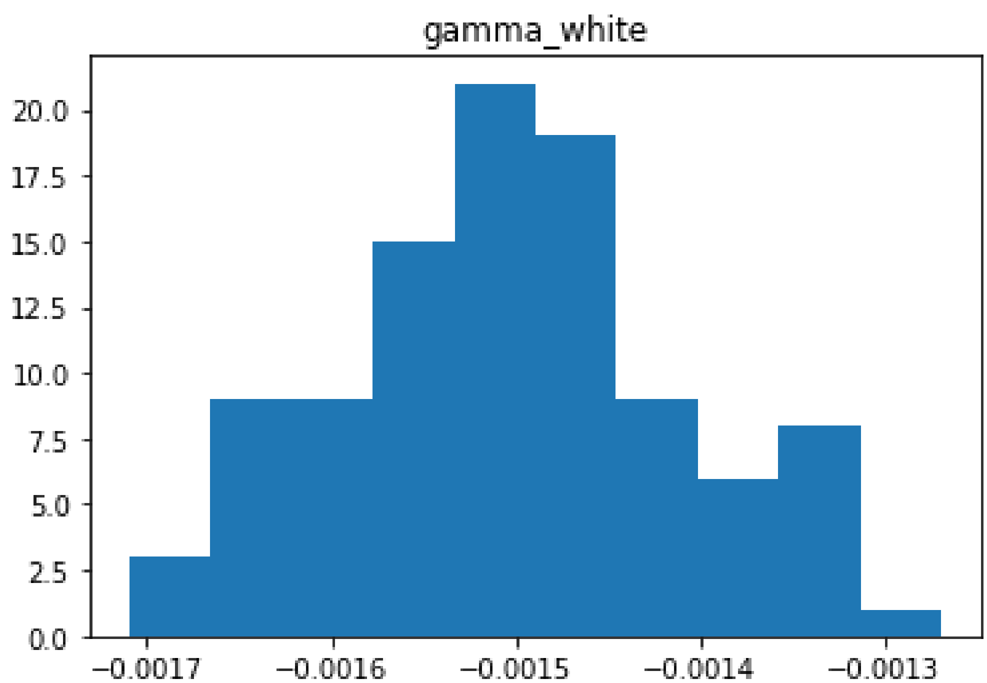
```
INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |██████████| 00:00 Sampling completed



INFO:cmdstanpy:CmdStan done processing.
```

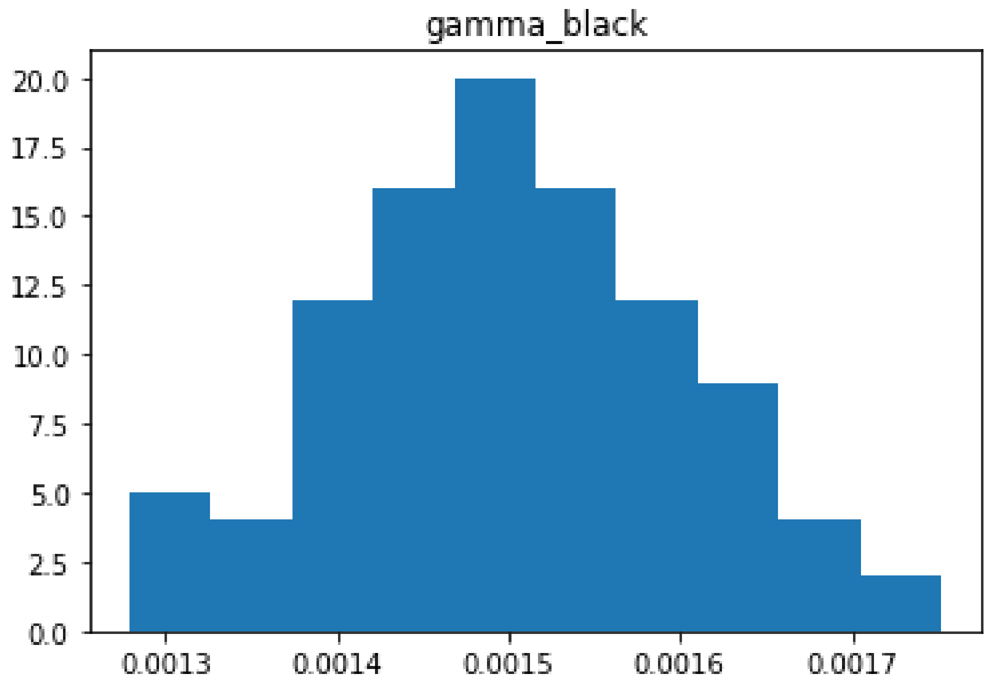
```
   lp__  accept_stat__  gamma_white  gamma_black
time_controll_coeff[1]  \
0   0.0            0.0    -0.001521     0.001432
1.17601
1   0.0            0.0    -0.001620     0.001312
1.33492
2   0.0            0.0    -0.001435     0.001665
1.37245
3   0.0            0.0    -0.001355     0.001473
1.11763
4   0.0            0.0    -0.001683     0.001416
1.40823

   time_controll_coeff[2]  time_controll_coeff[3]
mistakes_white[1]  \
0                0.811876                0.472156                5.0

1                0.662482                0.349655                5.0

2                0.737271                0.286275                5.0

3                0.726541                0.287589                3.0

4                0.743485                0.290130                4.0


   mistakes_white[2]  mistakes_white[3]  ...  mistakes_black[2018]  \
0                1.0                0.0  ...                   3.0
1                3.0                1.0  ...                   1.0
2                5.0                3.0  ...                   3.0
3                1.0                1.0  ...                   1.0
4                4.0                1.0  ...                   1.0

   mistakes_black[2019]  mistakes_black[2020]  mistakes_black[2021]  \
0                   1.0                   2.0                   2.0
1                   0.0                   3.0                   5.0
2                   4.0                   5.0                   4.0
```

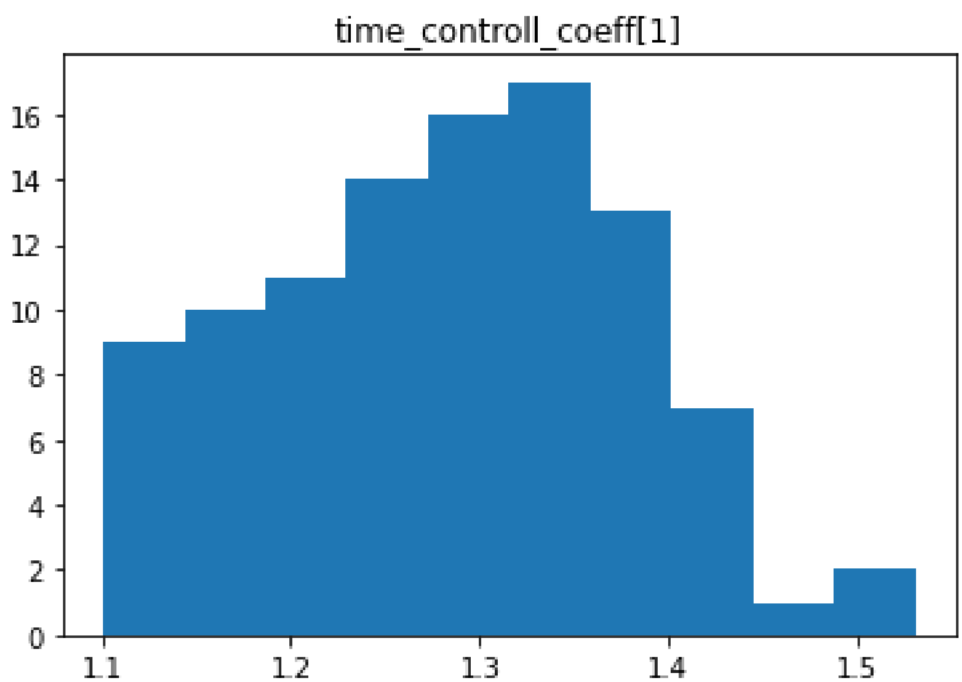
```
3                      3.0                    3.0                    4.0
4                      2.0                    5.0                    2.0

   mistakes_black[2022]  mistakes_black[2023]  mistakes_black[2024]  \
0                   2.0                   1.0                   3.0
1                   3.0                   5.0                   3.0
2                   4.0                   3.0                   3.0
3                   3.0                   4.0                   4.0
4                   3.0                   7.0                   4.0

   mistakes_black[2025]  mistakes_black[2026]  mistakes_black[2027]
0                   3.0                   3.0                   3.0
1                   8.0                   1.0                   0.0
2                   1.0                   0.0                   0.0
3                   4.0                   1.0                   2.0
4                   6.0                   2.0                   1.0

[5 rows x 4061 columns]
```

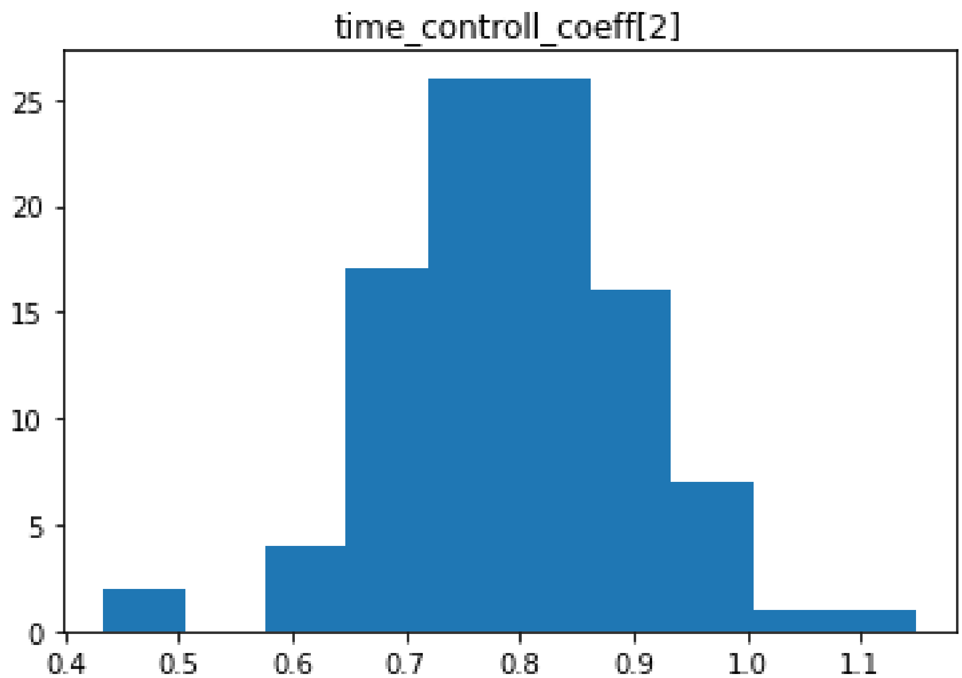
b) Prior predictive checks for parameters

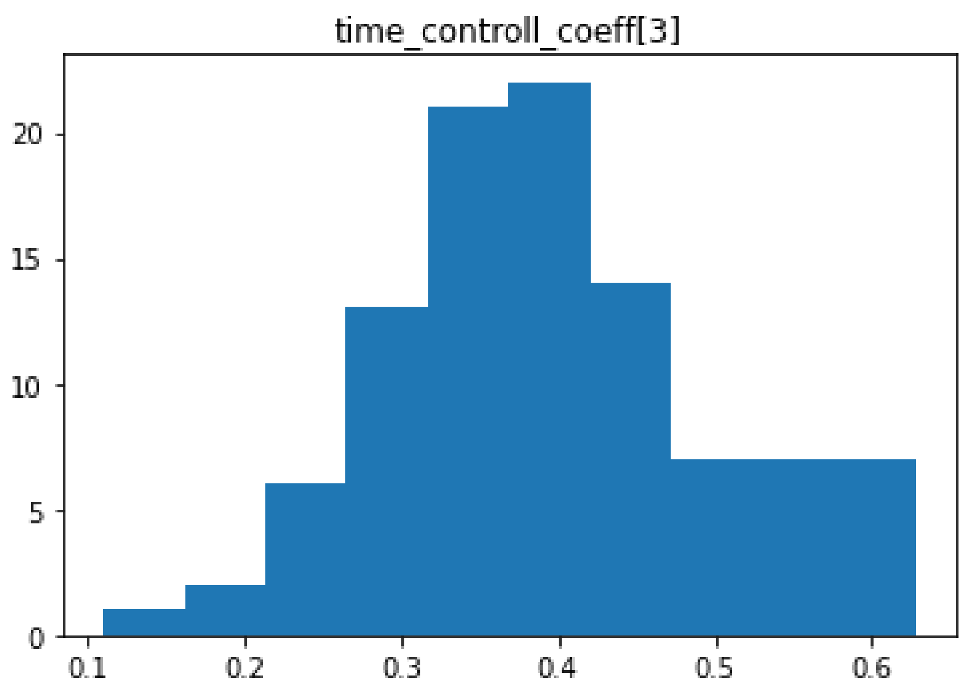
```
for parameter in ['gamma_white', 'gamma_black',
'time_controll_coeff[1]', 'time_controll_coeff[2]',
'time_controll_coeff[3]']:
    plt.hist(model_1_prior_df[parameter])
    plt.title(parameter)
    plt.show()
```




gamma_black
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
0.0013
0.0014
0.0015
0.0016
0.0017


time_controll_coeff[1]
16
14
12
10
8
6
4
2
0
1.1
1.2
1.3
1.4
1.5



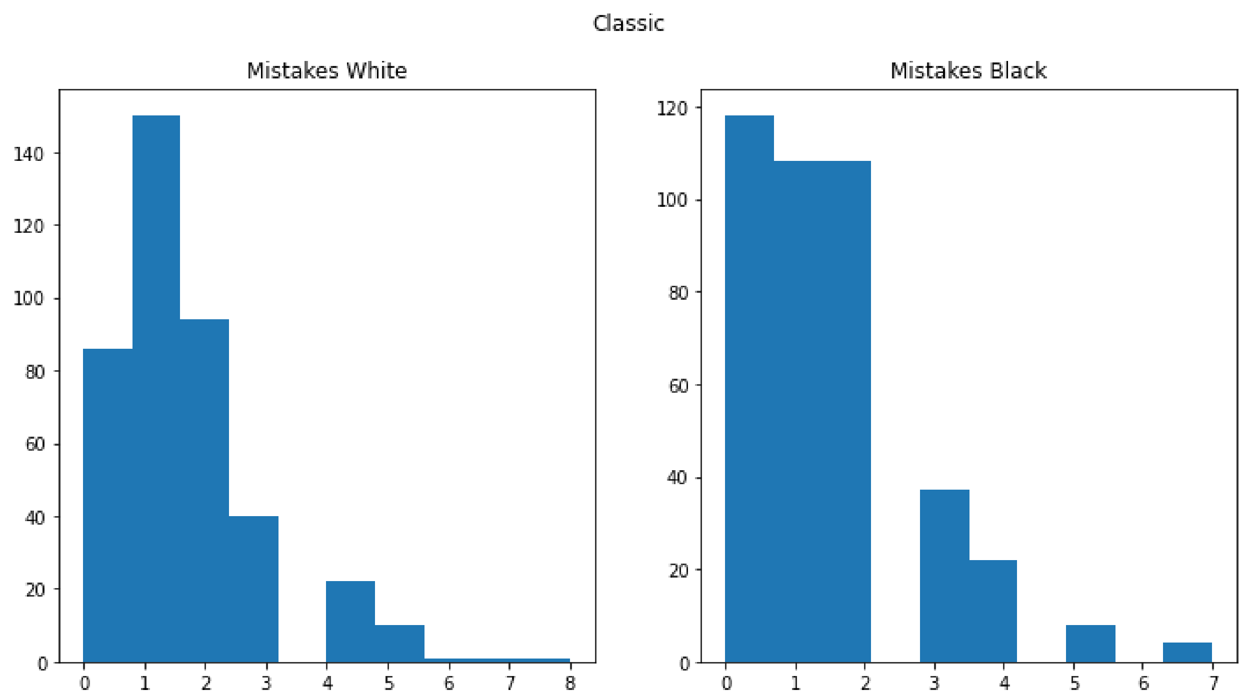


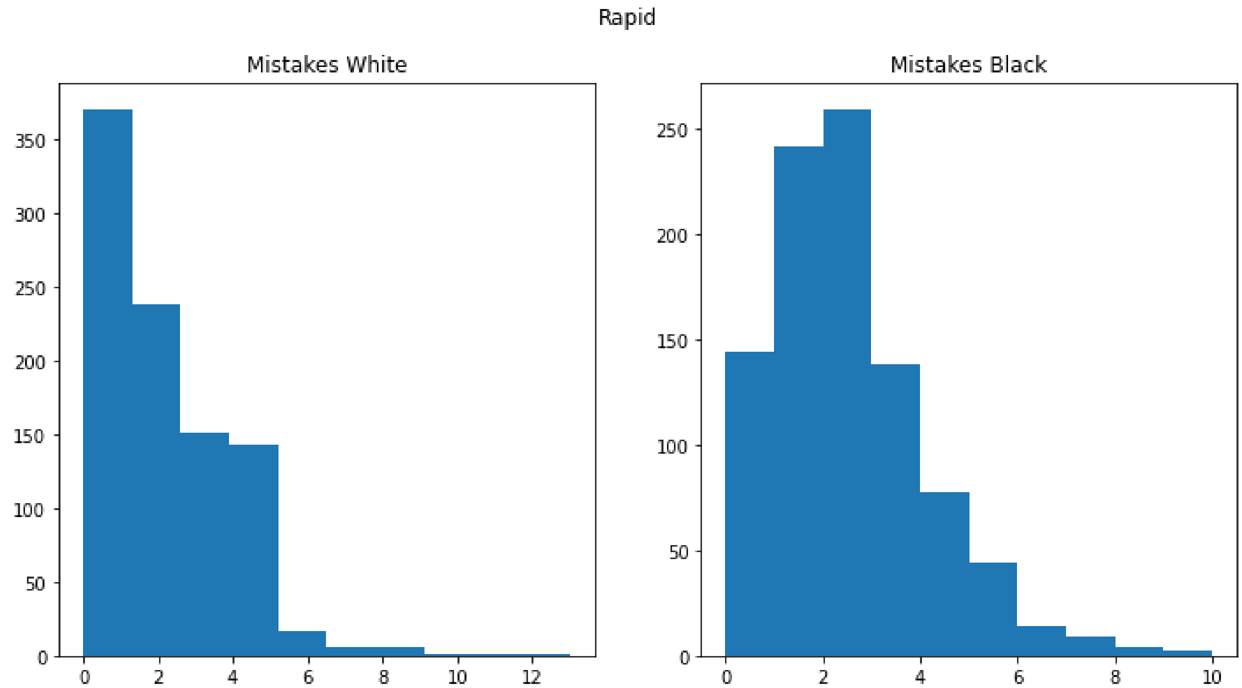
Parameters simulated from priors meet our expectations.

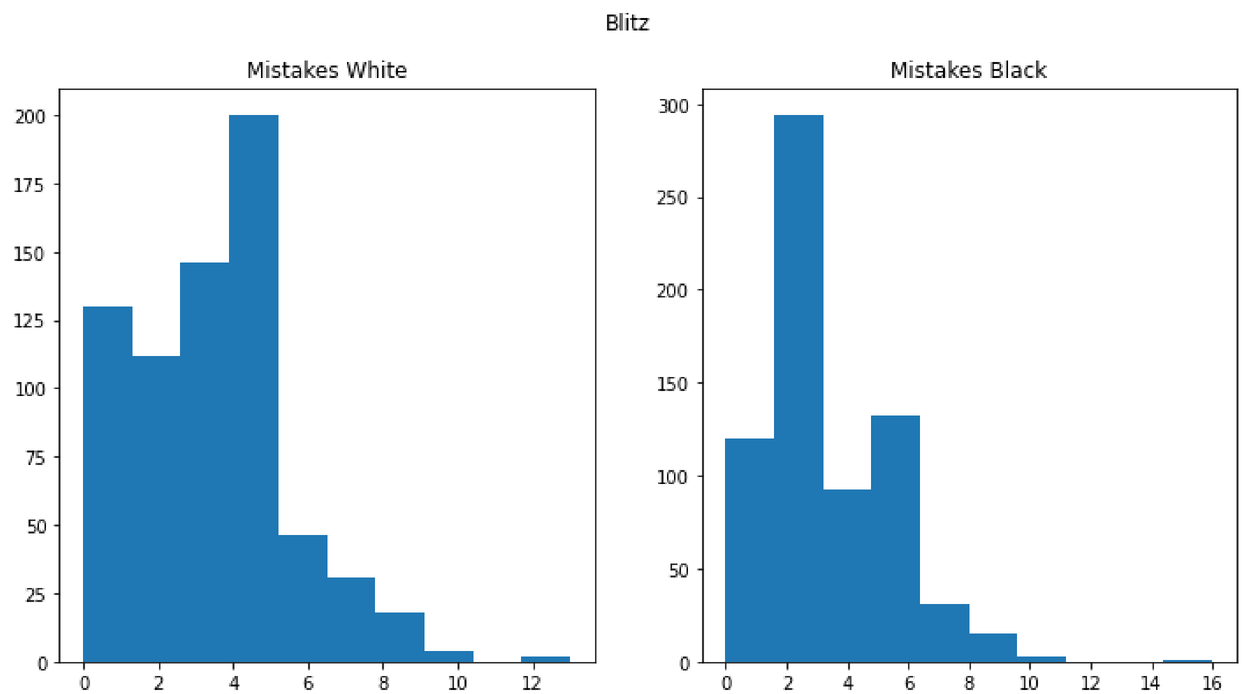
c) Prior predictive checks for measurements

```
mistakes_by_time_control = {
    'Classic': {'Black': [], 'White': []},
    'Rapid': {'Black': [], 'White': []},
    'Blitz': {'Black': [], 'White': []}
```

```
}

time_control_mapping = {
    3: 'Classic',
    2: 'Rapid',
    1: 'Blitz'
}

for i in range(N):
    tc = time_control_mapping[time_control_list[i]]
    for color in ['Black', 'White']:
        mistakes_by_time_control[tc]
[color].append(model_1_prior_df[f'mistakes_{color.lower()}
[{i+1}]'].values[59])

for tc in mistakes_by_time_control.keys():
    fig, axs = plt.subplots(1, 2, figsize=(12, 6))
    fig.suptitle(tc)
    for c_i, color in enumerate(['White', 'Black']):
        axs[c_i].hist(mistakes_by_time_control[tc][color])
        axs[c_i].set_title(f'Mistakes {color}')

    plt.show()
```







Measurements simulated from priors meet our expectations - as the time available for moves decreases, the number of errors increases

d) Prior parameters selection.

