챕터5 정리

Ch5: KARNAUGH MAPS

5.1 Minimum Forms of Switching Functions

· 기본적인 공식

- 1: Combine terms by using XY + X'Y = X (Dual: (X + Y)(X' + Y) = Y)

  Do this repeatedly to eliminates as many literals as possible.

  A given term may be used more than once because X + X = X

- 2: Eliminate redundant terms by using the consensus theorems.

  XY + X'Z + YZ = XY + X'Z

  직관적으로 와 닿지 않는 이러한 공식들을 카르노 맵을 이용하여 알 수 있다.

· Example (최소 항 찾기: Algebraic Simplification)

- 1: Find a minimum sum-of-products

$$F(a,b,c) = \sum m(0,1,2,5,6,7)$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + b'c + bc' + ab$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + bc' + ac$$

아래가 옳은 표현이지만 어떤 표현을 사용하냐에 따라서 불필요한 항이 생길 수도 있음

- 2: Find a minimum product-of-sums

$$(A+B'+C+D')(A+B'+C'+D')(A+B'+C'+D)(A'+B'+C'+D)(A+B+C'+D)(A'+B+C'+D)$$

$$= (A+B'+D')\quad(A+B'+C')\quad(B'+C'+D)\quad(B+C'+D)$$

$$= (A+B'+D')\quad\underbrace{(A+B'+C')}_{\text{Eliminate by consensus}}\quad(C'+D)$$

$$= (A+B'+D')(C'+D)$$

5.2 Two- and Three-Variable Karnaugh Maps

· Karnaugh Maps의 필요성

- Simplification using algebraic rules can be impossible, difficult, tedious

→ 기존 방식으로는 어렵고 복잡하며 경우에 따라서 불가능에 가까운 경우도 존재함

이러한 문제를 해결하기 위해 카르노 맵이 나옴

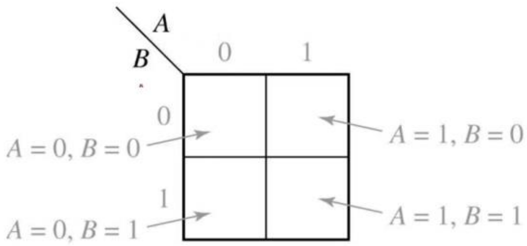
- Two-dimensional truth-table.

→ 카르노 맵은 2차원 진실 표임

- Karnaugh maps can be used up to 6 variables

→ 통상적으로 6개까지 처리가 가능함

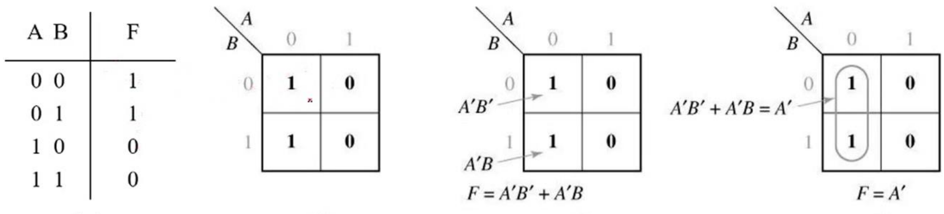
· Two-Variable Karnaugh Maps: 가장 단순한 버전



→ 통상적인 truth table을 이러한 형태로 바꾼 것이 카르노 맵임

하나하나의 칸이 전부 minterm임

- 예시

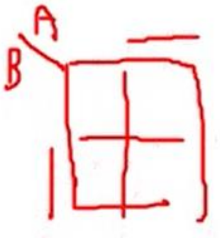


(a) (b) (c) (d)

두 개의 이웃한 것은 결합하여(grouping) 모은 후 관찰하면

→ 묶어도 불변하는 항만 살아남고 바뀌는 항은 소거됨

- 만일 카르노 맵이 이런 모양이라면..

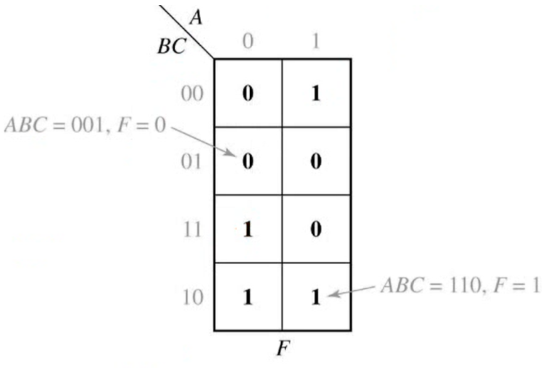


→ 이렇게 바 모양이 있는 형태라면 바가 있는 부분이 각 변수의 1인 영역임

· Truth Table and Karnaugh Map for Three-Variable Function

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

(a)



