Conditional Probability and Bayes Theorem

PRESENTED BY

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Contents

Learning Objectives

At the end of this lesson, you should be able to:

- Explain the concept of conditional probability
- Solve problems using Bayes' theorem

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Definition

Let A and B be any two events.

The conditional probability of the event A given that the event B has already occurred is:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \ P(B) > 0$$

Where,

$P(A \cap B)$ = Probability of occurrence of the events A and B

$P(A|B)$ = Probability of the event A given that the event B has already occurred (conditional probability)

$P(B)$ = Probability of the of event B and should always be greater than 0

Conditional Probabilities (contd.)

Symbolical representation

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B)P(A|B) = P(A \cap B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A)P(B|A) = P(A \cap B)$$

$$\therefore\ P(B|A) = P(A|B) \Rightarrow P(A)P(B|A) = P(B)P(A|B)$$

Dependence

Characteristics of dependent

- Result of one event **affects** the result of another

- Draw without replacement

Rule:

$$P(A \text{ and } B) = P(A) - P(B|A)$$

Characteristics of independent

- Result of one event **does not affect** the result of another

- Draw with replacement

Rule:

$$P(A \text{ and } B) = P(A) \,.\, P(B|A)$$

Q

What is the probability that a person selected has p(A) = blood group B given that he is p(B) = male?

Blood Group	Male	Female	Total
O	20	20	40
A	17	18	35
B	8	7	15
AB	5	5	10
Total	**50**	**50**	**100**

Conditional Probabilities: Example

A

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(8/100)}{(50/100)} = 0.16$$

Blood Group	Male	Female	Total
O	20	20	40
A	17	18	35
B	8	7	15
AB	5	5	10
Total	**50**	**50**	**100**

Unconditional Probability

If A and B are two independent events

Then $P(A \cap B) = P(A)P(B)$

We have:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(B)P(A|B) = P(B)P(A)$$

If A and B are independent then the conditional probability P(A B) will become unconditional probability P(A).

Bayes' Theorem

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Bayesian interpretation is an application of **conditional** probabilities.

Conditions:

- Sample space has mutually exclusive events: $\{A_1, A_2, \ldots A_k\}$
- Another event B exists with non-zero probability

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Purpose of Bayes' Theorem

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Bayes' theorem is most commonly used to estimate the state of a hidden, causal variable A based on the measured state of an observable variable B.

Bayes' equation

Likelihood

Prior

$$\therefore\ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

Posterior

Evidence

Purpose of Bayes' Theorem (contd.)

The posterior $P(A|B)$ is often difficult to estimate directly, reasonable models of the likelihood $P(B|A)$ can often be formed. This is typical because A is causal on B.

Thus, Bayes' theorem provides a means for estimating the posterior probability of the causal variable A based on observations B.

Calculating the Evidence

To calculate the evidence P(B) in Bayes' equation, we typically have to marginalise overall possible states of the causal variable A.

$$\therefore\ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \dots P(A_k \cap B)$$

$$= P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \dots P(B|A_k)P(A_k)$$

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Q

You are going to camp on a mountain tomorrow. It usually snows heavily only about five days in December.

There is heavy snow forecast for tomorrow. However, the weather report is known to come with some inaccuracy. On 90% of the days with heavy snow, there was a forecast of heavy snow. On 10% of the days without heavy snow, there was a forecast of heavy snow. You don't have a tent that can withstand heavy snow.

How likely are you to get stranded?

Probability of heavy snow	P(H)	5/31	0.16129
Probability of no snow	P(N)	26/31	0.83871
Probability of weatherman predicting heavy snow and it actually snows heavily	P (W | H)	90%	0.9
Probability of weatherman predicting heavy snow and it does not snow	P (W | N)	10%	0.1
Probability of heavy snow tomorrow	P (H | W)	?	?

Bayes' Theorem: Example I (contd.)

A

$$P(H \mid W) = \frac{P(W \mid H) * P(H)}{P(W \mid H) * P(H) + P(W \mid N) * P(N)}$$

$$= 63.38\%$$

BAD LUCK!

Probability of heavy snow	P(H)	5/31	0.16129
Probability of no snow	P(N)	26/31	0.83871
Probability of weatherman predicting heavy snow and it actually snows heavily	P (W | H)	90%	0.9
Probability of weatherman predicting heavy snow and it does not snow	P (W | N)	10%	0.1
Probability of heavy snow tomorrow	P (H | W)	?	?

Bayes' Theorem: Example II

Q

Four technicians regularly make repairs when breakdowns occur on an automated production line.