As prior parameters were selected those values for which the model results were most similar to the actual data

```
best_matching = -1
best_matching_ind = -1

for row_index, row in model_1_prior_df.iterrows():
    tmp_mistakes_white = []
    tmp_mistakes_black = []

    for i in range(N):
        tmp_mistakes_white.append(row[f'mistakes_white[{i+1}]'])
        tmp_mistakes_black.append(row[f'mistakes_black[{i+1}]'])

    number_of_matched_white = sum(1 for x, y in
zip(tmp_mistakes_white, mistakes_white_list) if x == y)
    number_of_matched_black = sum(1 for x, y in
zip(tmp_mistakes_black, mistakes_black_list) if x == y)
    acceptance_percent = (number_of_matched_white +
number_of_matched_black) / (len(mistakes_white_list) +
len(mistakes_black_list)) * 100

    if acceptance_percent > best_matching:
        best_matching = acceptance_percent
        best_matching_ind = row_index

chosen_row_in_prior_model_1 = best_matching_ind
prior_parameters_model_1 = {}

print('Prior parameters for Model 1:')
for col_name in ['gamma_white', 'gamma_black',
'time_controll_coeff[1]', 'time_controll_coeff[2]',
'time_controll_coeff[3]']:
    prior_parameters_model_1[col_name] =
model_1_prior_df.loc[chosen_row_in_prior_model_1, col_name]
    print(f"\t{col_name} =
{model_1_prior_df.loc[chosen_row_in_prior_model_1, col_name]}")
```

```
Prior parameters for Model 1:
	gamma_white = -0.00144638
	gamma_black = 0.00137461
	time_controll_coeff[1] = 1.23487
	time_controll_coeff[2] = 0.836987
	time_controll_coeff[3] = 0.295052
```

Model 2 - prior

a) Prior parameters values with explanation:

- gamma_white = normal_rng(-0.0015, 0.0001) - gamma_white is a coefficient that determines the effect of the ranking difference (rating_difference) on the number of mistakes made by a player playing with the white color (mistakes_white). The value of μ is negative, because a positive rating difference means an advantage for the white color (we assume that less mistakes are made when playing with a weaker player).

- gamma_black = normal_rng(0.0015, 0.0001) - gamma_black is a coefficient that determines the effect of the ranking difference (rating_difference) on the number of mistakes made by a player playing with the white black (mistakes_black). The value of μ is positive, because a positive rating difference means an advantage for the white color (we assume that more mistakes are made when playing with a stronger player).
- time_controll_coeff - time_controll_coeff is a vector of coefficients that determine the effect of the time controll (time_controll) on the number of mistakes made by both players (mistakes_white & mistakes_black). The assumption is that the faster game is the more mistakes are done.
  - time_controll_coeff[1] = normal_rng(1.3, 0.1) - coefficient for blitz games (μ is the greatest among time_controll_coefficients)
  - time_controll_coeff[2] = normal_rng(0.8, 0.1) - coefficient for rapid games (μ is greater than for classics but lower than for blitz)
  - time_controll_coeff[3] = normal_rng(0.4, 0.1) - coefficient for classic games (μ is the lowest among time_controll_coefficients)
- opening_coeff - opening_coeff is a vector of coefficients that determine the effect of the choosen opening on the number of mistakes made by both players (mistakes_white & mistakes_black). In a more aggressive opening, more mistakes will be made, while in a calm, tie-biased opening number of mistakes will be fewer.
  - opening_coeff[1] = normal_rng(0.6, 0.05) - coefficient for Sicilian opening (μ is the greatest among opening_coefficients)
  - opening_coeff[2] = normal_rng(0.25, 0.05) - coefficient for Petroff Defense opening (μ is the lowest among opening_coefficients)
  - opening_coeff[3] = normal_rng(0.35, 0.05) - coefficient for Queen Pawn opening (μ is greater than for Petroff Defense but lower than for Sicilian)
  - opening_coeff[4] = normal_rng(0.35, 0.05) - coefficient for Italian opening (μ is greater than for Petroff Defense but lower than for Sicilian)

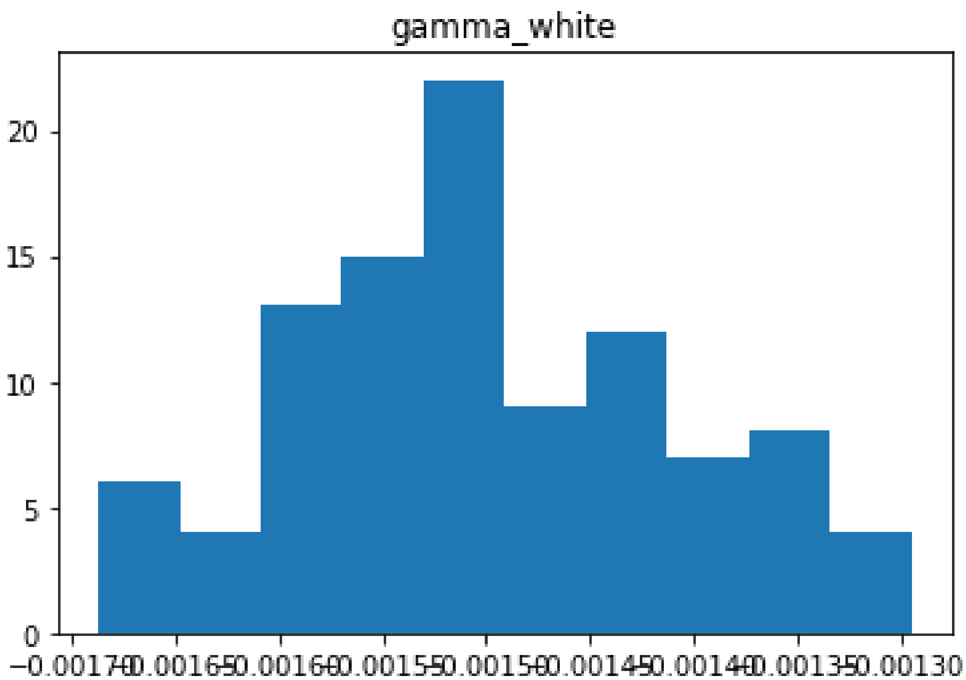
```
model_2_ppc = CmdStanModel(stan_file='stan_files/model_2_ppc.stan')

model_2_prior = model_2_ppc.sample(
    data={
        'rating_difference': rating_difference_list,
        'N': len(time_control_list),
        'time_control': time_control_list,
        'opening': opening_list
        },
    seed=19042023,
    fixed_param=True,
    iter_sampling=R,
    iter_warmup=0,
    chains=1
)

model_2_prior_df = model_2_prior.draws_pd()
model_2_prior_df.head()
```

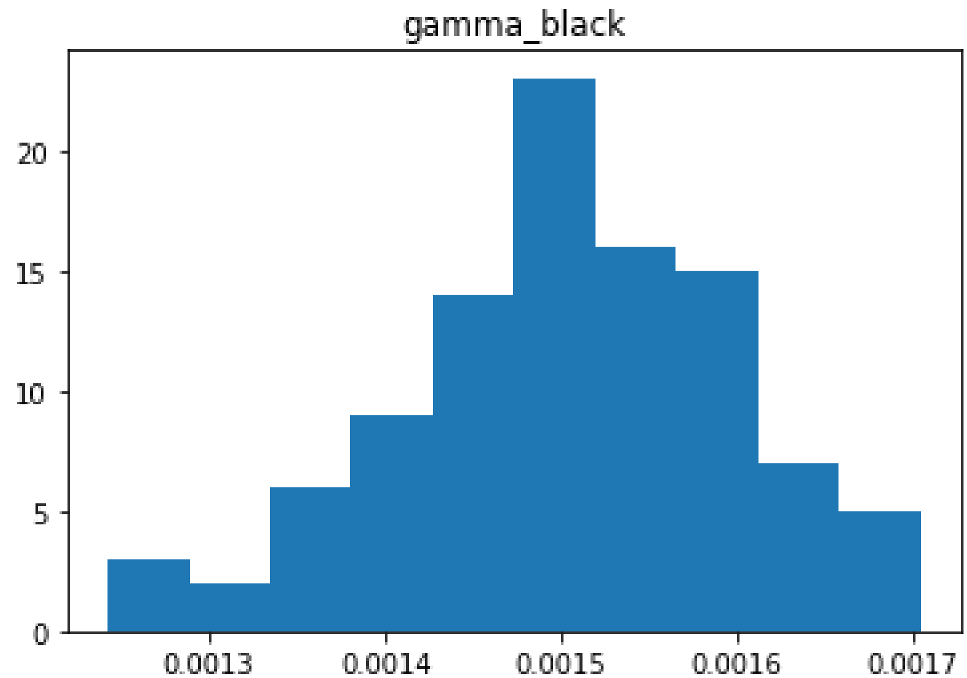
```
INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |██████████| 00:00 Sampling completed



INFO:cmdstanpy:CmdStan done processing.


   lp__  accept_stat__  gamma_white  gamma_black
time_controll_coeff[1]  \
0   0.0            0.0    -0.001521     0.001432
1.17601
1   0.0            0.0    -0.001424     0.001625
1.17026
2   0.0            0.0    -0.001515     0.001430
1.22963
3   0.0            0.0    -0.001307     0.001676
1.24910
4   0.0            0.0    -0.001488     0.001531
1.14947

   time_controll_coeff[2]  time_controll_coeff[3]  opening_coeff[1]  \
0                0.811876                0.472156          0.630683
1                0.821946                0.465057          0.648372
2                0.726746                0.449813          0.662715
3                0.803572                0.090108          0.592138
4                0.830487                0.169930          0.680737

   opening_coeff[2]  opening_coeff[3]  ...  mistakes_black[2018]  \
0          0.305196          0.272360  ...                   9.0
1          0.320042          0.354479  ...                   5.0
2          0.260256          0.326709  ...                   2.0
3          0.322077          0.397858  ...                   2.0
4          0.239092          0.391354  ...                   7.0

   mistakes_black[2019]  mistakes_black[2020]  mistakes_black[2021]  \
0                   2.0                  15.0                   6.0
1                   3.0                   6.0                   2.0
2                   5.0                   7.0                   4.0
3                   3.0                   6.0                  11.0
4                   3.0                  16.0                   4.0

   mistakes_black[2022]  mistakes_black[2023]  mistakes_black[2024]  \
0                   3.0                   3.0                   5.0
1                   7.0                   4.0                   3.0
2                   7.0                   6.0                   5.0
3                   2.0                  10.0                   1.0
4                   5.0                   5.0                   3.0
```

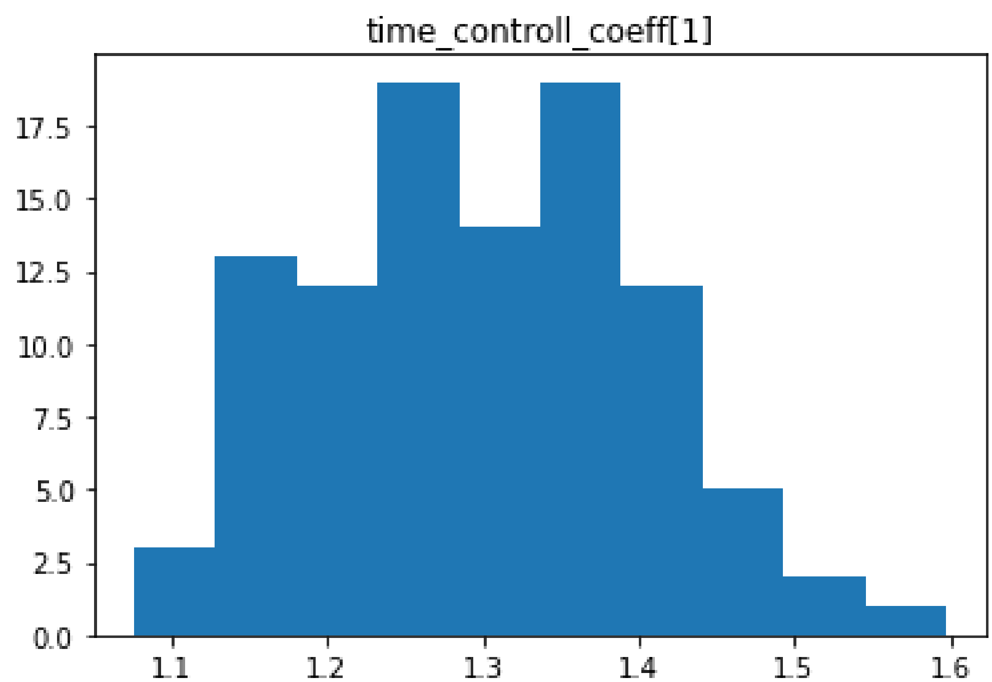
```
   mistakes_black[2025]  mistakes_black[2026]  mistakes_black[2027]
0                   5.0                   4.0                   4.0
1                   2.0                   4.0                   4.0
2                   4.0                   6.0                   2.0
3                   7.0                   6.0                   3.0
4                   3.0                   3.0                   3.0

[5 rows x 4065 columns]
```

b) Prior predictive checks for parameters

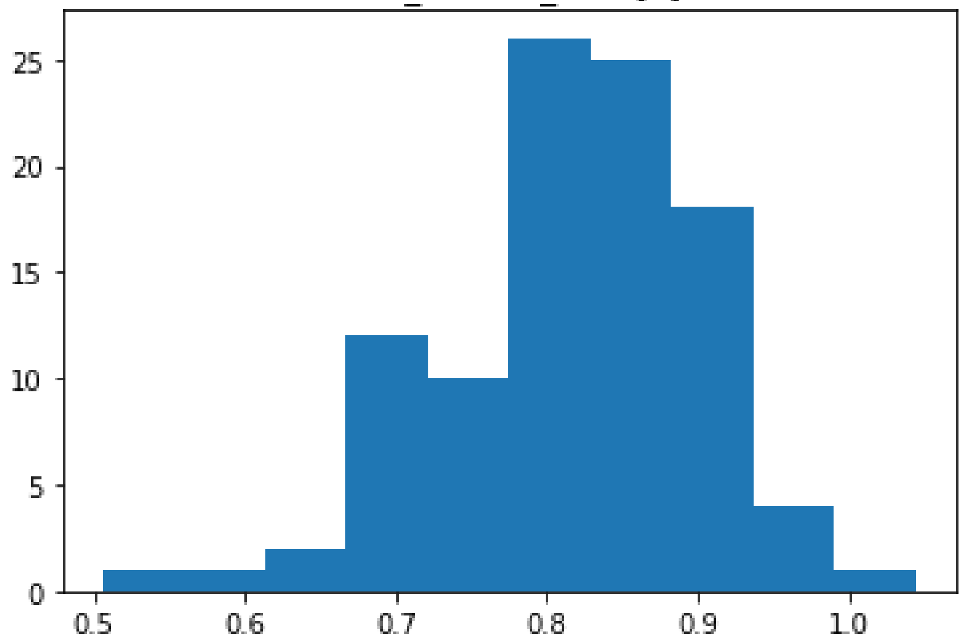
```
for parameter in ['gamma_white', 'gamma_black',
'time_controll_coeff[1]', 'time_controll_coeff[2]',
'time_controll_coeff[3]', 'opening_coeff[1]', 'opening_coeff[2]',
'opening_coeff[3]', 'opening_coeff[4]']:

plt.hist(model_2_prior_df[parameter].loc[model_2_prior_df[parameter] !
= 0])
    plt.title(parameter)
    plt.show()
```



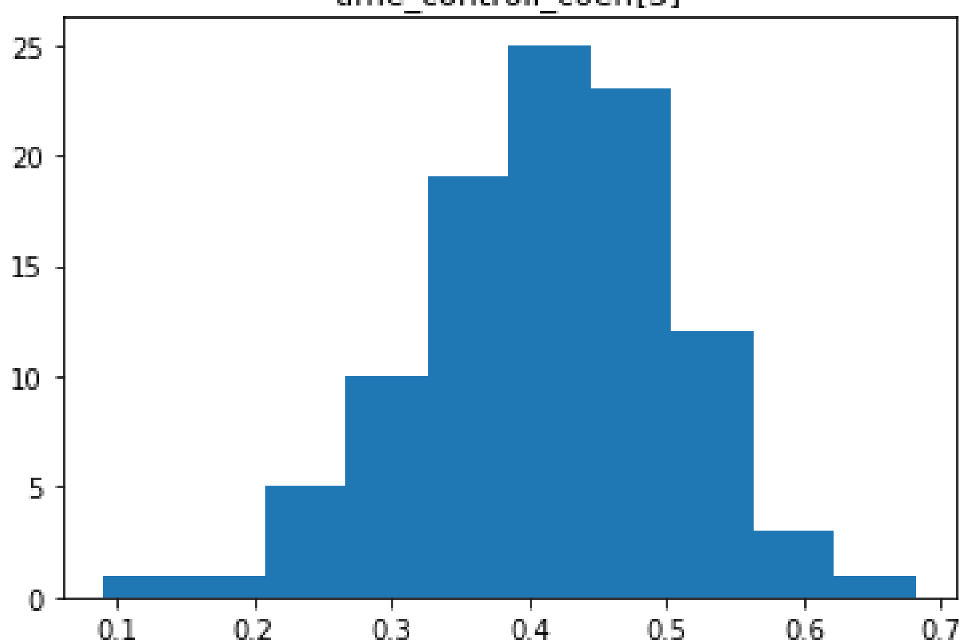

gamma_black
0
5
10
15
20
0.0013
0.0014
0.0015
0.0016
0.0017

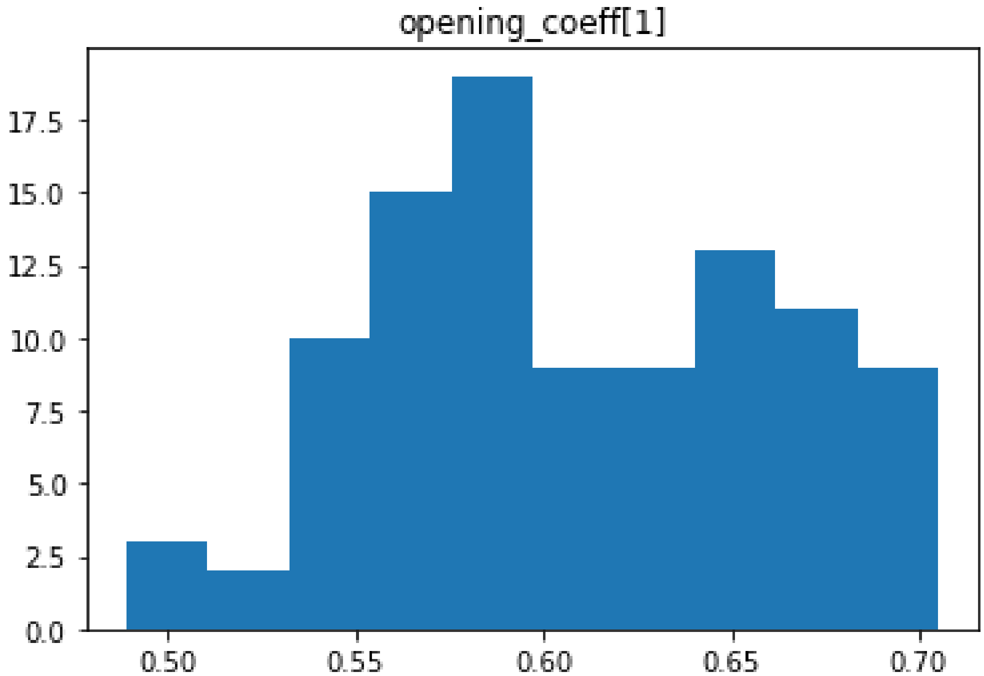

time_controll_coeff[1]
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
1.1
1.2
1.3
1.4
1.5
1.6

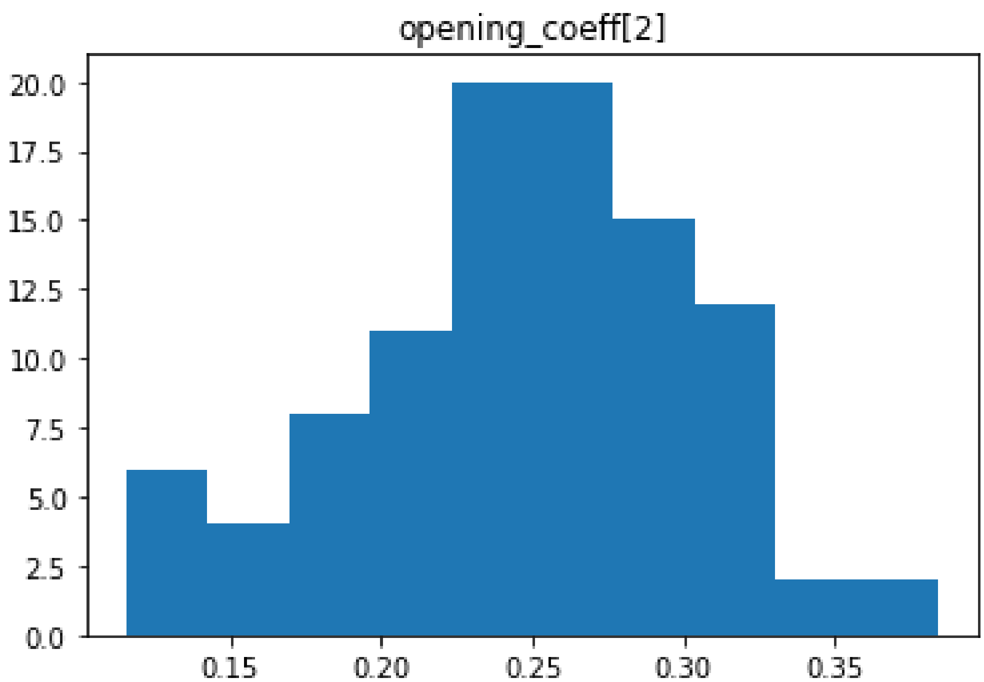
time_controll_coeff[2]

