# Econometric approach to efficiency analysis Project

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# Electricity production

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#### Abstract

The present study examines productivity in electricity generation across 40 companies over a period of six years, with a focus on the relationship between input variables like capital assets, workforce and maintenance, and fuel consumption in relation to the net generation of steam electric power. The research delves into the effects of various input costs, such as workforce and maintenance expenditures, fuel pricing, and the value of capital assets, on the operational efficiency of these firms. Leveraging a detailed dataset with company-specific information and a range of productivity evaluation techniques like cross-sectional and panel data analyses, the study aims to pinpoint the key factors that drive productivity in the power sector. The results will provide valuable guidance for policy suggestions and strategic choices to improve the overall performance and sustainability of the energy industry, benefiting policymakers, industry stakeholders, and academics alike.

Keywords: Econometric approach to efficiency analysis; Electricity Production Data Set

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#### 1 Introduction

A reliable and consistent electricity supply is essential for the growth and stability of the global economy. The effectiveness of electricity generation has attracted the attention of decision-makers, researchers, and industry participants, as it directly influences the overall performance and sustainability of the energy sector. This research project aims to perform an assessment of productivity in electricity generation across 40 firms over a 6-year period.

The focus of this analysis is to explore the connection between input variables such as capital assets, workforce and maintenance, and fuel, and their influence on the net production of steam electric power. Moreover, the research will delve into the impact of input costs, including labor and maintenance expenses, fuel prices, and capital asset values, on the efficiency of the firms' operations. This investigation will help to pinpoint the primary drivers of productivity and provide valuable insights for policy recommendations and strategic decisions to improve the overall performance of the power sector.

The dataset used for this research contains the following variables:

- Firm identifier (firm): A unique code for each of the 40 firms included in the dataset.
- Year (year): The year of observation, covering a 6-year period.
- Output net steam electric power (y): The net production of steam electric power generated by the firm, quantified in megawatt-hours (MWh).
- Capital assets (k): The input variable representing the capital assets employed in electricity generation.
- Workforce and maintenance (labor): The input variable representing the workforce and maintenance efforts for electricity generation.
- Fuel (fuel): The input variable representing the quantity of fuel utilized in electricity generation.
- Labor and maintenance cost (wl): The cost of the workforce and maintenance input.
- Fuel cost (wf): The cost of the fuel input.
- Capital asset value (wk): The value of the capital asset input.

Employing these variables, the research will utilize various productivity assessment methods, such as cross-sectional analysis, panel data analysis, to estimate the productivity of the electricity generation process for each firm and over time. The outcomes of this analysis will enhance our understanding of the factors affecting production efficiency in the power sector and offer valuable insights for decision-makers, industry participants, and researchers.

### 2 DEA Analysis - Cross-sectional

#### 2.1 Input and Output oriented Analysis

The efficiency summaries provided display one table demonstrating input-oriented efficiency and the other showcasing output-oriented efficiency. Each table lists firms along with their efficiency measurements: constant returns to scale (CRSTE), variable returns to scale (VRSTE), and scale efficiency (scale). Additionally, the tables show whether the firms experience increasing returns to scale (IRS), decreasing returns to scale (DRS), or constant returns to scale (-) based on their scale efficiency.

Input-oriented efficiency evaluates how effectively a company can reduce its inputs while maintaining a steady output level. In contrast, output-oriented efficiency measures how well a company can increase its outputs while keeping its inputs constant. The primary distinction between the two methods is whether the focus is on minimizing inputs or maximizing outputs to achieve efficiency.

Analyzing the input-oriented efficiency summary:

The average CRSTE, VRSTE, and scale efficiency values are 0.813, 0.867, and 0.939, respectively. This suggests that firms can potentially decrease their inputs by about 18.7% (1 - 0.813) on average, while keeping output levels unchanged. The most efficient firms have CRSTE, VRSTE, and scale efficiency values of 1. These companies are already functioning at optimal efficiency levels (for example, firms 25, 29, 31, 34, and 37). Companies with DRS (such as firm 1) can enhance their efficiency by reducing their scale of operations, while those with IRS (such as firm 3) can improve efficiency by expanding their scale of operations.