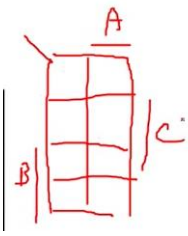
(b)

→ 예시. 통상적인 순서와는 다르게 11 후에 10이 나온다. 순서가 이렇다는 거, 주의하자

이웃한 minterm(square) 사이에 결합이 가능하도록 하기 위함이다. 하나만 바뀌게끔

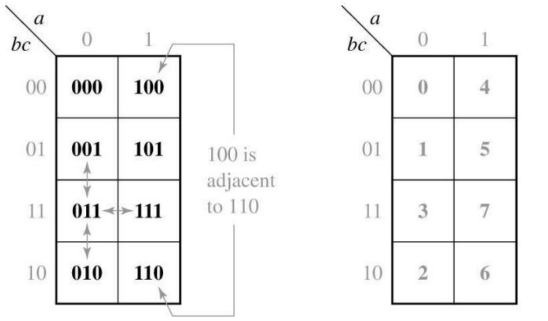
(A = 0일 때 01에서 10으로가면 둘 다 바뀌느라 결합 가능성 없음, 01에서 11로 가면 하나만)

한쪽 끝과 반대쪽 끝도 결합 가능성이 있다. 인접했다고 봐야한다



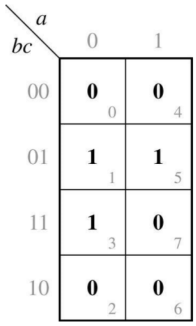
→ 이렇게 표현하기도 한다

- Location of Minterms on a Three-Variable Karnaugh Map



(a) Binary notation (b) Decimal notation

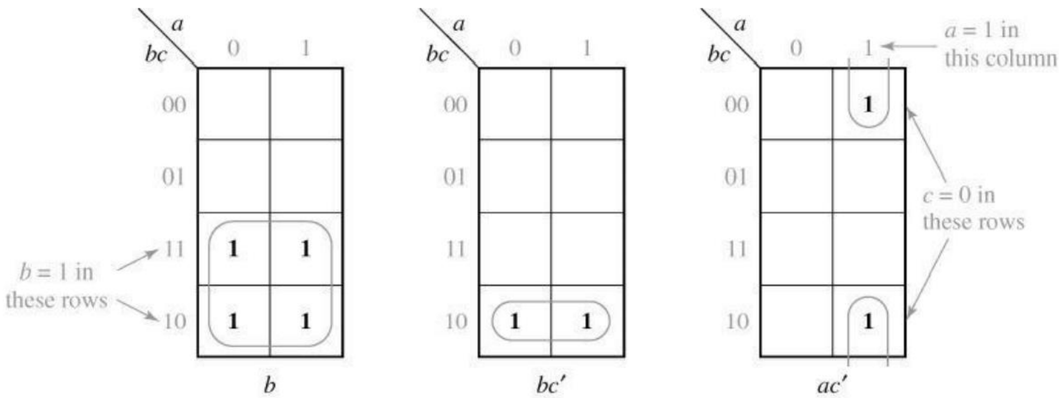
- Karnaugh Map of F(a, b, c) = $\sum m(1, 3, 5) = \prod M(0, 2, 4, 6, 7)$



$F(a, b, c) = m_1 + m_3 + m_5 = M_0M_2M_4M_6M_7$

밑부분에 minterm 숫자를 적어두면 편하다 – 1은 min, 0은 max

- Karnaugh Maps for Product Terms



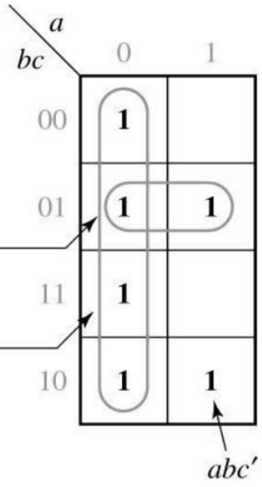
Grouping을 통해 왔다갔다하는 부분을 전부 삭제하고 고정된 부분만 살리며 결합한다