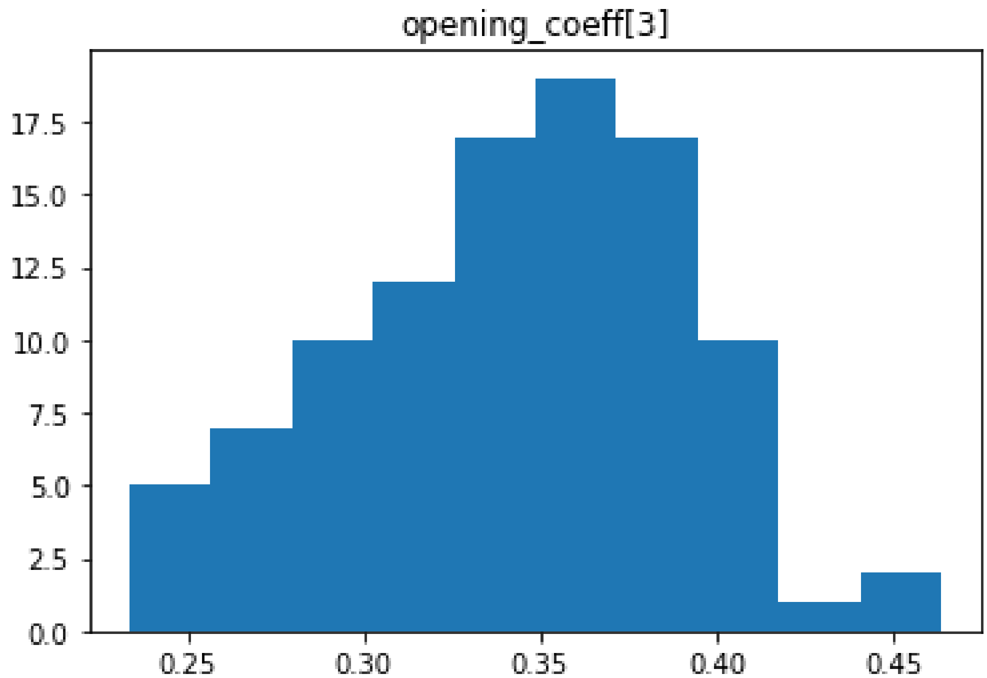


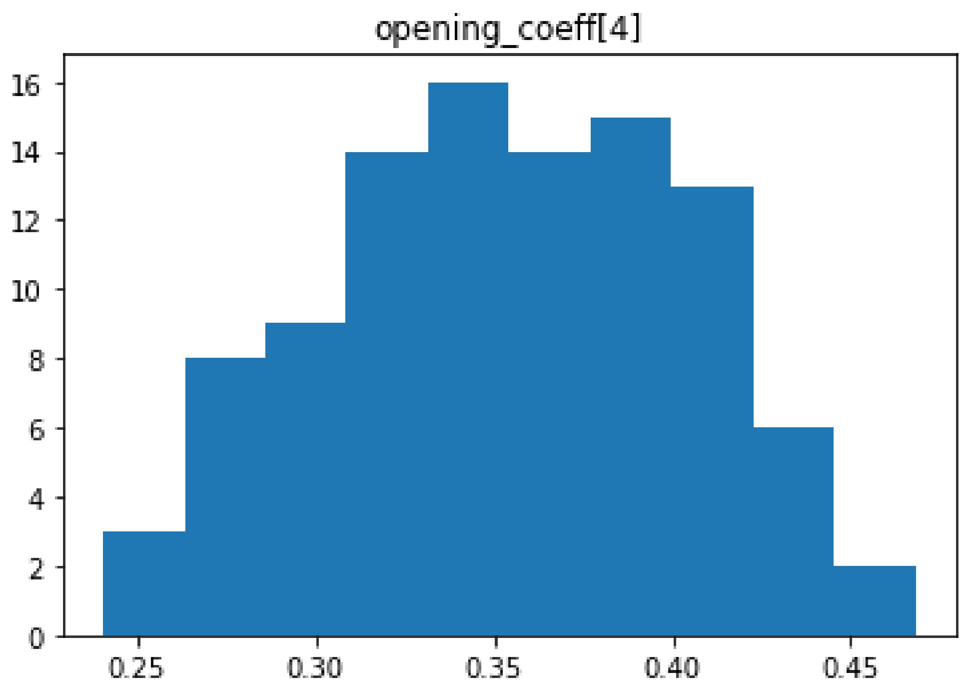
time_controll_coeff[3]




opening_coeff[1]
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.50
0.55
0.60
0.65
0.70


opening_coeff[2]
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
0.15
0.20
0.25
0.30
0.35





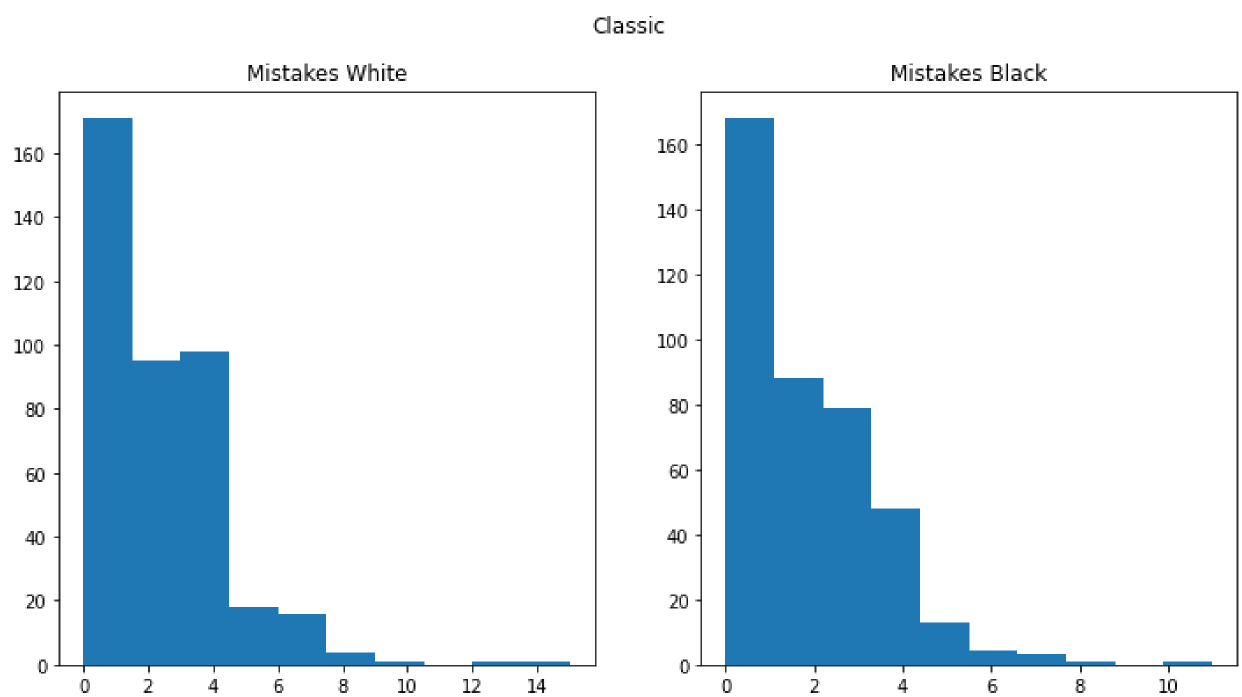
Parameters simulated from priors meet our expectations.

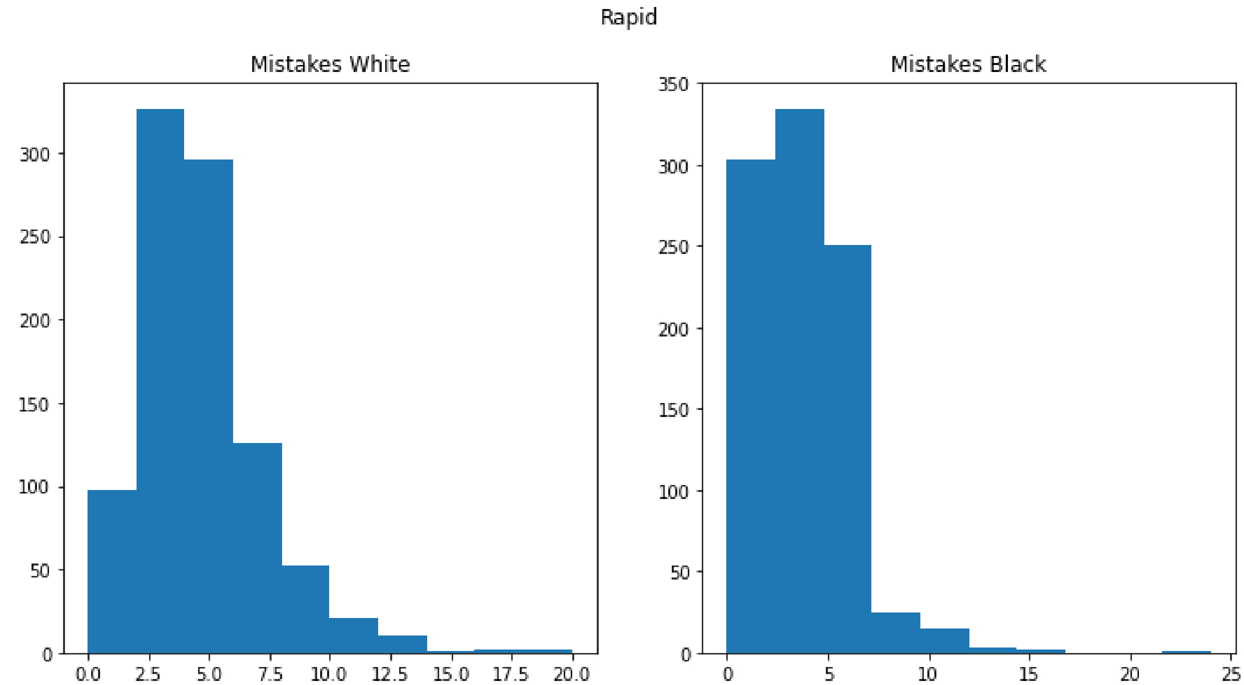
c) Prior predictive checks for measurements

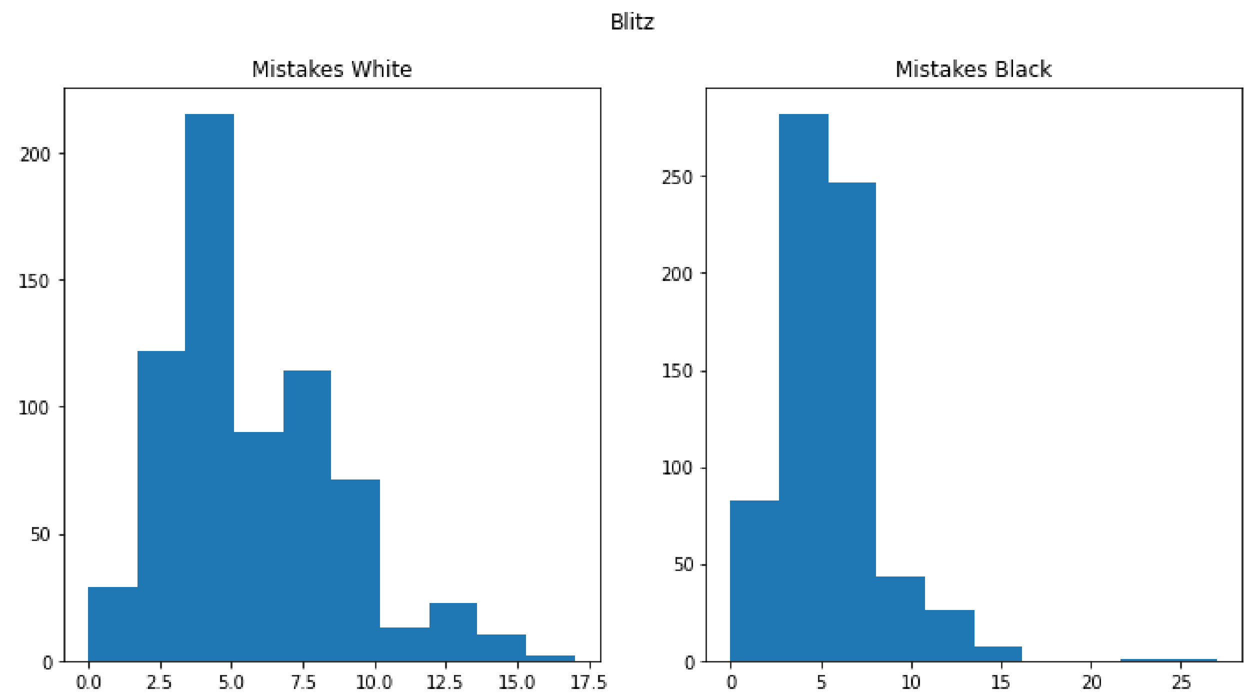
```
mistakes_by_time_control = {
    'Classic': {'Black': [], 'White': []},
    'Rapid': {'Black': [], 'White': []},
    'Blitz': {'Black': [], 'White': []}
```

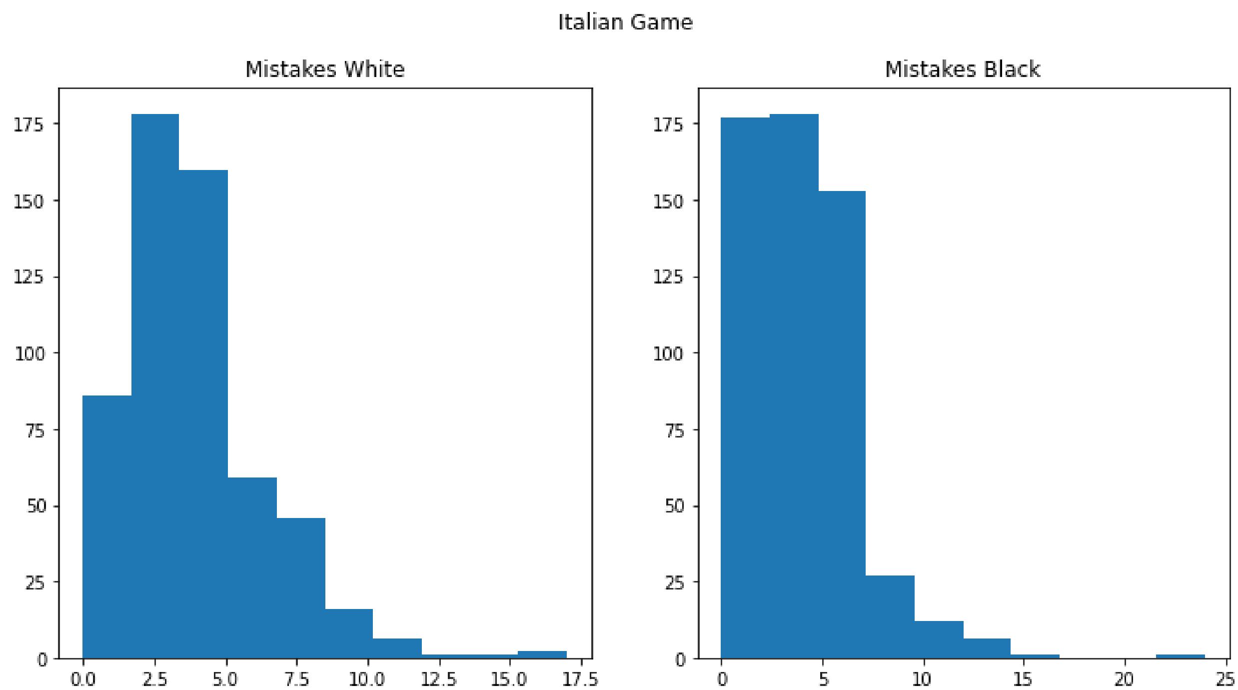
```
}

mistakes_by_opening = {
    'Italian Game': {'Black': [], 'White': []},
    'Queen Pawn Game': {'Black': [], 'White': []},
    'Petroff Defense': {'Black': [], 'White': []},
    'Sicilian Defense': {'Black': [], 'White': []}
}

mistakes_by_tc_and_opening = {
    'Italian Game': deepcopy(mistakes_by_time_control),
    'Queen Pawn Game': deepcopy(mistakes_by_time_control),
    'Petroff Defense': deepcopy(mistakes_by_time_control),
    'Sicilian Defense': deepcopy(mistakes_by_time_control)
}

time_control_mapping = {
    3: 'Classic',
    2: 'Rapid',
    1: 'Blitz'
}

opening_mapping = {
    4: 'Italian Game',
    3: 'Queen Pawn Game',
    2: 'Petroff Defense',
    1: 'Sicilian Defense'
}

for i in range(N):
    tc = time_control_mapping[time_control_list[i]]
    op = opening_mapping[opening_list[i]]

    for color in ['Black', 'White']:
        mistakes_by_time_control[tc]
[color].append(model_2_prior_df[f'mistakes_{color.lower()}
[{i+1}]'].values[19])
        mistakes_by_opening[op]
[color].append(model_2_prior_df[f'mistakes_{color.lower()}
[{i+1}]'].values[19])
        mistakes_by_tc_and_opening[op][tc]
[color].append(model_2_prior_df[f'mistakes_{color.lower()}
[{i+1}]'].values[19])

for tc in mistakes_by_time_control.keys():
    fig, axs = plt.subplots(1, 2, figsize=(12, 6))
    fig.suptitle(tc)
    for c_i, color in enumerate(['White', 'Black']):
        axs[c_i].hist(mistakes_by_time_control[tc][color])
        axs[c_i].set_title(f'Mistakes {color}')
```

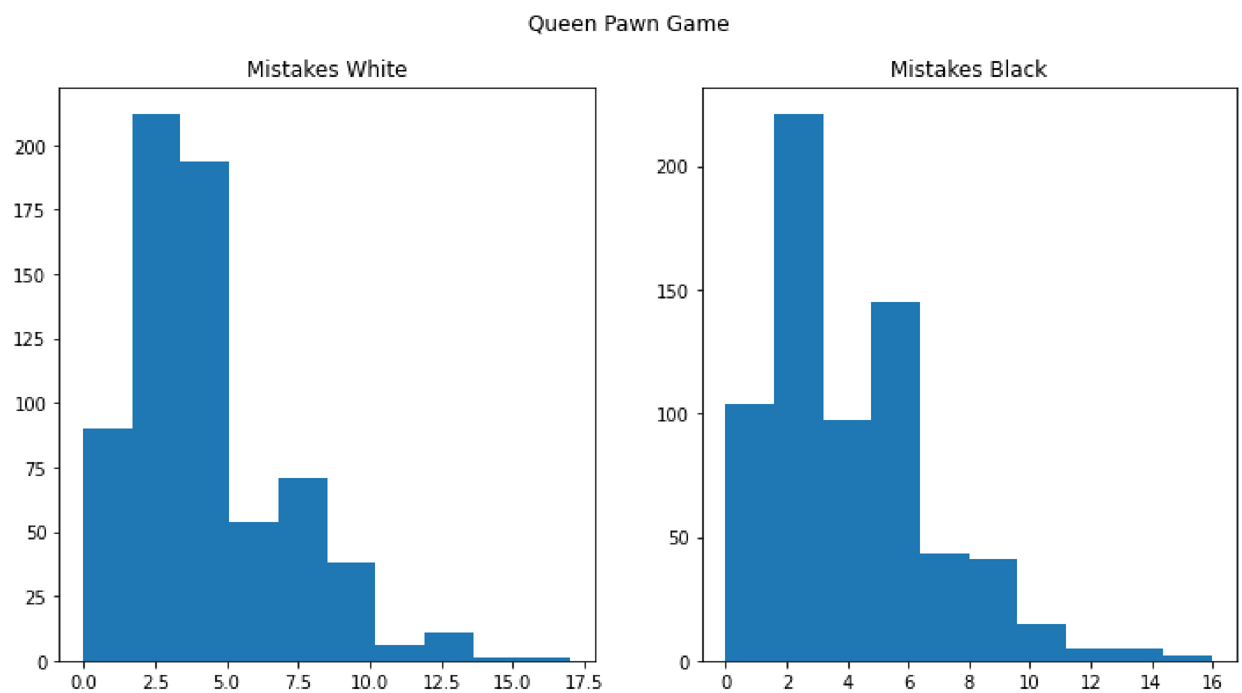
```
    plt.show()

for op in mistakes_by_opening.keys():
    fig, axs = plt.subplots(1, 2, figsize=(12, 6))
    fig.suptitle(op)
    for c_i, color in enumerate(['White', 'Black']):
        axs[c_i].hist(mistakes_by_opening[op][color])
        axs[c_i].set_title(f'Mistakes {color}')

    plt.show()

for op in mistakes_by_opening.keys():
    for tc in mistakes_by_time_control.keys():
        fig, axs = plt.subplots(1, 2, figsize=(12, 6))
        fig.suptitle(f'{op} - {tc}')
        for c_i, color in enumerate(['White', 'Black']):
            axs[c_i].hist(mistakes_by_tc_and_opening[op][tc][color])
            axs[c_i].set_title(f'Mistakes {color}')

        plt.show()
```