Examining the output-oriented efficiency summary:

The average CRSTE, VRSTE, and scale efficiency values are 0.813, 0.885, and 0.917, respectively. This implies that firms can potentially increase their outputs by approximately 18.7% (1 - 0.813) on average while maintaining constant input levels. The most efficient firms have CRSTE, VRSTE, and scale efficiency values of 1. These companies are already functioning at optimal efficiency levels (for example, firms 25, 29, 31, 34, and 37). The key difference between the two tables is the emphasis of the efficiency measure. Input-oriented efficiency prioritizes minimizing inputs to reach efficiency, while output-oriented efficiency centers on maximizing outputs. Nevertheless, in both tables, the overall interpretation of the efficiency measurements is consistent. Factories with efficiency values of 1 are operating at their best, while others can improve their efficiency by adjusting their scale of operations.

Table 1: Efficiency Summary (Input and Output Oriented)

	I	nput Orio	ented		Outpu	t Oriente	$\operatorname{ed}$
$\operatorname{Firm}$	CRSTE	VRSTE	Scale	$\operatorname{Firm}$	CRSTE	VRSTE	Scale
1	0.864	0.884	$0.977~\mathrm{drs}$	1	0.864	0.895	$0.965~\mathrm{drs}$
2	0.780	0.791	$0.986~\mathrm{drs}$	2	0.780	0.862	$0.905~\mathrm{drs}$
3	0.928	0.930	0.998 irs	3	0.928	0.928	0.999 irs
4	0.804	0.806	0.998 irs	4	0.804	0.807	$0.996~\mathrm{drs}$
5	0.807	0.869	$0.928~\mathrm{drs}$	5	0.807	0.879	$0.918~\mathrm{drs}$
6	0.860	0.931	$0.923~\mathrm{drs}$	6	0.860	0.935	$0.919~\mathrm{drs}$
7	0.863	0.908	$0.951~\mathrm{drs}$	7	0.863	0.913	$0.946~\mathrm{drs}$
8	0.548	0.805	$0.682~\mathrm{drs}$	8	0.548	0.858	$0.639~\mathrm{drs}$
9	0.691	0.760	$0.909~\mathrm{drs}$	9	0.691	0.853	$0.810~\mathrm{drs}$
10	0.813	0.820	0.992 irs	10	0.813	0.817	0.995 irs
11	0.794	1.000	$0.794~\mathrm{drs}$	11	0.794	1.000	$0.794~\mathrm{drs}$
12	0.877	1.000	$0.877~\mathrm{drs}$	12	0.877	1.000	$0.877~\mathrm{drs}$
13	0.874	0.905	$0.966~\mathrm{drs}$	13	0.874	0.937	$0.933~\mathrm{drs}$
14	0.691	0.898	$0.769~\mathrm{drs}$	14	0.691	0.928	$0.744~\mathrm{drs}$
15	0.740	0.894	$0.828~\mathrm{drs}$	15	0.740	0.904	$0.819~\mathrm{drs}$
16	0.680	0.694	$0.979~\mathrm{drs}$	16	0.680	0.733	$0.928~\mathrm{drs}$
17	0.603	0.608	0.992 irs	17	0.603	0.604	0.998 irs
18	0.729	0.739	0.987 irs	18	0.729	0.734	0.994 irs
19	0.401	0.447	$0.897~\mathrm{drs}$	19	0.401	0.608	$0.660~\mathrm{drs}$
20	0.966	1.000	0.966  irs	20	0.966	1.000	0.966 irs
21	0.981	1.000	$0.981~\mathrm{drs}$	21	0.981	1.000	$0.981~\mathrm{drs}$
22	0.884	0.958	$0.923~\mathrm{drs}$	22	0.884	0.960	$0.921~\mathrm{drs}$
23	0.981	1.000	0.981 irs	23	0.981	1.000	0.981 irs
24	0.847	1.000	$0.847~\mathrm{drs}$	24	0.847	1.000	$0.847~\mathrm{drs}$
25	1.000	1.000	1.000 -	25	1.000	1.000	1.000 -
26	0.586	0.587	0.998 irs	26	0.586	0.634	$0.924~\mathrm{drs}$
27	0.861	1.000	0.861  irs	27	0.861	1.000	0.861 irs
28	0.993	0.999	0.994 irs	28	0.993	0.999	0.994 irs
29	1.000	1.000	1.000 -	29	1.000	1.000	1.000 -
30	0.782	0.789	$0.992 \mathrm{\ drs}$	30	0.782	0.805	$0.971 \mathrm{\ drs}$
31	1.000	1.000	1.000 -	31	1.000	1.000	1.000 -
32	0.791	0.822	$0.963 \ \mathrm{drs}$	32	0.791	0.916	$0.864~\mathrm{drs}$
33	0.560	0.562	0.996 irs	33	0.560	0.588	$0.953~\mathrm{drs}$
34	1.000	1.000	1.000 -	34	1.000	1.000	1.000 -
35	0.760	1.000	$0.760 \ \mathrm{drs}$	35	0.760	1.000	$0.760 \ \mathrm{drs}$
36	0.705	0.781	$0.903~\mathrm{drs}$	36	0.705	0.798	$0.883~\mathrm{drs}$
37	1.000	1.000	1.000 -	37	1.000	1.000	1.000 -
38	0.819	0.827	0.990 irs	38	0.819	0.824	0.993 irs
39	0.671	0.673	0.998 irs	39	0.671	0.696	$0.965~\mathrm{drs}$
40	0.971	0.996	$0.975  \mathrm{drs}$	40	0.971	0.996	$0.975  \mathrm{drs}$
mean	0.813	0.867	0.939	mean	0.813	0.885	0.917