2개가 그루핑이 되면 변수 1개가 탈락함. 4개는 변수 2개 탈락 → 2의 배수배로 그루핑

- 반대의 경우: 함수가 주어졌을 때 카르노 맵으로 나타내기

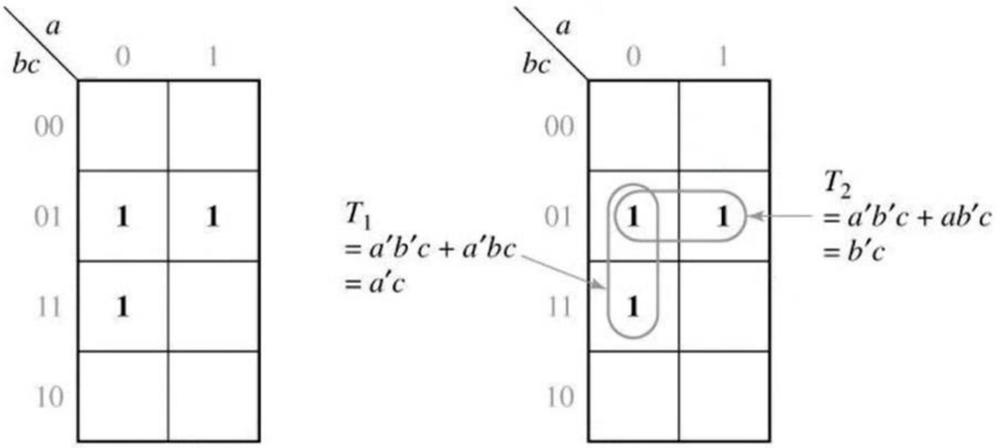
$f(a,b,c) = abc' + b'c + a'$

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1's in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1's in all the squares of the $a = 0$ column of the map. (Note: Since there already is a 1 in the $abc = 001$ square, we do not have to place a second 1 there because $x + x = x$.)



→ 쉽죠?

- Simplification of a Three-Variable Function



$F = \Sigma m\ (1, 3, 5)$

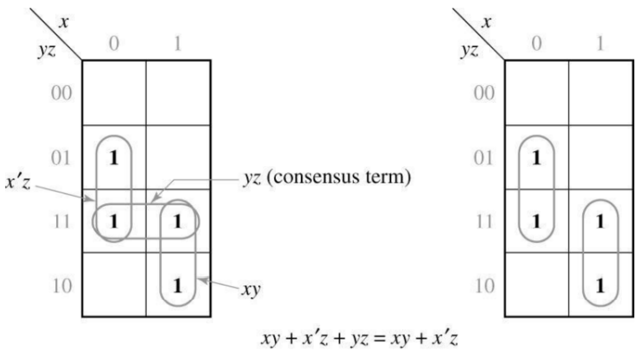
(a) Plot of minterms

$F = a'c + b'c$

(b) Simplified form of F

$F = T_1 + T_2 = a'c + b'c$

- Karnaugh Maps Which Illustrate the Consensus Theorem



$xy + x'z + yz = xy + x'z$

↑

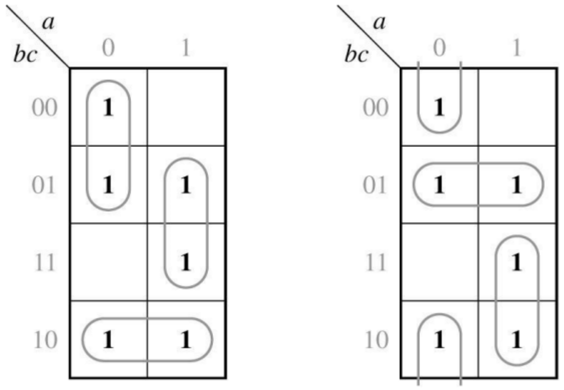
Consensus term is redundant

→ 카르노 그는 천재인가?

가운데 그루핑은 쓸모가 없음 양 쪽으로도 충분히 다 커버가 되니까

- Function with Two Minimal Forms

$$F = \sum m(0,1,2,5,6,7)$$



$F = a'b' + bc' + ac$ $\qquad$ $F = a'c' + b'c + ab$

최적의 그루핑을 통해 간소화했는데도 경우의 수가 여러 가지일 때 → 둘 다 맞음

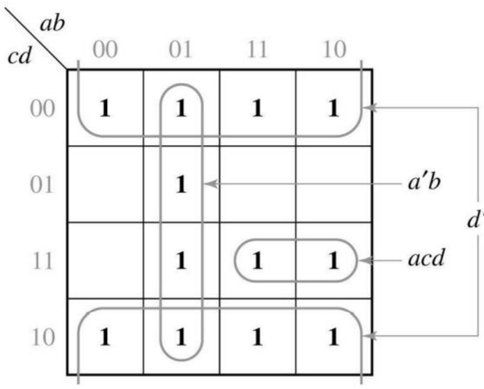
5.3 Four-Variable Karnaugh Maps

· Location of Minterms on Four-Variable Karnaugh Map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