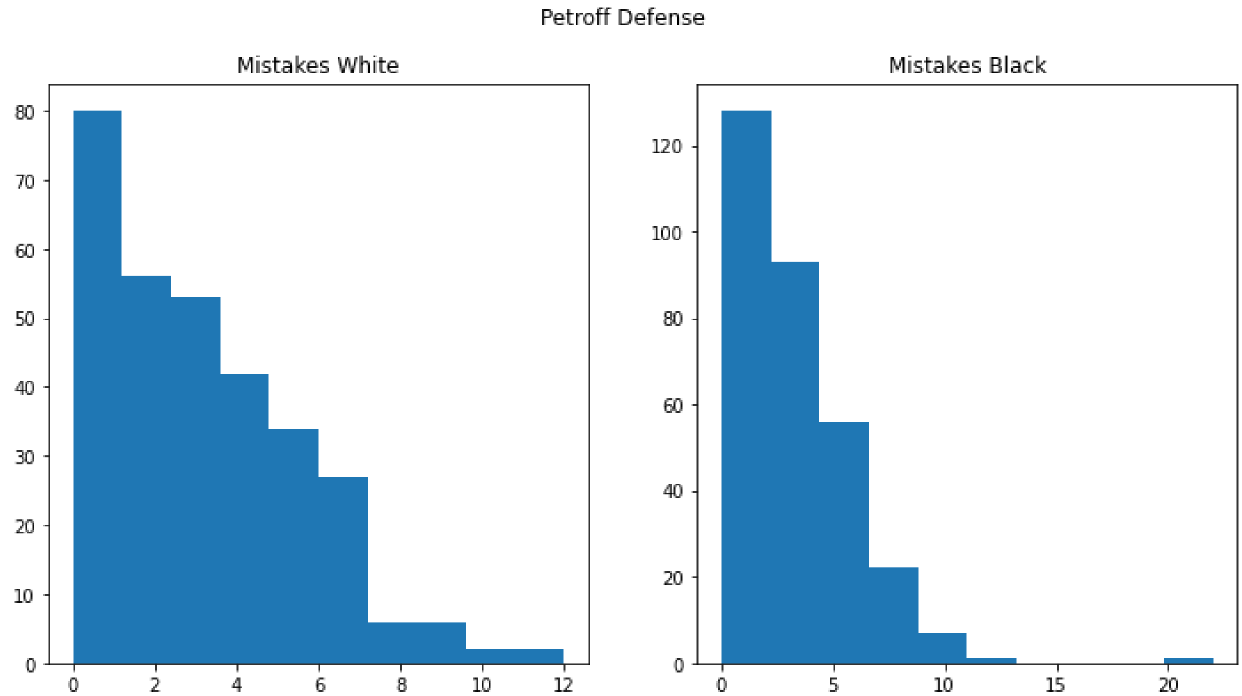


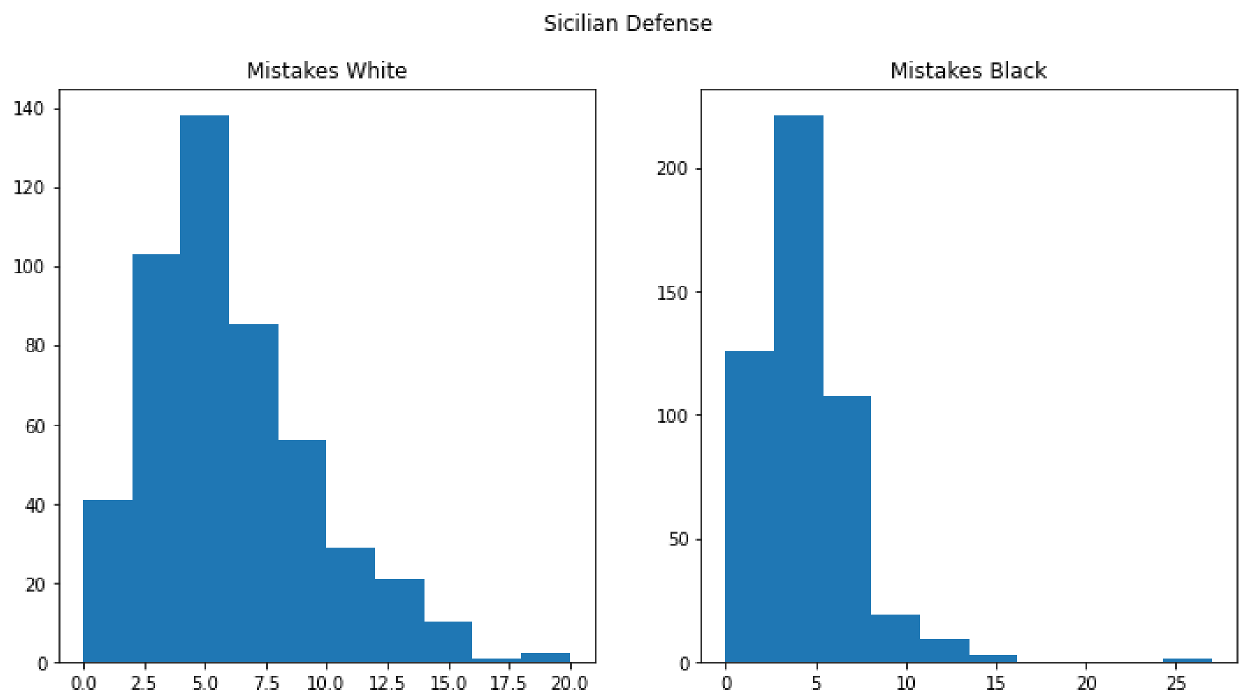

Rapid
Mistakes White
Mistakes Black

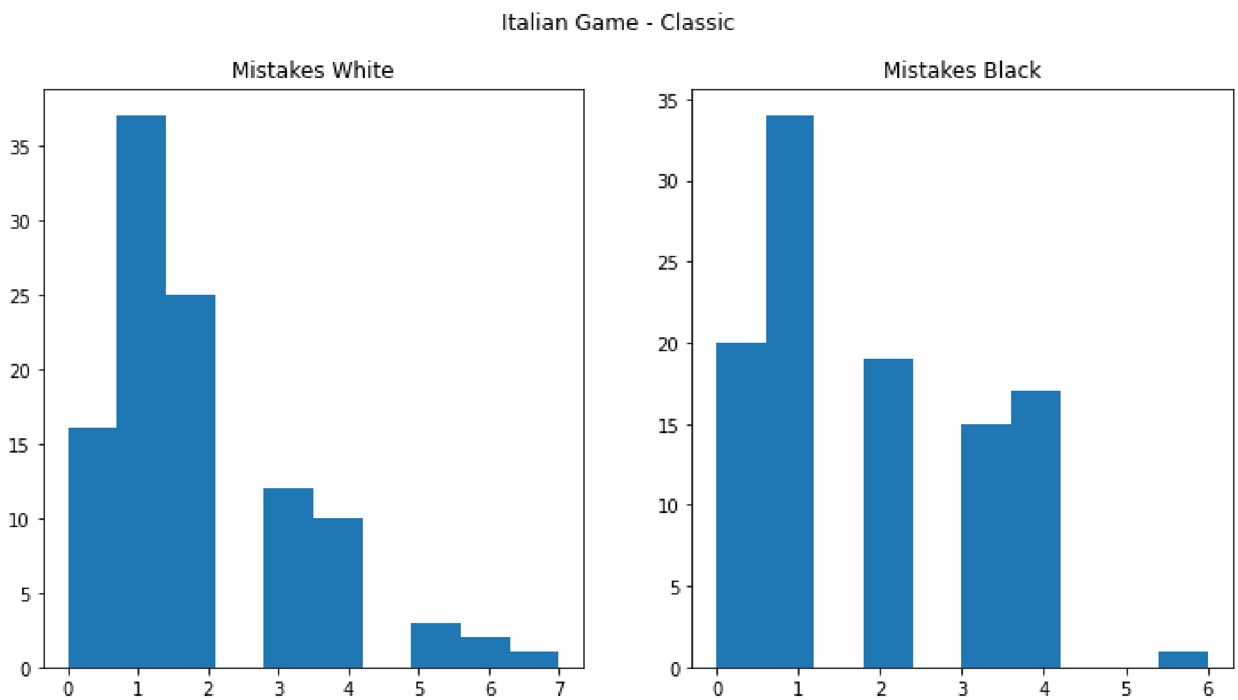

Blitz
Mistakes White
Mistakes Black

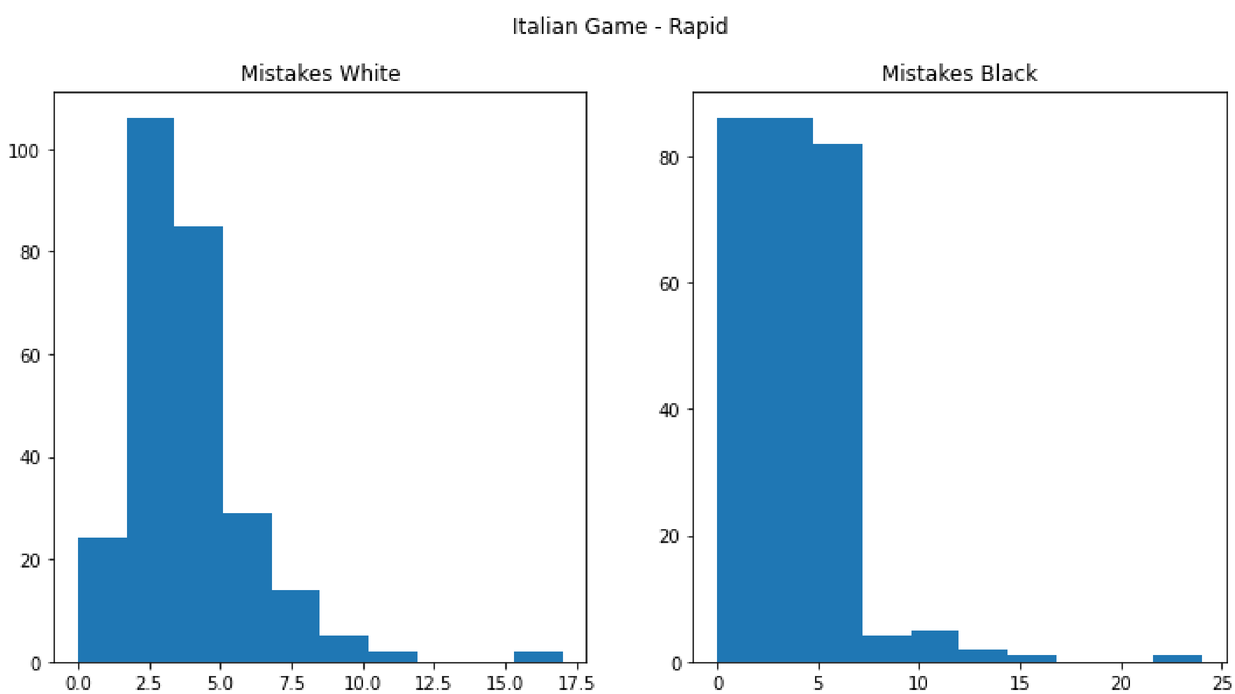

Italian Game
Mistakes White
Mistakes Black
0 25 50 75 100 125 150 175
0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5
0 5 10 15 20 25

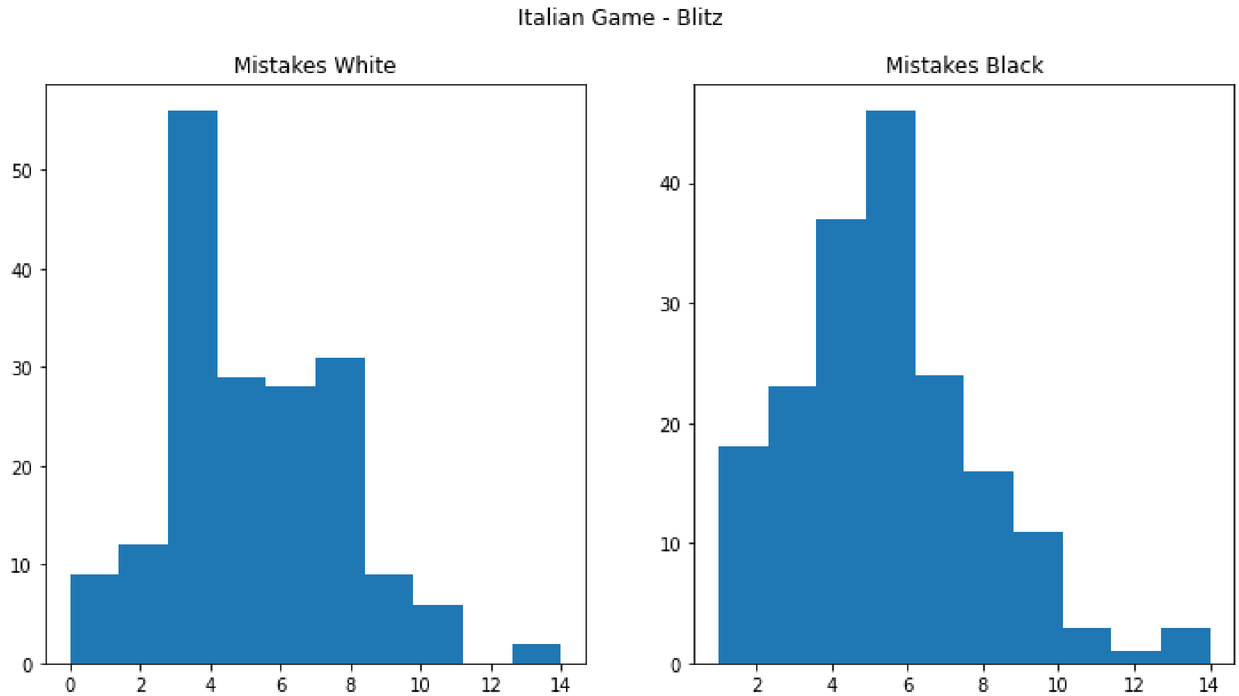

Queen Pawn Game
Mistakes White
Mistakes Black
0 25 50 75 100 125 150 175 200
0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5
0 50 100 150 200
0 2 4 6 8 10 12 14 16

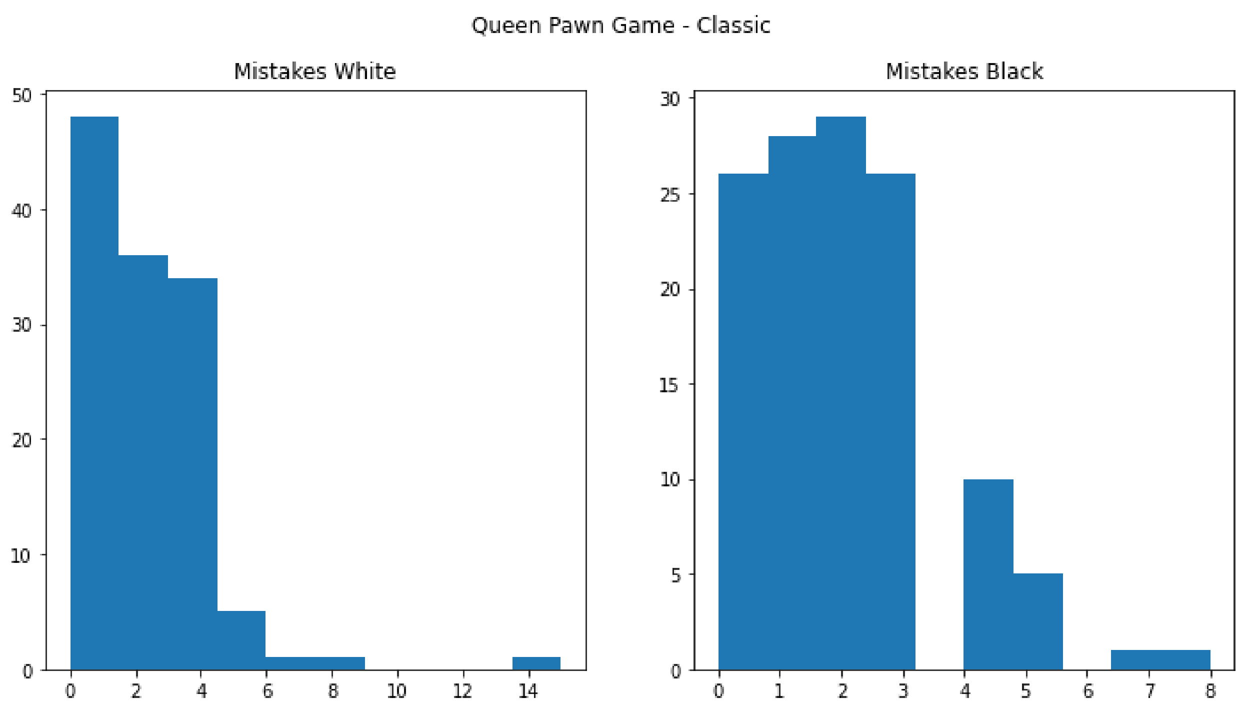

Petroff Defense
Mistakes White
Mistakes Black
0
10
20
30
40
50
60
70
80
2
4
6
8
12
100
120
5
15

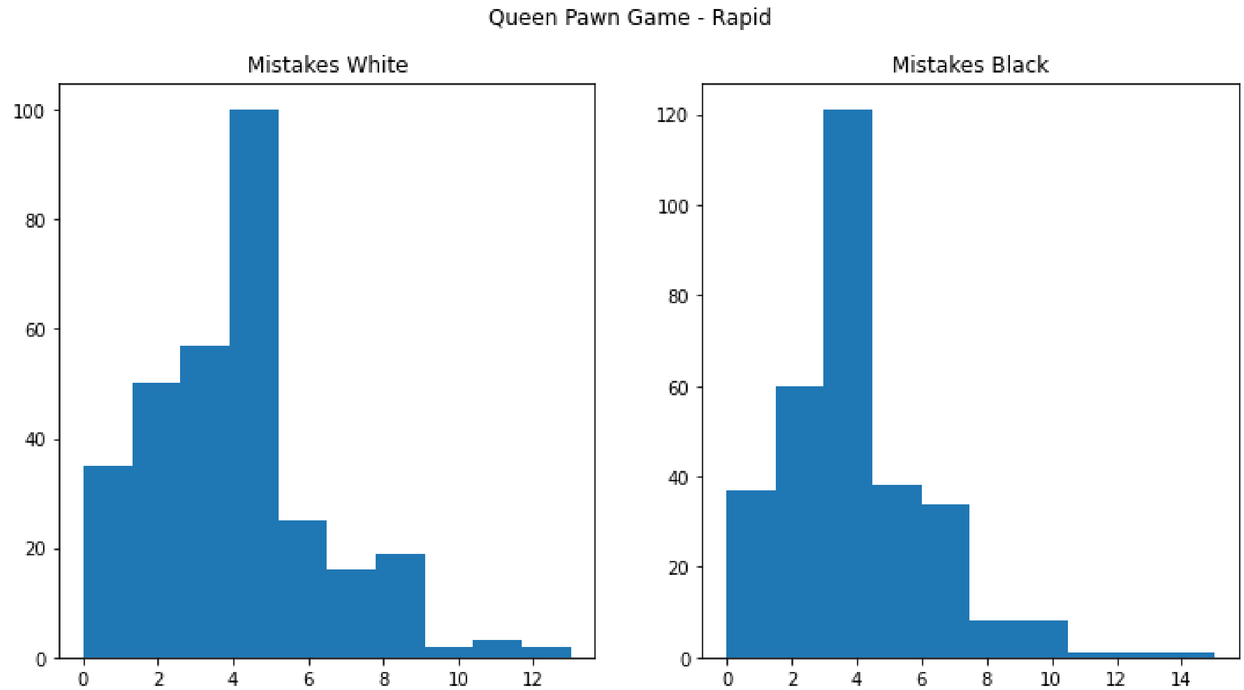

Sicilian Defense
Mistakes White
Mistakes Black
0
20
40
60
80
100
120
140
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
50
150
200
5
10
15
20
25

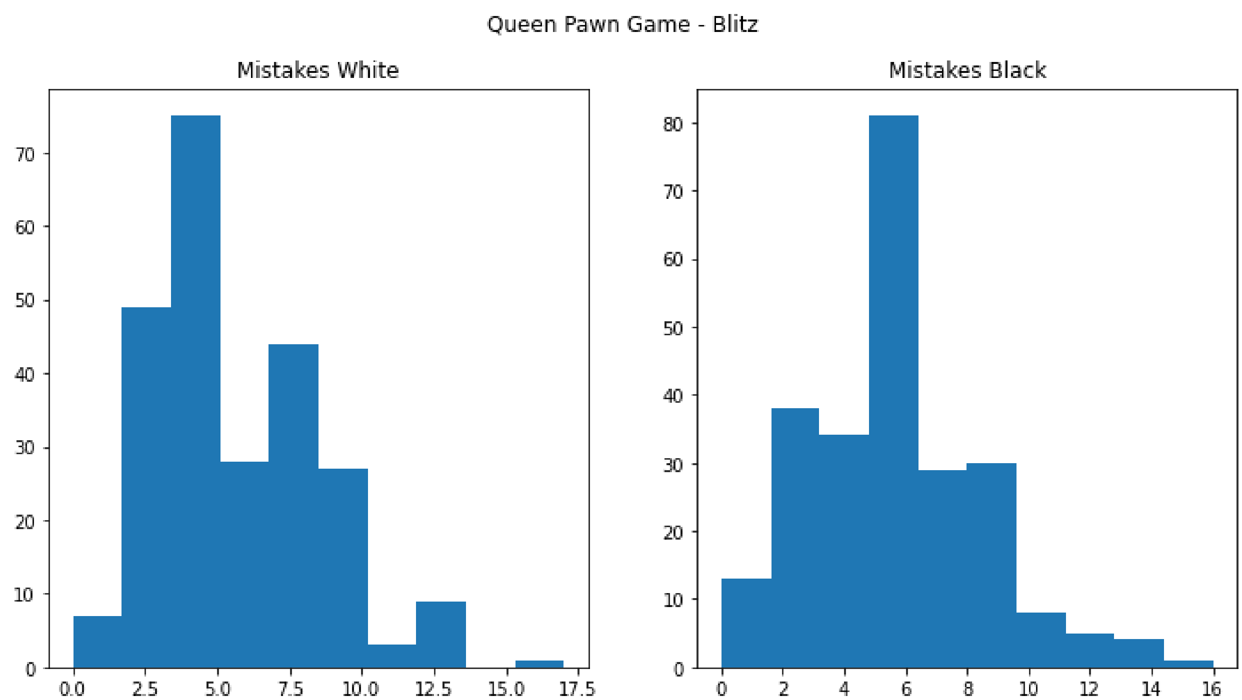

Italian Game - Classic
Mistakes White
Mistakes Black
0
5
10
15
20
25
30
35
0 1 2 3 4 5 6 7
0 1 2 3 4 5 6

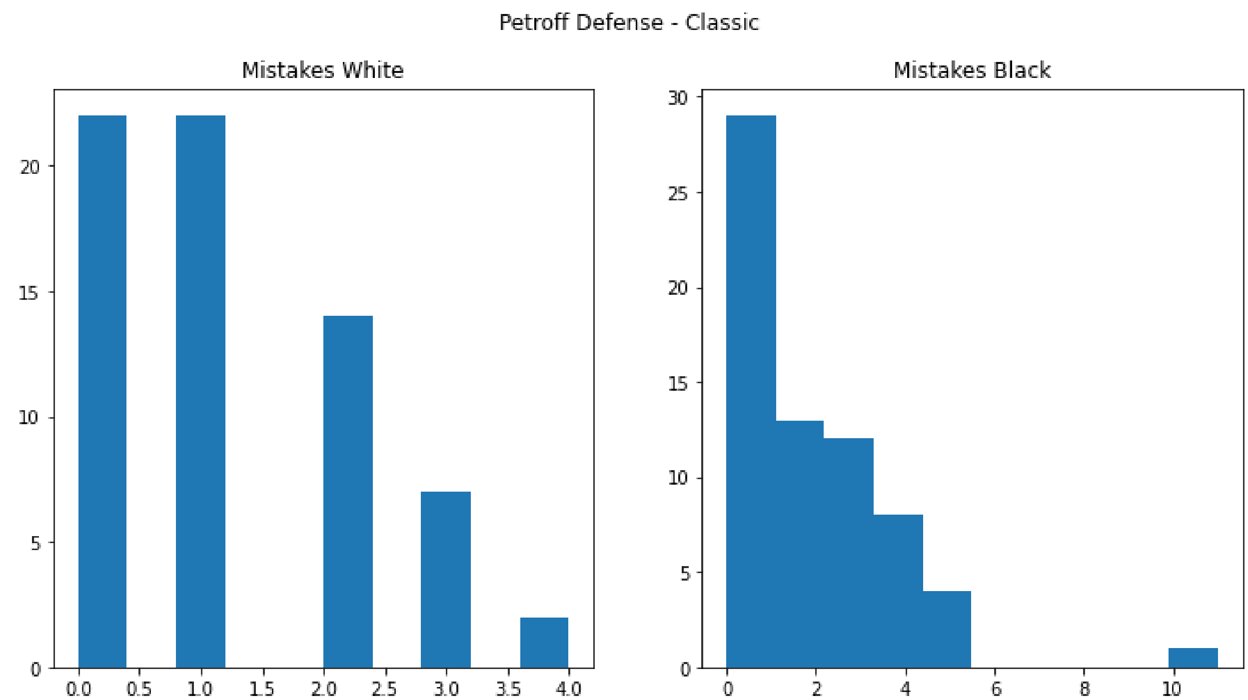

Italian Game - Rapid
Mistakes White
Mistakes Black
0
20
40
60
80
100
0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5
0 5 10 15 20 25