The software give us the summary of output slacks. It reveals that the value of 0.000 for all firms denotes that none of them are experiencing output slack. This implies that all firms are efficiently utilizing their outputs without any waste, and this is applicable for both input and output-oriented methods. (We will not display it because we found it very useless)

The table 2 summarize the input slacks for a set of firms using input-oriented and output-oriented approaches.

In the input-oriented approach, the value of input slack represents the amount by which a firm's input usage could be reduced without decreasing its output. A slack value of zero indicates that the firm is using its inputs efficiently without any waste. From the table, we can see that some firms have input slack in some inputs, such as firm 1 and firm 2 in input 3. On average, the firms have input slack of 3453.447 in input 1 and 6282.543 in input 3.

On the other hand, the output-oriented approach measures the amount by which a firm's output could be increased without increasing its input usage. A slack value of zero means that the firm is producing its output efficiently. From the second table, we can see that some firms have output slack in some outputs, such as firm 14 in output 2 and firm 32 in output 1. On average, the firms have output slack of 5344.844 in output 1 and 11825.167 in output 3.

Overall, these tables provide insights into the input and output usage efficiency of the firms and can be useful for identifying areas where improvements can be made to enhance the firms' productivity.

These tables (5 & 6) show the peer count for each firm from two different approaches, output-oriented and input-oriented. The peer count indicates how many times a firm is considered a benchmark by its peers.

The output-oriented approach highlights firms that are seen as benchmarks in terms of output performance, with firm 37 having the highest peer count of 22. In contrast, the input-oriented approach highlights firms that are seen as benchmarks in terms of input usage efficiency, with firms 25, 31, and 37 having the highest peer count of 12, 14, and 23, respectively.