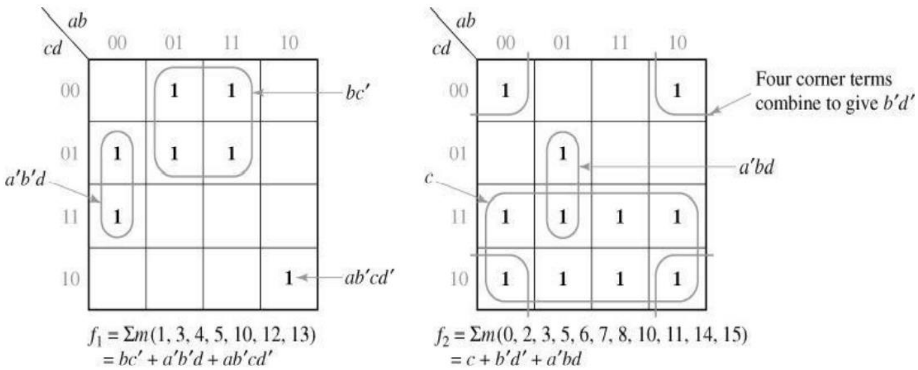
→ 순서에 유의하자! 00 01 11 10순이다

- Plot of acd + a’b + d’



→ f(a, b, c, d) = acd + a'b + d'

· Simplification of Four-Variable Functions

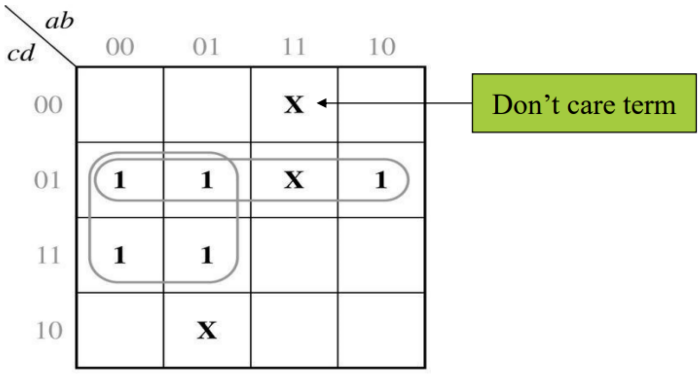


$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$
(a)

$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$
(b)

→ 네 귀퉁이도 그루핑이 가능하다!

· Simplification of an Incompletely Specified Function

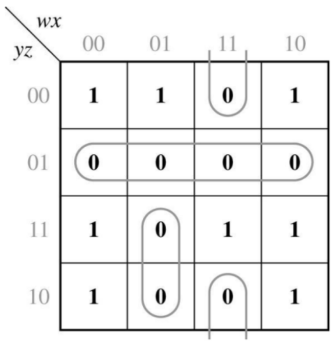


$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

→ 카르노 그는 진짜 신이 맞는 듯하다 정말 쉽게 표현 가능함

· 만약에 maxterm expansion을 통한 POS를 얻고 싶다면

Figure 5-14



1's of f
$$f = x'z' + wyz + w'y'z' + x'y$$

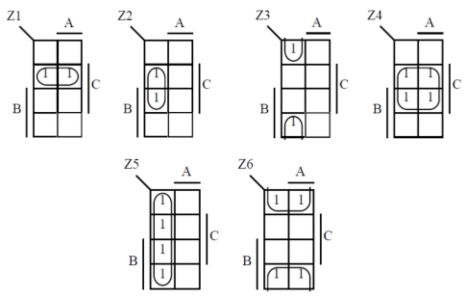
0's of f
$$f' = y'z + wxz' + w'xy$$

$$f = (y + z')(w' + x' + z)(w + x' + y')$$
minimum product of sums for f

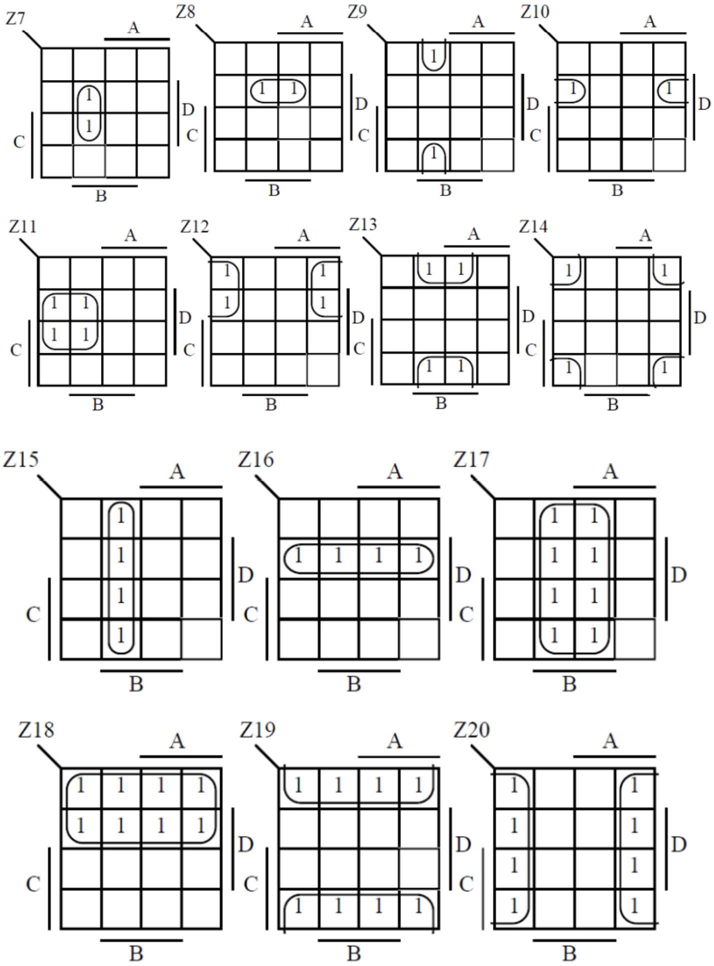
→ 0을 기준으로 그루핑한 후 f'의 SOF를 얻은 후 드모르간 취해준다

