Statistical Technique for Data Science

Testing of Hypothesis

Prof.Gangaboraiah PhD (Stats)

Basics of testing of hypothesis

Testing of Hypothesis → What's in it for Me?

Here's what you will learn in the entire lesson:

1	Basics of hypothesis, null and alternative hypothesis
2	Definition of Type-I and Type-II error
3	Definition α-errors, β-errors, confidence level
4	Definition power of the test and P-value
5	Concepts of Parametric and Non-parametric tests
6	Example to each of the concepts

Testing of Hypothesis → Learning Objectives

At the end of this lesson you will be able to:

01 Explain the concepts of hypothesis, null and alternative hypothesis

02 Define Type-I and Type-II errors

03 Define α-errors, β-errors, confidence level

04 Define power of the test and P-value

05 Concepts of Parametric and Non-parametric tests

06 Give example for each of the concepts

Testing of Hypothesis → Example

A suspected criminal is produced before Jury. The Jury has to decide whether the defendant is

Innocence

Guilty

Testing of Hypothesis → Non-statistical example

A suspected criminal is produced before jury. The Jury has to decide whether the defendant is innocent or guilty.

Jury must decide between two hypotheses

The null hypothesis → H_0: The defendant is innocent

The alternative hypothesis → H_1: The defendant is guilty

Testing of Hypothesis → Definition

Hypothesis

A statement on the parameter(s) which is yet to be proved or established

Null Hypothesis

Hypothesis of no difference or neutral or may be due to Sampling variation

Alternative Hypothesis

Hypothesis of difference which is yet to be proved/ established

Testing of Hypothesis → Example: Formulate hypothesis

It is claimed that sports-car owners drive on the average 17897 kms per year. A consumer firm believes that the average milage is probably different. To check, the consumer firm obtained information from randomly selected 40 sports-car owners that resulted in a sample mean of 17525 kms with a population standard deviation of 1348 kms. At 5% level of significance what can be concluded about this claim?

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The average milage of sports-car as claimed and the sample average milage may be same

$$H_0 : \mu = \mu_0$$

H_1 → The average milage of sports-car as claimed may be **less than** the sample average milage

$$H_1 : \mu < \mu_0$$

Testing of Hypothesis → Formulation of Statistical hypothesis

H_1

The average milage of sports-car as claimed may be **more than** the sample average milage

$$H_1 : \mu > \mu_0$$

H_1

The average milage of sports-car as claimed and the sample average milage may be different

$$H_1 : \mu \neq \mu_0$$

Testing of Hypothesis → Example on Statistical hypothesis

Regular Tires

Radial Tyres

The variable measured fuel consumption

Quantitative

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The mean fuel consumption in cars fitted with radial tires and regular belted tires will be same

$$H_0 : \mu_1 = \mu_2$$

H_1 → The mean fuel consumption in cars fitted with radial tires may be inferior to regular belted tires

$$H_1 : \mu_1 < \mu_2$$

Testing of Hypothesis → Formulation of Statistical hypothesis

H_1 → The mean fuel consumption in cars fitted with radial tires may be better than regular belted tires

$$H_1 : \mu_1 > \mu_2$$

H_1 → The mean fuel consumption in cars fitted with radial tires and regular belted tires may be different

$$H_1 : \mu_1 \neq \mu_2$$

Testing of Hypothesis → Example: Formulate hypothesis

A manufacturer of submersible pumps claims that at most 30% of the pumps require repairs within the first 5 years of operation. If a random sample of 120 of the pumps includes 47 which required repairs within the first 5 years. Test the hypothesis at $\alpha = 0.05$, whether the claim made can be acceptable?

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The proportion of repairs of submersible pumps as claimed and proportion of sample pumps may be same

$$H_0 : P = P_0$$

H_1 → The proportion of repairs of submersible pumps as claimed may be **lower than** proportion of sample pumps

$$H_1 : P < P_0$$

H_1 → The proportion of repairs of submersible pumps as claimed may be **higher than** proportion of sample pumps

$$H_1 : P > P_0$$

H_1 → The proportion of repairs of submersible pumps as claimed and proportion of sample pumps may be different

$$H_1 : P \neq P_0$$

Testing of Hypothesis → Example on Statistical hypothesis

Judge 1

Judge 2

Suppose based on evidences, if we are interested in finding **proportion of false positivity** in the judgment.

Formulate the hypotheses

???

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The proportion of false positive judgements by Judge 1 and Judge 2 will be same

$$H_0 : P_1 = P_2$$

H_1 → The proportion of false positive judgement by Judge 1 may be less compared Judge 2

$$H_1 : P_1 < P_2$$

Testing of Hypothesis → Formulation of Statistical hypothesis

H_1 → The proportion of false positive judgement by Judge 1 may be more compared Judge 2

$$H_1 : P_1 > P_2$$

H_1 → The proportion of false positive judgements by Judge 1 and Judge 2 may be different

$$H_1 : P_1 \neq P_2$$

Exercise: Formulate hypothesis

A product developer is interested in reducing the drying time of a premier paint. Two formulations of the paint are tested: formulation 1 is standard chemistry and formulation 2 has a new drying ingredient that should reduce the dry time. From experience, it is known that the population standard deviation of drying time is 8 and 6 minutes respectively and this inherent variability should be unaffected by the addition of the new ingredient. Both formulations are separately tested for 20 specimens each. The respective average dry time of formulation 1 and formulation 2 is 121 minutes and 112 minutes. The developer is interested to know whether the formulation 2 is better in reducing the drying time as compared to formulation 1 at 5% level of significance?