Table 2: Summary of Input and Output Slacks

	Inpi	ut Orie	$\operatorname{nted}$		Out	tput Orien	ited
$\mathbf{Firm}$	$\overline{1}$	<b>2</b>	3	Firm	1	2	3
1	0.000	0.000	27733.605	1	0.000	0.000	36920.155
2	0.000	0.000	37560.673	2	0.000	0.000	55211.491
3	0.000	0.000	0.000	3	0.000	0.000	0.000
4	138137.883	0.000	0.000	4	155732.977	0.000	0.000
5	0.000	0.000	0.000	5	0.000	0.000	0.000
6	0.000	0.000	0.000	6	0.000	0.000	0.000
7	0.000	0.000	0.000	7	0.000	0.000	0.000
8	0.000	0.000	107590.967	8	0.000	0.000	163357.744
9	0.000	0.000	0.000	9	0.000	0.000	18676.879
10	0.000	0.000	36624.754	10	0.000	0.000	44799.994
11	0.000	0.000	0.000	11	0.000	0.000	0.000
12	0.000	0.000	0.000	12	0.000	0.000	0.000
13	0.000	0.000	32066.845	13	0.000	0.000	38701.961
14	0.000	0.000	0.000	14	0.000	198.931	0.000
15	0.000	0.000	0.000	15	0.000	0.000	0.000
16	0.000	0.000	0.000	16	0.000	0.000	0.000
17	0.000	0.000	0.000	17	0.000	0.000	0.000
18	0.000	0.000	0.000	18	0.000	0.000	0.000
19	0.000	0.000	0.000	19	0.000	2240.009	63507.174
20	0.000	0.000	0.000	20	0.000	0.000	0.000
21	0.000	0.000	0.000	21	0.000	0.000	0.000
22	0.000	0.000	0.000	22	0.000	0.000	0.000
23	0.000	0.000	0.000	23	0.000	0.000	0.000
24	0.000	0.000	0.000	24	0.000	0.000	0.000
25	0.000	0.000	0.000	25	0.000	0.000	0.000
26	0.000	0.000	6334.277	26	0.000	0.000	23688.204
27	0.000	0.000	0.000	27	0.000	0.000	0.000
28	0.000	0.000	0.000	28	0.000	0.000	0.000
29	0.000	0.000	0.000	29	0.000	0.000	0.000
30	0.000	0.000	0.000	30	0.000	0.000	0.000
31	0.000	0.000	0.000	31	0.000	0.000	0.000
32	0.000	0.000	0.000	32	58060.788	0.000	23922.367
33	0.000	0.000	0.000	33	0.000	0.000	0.000
34	0.000	0.000	0.000	34	0.000	0.000	0.000
35	0.000	0.000	0.000	35	0.000	0.000	0.000
36	0.000	0.000	0.000	36	0.000	0.000	0.000
37	0.000	0.000	0.000	37	0.000	0.000	0.000
38	0.000	0.000	3390.585	38	0.000	0.000	4220.697
39	0.000	0.000	0.000	39	0.000	0.000	0.000
40	0.000	0.000	0.000	40	0.000	0.000	0.000
2*Mean	3453.447	0.000	6282.543	2*Mean	5344.844	60.974	11825.167

#### 2.2 Cost efficiency analysis:

The table 3 displays the technical efficiency (te), allocative efficiency (ae), and cost efficiency (ce) scores for each firm in the analysis. The technical efficiency score indicates how efficiently a firm is utilizing its inputs to produce outputs, with a score of 1 indicating perfect efficiency. The allocative efficiency score assesses how effectively a firm is allocating its resources to minimize costs, with a score of 1 indicating optimal efficiency. The cost efficiency score is calculated by multiplying the technical and allocative efficiency scores.

The average cost efficiency score across all firms is 0.797, which implies that the firms could reduce their costs by 20.3% on average. Firm 11 has the highest score in all three categories, signifying that it is the most efficient firm in the analysis. Conversely, firm 19 has the lowest technical efficiency score of 0.447, implying that it could enhance its input utilization to produce outputs more efficiently.

The table 4 summarizes the cost-minimizing input quantities for each firm in the analysis. The inputs are represented by three variables labeled 1, 2, and 3. The values in the table represent the minimum quantity of each input required to produce a given level of output while minimizing costs.

By analyzing the table, we can see that the input quantities required vary widely across firms. For example, firm 8 requires a significantly higher quantity of input 1 than any other firm, while firm 27 requires the least quantity of input 2.

It is important to note that the table only represents the minimum input quantities required to achieve a given level of output. The efficiency of each firm in terms of input usage is not directly represented in this table. However, these input quantities can be used to calculate the cost efficiency of each firm when combined with the output data.