· Basic Karnaugh Map Groupings

- For Three-Variable Maps



- For Four-Variable Maps



5.4 Determination of Minimum Expressions

· 카르노 맵을 이용한 단순화 과정에서의 좀 더 체계화

- Implicants of F

  : Any single '1' or any group of "1's which can be combined together on a Map → each grouping of any size is thus an implicant

  싱글 1이거나 결합된 덩어리 하나하나를 말함

- Prime Implicants of F

  : A product term if it cannot be combined with other terms to eliminate variable → a largest possible grouping

  Implicants 중에서 가장 큰 크기의 그루핑

- Essential Prime Implicants of F (EPI)

  : A prime implicant that is the ONLY cover for some 1's on the map (essential is relative to a particular minterm) → always look for E.P.I. first in simplication

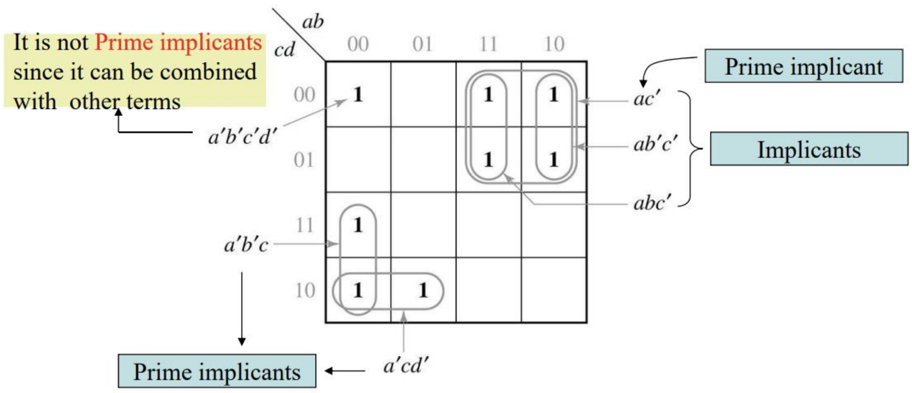
  Prime Implicants 중에서 특정한 일을 커버 가능한 유일한 Prime Implicants

- Simplification Procedure

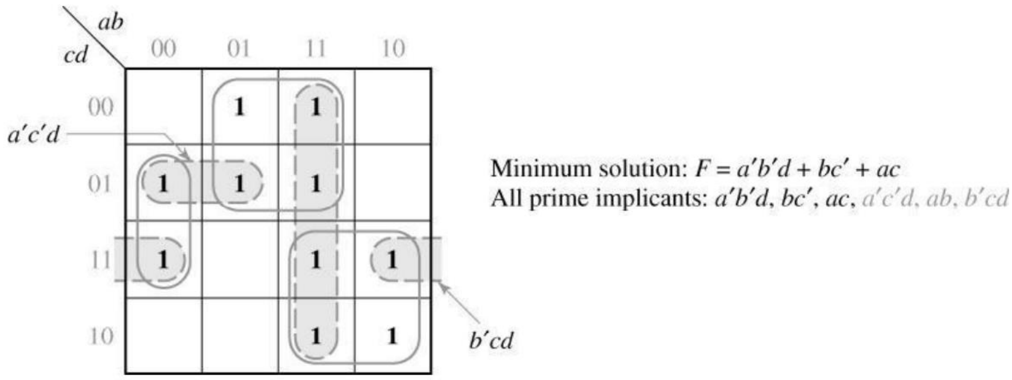
  Step 1) Identify those groupings that are maximal

  Step 2) Use the fewest possible number of maximal groupings

- 그림 예시



- Determination of All Prime Implicants



점선, 실선 모두 다 prime implicant라는 점에서는 맞음.

