

PROBLEMS UNIT 1

3) Analyze the efficiency of the following code:

We assume the cost of $\text{Mínimo}(i,j) = 1$

$$T(n) \rightarrow O(n^{\log(n)})$$

Handwritten derivation of the time complexity $T(n)$:

$$\begin{aligned}
 T(n) &= 1 + \sum_{i=1}^n \left(1 + \sum_{j=1}^n \left(\sum_{k=1}^n (1+1) \right) + 1 + t(n/2) \right) \\
 &= 1 + \sum_{i=1}^n \left(1 + \sum_{j=1}^n 2n + 1 + t(n/2) \right) \\
 &= 1 + \sum_{i=1}^n \left(2 + 2n^2 + t(n/2) \right) \\
 &= 1 + 2n + 2n^3 + 2t(n/2) \\
 n &= 2^k \\
 T(2^k) &= 1 + 2 \cdot 2^k + 2 \cdot (2^k)^3 + 2^k \cdot t(2^k/2) \\
 T(2^k) &= x^k \\
 x^k &= 1 + 2 \cdot 2^k + 2 \cdot 8^k + 2^k \cdot x^{k-1} \\
 x^k - 2^k \cdot x^{k-1} &= 1 + 2 \cdot 2^k + 2 \cdot 8^k \\
 \text{Homogeneous: } x^{k-1} \cdot (x - 2^k) &= 0 & \text{Particular: } x^{k-1} \cdot (x - 2^k) &= 1 + 2 \cdot 2^k + 2 \cdot 8^k \\
 \text{roots: } x &= 2^k & \text{roots: } x &= 1, x = 2, x = 8 \\
 x^{(H)} &= A(2^k)^k & x^{(P)} &= B + C2^k + D8^k \\
 x &= x^{(H)} + x^{(P)} = A(2^k)^k + B + C2^k + D8^k = A n^{\log_2 n} + B + Cn + Dn^3 \\
 & & n=2^k & k=\log_2(n) \\
 T(n) &\rightarrow O(n^{\log(n)})
 \end{aligned}$$

6) Program a function to determine if a number received as parameter is prime. Analyze the efficiency and complexity.

```

public static boolean isPrime(int num) {
    int temp;
    boolean isPrime = true;
    if (num == 0 || num == 1) // Both 0 and 1 are not prime numbers
    {
        isPrime = false;
    }
    else
    {
        for(int i = 2; i <= num/2; i++) // num/2 because we are going through the possible factors,
        {                               // so we don't want to repeat them.
            temp = num % i;
            if(temp == 0)
            {
                isPrime = false; // If we find a factor different from 0 or num then
                break;           // num is not prime.
            }
        }
    }
    return isPrime;
}

```

$$T(n) = 1 + 1 + 1 + 1$$

$$= 1 + 3 \cdot n/2$$

$$= 1 + 1,5n$$

$$= O(n)$$

8) Program a recursive procedure to obtain the inverse number of a given one. Example : 627 -> 726. Analyze the efficiency and complexity.

```
public static void reverseNumber(int num, int n, int aux) {
    if (num < 10)
    {
        aux = aux * 10 + num;
        System.out.println(aux);
    }
    else
    {
        aux = aux * 10 + (num % 10 );
        reverseNumber(num/10,n++,aux);
    }
}
```

$$T(n) = 1 + 1 + t(n/10)$$

$$N = 10^k$$

$$X^k = 2 + x^{k-1}$$

$$X^{k-1}(x - 1) = 0 \quad x(H) = A$$

$$X^{k-1}(x - 1) = 2 \quad x(P) = Bk$$

$$X = A + Bk = A + \log(n)B$$

$$= O(\log(n))$$