Table 3: Efficiency Summary

Firm	Technical Efficiency (TE)	Allocative Efficiency (AE)	Cost Efficiency (CE) height1
0.884	0.814	0.720	
2	0.791	0.848	0.670
3	0.930	0.983	0.914
4	0.806	0.813	0.655
5	0.869	0.963	0.837
6	0.931	0.981	0.914
7	0.908	0.982	0.892
8	0.805	0.821	0.661
9	0.760	0.982	0.747
10	0.820	0.557	0.457
11	1.000	1.000	1.000
12	1.000	0.905	0.905
13	0.905	0.851	0.770
14	0.898	0.902	0.810
15	0.894	0.927	0.829
16	0.694	0.957	0.664
17	0.608	0.929	0.565
18	0.739	0.937	0.692
19	0.447	0.939	0.420
20	1.000	0.865	0.865
21	1.000	0.997	0.997
22	0.958	0.960	0.920
23	1.000	0.831	0.831
24	1.000	0.928	0.928
25	1.000	0.802	0.802
26	0.587	0.935	0.549
27	1.000	1.000	1.000
28	0.999	0.921	0.920
29	1.000	0.939	0.939
30	0.789	0.992	0.782
31	1.000	1.000	1.000
32	0.822	0.999	0.821
33	0.562	0.971	0.546
34	1.000	1.000	1.000
35	1.000	0.847	0.847
36	0.781	0.886	0.692
37	1.000	1.000	1.000
38	0.827	0.856	0.708
39	0.673	0.941	0.633
40	0.996	0.968	0.963
Mean	0.867	0.918	0.797

Table 4: Cost minimising input quantities:

$\mathbf{firm}$	$\mathbf{input} \ 1$	$\mathbf{input} \ 2$	input 3 height1
620753.581	1245.208	114523.150 2	756895.342
1481.624	142434.985 3	165924.669	2677.917
$26335.721  ext{ } 4$	174565.422	3141.313	29128.447 5
760842.789	1488.479	143244.292 6	519188.169
3622.827	101733.287 7	392263.544	3491.969
75093.053 8	2364015.870	5445.717	257029.5799
1089917.177	2247.839	176293.474 10	166714.114
2720.254	26590.873 11	3720749.000	8851.000
343002.000 12	589847.114	1191.537	108186.694 $13$
741635.410	1455.125	139306.388 14	1737318.638
3872.761	217317.511 15	742729.823	1457.025
$139530.765 \ 16$	626300.382	1254.840	$115660.357 \ 17$
336773.195	752.062	56301.384 18	235645.061
576.449	35568.056 19	967377.985	1940.276
168528.505 20	117015.935	377.814	$12264.121 \ 21$
1043037.271	2130.175	173322.824 22	632681.339
1265.921	116968.583 23	141141.336	1348.809
18325.648 24	470614.911	984.485	83741.663 25
288367.255	668.003	46377.180 26	559580.389
1138.978	101981.399 27	75393.000	356.000
10697.000 28	159920.855	2355.938	24395.264 29
300293.679	688.714	$48822.340 \ 30$	270188.001
636.434	$42650.063 \ 31$	618291.000	3725.000
122534.000 32	1039628.194	2121.618	173106.800 33
138621.117	1213.652	17511.103 34	123095.000
381.000	12493.000 35	1674261.928	3714.494
213321.782 36	166286.458	2697.319	26452.653 37
846945.000	1638.000	160897.000 38	260400.575
619.438	40643.441 39	141344.209	1359.689
18391.218 40	194534.032	3288.113	33591.568

### 3 SFA Analysis - Cross-sectional

```
Iteration 0:    log likelihood = 13.090804
Iteration 1:    log likelihood = 13.630228
Iteration 2:    log likelihood = 13.648624
Iteration 3:    log likelihood = 13.648636
Iteration 4:    log likelihood = 13.648636
```

Stoc. frontier normal/half-normal model Number of obs = 40 Wald chi2(3) = 856.76 Log likelihood = 13.648636 Prob > chi2 = 0.0000

ly	Coef.	Std. Err.	Z	P> z	[95% Conf.	Interval]
llab	.2336621	.0574575	4.07	0.000	.1210476	.3462767
lk	.3521837	.0721876	4.88	0.000	.2106986	.4936689
lfuel	.324796	.0907284	3.58	0.000	.1469717	.5026204
_cons	6.151098	.3600706	17.08	0.000	5.445372	6.856823
/lnsig2v	-4.9063	.6854827	-7.16	0.000	-6.249821	-3.562778
/lnsig2u	-2.687113	.3809604	-7.05		-3.433781	-1.940444
sigma_v sigma_u sigma2 lambda	.0860222 .2609161 .075477 3.033125	.0294834 .0496994 .0235255 .0703788			.0439409 .1796238 .0293679 2.895185	.1684041 .3789989 .1215862 3.171065

Likelihood-ratio test of sigma\_u=0: chibar2(01) = 4.10 Prob>=chibar2 = 0.021

The results displayed here are derived from a Stochastic Frontier Analysis (SFA) using a cross-sectional method to examine efficiency. SFA is employed to determine production functions and gauge the technical efficiency of firms, industries, or other entities. In this instance, a normal/half-normal model is applied.