하지만 최소의 표현식을 얻는다는 관점에서 보면 중앙 상단, 우측 하단 실선이 EPI임

4번과 15번을 커버할 수 있는 유일한 수단이기 때문

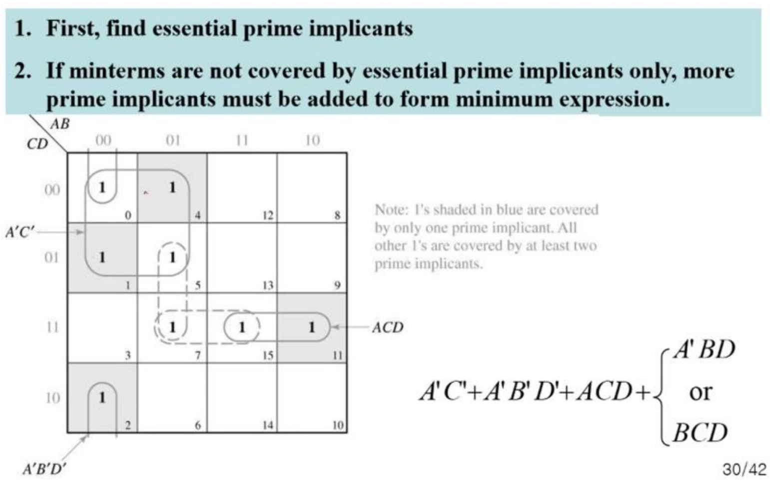
EPI를 먼저 취한 후 커버 안된 그루핑을 만든다

Because all of the prime implicants of a function are generally not needed in forming the minimum sum of products, selecting prime implicants is needed.

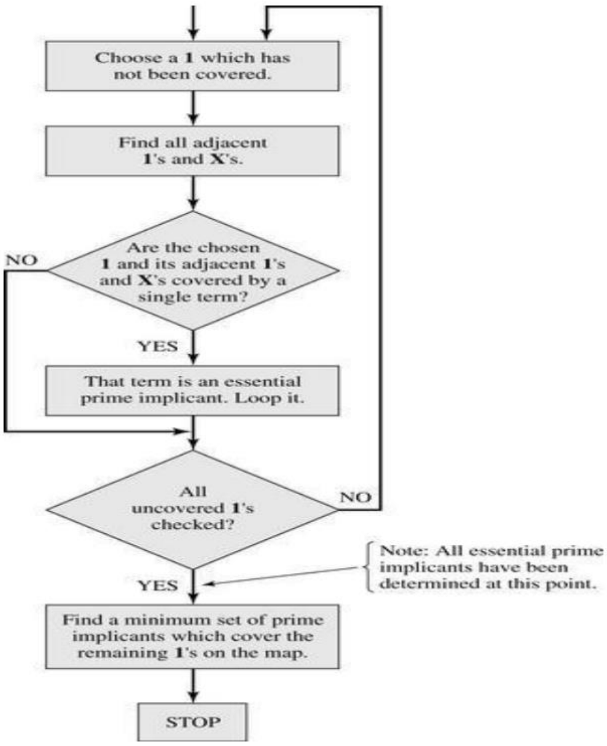
- 절차

1. First, find essential prime implicants

2. If minterms are not covered by essential prime implicants only, more prime implicants must be added to form minimum expression.

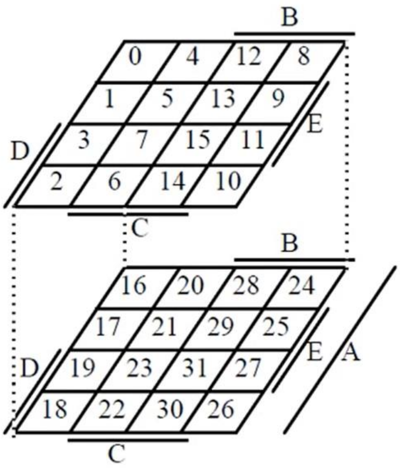


· Flowchart for Determining a Minimum Sum of Products Using a Karnaugh Map

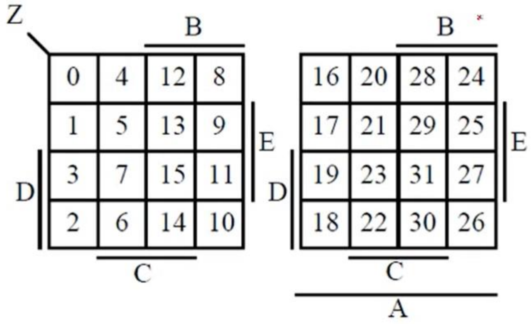

Choose a 1 which has not been covered.
Find all adjacent 1's and X's.
Are the chosen 1 and its adjacent 1's and X's covered by a single term?
NO
YES
That term is an essential prime implicant. Loop it.
All uncovered 1's checked?
NO
YES
Note: All essential prime implicants have been determined at this point.
Find a minimum set of prime implicants which cover the remaining 1's on the map.
STOP