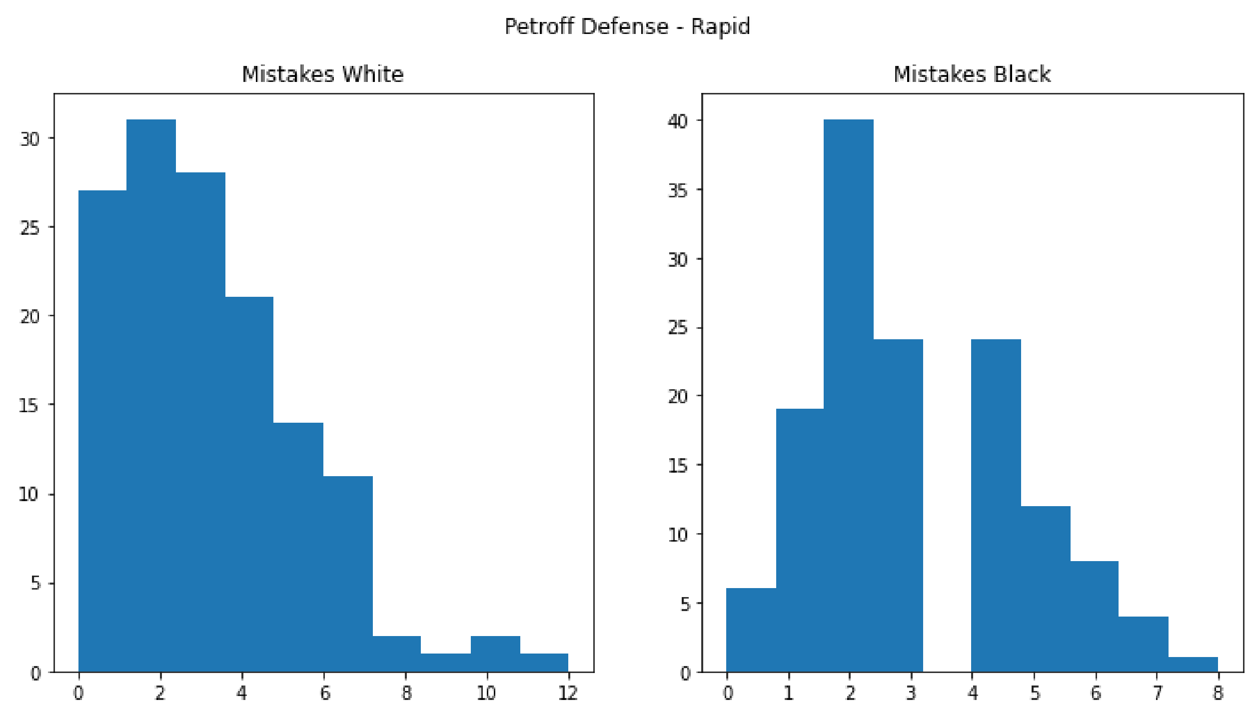

Italian Game - Blitz
Mistakes White
Mistakes Black
0
10
20
30
40
50
0
2
4
6
8
10
12
14
0
10
20
30
40
2
4
6
8
10
12
14


Queen Pawn Game - Classic
Mistakes White
Mistakes Black
0
10
20
30
40
50
0
2
4
6
8
10
12
14
0
5
10
15
20
25
30
0
1
2
3
4
5
6
7
8

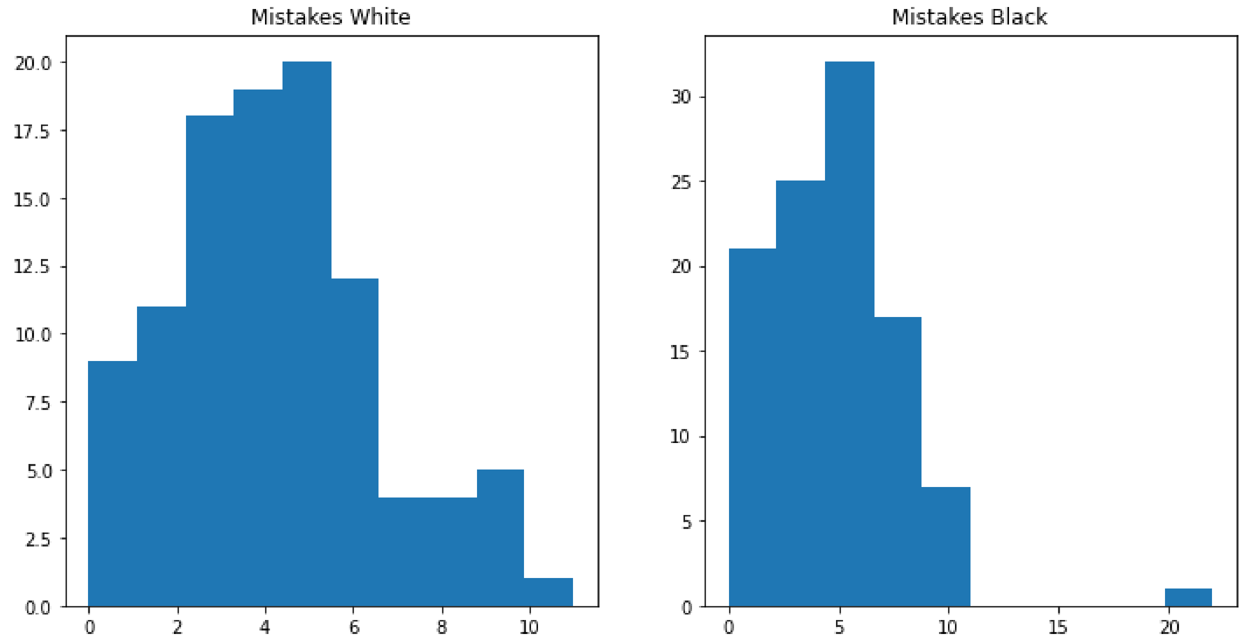

Queen Pawn Game - Rapid
Mistakes White
Mistakes Black
0
20
40
60
80
100
120
0
2
4
6
8
10
12
14


Queen Pawn Game - Blitz
Mistakes White
Mistakes Black
0
10
20
30
40
50
60
70
80
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0
2
4
6
8
10
12
14
16

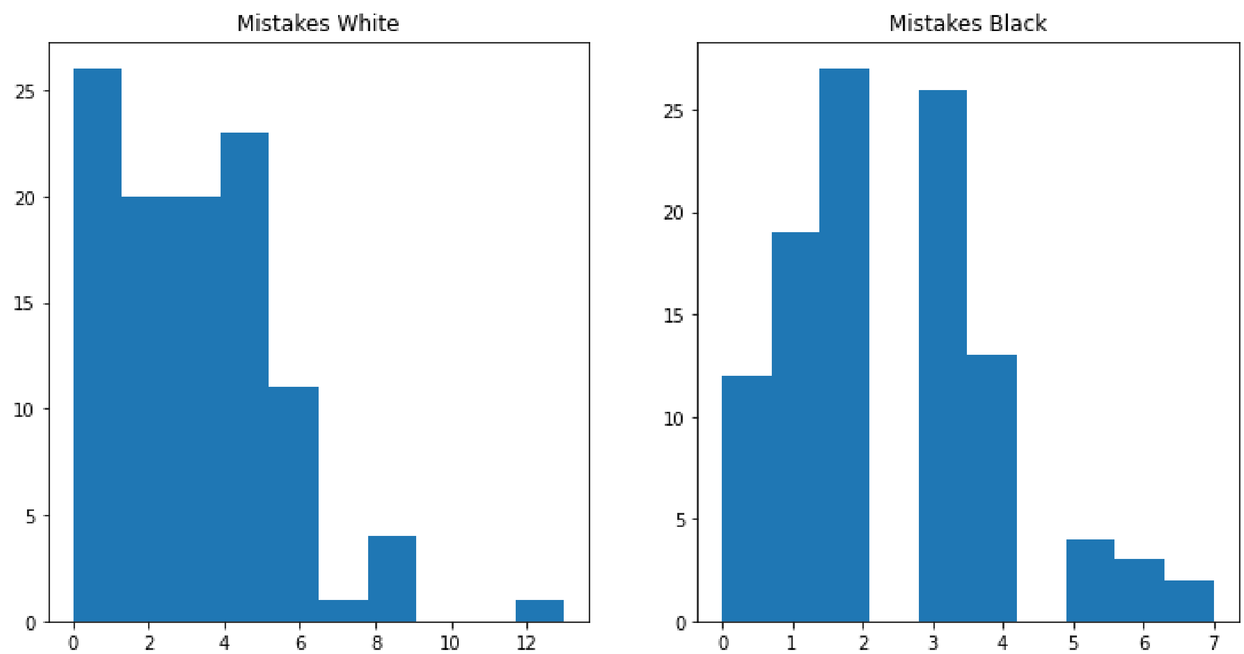

Petroff Defense - Classic
Mistakes White
Mistakes Black
0
5
10
15
20
25
30
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
2
4
6
8
10

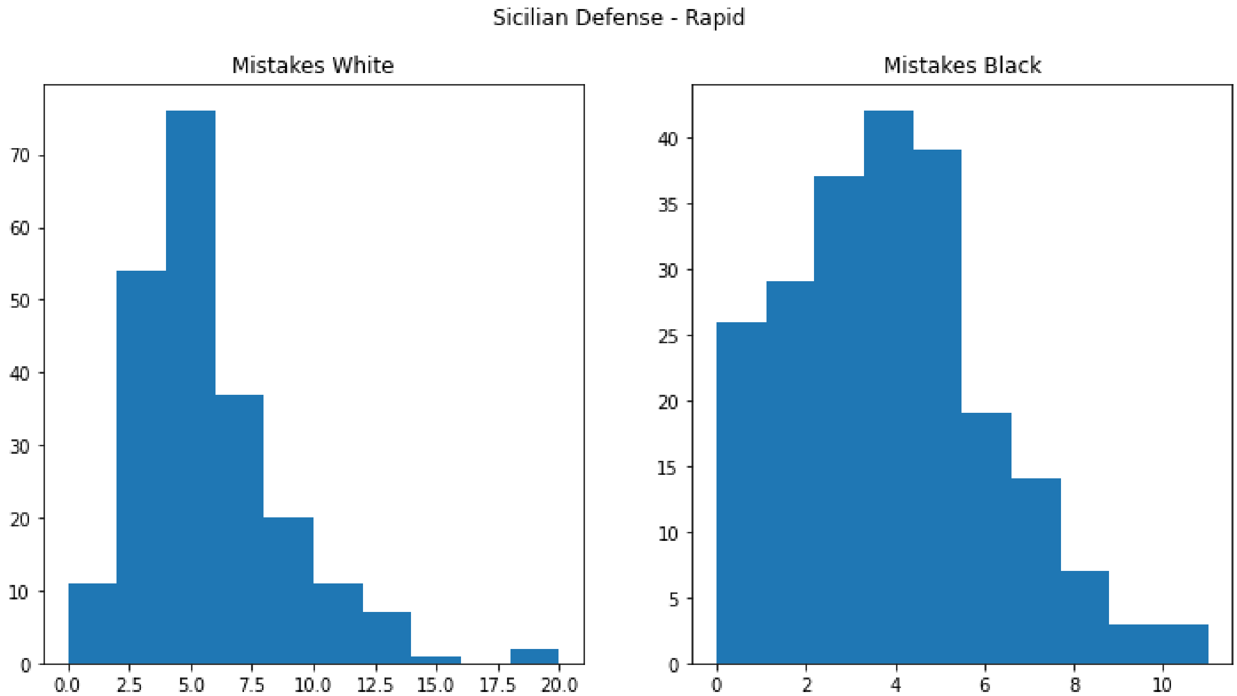

Petroff Defense - Rapid
Mistakes White
Mistakes Black
0
5
10
15
20
25
30
35
40
2
4
6
8
10
12
1
3
5
7

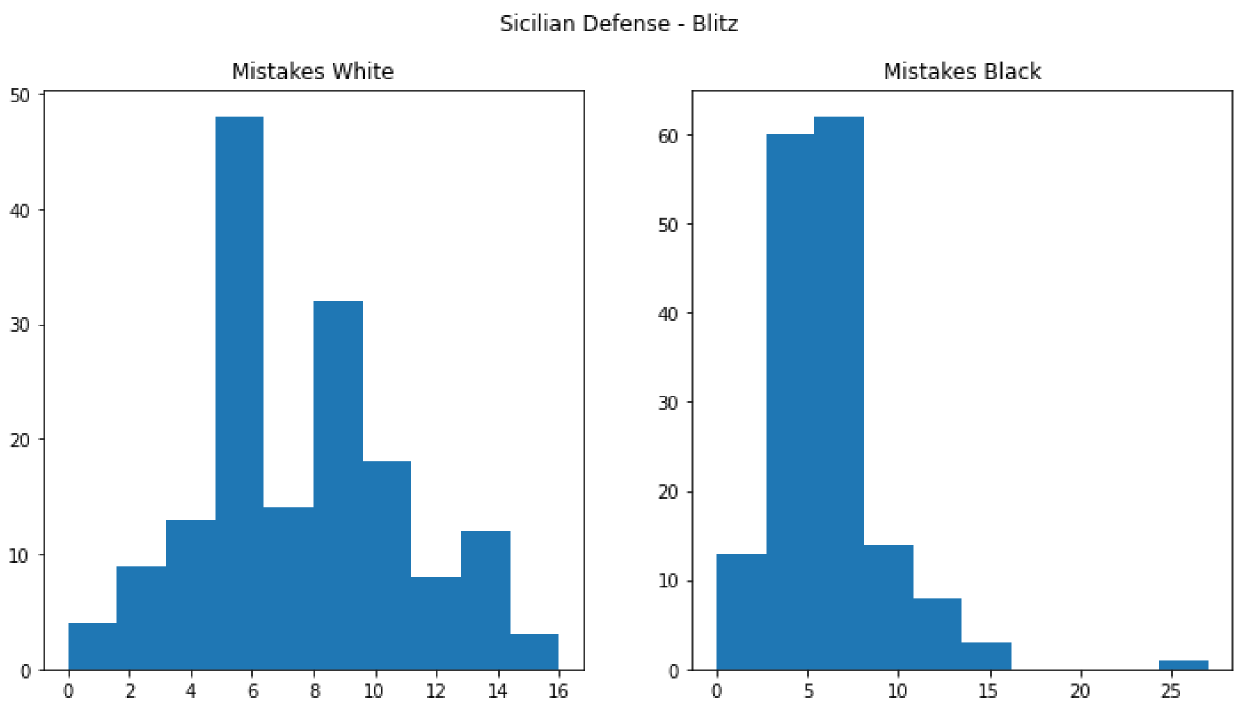
Petroff Defense - Blitz



Sicilian Defense - Classic







Measurements simulated from priors meet our expectations, because:

- as the time available for moves decreases, the number of errors increases,
- openings consiedered as more agressive lead to higher number of mistakes.

d) Prior parameters selection.

As prior parameters were selected those values for which the model results were most similar to the actual data

```
best_matching = -1
best_matching_ind = -1

for row_index, row in model_2_prior_df.iterrows():
    tmp_mistakes_white = []
    tmp_mistakes_black = []

    for i in range(N):
        tmp_mistakes_white.append(row[f'mistakes_white[{i+1}]'])
        tmp_mistakes_black.append(row[f'mistakes_black[{i+1}]'])

    number_of_matched_white = sum(1 for x, y in
zip(tmp_mistakes_white, mistakes_white_list) if x == y)
    number_of_matched_black = sum(1 for x, y in
zip(tmp_mistakes_black, mistakes_black_list) if x == y)
    acceptance_percent = ((number_of_matched_white +
number_of_matched_black) / (len(mistakes_white_list) +
len(mistakes_black_list))) * 100

    if acceptance_percent > best_matching:
        best_matching = acceptance_percent
        best_matching_ind = row_index

chosen_row_in_prior_model_2 = best_matching_ind
prior_parameters_model_2 = {}

print('Prior parameters for Model 2:')
for col_name in ['gamma_white', 'gamma_black',
'time_controll_coeff[1]', 'time_controll_coeff[2]',
'time_controll_coeff[3]',
                 'opening_coeff[1]', 'opening_coeff[2]',
'opening_coeff[3]', 'opening_coeff[4]']:
    prior_parameters_model_2[col_name] =
model_2_prior_df.loc[chosen_row_in_prior_model_2, col_name]
    print(f"\t{col_name} =
{model_2_prior_df.loc[chosen_row_in_prior_model_2, col_name]}")
```

```
Prior parameters for Model 2:
	gamma_white = -0.00141787
	gamma_black = 0.00154708
	time_controll_coeff[1] = 1.28198
	time_controll_coeff[2] = 0.840261
	time_controll_coeff[3] = 0.269627
	opening_coeff[1] = 0.675352
	opening_coeff[2] = 0.305338
```

```
    opening_coeff[3] = 0.389296
    opening_coeff[4] = 0.288916
```

5. Posterior analysis (Model 1)

Parameters chosen in prior

```
prior_parameters_model_1
```

```
{'gamma_white': -0.00144638,
 'gamma_black': 0.00137461,
 'time_controll_coeff[1]': 1.23487,
 'time_controll_coeff[2]': 0.836987,
 'time_controll_coeff[3]': 0.295052}
```

a) Model trainning and sampling

```
mistakes_white_model = deepcopy(mistakes_white_list)
mistakes_black_model = deepcopy(mistakes_black_list)

model_1_posterior =
CmdStanModel(stan_file='stan_files/model_1_fit.stan')

model_1_fit = model_1_posterior.sample(
    data={
        'rating_difference': rating_difference_list,
        'N': len(time_control_list),
        'time_control': time_control_list,
        'mistakes_white_model': mistakes_white_model,
        'mistakes_black_model': mistakes_black_model
        }
)
```

```
INFO:cmdstanpy:compiling stan file
/home/projekt/stan_files/model_1_fit.stan to exe file
/home/projekt/stan_files/model_1_fit
INFO:cmdstanpy:compiled model executable:
/home/projekt/stan_files/model_1_fit
WARNING:cmdstanpy:Stan compiler has produced 9 warnings:
WARNING:cmdstanpy:
--- Translating Stan model to C++ code ---
bin/stanc  --o=/home/projekt/stan_files/model_1_fit.hpp
/home/projekt/stan_files/model_1_fit.stan
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 2, column
12: Comments
    beginning with # are deprecated and this syntax will be removed in
Stan
    2.32.0. Use // to begin line comments; this can be done
automatically
```

```
    using the auto-format flag to stanc
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 3, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 4, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 5, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 6, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 19,
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 20,
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
```

```
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 48,
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_1_fit.stan', line 49,
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc

--- Compiling, linking C++ code ---
g++ -std=c++1y -pthread -D_REENTRANT -Wno-sign-compare -Wno-ignored-
attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I
src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I
stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I
stan/lib/stan_math/lib/boost_1.75.0 -I
stan/lib/stan_math/lib/sundials_6.0.0/include -I
stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -
DBOOST_DISABLE_ASSERTS          -c -Wno-ignored-attributes   -x c++ -o
/home/projekt/stan_files/model_1_fit.o
/home/projekt/stan_files/model_1_fit.hpp
g++ -std=c++1y -pthread -D_REENTRANT -Wno-sign-compare -Wno-ignored-
attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I
src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I
stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I
stan/lib/stan_math/lib/boost_1.75.0 -I
stan/lib/stan_math/lib/sundials_6.0.0/include -I
stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -
DBOOST_DISABLE_ASSERTS
-Wl,-L,"/opt/cmdstan-2.29.0/stan/lib/stan_math/lib/tbb"
-Wl,-rpath,"/opt/cmdstan-2.29.0/stan/lib/stan_math/lib/tbb"
/home/projekt/stan_files/model_1_fit.o src/cmdstan/main.o        -Wl,-
L,"/opt/cmdstan-2.29.0/stan/lib/stan_math/lib/tbb"
-Wl,-rpath,"/opt/cmdstan-2.29.0/stan/lib/stan_math/lib/tbb"
```

```
stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_nvecserial.a
stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_cvodes.a
stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_idas.a
stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_kinsol.a
stan/lib/stan_math/lib/tbb/libtbb.so.2 -o
/home/projekt/stan_files/model_1_fit
rm -f /home/projekt/stan_files/model_1_fit.o

INFO:cmdstanpy:CmdStan start processing
chain 1 |          | 00:00 Status
▌         | 00:00 Status
chain 1 |███       | 00:00 Iteration:  500 / 2000 [ 25%]  (Warmup)

██████    | 00:00 Iteration: 1001 / 2000 [ 50%]  (Sampling)

███████   | 00:05 Iteration: 1300 / 2000 [ 65%]  (Sampling)
chain 1 |████████  | 00:06 Iteration: 1500 / 2000 [ 75%]  (Sampling)

████████▌ | 00:07 Iteration: 1600 / 2000 [ 80%]  (Sampling)
█████████ | 00:07 Iteration: 1700 / 2000 [ 85%]  (Sampling)
chain 1 |█████████▌| 00:08 Iteration: 1800 / 2000 [ 90%]  (Sampling)

██████████| 00:08 Iteration: 1900 / 2000 [ 95%]  (Sampling)
chain 1 |██████████| 00:09 Sampling completed
chain 2 |██████████| 00:09 Sampling completed
chain 3 |██████████| 00:09 Sampling completed
chain 4 |██████████| 00:09 Sampling completed



INFO:cmdstanpy:CmdStan done processing.
```

There were no issues with the sampling.

b) Analysis of samples obtained from the predictive distribution.