A Cobb-Douglas production function is estimated by the model:

$$log(y) = \beta 0 + \beta_1 * log(lab) + \beta_2 * log(k) + \beta_3 * log(fuel) + \epsilon$$

Here, ly denotes the logarithm of output, llab is the log of labor input, lk signifies the log of capital input, and lfuel represents the log of fuel input.  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  are coefficients to be calculated.  $\epsilon$  is the error term consisting of two components: a symmetric error term (v) indicating statistical noise and an asymmetric error term (u) signifying technical inefficiency.

The coefficients estimated are as follows:

 $\beta_1(\text{llab}) = 0.2336621$ : A 1% rise in labor input results in around a 0.23% increase in output, with other factors held constant.  $\beta_2(\text{lk}) = 0.3521837$ : A 1% rise in capital input results in around a 0.35% increase in output, with other factors held constant.  $\beta_3(\text{lfuel}) = 0.324796$ : A 1% rise in fuel input results in around a 0.32% increase in output, with other factors held constant.

The components of the error term are modeled in the following manner:

lnsig2v: The log of the variance of the symmetric error term () is estimated to be -4.91, while the log of the variance of the inefficiency error term is estimated to be -2.687113.

An interesting parameter is lambda, which takes the value 3.03, and it represents the ratio of the standard deviations of the inefficiency term  $(sigma_u)$  and the statistical noise term  $(sigma_v)$ . A greater lambda value suggests that the inefficiency component is more prevalent in the error term.

The likelihood-ratio test for  $sigma_u = 0$  checks the null hypothesis that there is no technical inefficiency in the model (i.e.,  $sigma_u$  equals zero). The p-value is 0.021, which is below the standard significance level of 0.05, and we thu reject the null hypothesis and conclude that there is evidence of technical inefficiency in the model.

In conclusion, this SFA cross-sectional study calculates the production function coefficients and assesses the technical inefficiency of decision-making units. The findings indicate that labor, capital, and fuel inputs all have a positive effect on output, and there is evidence of technical inefficiency in the sample.

In the following table, we showcase the calculated inefficiency as determined through the application of Cross-Sectional Stochastic Frontier Analysis (SFA). The values, denoted as ef\_cs7, reveal the degree of deviation from the optimal efficiency frontier for each DMU.

This first basic frontier analysis, where we do not take into account heteroskedasticity or other more complex distributions for the inefficiency term, allows us to get a first glimpse at the technical inefficiencies of the firms in the sample.

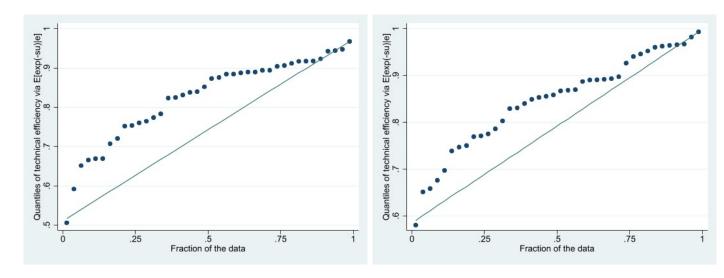
DMU 37 is the most efficient entity, as evidenced by its inefficiency value of 0.97, signifying a position in close proximity to the optimal efficiency frontier. In contrast, DMU 19 is the least efficient entity, displaying an inefficiency value of 0.51, indicating that it is relatively further from the optimal efficiency frontier when compared to other entities. The inefficiency values exhibit a considerable range, spanning from roughly 0.5 to 0.97, suggesting a diverse degree of efficiency among the entities. The mean technical inefficiency score is of .82, and using a quantile plot we can see that around twenty five percent of the sample has an efficiency score above 0.9.