5.5 Five-Variable Karnaugh Maps

· Five-Variable Karnaugh Maps



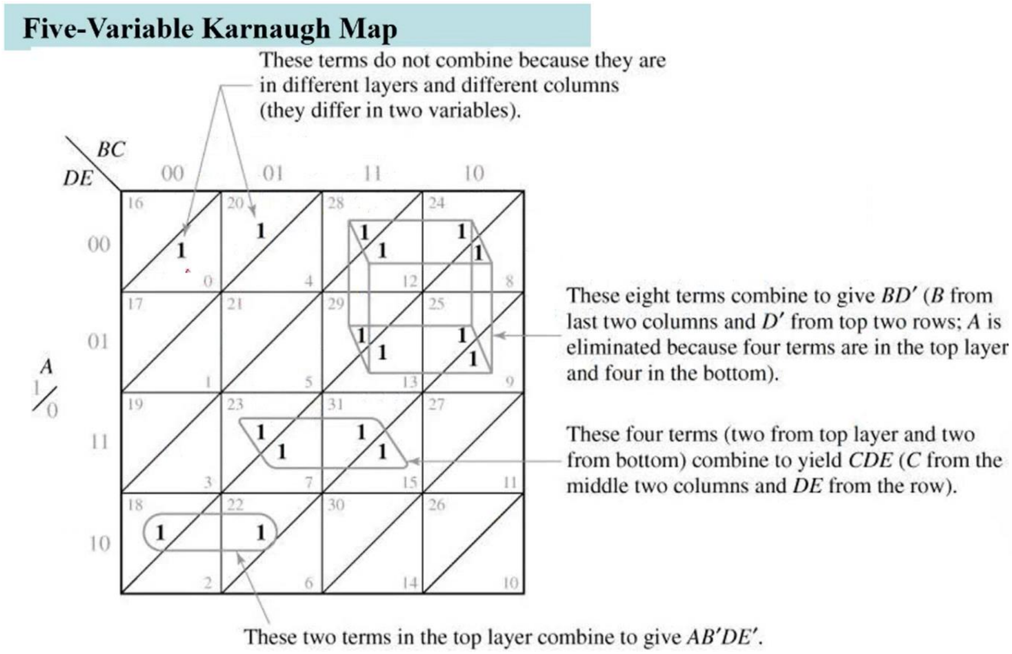
Five-Variable Map Structure



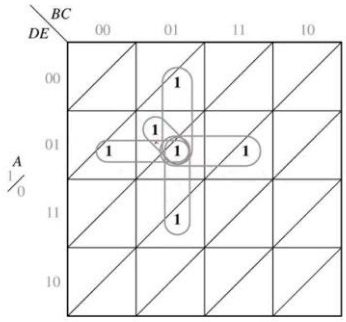
Alternate Version of Five-Variable Map

→ 36개의 minterm. Top/Bottom layer 표기 혹은 좌우 표기 방식

인접끼리, 좌우끼리, 위아래끼리, 귀퉁이끼리 그루핑 가능함 (8과 16도, 8과 24도 가능)