Name of technician	Breakdowns serviced (%)	Times of incomplete repairs
Janet	20%	1 time in 20
Tom	60%	1 time in 10
Georgia	15%	1 time in 10
Peter	5%	1 time in 20

For a subsequent problem with the production line, it was diagnosed as being due to an initial repair that was incomplete.

What is the probability that this initial repair was made by Janet?

Bayes' Theorem: Example II (contd.)

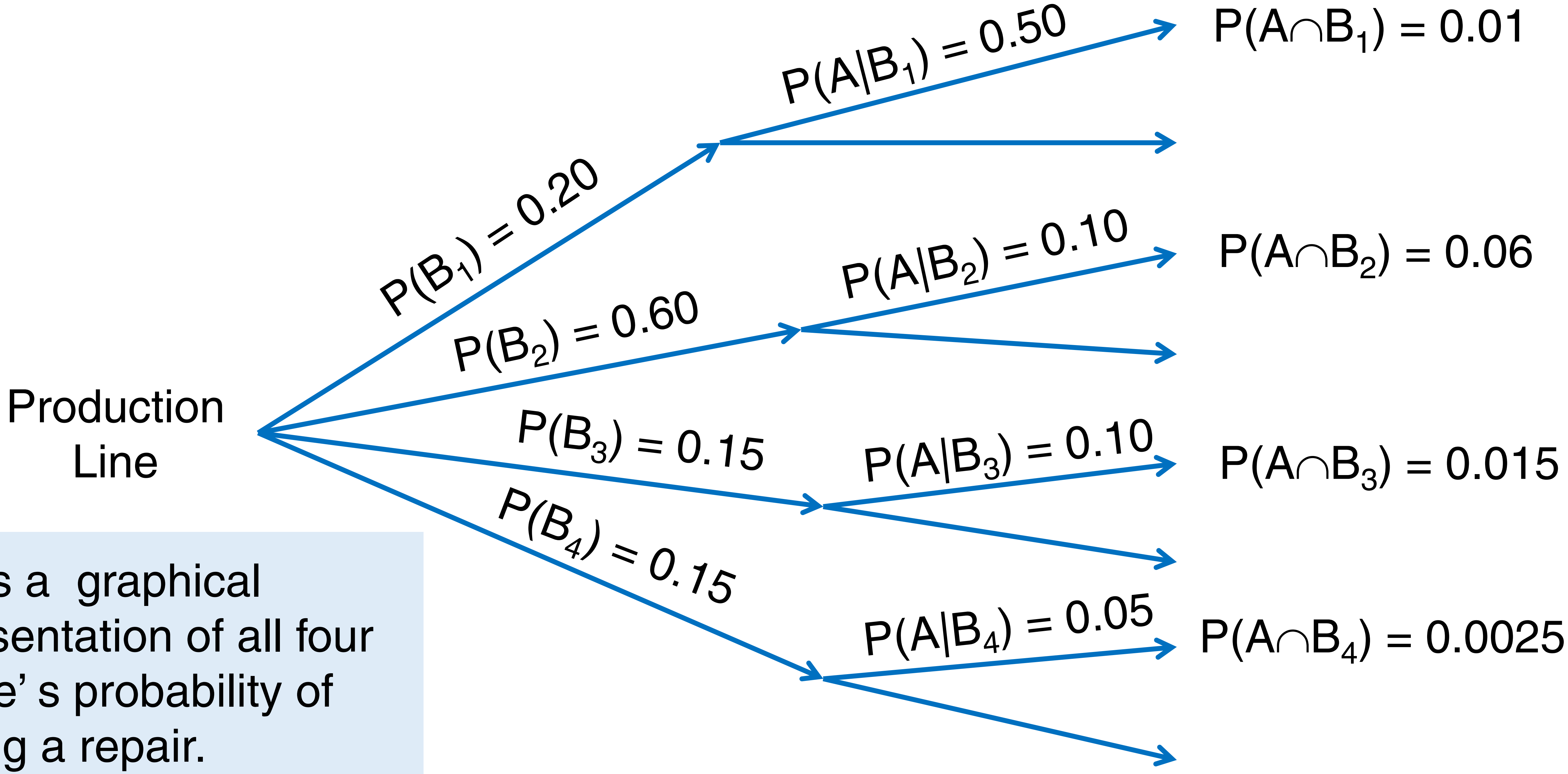
A

Let,

Event	Description
A	The event that the initial repair was incomplete
B_1	An event that the initial repair was made by Janet
B_2	An event that the initial repair was made by Tom
B_3	An event that the initial repair was made by Georgia
B_4	An event that the initial repair was made by Peter

Bayes' Theorem: Example II (contd.)

A



This is a graphical representation of all four people's probability of making a repair.

