챕터3 정리

Ch3: BOOLEAN ALGEBRA (continued)

3.1 Multiplying Out and Factoring Expressions

· Multiplying Out and Factoring Expressions

- 2장 정리

  SOF 형태를 얻기 위해 전개를 해서 단순화하면 얻을 수 있음

  하지만 이것을 더욱 효율적으로 하는 방법이 있다

  $X(Y + Z) = XY + XZ$

  $(X + Y)(X + Z) = X + YZ$

  이 식을 활용하여 단순한 결과값을 빠르게 얻음.

- Theorem for multiplying out

  $(X + Y)(X' + Z) = XZ + X'Y$

  $X = 0$이면 위의 식 (좌변) = $Y(1 + Z) = Y * 1 = Y$, (우변) = $0 + Y = Y$

  $X = 1$이면 위의 식 (좌변) = $(1 + Y)Z = 1 * Z = Z$, (좌변) = $Z + 0 = Z$

  따라서 두 경우 모두 성립하므로 성립함.

- Theorem for factoring

  위의 식을 역이용하여

  $AB + A'C = (A + C)(A' + B)$

- 식의 응용

  i) Multiplying out using Theorem: $(Q + AB')(C'D + Q') = QC'D + Q'AB'$

  ii) Multiplying out using distributive laws:

  $(Q + AB')(C'D + Q') = QC'D + QQ' + AB'C'D + AB'Q'$

  $QQ' + AB'C'D$: Redundant terms, 즉 불필요한 부분

  $AB'C'D$가 불필요한 이유는 나중에 기술

- Multiplying out의 방식

  1) distributive laws

2) $(X + Y)(X' + Z) = XZ + X'Y$

3.2 Exclusive-OR and Equivalence Operations

· Exclusive-OR

- Operations

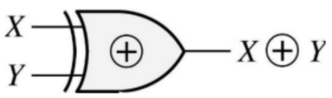
$0 \oplus 0 = 0,\ 0 \oplus 1 = 1,\ 1 \oplus 0 = 1,\ 1 \oplus 1 = 0$

- Truth Table

XY	$X \oplus Y$
0 0	0
0 1	1
1 0	1
1 1	0

엇갈릴 때만 1, 아닐땐 0

- Symbol



$$\boxed{X \oplus Y = X'Y + XY'}$$

- Theorems for Exclusive-OR

$X \oplus 0 = X,\ X \oplus 1 = X',\ X \oplus X = 0,\ X \oplus X' = 1,$

$X \oplus Y = Y \oplus X$ (commutative law)

$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$ (associative law)

$X(Y \oplus Z) = XY \oplus XZ$ (distributive law)

$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$

· Exclusive-NOR

- Operations

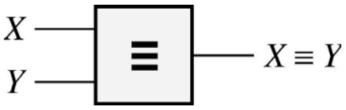
$(0 \equiv 0) = 1,\ (0 \equiv 1) = 0,\ (1 \equiv 0) = 0,\ (1 \equiv 1) = 1$

- Truth Table

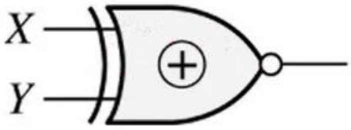
XY	$X \equiv Y$
0 0	1
0 1	0
1 0	0
1 1	1

같을 때만 1, 다르면 0

- Symbol



$$\boxed{(X \equiv Y) = XY + X'Y'}$$



$$(X \oplus Y)' = (X \equiv Y)$$

- Useful Theorem

(XY'+ X'Y)' = XY + X'Y'

3.3 The Consensus Theorem

· Consensus Theorem

- Theorem

XY + X'Z + YZ = XY + X'Z

- Proof

$$
\begin{aligned}
XY + X'Z + YZ &= XY + X'Z + (X + X')YZ \\
&= (XY + XYZ) + (X'Z + X'YZ) \\
&= XY(1+Z) + X'Z(1+Y) = XY + X'Z
\end{aligned}
$$

- Dual form of consensus theorem

(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)

3.4 Algebraic Simplification of Switching Expressions

· Combining terms

$XY + XY' = X$

· Eliminating terms

$X + XY = X$, $XY + X'Z + YZ = XY + X'Z$

· Eliminating literals

$X + X'Y = X + Y$

· Adding redundant terms

Adding xx', multiplying $(x + x')$, adding yz to $xy + x'z$, adding xy to x, etc…

3.5 Proving the Validity of an Equation Programmed Exercises Problems

· Proving an equation valid

1) Construct a truth table and evaluate both sides – tedious, not elegant method

→ Truth Table 써가며 비교하기

2) Manipulate one side by applying theorems until it is the same as the other side

→ 한 쪽 식을 여러가지 정리를 적용해서 다른 식이 되도록 유도

3) Reduce both sides of the equation independently

→ 양 쪽 식 다 줄여서 같음을 보이는 방법

4) Apply same operation in both sides (complement both sides, add 1 or 0)

→ 양 쪽 식에 동일한 계산을 적용해 비교

· Some of Boolean Algebra are not true for ordinary algebra

Example: If $x+y=x+z$, then $y=z$ **True in ordinary algebra**

$1+0=1+1$ but $0 \neq 1$ **Not True in Boolean algebra**

Example: If $xy=xz$, then $y=z$ **True in ordinary algebra** **Not True in Boolean algebra**

- But the converses are True

Example: If $y=z$, then $x+\mathrm{y}=\mathrm{x}+\mathrm{z}$ **True in ordinary algebra**

If $y=z$, then $x\mathrm{y}=\mathrm{xz}$ **True in Boolean algebra**

- Reason: Subtraction and Division is not defined in Boolean Algebra