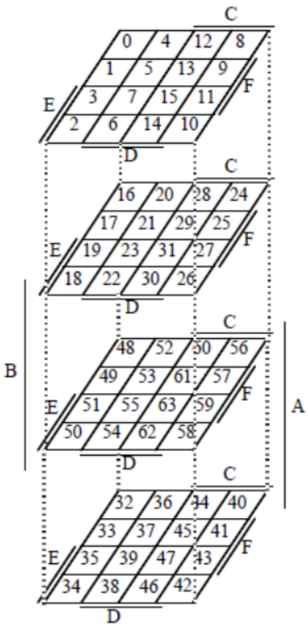


→ 이런 식도 가능하다. EPI 구하는 것을 제일 중요하게 생각하자

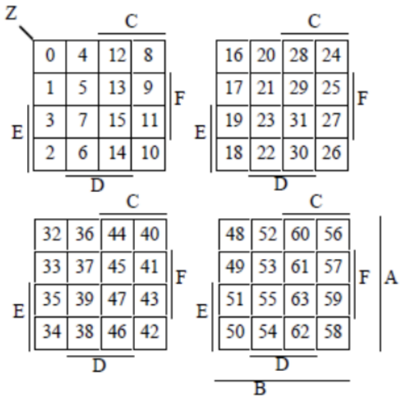


→ 상하좌우 매핑 가능

· Six-Variable Karnaugh Map



Six Variable Map Structure



Alternate Version of Six-Variable Map

→ minterm이 64개, 배치 순서 조심합시다

5.6 Other Uses of Karnaugh Maps

· Minterm, Maxterm expansion을 할 수 있다

minterm expansion of f is $f = \sum m(0,2,3,4,8,10,11,15)$

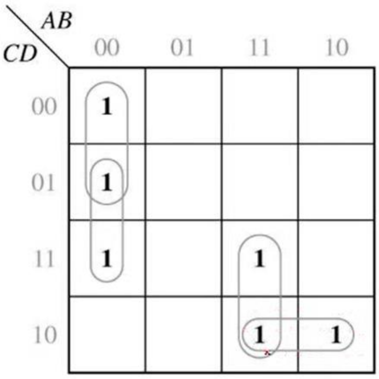
maxterm expansion of f is $f = \prod M(1,5,6,7,9,12,13,14)$

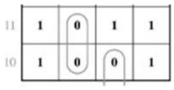
same

- minterm: 1 초점

- maxterm: 0 초점

· 묶어서 표현 가능하다





$F = A'B'(C' + D) + AC(B + D')$

· 고난도의 simplification: consensus

Figure 5-26

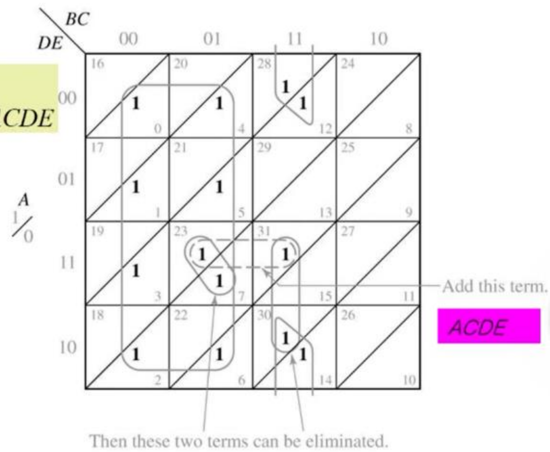
$F = ABCD + B'CDE + A'B' + BCE'$

Using the consensus theorem :

$F = ABCD + B'CDE + A'B' + BCE' + ACDE$

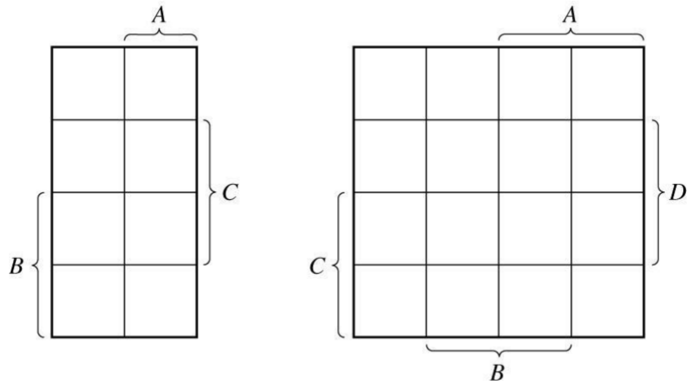
minimum solution :

$F = A'B' + BCE' + ACDE$

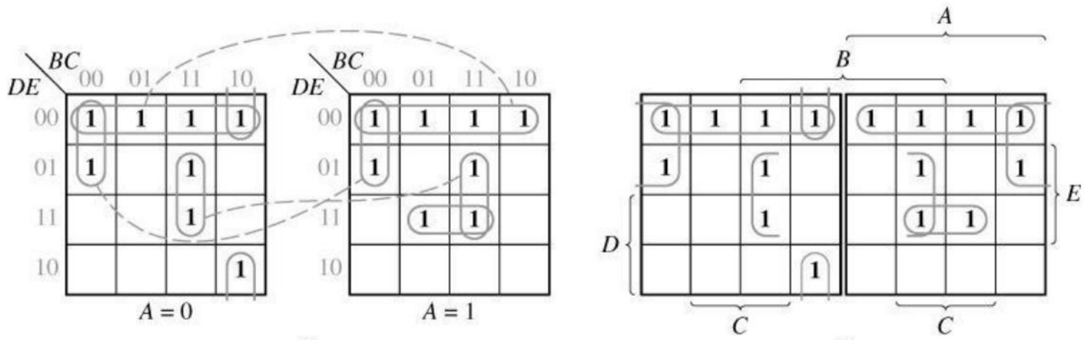


5.7 Other Forms of Karnaugh Maps Programmed Exercises Problems

· Veitch Diagrams



· Other Forms of Five-Variable Karnaugh Maps



(a) (b)

$F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE$

→ 그루핑 표기의 방식: 선을 연결함(왼쪽)

→ 오른쪽: 뒤집어 놓은 것처럼 minterm 배치(거울처럼), 자주 사용하진 않음