A

Understanding the formula:

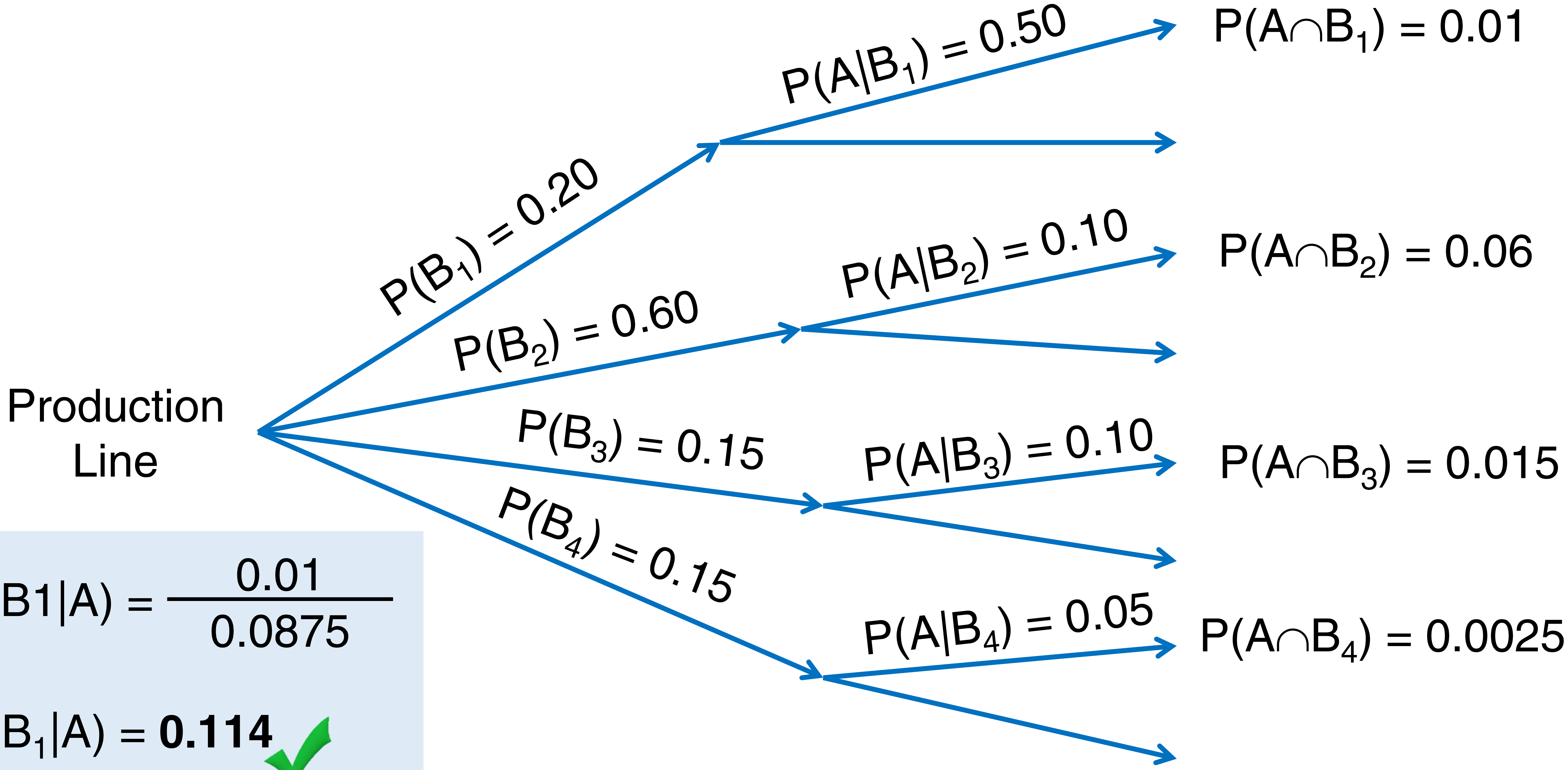
Numerator comes directly from $P(A \cap B_1) = 0.01$

$$P(B1|A) = \frac{0.01}{0.0875}$$

Denominator is the sum of $0.01+0.06+0.015+0.0025 = 0.0875$

Bayes' Theorem: Example II (contd.)

A



$$P(B1|A) = \frac{0.01}{0.0875}$$

$$\therefore P(B_1|A) = \mathbf{0.114}$$

Summary

In this unit, you learnt that:

- Conditional probability of an event occurs when another event has already occurred
- According to Bayes' theorem, the probability of an event is based on previous knowledge of conditions that may influence the event

THANK YOU!