Model summary:

```
summary = model_1_fit.summary()
summary.head(6)

                                  Mean          MCSE    StdDev
5%  \
name

lp__                         4600.0000  4.700000e-02  1.800000
4600.0000
gamma_white                    -0.0014  9.300000e-08  0.000006    -
```

```
0.0014
gamma_black                         0.0014  8.900000e-08  0.000006
0.0014
time_controll_coeff_blitz           1.1000  3.100000e-06  0.000220
1.1000
time_controll_coeff_rapid           0.7800  2.400000e-05  0.001700
0.7700
time_controll_coeff_classic         0.2100  7.600000e-05  0.005200
0.2000

                                       50%        95%   N_Eff  N_Eff/s
R_hat
name

lp__                             4600.0000  4600.0000  1400.0     24.0
1.0
gamma_white                        -0.0014    -0.0014  4500.0     74.0
1.0
gamma_black                         0.0014     0.0014  4700.0     77.0
1.0
time_controll_coeff_blitz           1.1000     1.1000  4900.0     81.0
1.0
time_controll_coeff_rapid           0.7800     0.7800  4800.0     79.0
1.0
time_controll_coeff_classic         0.2100     0.2200  4700.0     77.0
1.0
```

Parameters values are similiar to those from priors but not the same.

```
model_1_fit_df = model_1_fit.draws_pd()
model_1_fit_df.head()

      lp__  accept_stat__  stepsize__  treedepth__  n_leapfrog__
divergent__  \
0  4569.74       0.814108    0.458273          3.0           7.0
0.0
1  4571.23       0.996769    0.458273          3.0           7.0
0.0
2  4570.93       0.896218    0.458273          3.0           7.0
0.0
3  4571.74       0.985942    0.458273          3.0           7.0
0.0
4  4568.84       0.771899    0.458273          3.0           7.0
0.0

   energy__  gamma_white  gamma_black  time_controll_coeff_blitz  ...
\
0  -4566.99    -0.001413     0.001402                    1.13997  ...

1  -4569.26    -0.001402     0.001405                    1.13966  ...
```

```
2  -4569.24    -0.001406     0.001405                          1.13997  ...

3  -4570.28    -0.001407     0.001405                          1.13958  ...

4  -4567.99    -0.001406     0.001407                          1.13996  ...


   mistakes_black_pred[2018]  mistakes_black_pred[2019]  \
0                        3.0                        2.0
1                        3.0                        1.0
2                        3.0                        1.0
3                        0.0                        3.0
4                        2.0                        1.0

   mistakes_black_pred[2020]  mistakes_black_pred[2021]  \
0                        2.0                        1.0
1                        1.0                        6.0
2                        2.0                        3.0
3                        2.0                        2.0
4                        5.0                        1.0

   mistakes_black_pred[2022]  mistakes_black_pred[2023]  \
0                        1.0                        3.0
1                        1.0                        1.0
2                        4.0                        1.0
3                        5.0                        3.0
4                        3.0                        5.0

   mistakes_black_pred[2024]  mistakes_black_pred[2025]  \
0                        2.0                        3.0
1                        2.0                        4.0
2                        1.0                        4.0
3                        3.0                        2.0
4                        5.0                        4.0

   mistakes_black_pred[2026]  mistakes_black_pred[2027]
0                        4.0                        3.0
1                        4.0                        1.0
2                        3.0                        2.0
3                        1.0                        4.0
4                        3.0                        2.0

[5 rows x 4071 columns]
```

Choosing sample with best matching to reality.

```
best_matching = -1
best_matching_ind = -1
```

```
for row_index, row in model_1_fit_df.iterrows():
    tmp_mistakes_white = []
    tmp_mistakes_black = []

    for i in range(N):
        tmp_mistakes_white.append(row[f'mistakes_white_pred[{i+1}]'])
        tmp_mistakes_black.append(row[f'mistakes_black_pred[{i+1}]'])

    number_of_matched_white = sum(1 for x, y in
zip(tmp_mistakes_white, mistakes_white_list) if x == y)
    number_of_matched_black = sum(1 for x, y in
zip(tmp_mistakes_black, mistakes_black_list) if x == y)
    acceptance_percent = ((number_of_matched_white +
number_of_matched_black) / (len(mistakes_white_list) +
len(mistakes_black_list))) * 100

    if acceptance_percent > best_matching:
        best_matching = acceptance_percent
        best_matching_ind = row_index

chosen_row_in_posterior_model_1 = best_matching_ind
```

Creating pandas DataFrame with chosen posterior predictive samples

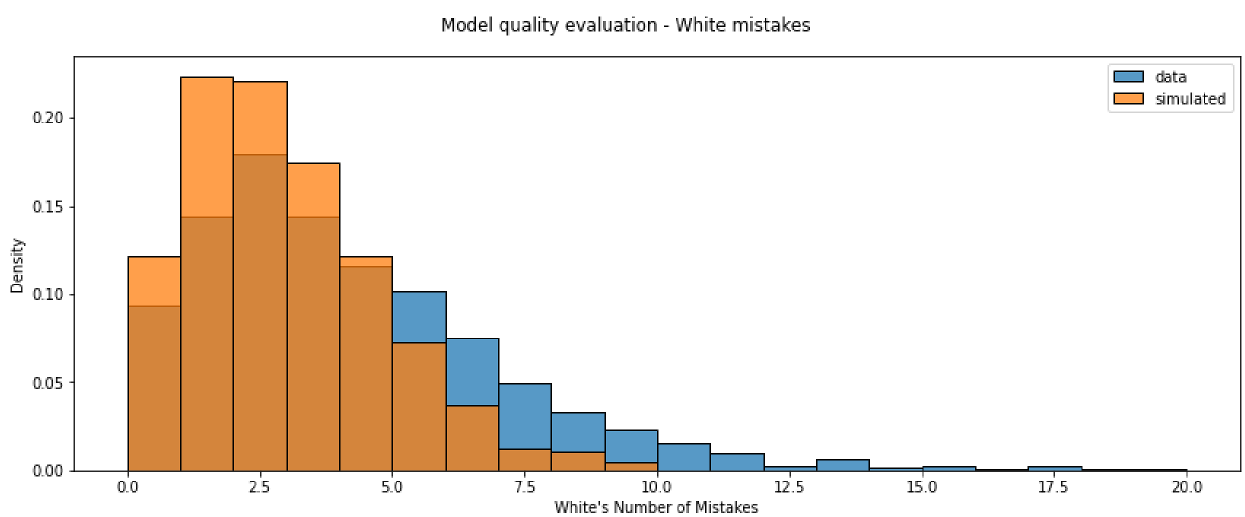
```
mistakes_white_pred = []
mistakes_black_pred = []

for i in range(N):

mistakes_white_pred.append(model_1_fit_df[f'mistakes_white_pred[{i+1}]
'].values[chosen_row_in_posterior_model_1])

mistakes_black_pred.append(model_1_fit_df[f'mistakes_black_pred[{i+1}]
'].values[chosen_row_in_posterior_model_1])

df_model_1_fit_transposed = pd.DataFrame({'mistakes_white_pred':
mistakes_white_pred, 'mistakes_black_pred': mistakes_black_pred})
df_model_1_fit_transposed
```

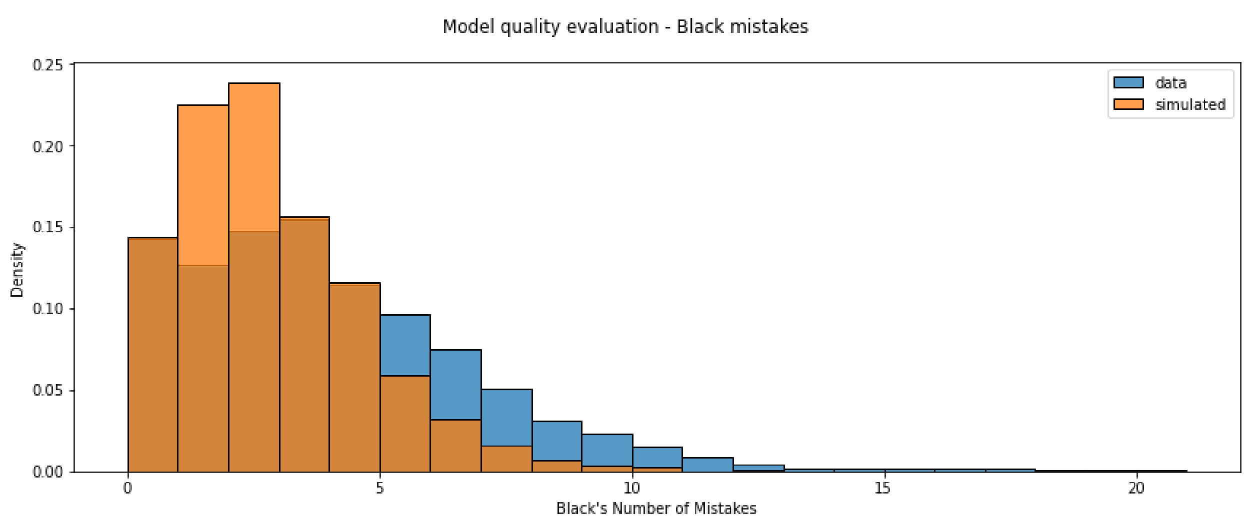
	mistakes_white_pred	mistakes_black_pred
0	2.0	1.0
1	4.0	6.0
2	3.0	2.0
3	2.0	0.0
4	3.0	0.0
...	...	...
2022	2.0	4.0
2023	2.0	2.0
2024	6.0	3.0
2025	3.0	0.0

```
2026                    1.0                     3.0

[2027 rows x 2 columns]
```

c) Analysis of posterior predictive samples

```
for color in ['White', 'Black']:
    fig, ax = plt.subplots(figsize=(12,5))
    fig.suptitle(f"Model quality evaluation - {color} mistakes")
    sns.histplot(data=df, x=f"{color}'s Number of Mistakes",
binwidth=1, stat="density", ax=ax, label='data')
    sns.histplot(data=df_model_1_fit_transposed,
x=f"mistakes_{color.lower()}_pred", binwidth=1, stat="density", ax=ax,
label='simulated')
    plt.legend()
    plt.tight_layout()
    plt.plot()
    plt.show()
```





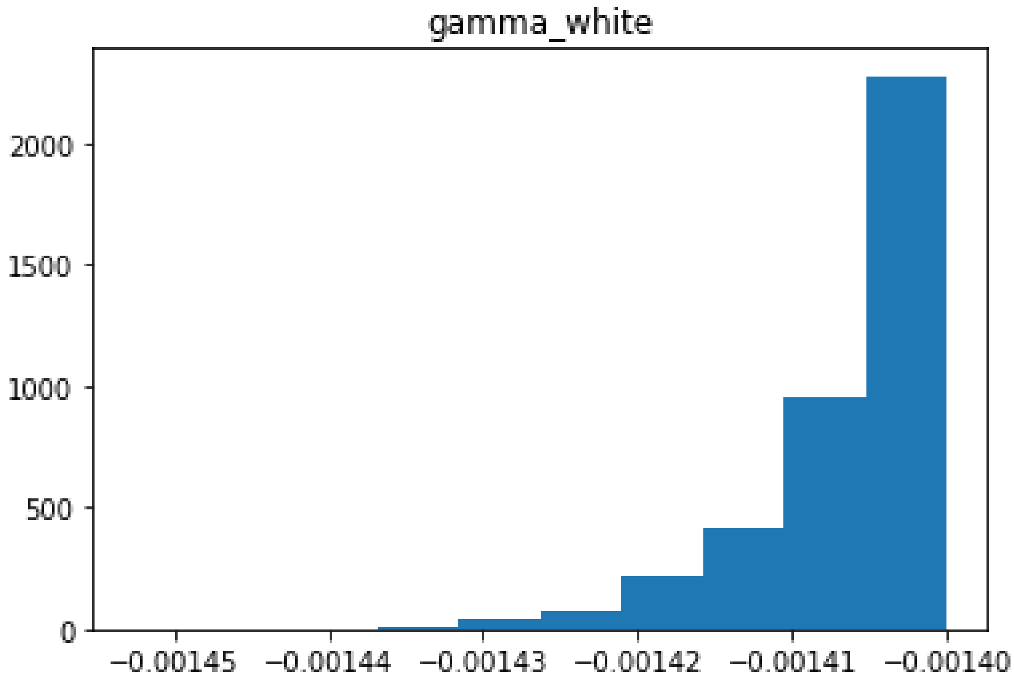
Posterior predictive samples are more or less consistent with the data. This is the result of the first (simpler) model, so we expect these results to improve in the second (more complex) model. Nevertheless, the results returned by the model considering only time control and rating difference are very satisfactory. Below we calculated overlap of posterior predictive samples and data.

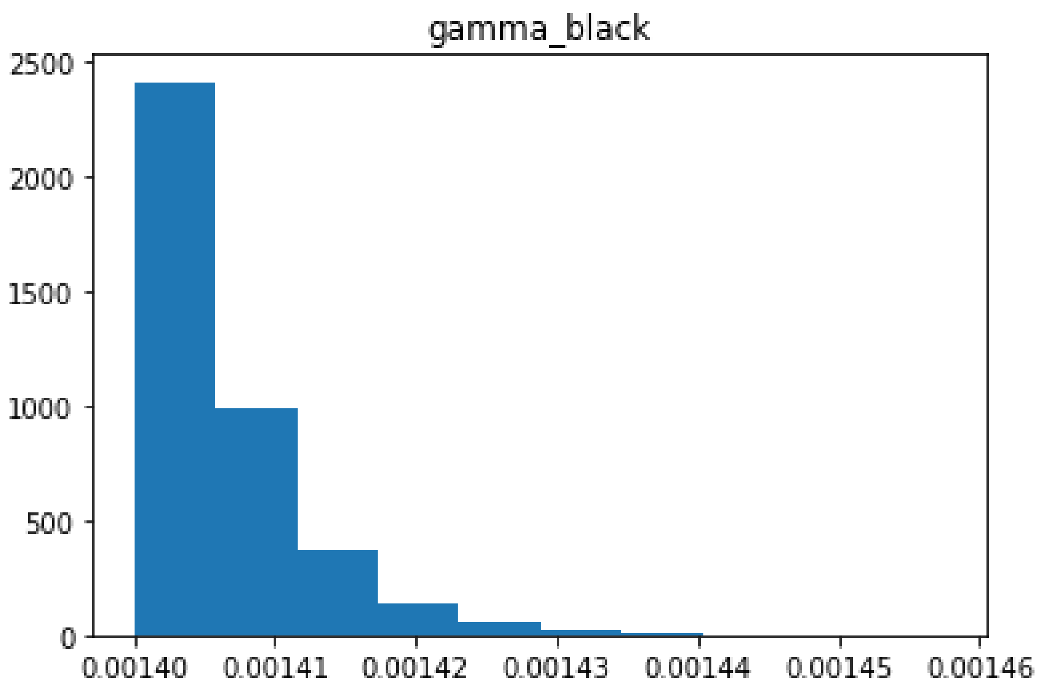
```
def calculate_hist_overlap(df_data, df_fit):
    for color in ['White', 'Black']:
        overlap_coefficient = 0
        hist_data = []
        hist_fit = []
        for n in range(int(max(df_data[f"{color}'s Number of Mistakes"].max(), df_fit[f"mistakes_{color.lower()}_pred"].max()))):
            bar_data = np.count_nonzero(np.array(df_data[f"{color}'s Number of Mistakes"]) == n)
            bar_fit = np.count_nonzero(np.array(df_fit[f"mistakes_{color.lower()}_pred"]) == n)
            hist_data.append(bar_data)
            hist_fit.append(bar_fit)

        overlap_coefficient = np.minimum(hist_data, hist_fit).sum() / np.maximum(hist_data, hist_fit).sum()

        print(f"Overlap of posterior predictive samples and data for mistakes {color.lower()} :\t{np.round(100 * overlap_coefficient, 2)}%")

calculate_hist_overlap(df, df_model_1_fit_transposed)
```

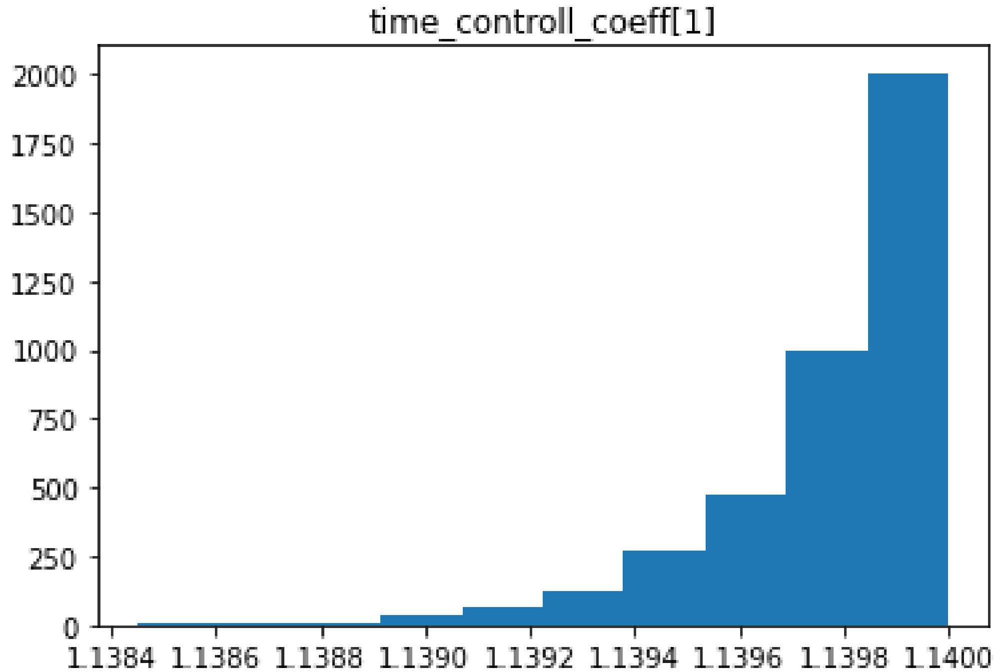
```
Overlap of posterior predictive samples and data for mistakes white :
     68.66%
Overlap of posterior predictive samples and data for mistakes black :
     67.48%
```

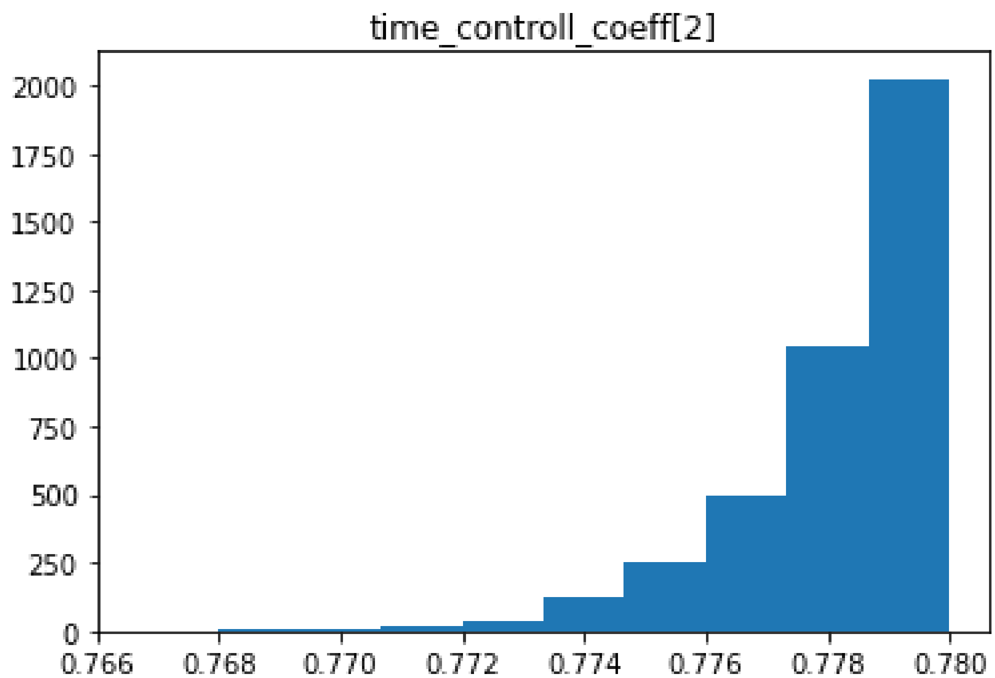
d) Analysis of parameter marginal disrtibutions

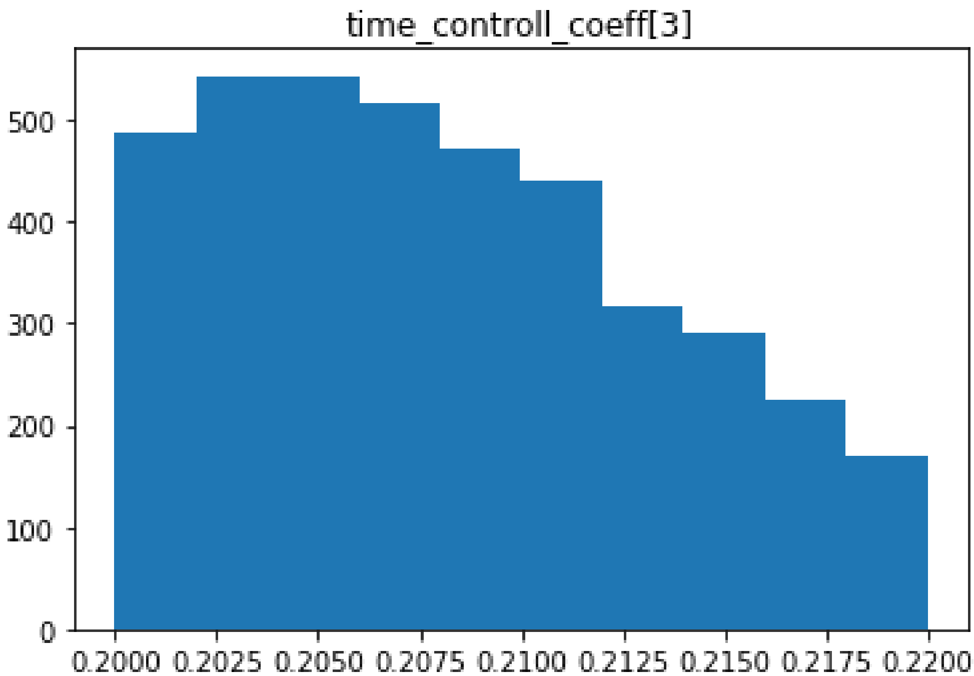
```
for parameter in ['gamma_white', 'gamma_black', 'time_controll_coeff[1]', 'time_controll_coeff[2]', 'time_controll_coeff[3]']:
    plt.hist(model_1_fit_df[parameter])
    plt.title(parameter)
    plt.show()
```


gamma_white
2000
1500
1000
500
0
−0.00145
−0.00144
−0.00143
−0.00142
−0.00141
−0.00140


gamma_black
2500
2000
1500
1000
500
0
0.00140
0.00141
0.00142
0.00143
0.00144
0.00145
0.00146


time_controll_coeff[1]
2000
1750
1500
1250
1000
750
500
250
0
1.1384
1.1386
1.1388
1.1390
1.1392
1.1394
1.1396
1.1398
1.1400


time_controll_coeff[2]
2000
1750
1500
1250
1000
750
500
250
0
0.766
0.768
0.770
0.772
0.774
0.776
0.778
0.780



Comment: The parameter values are concentrated and quite close to those obtained by prior analysis.