Table 5: Estimated inefficiency - Cross-Sectional DFA  $\,$ 

$\mathbf{dmu}$	$ m ef\_cs7$	dmu	$ef_cs7$
1	0.8521737	21	0.9476904
2	0.8236679	22	0.9234803
3	0.8898154	23	0.893849
4	0.760581	24	0.8900046
5	0.8874036	25	0.9181713
6	0.8940104	26	0.6696653
7	0.8845149	27	0.7644562
8	0.7207142	28	0.9118798
9	0.8310308	29	0.9430065
10	0.6655822	30	0.7742042
11	0.9067556	31	0.9446261
12	0.8756911	32	0.9040266
13	0.8841907	33	0.591888
14	0.8250926	34	0.9171332
15	0.8398703	35	0.873402
16	0.7521638	36	0.7071065
17	0.6515611	37	0.9680015
18	0.753739	38	0.7831885
19	0.5057587	39	0.6693134
20	0.8379075	40	0.9169886

In this basic framework, we can test for the parameters of labour, capital and fuel in this log-log specification to have estimates which sum up to 1. This would test for the fact that we are in fact in the presence of a Cobb Douglas specification, and we reject the null hypothesis at the level five percent, which means that we are not in a the case of a Cobb Douglas specification.

We can then expand our analysis to the case of heteroskedasticity. We parameterize sequentially the error term  $\nu$  to be a function of all the different combinations of the logarithm of inputs. The logarithm of fuel is always significant at the 5% level, while the logarithm of capital is significant considered alone but not anymore when we combine it with the other controls. We will thus use a specification of  $\nu$  as a function of the logarithm of the fuel input in the following analysis. We reproduce the same analysis for the u component. No combination of the controls significantly explains variation in the inefficiency component u. We estimate again the efficiency scores taking into account the heteroskedasticity in  $\nu$ . The distribution of those scores does seem to change with this corrected heteroskedasticity, and the scores seems shifted upwards. Finally, we try to estimate other distributions for the inefficiency term u. We first try a specification where we u is parameterized as an exponential distribution. Although the values of the inefficiency score does change, the ranking of the DMU's seems to be unchanged. We find the same results for the truncated normal parameterization of the inefficiency term.



## 4 SFA Analysis - Panel Data

In this second SFA estimation, we are now interested in using the full panel of data at hand in order to exploit the additional variation obtained compared to the previous cross-sectional case. The inefficiency term is parameterized to follow a half-normal distribution in this first estimation. The coefficients are reported in the following table, where the first specification concerns time invarying efficiency scores. All our inputs are positively correlated with the logarithm of the output, with labour and capital having similar returns while the fuel input is much more determining in the production of electricity in this specification. We compute once again the basic specification of the log-log production function, and estimate the inefficiency scores making first the assumption that they are time invariant. We can observe that their distribution is similar to the one we found previously in the analog framework applied to the cross-section sample.

Now, we turn to the analysis of the inefficiency score in a time-varying framework. We can observe that the coefficients associated with the inputs, summarized in column 2 of the following estimation results, change quite substantially. Labour seems to be much less productive, and capital almost doubled its productivity. On the other hand, fuel seems to keep a constant productivity in both specifications. Turning to the analysis of the time-varying inefficiency scores, we observe throughout that there is not much variation for firms throughout the duration of the panel of their inefficiency scores, as the mean individual standard deviation over the six years is of about 0.02. We observe heterogeneity still over the years and between individuals, as the between standard variation is of about 0.12 per year, where the mean efficiency score is of about 0.72 every year.

	(1)	(2)
	ly	ly
ly		
llab	0.121***	0.0839**
	(4.90)	(2.77)
lk	0.174***	0.205***
	(4.95)	(6.69)
lfuel	0.564***	0.521***
	(14.88)	(14.85)
_cons	6.683***	7.085***
	(27.26)	(27.61)
lnsigma2		
_cons	-2.219***	-2.184***
	(-8.69)	(-7.42)
lgtgamma		
_cons	2.977***	3.264***
	(9.96)	(9.51)
mu		
_cons	0	0
	(.)	(.)
eta		
_cons		0.0477***
		(6.38)
N	240	240

t statistics in parentheses

<sup>\*</sup> p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

