Ch.3: Boolean Algebra (Continued)

3.1 Multiplying Out and Factoring Expressions

- 2강 정리

SOP form 만들 위해 → Multiplying out using distributive laws

$$\begin{cases} X(Y+Z) = XY + XZ \\ (X+Y)(X+Z) = X + YZ \end{cases}$$

- Theorem for multiplying out.

$(X+Y)(X'+Z) = XZ + X'Y$

→ X 값 둘 다 넣어봤을 때 전부 같음 ∴ 성립.

- Theorem for factoring.

$AB + A'C = (A+C)(A'+B)$ → 위의 식 역이용.

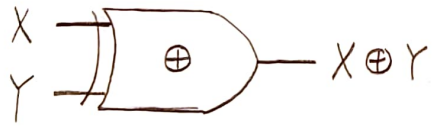
- Multiplying out 방법.

1. Theorem for multiplying out.
2. Distributive laws. → Redundant terms 발생 가능성 존재.

3.2 Exclusive-OR and Equivalence Operations

- Exclusive-OR

$0 \oplus 0 = 0$, $0 \oplus 1 = 1$, $1 \oplus 0 = 1$, $1 \oplus 1 = 0$



$X \oplus Y = X'Y + XY'$

- Theorems for Exclusive-OR

$X \oplus 0 = X$. $X \oplus 1 = X'$. $X \oplus X = 0$, $X \oplus X' = 1$

$X \oplus Y = Y \oplus X$ (Commutative law)

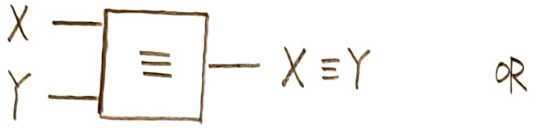
$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$ (associative law)

$X(Y \oplus Z) = XY \oplus XZ$ (distributive law)

$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$

- Equivalence operation (Exclusive - NOR)

$(0 \equiv 0) = 1$, $(0 \equiv 1) = 0$, $(1 \equiv 0) = 0$, $(1 \equiv 1) = 1$



OR

$(X \equiv Y) = XY + X'Y'$

- Useful theorem

$(XY' + X'Y)' = XY + X'Y'$

$XY' + X'Y$: $X \oplus Y$, $XY + X'Y'$: $X \equiv Y$

3.3 The Consensus Theorem

- Consensus Theorem

$XY + X'Z + YZ = XY + X'Z$

- Proof

$XY + X'Z + YZ = XY + X'Z + (X + X')YZ$

$= (XY + XYZ) + (X'Z + X'YZ) = XY(1 + Z) + X'Z(1 + Y) = XY + X'Z$

- Dual form of Consensus theorem

$(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$

3.4 Algebraic Simplification of Switching Expressions.

- Combining terms

$XY + XY' = X$

+) Adding terms using $X + X = X$

- Eliminating terms

$X + XY = X$

$XY + X'Z + YZ = XY + X'Z$

- Eliminating literals

$X + X'Y = X + Y$

- Adding redundant terms.

Adding xx', multiplying $(x + x')$, adding yz to $(xy + x'z)$, adding xy to x ~~등등~~

3.5 Proving Validity of an Equation

- Proving an equation valid
  1. Truth Table 써서 양쪽 다 나오기 → tedious, not elegant
  2. 한 쪽 식을 여러 정리를 사용해서 조작 후 다른 쪽 식이 되도록 유도
  3. 양쪽 식을 다 풀어서 같음을 보임
  4. 양쪽 식에 동일한 계산을 적용해 비교 (예: 양쪽 inverse, 1 or 0 대하기)
- Warning : Some of Boolean Algebra are not true for ordinary algebra

ex 1) If $x+y = x+z$, then $y = z$. } True in ordinary algebra,
ex 2) If $xy = yz$, then $y = z$ } Not true in Boolean algebra

* But the Converses are True.

ex 1-1) If $y = z$, then $x+y = y+z$
ex 2-1) If $y = z$, then $xy = yz$

→ Reason : Subtraction and Division is not defined in Boolean Algebra