6. Posterior analysis (Model 2)

Parameters chosen in prior

```
prior_parameters_model_2

{'gamma_white': -0.00141787,
 'gamma_black': 0.00154708,
 'time_controll_coeff[1]': 1.28198,
 'time_controll_coeff[2]': 0.840261,
 'time_controll_coeff[3]': 0.269627,
 'opening_coeff[1]': 0.675352,
 'opening_coeff[2]': 0.305338,
 'opening_coeff[3]': 0.389296,
 'opening_coeff[4]': 0.288916}
```

a) Model trainning and sampling

```
mistakes_white_model = deepcopy(mistakes_white_list)
mistakes_black_model = deepcopy(mistakes_black_list)

model_2_posterior =
CmdStanModel(stan_file='stan_files/model_2_fit.stan')

model_2_fit = model_2_posterior.sample(
    data={
```

```
        'rating_difference': rating_difference_list,
        'N': len(time_control_list),
        'time_control': time_control_list,
        'mistakes_white_model': mistakes_white_model,
        'mistakes_black_model': mistakes_black_model,
        'opening': opening_list
        }
)
```

```
INFO:cmdstanpy:compiling stan file
/home/projekt/stan_files/model_2_fit.stan to exe file
/home/projekt/stan_files/model_2_fit
INFO:cmdstanpy:compiled model executable:
/home/projekt/stan_files/model_2_fit
WARNING:cmdstanpy:Stan compiler has produced 11 warnings:
WARNING:cmdstanpy:
--- Translating Stan model to C++ code ---
bin/stanc  --o=/home/projekt/stan_files/model_2_fit.hpp
/home/projekt/stan_files/model_2_fit.stan
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 2, column
12: Comments
    beginning with # are deprecated and this syntax will be removed in
Stan
    2.32.0. Use // to begin line comments; this can be done
automatically
    using the auto-format flag to stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 3, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 4, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 5, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
```

```
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 6, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 7, column
4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 25,
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 26,
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 27,
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 66,
```

```
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc
Warning in '/home/projekt/stan_files/model_2_fit.stan', line 67,
column 4: Declaration
    of arrays by placing brackets after a variable name is deprecated
and
    will be removed in Stan 2.32.0. Instead use the array keyword
before the
    type. This can be changed automatically using the auto-format flag
to
    stanc

--- Compiling, linking C++ code ---
g++ -std=c++1y -pthread -D_REENTRANT -Wno-sign-compare -Wno-ignored-
attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I
src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I
stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I
stan/lib/stan_math/lib/boost_1.75.0 -I
stan/lib/stan_math/lib/sundials_6.0.0/include -I
stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -
DBOOST_DISABLE_ASSERTS          -c -Wno-ignored-attributes   -x c++ -o
/home/projekt/stan_files/model_2_fit.o
/home/projekt/stan_files/model_2_fit.hpp
g++ -std=c++1y -pthread -D_REENTRANT -Wno-sign-compare -Wno-ignored-
attributes      -I stan/lib/stan_math/lib/tbb_2020.3/include    -O3 -I
src -I stan/src -I lib/rapidjson_1.1.0/ -I lib/CLI11-1.9.1/ -I
stan/lib/stan_math/ -I stan/lib/stan_math/lib/eigen_3.3.9 -I
stan/lib/stan_math/lib/boost_1.75.0 -I
stan/lib/stan_math/lib/sundials_6.0.0/include -I
stan/lib/stan_math/lib/sundials_6.0.0/src/sundials    -
DBOOST_DISABLE_ASSERTS
-Wl,-L,"/opt/cmdstan-2.29.0/stan/lib/stan_math/lib/tbb"
-Wl,-rpath,"/opt/cmdstan-2.29.0/stan/lib/stan_math/lib/tbb"
/home/projekt/stan_files/model_2_fit.o src/cmdstan/main.o        -Wl,-
L,"/opt/cmdstan-2.29.0/stan/lib/stan_math/lib/tbb"
-Wl,-rpath,"/opt/cmdstan-2.29.0/stan/lib/stan_math/lib/tbb"
stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_nvecserial.a
stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_cvodes.a
stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_idas.a
stan/lib/stan_math/lib/sundials_6.0.0/lib/libsundials_kinsol.a
stan/lib/stan_math/lib/tbb/libtbb.so.2 -o
/home/projekt/stan_files/model_2_fit
rm -f /home/projekt/stan_files/model_2_fit.o
```

```
INFO:cmdstanpy:CmdStan start processing
chain 1 |          | 00:00 Status
▍          | 00:00 Status
chain 1 |█         | 00:00 Iteration:    1 / 2000 [  0%]  (Warmup)

██         | 00:00 Iteration:  300 / 2000 [ 15%]  (Warmup)

███        | 00:00 Iteration:  500 / 2000 [ 25%]  (Warmup)

█████      | 00:01 Iteration: 1001 / 2000 [ 50%]  (Sampling)

██████▌    | 00:03 Iteration: 1300 / 2000 [ 65%]  (Sampling)

███████▌   | 00:04 Iteration: 1500 / 2000 [ 75%]  (Sampling)

████████   | 00:05 Iteration: 1600 / 2000 [ 80%]  (Sampling)
████████▌  | 00:06 Iteration: 1700 / 2000 [ 85%]  (Sampling)
█████████  | 00:06 Iteration: 1800 / 2000 [ 90%]  (Sampling)
█████████▌ | 00:07 Iteration: 1900 / 2000 [ 95%]  (Sampling)
chain 1 |██████████| 00:07 Sampling completed
chain 2 |██████████| 00:07 Sampling completed
chain 3 |██████████| 00:07 Sampling completed
chain 4 |██████████| 00:07 Sampling completed



INFO:cmdstanpy:CmdStan done processing.
```

There were no issues with the sampling.

b) Analysis of samples obtained from the predictive distribution.

Model summary:

```
summary = model_2_fit.summary()
summary.head(10)

                                  Mean          MCSE    StdDev
5%  \
name

lp__                         5300.0000  7.400000e-02  2.500000
5300.0000
gamma_white                    -0.0013  5.100000e-08  0.000004    -
0.0013
gamma_black                     0.0013  7.000000e-08  0.000005
0.0013
```

```
time_controll_coeff_blitz       1.3000  3.600000e-05  0.002600
1.3000
time_controll_coeff_rapid       0.4800  4.100000e-06  0.000310
0.4800
time_controll_coeff_classic     0.3500  6.200000e-06  0.000440
0.3500
opening_coeff_sicilian          0.5700  2.600000e-05  0.001900
0.5700
opening_coeff_petroff           0.3600  7.400000e-05  0.005500
0.3500
opening_coeff_queen_pawn        0.3400  2.000000e-05  0.001500
0.3300
opening_coeff_italian           0.3400  2.700000e-05  0.002000
0.3300

                                   50%        95%   N_Eff  N_Eff/s
R_hat
name

lp__                         5300.0000  5300.0000  1100.0     23.0
1.0
gamma_white                    -0.0013    -0.0013  5600.0    110.0
1.0
gamma_black                     0.0013     0.0013  5200.0    110.0
1.0
time_controll_coeff_blitz       1.3000     1.3000  5300.0    110.0
1.0
time_controll_coeff_rapid       0.4800     0.4800  5700.0    120.0
1.0
time_controll_coeff_classic     0.3500     0.3500  5100.0    100.0
1.0
opening_coeff_sicilian          0.5700     0.5800  5800.0    120.0
1.0
opening_coeff_petroff           0.3600     0.3700  5500.0    110.0
1.0
opening_coeff_queen_pawn        0.3400     0.3400  5700.0    120.0
1.0
opening_coeff_italian           0.3400     0.3400  5700.0    120.0
1.0
```

Parameters values are similiar to those from priors but not the same.

```
model_2_fit_df = model_2_fit.draws_pd()
model_2_fit_df.head()

      lp__  accept_stat__  stepsize__  treedepth__  n_leapfrog__
divergent__  \
0  5292.56       0.876941    0.455339          3.0           7.0
0.0
```

```
1  5290.63       0.804896    0.455339         3.0           7.0
0.0
2  5288.97       0.884348    0.455339         3.0           7.0
0.0
3  5284.28       0.941027    0.455339         3.0           7.0
0.0
4  5290.04       0.997787    0.455339         3.0           7.0
0.0

   energy__  gamma_white  gamma_black  time_controll_coeff_blitz  ...
\
0  -5288.18    -0.001301     0.001302                    1.26627  ...

1  -5285.84    -0.001313     0.001311                    1.26986  ...

2  -5287.30    -0.001300     0.001301                    1.26060  ...

3  -5280.57    -0.001302     0.001300                    1.26007  ...

4  -5281.52    -0.001302     0.001300                    1.26841  ...


   mistakes_black_pred[2018]  mistakes_black_pred[2019]  \
0                        1.0                        1.0
1                        3.0                        1.0
2                        2.0                        0.0
3                        2.0                        1.0
4                        4.0                        2.0

   mistakes_black_pred[2020]  mistakes_black_pred[2021]  \
0                        8.0                        3.0
1                        7.0                        2.0
2                        5.0                        3.0
3                        7.0                        4.0
4                        7.0                        3.0

   mistakes_black_pred[2022]  mistakes_black_pred[2023]  \
0                        1.0                        5.0
1                        3.0                        6.0
2                        8.0                        8.0
3                        8.0                        6.0
4                        3.0                        6.0

   mistakes_black_pred[2024]  mistakes_black_pred[2025]  \
0                        3.0                        1.0
1                        0.0                        4.0
2                        3.0                        7.0
3                        2.0                        8.0
4                        2.0                        5.0
```

```
   mistakes_black_pred[2026]  mistakes_black_pred[2027]
0                        1.0                        1.0
1                        1.0                        3.0
2                        1.0                        6.0
3                        2.0                        3.0
4                        3.0                        8.0

[5 rows x 4079 columns]
```

Choosing sample with best matching to reality.

```
best_matching = -1
best_matching_ind = -1

for row_index, row in model_2_fit_df.iterrows():
    tmp_mistakes_white = []
    tmp_mistakes_black = []

    for i in range(N):
        tmp_mistakes_white.append(row[f'mistakes_white_pred[{i+1}]'])
        tmp_mistakes_black.append(row[f'mistakes_black_pred[{i+1}]'])

    number_of_matched_white = sum(1 for x, y in
zip(tmp_mistakes_white, mistakes_white_list) if x == y)
    number_of_matched_black = sum(1 for x, y in
zip(tmp_mistakes_black, mistakes_black_list) if x == y)
    acceptance_percent = ((number_of_matched_white +
number_of_matched_black) / (len(mistakes_white_list) +
len(mistakes_black_list))) * 100

    if acceptance_percent > best_matching:
        best_matching = acceptance_percent
        best_matching_ind = row_index

chosen_row_in_posterior_model_2 = best_matching_ind
```

Creating pandas DataFrame with chosen posterior predictive samples

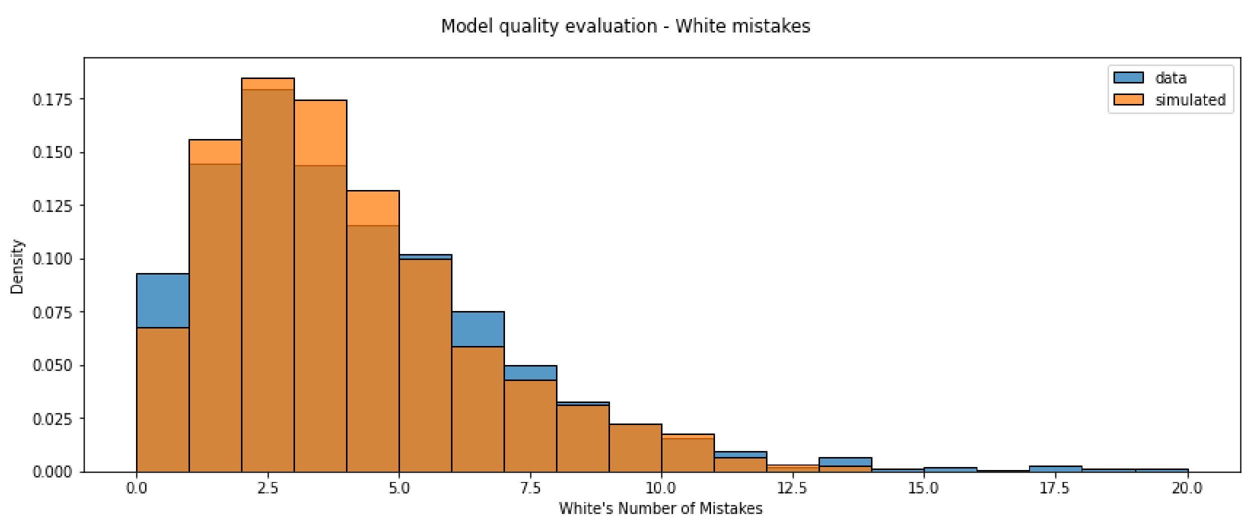
```
mistakes_white_pred = []
mistakes_black_pred = []

for i in range(N):

mistakes_white_pred.append(model_2_fit_df[f'mistakes_white_pred[{i+1}]
'].values[chosen_row_in_posterior_model_2])

mistakes_black_pred.append(model_2_fit_df[f'mistakes_black_pred[{i+1}]
'].values[chosen_row_in_posterior_model_2])

df_model_2_fit_transposed = pd.DataFrame({'mistakes_white_pred':
```

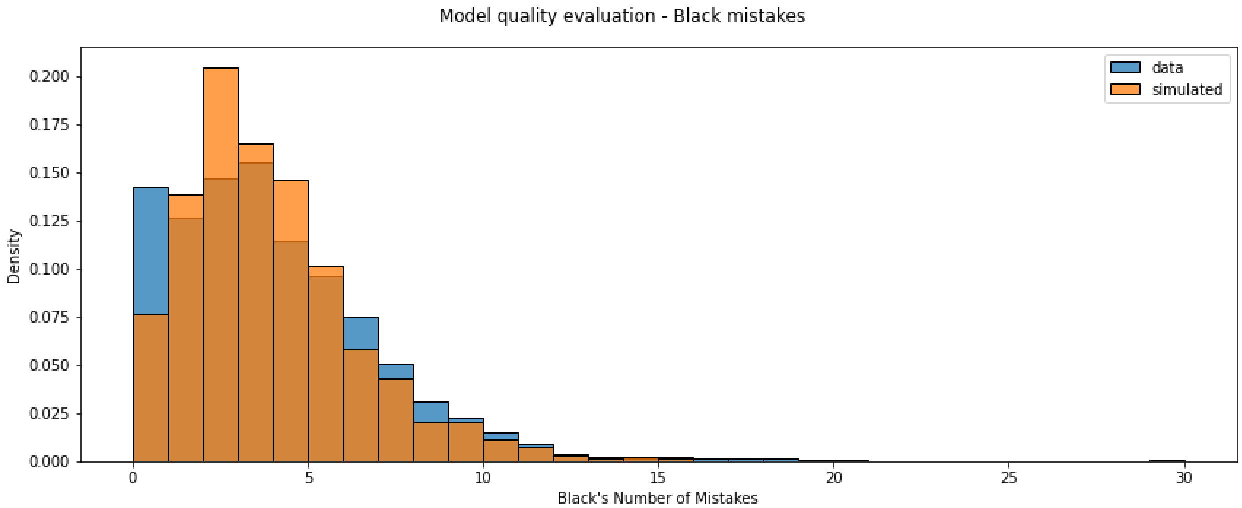
```
mistakes_white_pred, 'mistakes_black_pred': mistakes_black_pred})
df_model_2_fit_transposed
```

```
      mistakes_white_pred  mistakes_black_pred
0                     5.0                  2.0
1                     6.0                  4.0
2                     2.0                  2.0
3                     2.0                  2.0
4                     3.0                  3.0
...                   ...                  ...
2022                  7.0                  8.0
2023                  2.0                  5.0
2024                  2.0                  4.0
2025                  2.0                  3.0
2026                  2.0                  5.0

[2027 rows x 2 columns]
```

c) Analysis of posterior predictive samples

```
for color in ['White', 'Black']:
    fig, ax = plt.subplots(figsize=(12,5))
    fig.suptitle(f"Model quality evaluation - {color} mistakes")
    sns.histplot(data=df, x=f"{color}'s Number of Mistakes",
binwidth=1, stat="density", ax=ax, label='data')
    sns.histplot(data=df_model_2_fit_transposed,
x=f"mistakes_{color.lower()}_pred", binwidth=1, stat="density", ax=ax,
label='simulated')
    plt.legend()
    plt.tight_layout()
    plt.plot()
    plt.show()
```



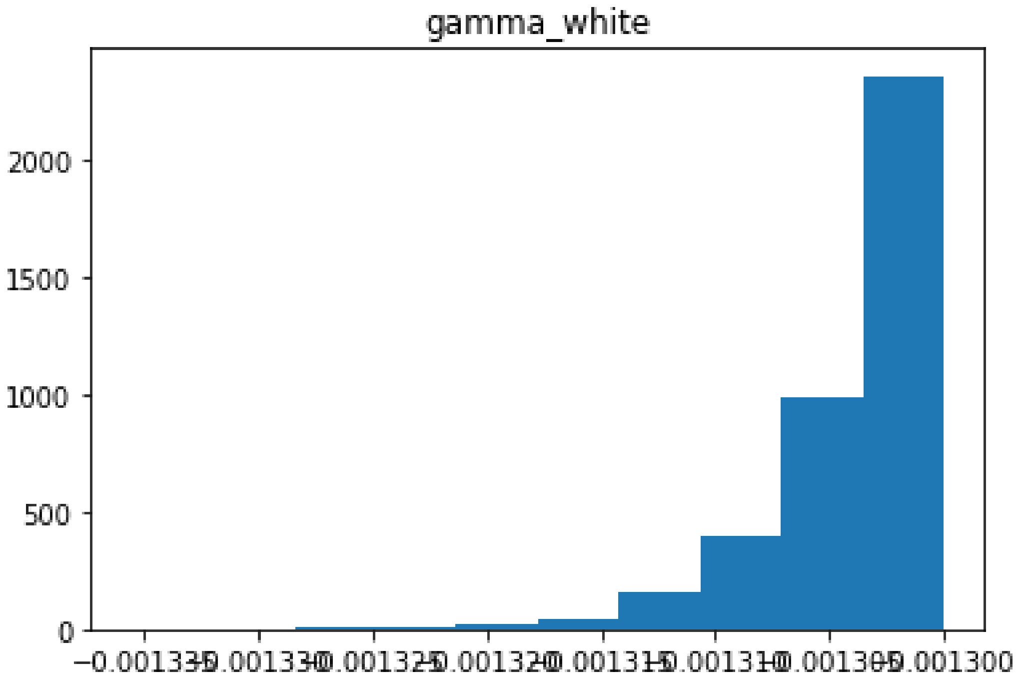


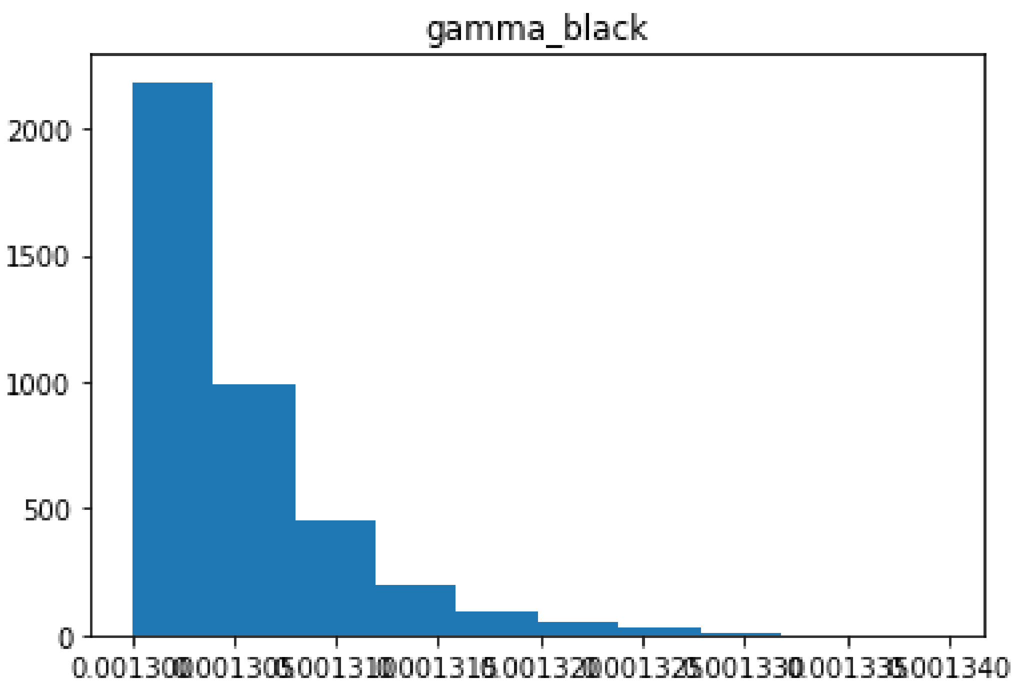
Posterior predictive samples are consistent with the data. This is the result of the second (more complex) model and (as we xpected) it is better than result from first (simpler) model. The results returned by the model considering time control, rating difference and chosen opening are very satisfactory. Below we calculated overlap of posterior predictive samples and data.

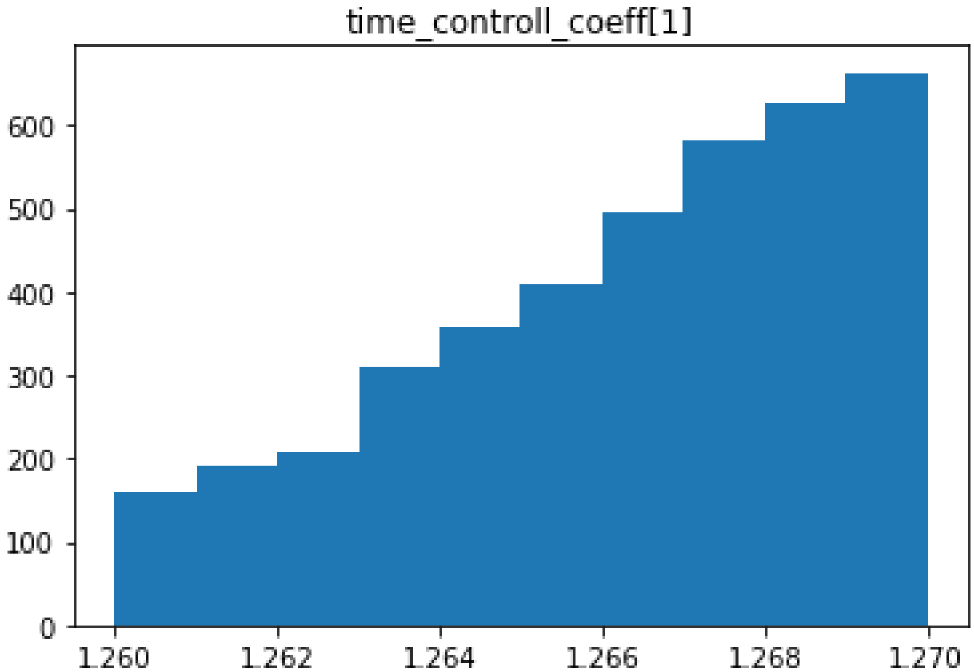
```
calculate_hist_overlap(df, df_model_2_fit_transposed)

Overlap of posterior predictive samples and data for mistakes white :
     87.29%
Overlap of posterior predictive samples and data for mistakes black :
     79.02%
```

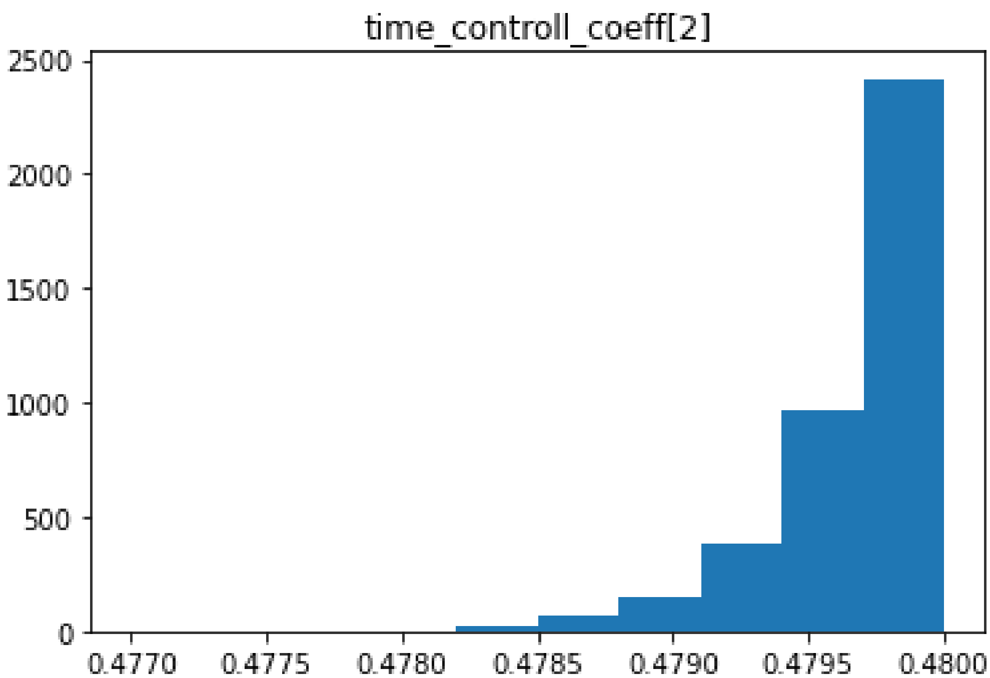
d) Analysis of parameter marginal disrtibutions

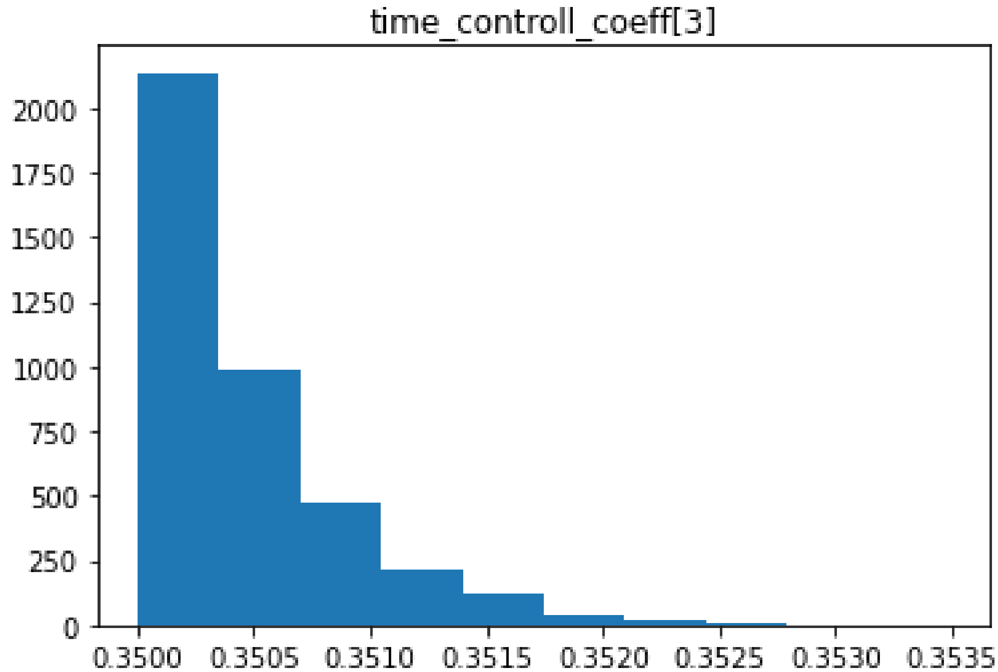
```
for parameter in ['gamma_white', 'gamma_black',
'time_controll_coeff[1]', 'time_controll_coeff[2]',
'time_controll_coeff[3]',
                 'opening_coeff[1]', 'opening_coeff[2]',
'opening_coeff[3]', 'opening_coeff[4]']:
    plt.hist(model_2_fit_df[parameter])
    plt.title(parameter)
    plt.show()
```

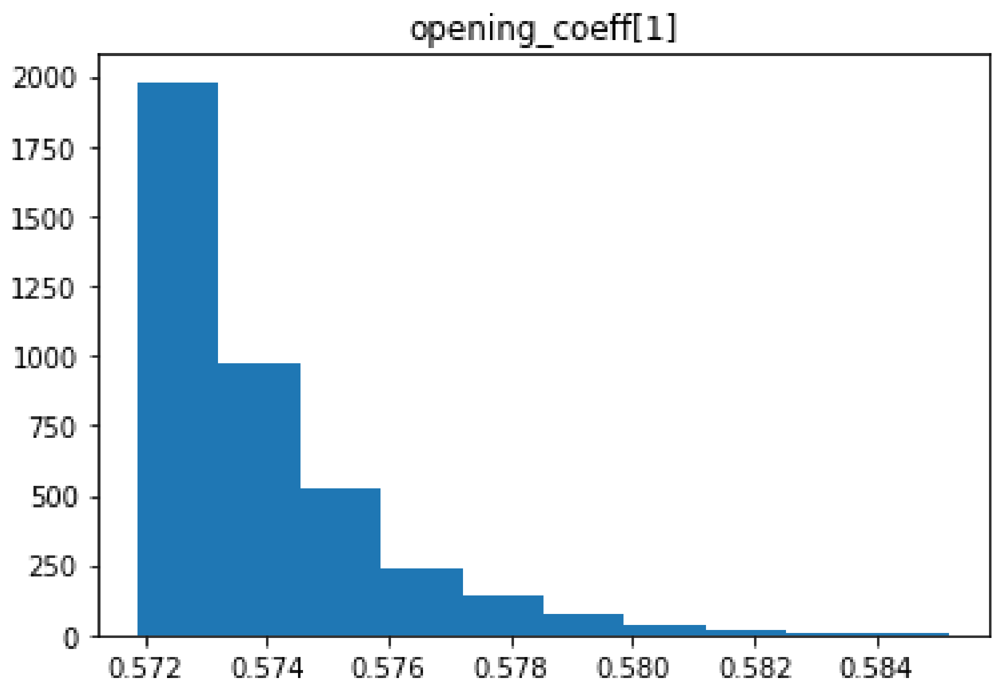

gamma_white
0
500
1000
1500
2000
−0.001335
−0.001330
−0.001325
−0.001320
−0.001315
−0.001310
−0.001305
−0.001300

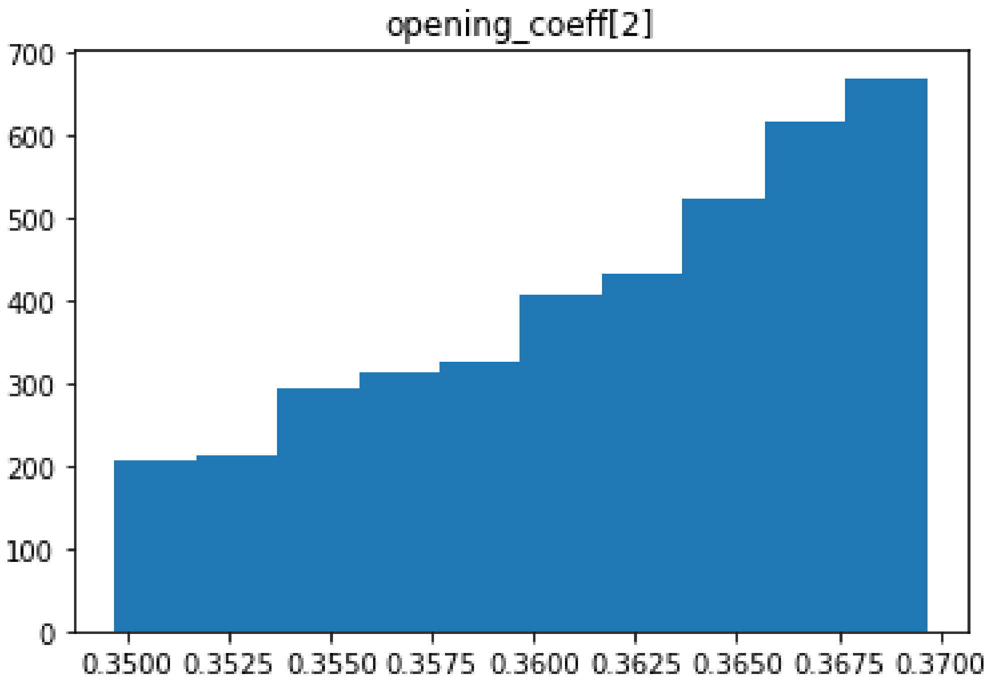

gamma_black
0
500
1000
1500
2000
0.001300
0.001305
0.001310
0.001315
0.001320
0.001325
0.001330
0.001335
0.001340

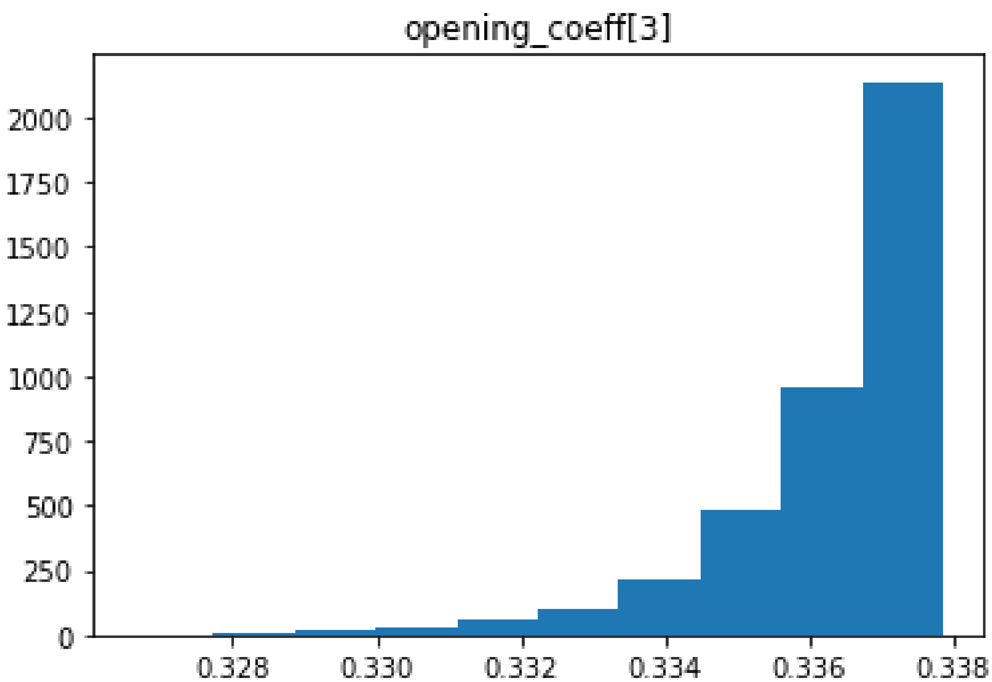

time_controll_coeff[1]
600
500
400
300
200
100
0
1.260
1.262
1.264
1.266
1.268
1.270

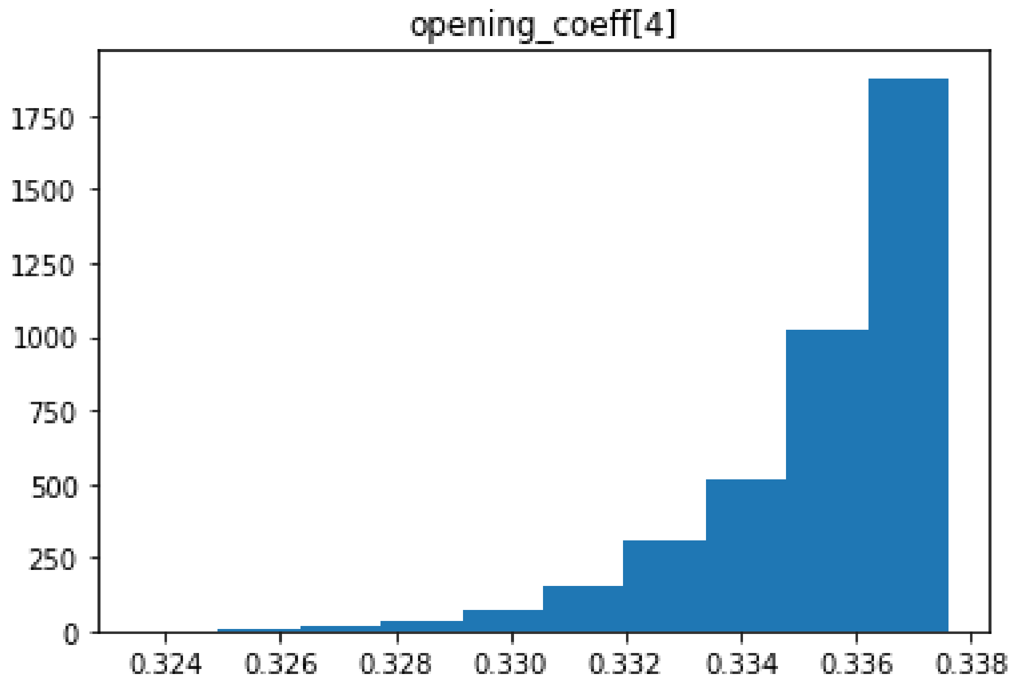

time_controll_coeff[2]
2500
2000
1500
1000
500
0
0.4770
0.4775
0.4780
0.4785
0.4790
0.4795
0.4800


time_controll_coeff[3]
2000
1750
1500
1250
1000
750
500
250
0
0.3500
0.3505
0.3510
0.3515
0.3520
0.3525
0.3530
0.3535


opening_coeff[1]
2000
1750
1500
1250
1000
750
500
250
0
0.572
0.574
0.576
0.578
0.580
0.582
0.584


opening_coeff[2]
0
100
200
300
400
500
600
700
0.3500
0.3525
0.3550
0.3575
0.3600
0.3625
0.3650
0.3675
0.3700


opening_coeff[3]
0
250
500
750
1000
1250
1500
1750
2000
0.328
0.330
0.332
0.334
0.336
0.338



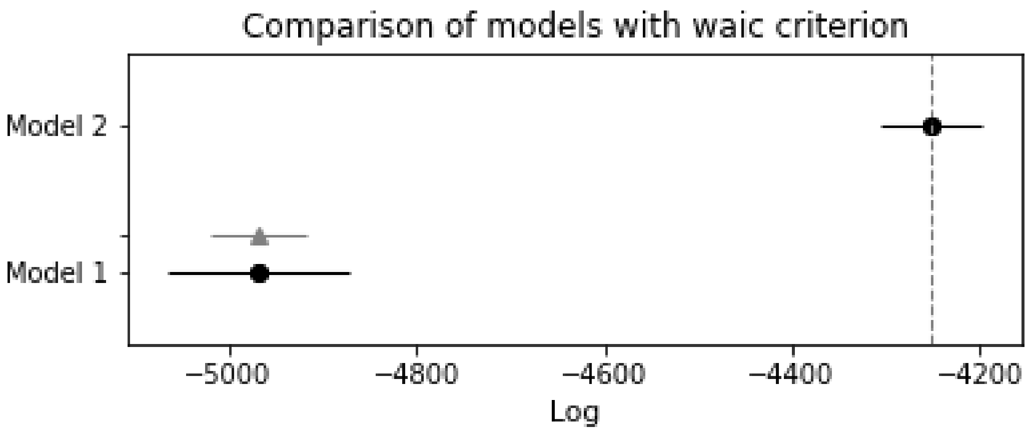
Comment: The parameter values are concentrated and quite close to those obtained by prior analysis.

7. Model comaprison

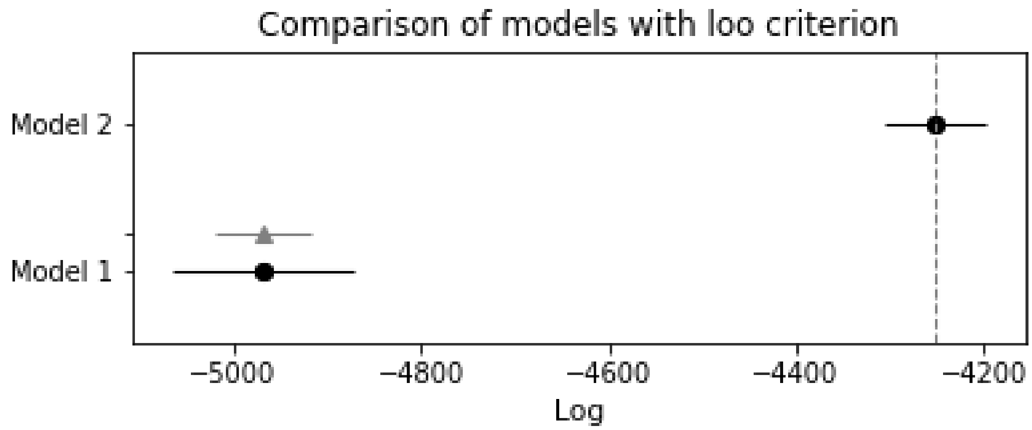
a) Comparison using information criteria

```
compare_dict = {
    "Model 1": az.from_cmdstanpy(posterior=model_1_fit),
    "Model 2": az.from_cmdstanpy(posterior=model_2_fit)
    }

for information_criteria in ["waic", "loo"]:
    diff = az.compare(compare_dict=compare_dict,
ic=information_criteria)
    print(f"{information_criteria.upper()}")
    print("="*100)
    display(diff)
    ax = az.plot_compare(diff)
    ax.set_title(f"Comparison of models with {information_criteria}
criterion")
    plt.show()
    print("="*100)
    if information_criteria == "waic":
        print('\n\n\n')

WAIC
======================================================================
==============================
```

```
         rank          waic    p_waic      d_waic    weight         se  \
Model 2     0 -4252.434847  0.129815    0.000000  0.974588  54.040376   

Model 1     1 -4968.331503  0.044622  715.896655  0.025412  95.905208   


               dse  warning waic_scale
Model 2   0.000000    False        log
Model 1  51.299359    False        log
```




```
======================================================================
==============================




LOO
======================================================================
==============================

         rank          loo     p_loo       d_loo    weight         se  \
Model 2     0 -4252.434849  0.129816    0.000000  0.974588  54.040376   

Model 1     1 -4968.331510  0.044628  715.896661  0.025412  95.905209   


              dse  warning loo_scale
Model 2   0.00000    False       log
Model 1  51.29936     True       log
```




```
===============================================================================
==============================
```

b) Conclusions and comments drawn from the result for WAIC

The results obtained for the Watanabe-Akaike Information Criterio (WAIC) ratio allow comparing the "Model 1" and "Model 2" models on the basis of predictive evaluation.

- Model 2 has a "rank" of 0, indicating that it is better than Model 1, which has a rank of 1.
- "waic" refers to the Watanabe-Akaike Information Criterion (WAIC) value for each model. Model 2 has a WAIC value of -4252.434847, while Model 1 has a WAIC value of -4968.331503.
- "p_waic" represents the estimated value of the WAIC estimate along with the standard error. For Model 2, it is 0.129815, and for Model 1, it is 0.044622.
- "d_waic" shows the difference in WAIC between Model 2 and Model 1. In this case, it is 715.896655, indicating that Model 2 has a lower WAIC by that amount compared to Model 1.
- "weight" represents the weight assigned to each model based on WAIC comparisons. For Model 2, the weight is 0.974588, and for Model 1, it is 0.025412. A higher weight implies a higher probability that the model is well-fitted to the data.
- "se" indicates the standard error for the model weights. For Model 2, it is 54.040376, and for Model 1, it is 95.905208.
- "dse" represents the standard error for the difference in WAIC between the models. For Model 2, it is 0.000000, and for Model 1, it is 51.299359.
- "warning" indicates whether any warnings occurred during the calculations. Both models have a value of False, suggesting that no warnings were raised.
- "waic_scale" specifies the scale used for WAIC calculation. In this case, it is "log" for both models.

In summary, based on the results from the az.compare function, it can be concluded that Model 2 has a lower WAIC, lower standard error, higher weight, and no warnings compared to Model 1. This suggests that Model 2 is better fitted to the data and more reliable according to the Watanabe-Akaike Information Criterion.

c) Conclusions and comments drawn from the result for LOO

The results obtained for the Leave-One-Out (LOO) ratio allow comparing the "Model 1" and "Model 2" models on the basis of predictive evaluation.

- Model 2 has a "rank" of 0, which means it is better than Model 1, which has a rank of 1.
- "loo" refers to the Leave-One-Out (LOO) value for each model. Model 2 has a LOO value of -4252.434849, while Model 1 has a LOO value of -4968.331510.
- "p_loo" represents the estimated value of the LOO estimate along with the standard error. For Model 2, it is 0.129816, and for Model 1, it is 0.044628.
- "d_loo" shows the difference in LOO between Model 2 and Model 1. In this case, it is 715.896661, indicating that Model 2 has a lower LOO by that amount compared to Model 1.
- "weight" represents the weight assigned to each model based on LOO comparisons. For Model 2, the weight is 0.974588, and for Model 1, it is 0.025412. A higher weight implies a higher probability that the model is well-fitted to the data.
- "se" indicates the standard error for the model weights. For Model 2, it is 54.040376, and for Model 1, it is 95.905209.
- "dse" represents the standard error for the difference in LOO between the models. For Model 2, it is 0.00000, and for Model 1, it is 51.29936.
- "warning" indicates whether any warnings occurred during the calculations. For Model 2, the value is False, and for Model 1, it is True.
- "loo_scale" specifies the scale used for LOO estimation. In this case, it is "log" for both models.

In summary - Model 2 has a lower LOO, lower standard error, higher weight, and no warning compared to Model 1. This suggests that Model 2 is better fitted to the data and more reliable.

d) Final conclusions

The comparison made above using information criteria confirmed our earlier conclusions. Model 2 proved to be significantly better, showing that the impact of chess opening selection on the course of the game is significant. As chess players, we predicted that we would get such results after comparing these models.