Testing of Hypothesis → **Formulation of Statistical hypothesis**

H_0 → The mean drying time of formulation 1 and formulation 2 will be same

$$H_0 : \mu_1 = \mu_2$$

H_1 → The mean drying time of formulation 2 may be better as compared to formulation 1

$$H_1 : \mu_1 > \mu_2$$

Testing of Hypothesis → **Exercise: Formulate hypothesis**

A private ambulance services claim that 40% of its calls are life-threatening emergencies. A random sample 150 was taken from its files and found that 49 of them were life threatening. At α = 0.05, can their claim be acceptable?

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The proportion of life threatening calls as claimed and proportion of life threatening calls taken from sample files may be same

$$H_0 : P = P_0$$

H_1 → The proportion of life threatening calls as claimed and proportion of life threatening calls taken from sample files may be same

$$H_1 : P \neq P_0$$

Testing of Hypothesis → **Exercise: Formulate hypothesis**

Because of the impact of global economy on a high-wage country such as USA, it is claimed that the domestic content in manufacturing industries fell between 2001 and 2008. A survey of 36 randomly selected US companies each for 2001 and 2008 showed that the domestic content total manufacturing in 2001 was 37% where as in 2008 it was 21%. At 1% level of significance, test the claim that the domestic content really fell during the period 2001 – 2008.

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The proportion of domestic content of the years 2001 and 2008 will be same

$$H_0 : P_1 = P_2$$

H_1 → The proportion of domestic content of the year 2008 may be fallen compared to the year 2001

$$H_1 : P_1 > P_2$$

Testing of Hypothesis → Statistical test

It is a statistical rule which decides whether to accept the null hypothesis or not ?

Warning

Decision is made based on the sample not on the population

Leads to possibility of error between the decision made and the reality

Statistical tests

Testing of Hypothesis → Statistical tests

A statistical rule which decides whether to accept or reject the null hypothesis on the basis of data

Parametric tests	Non-parametric tests
Based on the assumption of some probability distribution	Not based on any assumption of probability distribution

Testing of Hypothesis → Parametric statistical test

It is assumed that the data do follow some probability distribution which is characterized by any parameters.

- **Large Sample Test** → $n \geq 30$
- **Standard Normal Test** → Z-Test
- **Small Sample Test** → $n < 30$
- **Student's t-test** → Unpaired t-Test, Paired t-Test
- **Analysis of Variance** → ANOVA, Rm ANOVA

Testing of Hypothesis → Non-parametric Statistical test

It is assumed that the data do not follow any probability distribution which is not characterized by any parameters.

→ **Distribution - free tests**

- Chi - Square Test
- Fisher's exact probabilities
- Mann – Whitney U test
- Wilcoxon Signed Rank Test
- Kruskal - WallisTest
- Friedman'sTest

Testing of Hypothesis → Steps involved in testing of hypothesis

State null and alternative hypotheses

↓

Specify the level of significance ‘α’

↓

Define the probability distribution the data follows

↓

Compute the test statistic based defined population

↓

Define the rejection criteria/critical regional

↓

Conclusion

Computation of a test statistic - example

Based on the sample data the test-statistic should be computed using

$$t = \frac{\text{Mean 1 - Mean 2}}{\text{SE (Diff. b'n mean)}}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \approx t_{(\alpha; n_1 + n_2 - 2)}$$

Based on the computed value of test-statistic the H_0 will either be accepted or rejected

Errors in Decision Making

Testing of Hypothesis → Errors

Jury Example (Not based on data)
Non - Statistical Example

Verdict	Actual Situation (H_0)	
	Innocent (True)	Guilty (False)
Innocent		
Guilty		

Any example based on data
Statistical Example

Decision	Null Hypothesis (H_0)	
	True	False
Do not Reject		
Reject		

Testing of Hypothesis → Errors

Jury Example (Not based on data)		
Non - Statistical Example		
Verdict	Actual Situation (H_0)	
	Innocent (True)	Guilty (False)
Innocent	Correct Decision	Wrong Decision
Guilty	Wrong Decision	Correct Decision

Any example based on data		
Statistical Example		
Decision	Actual Situation (H_0)	
	True	False
Do not Reject	Correct Decision	Wrong Decision
Reject	Wrong Decision	Correct Decision

Testing of Hypothesis → Errors

Jury Example (Not based on data)

Non - Statistical Example

Verdict	Actual Situation (H_0)	
	Innocent (True)	Guilty (False)
Innocent		Type II-error
Guilty	Type I-error	

Any example based on data

Statistical Example

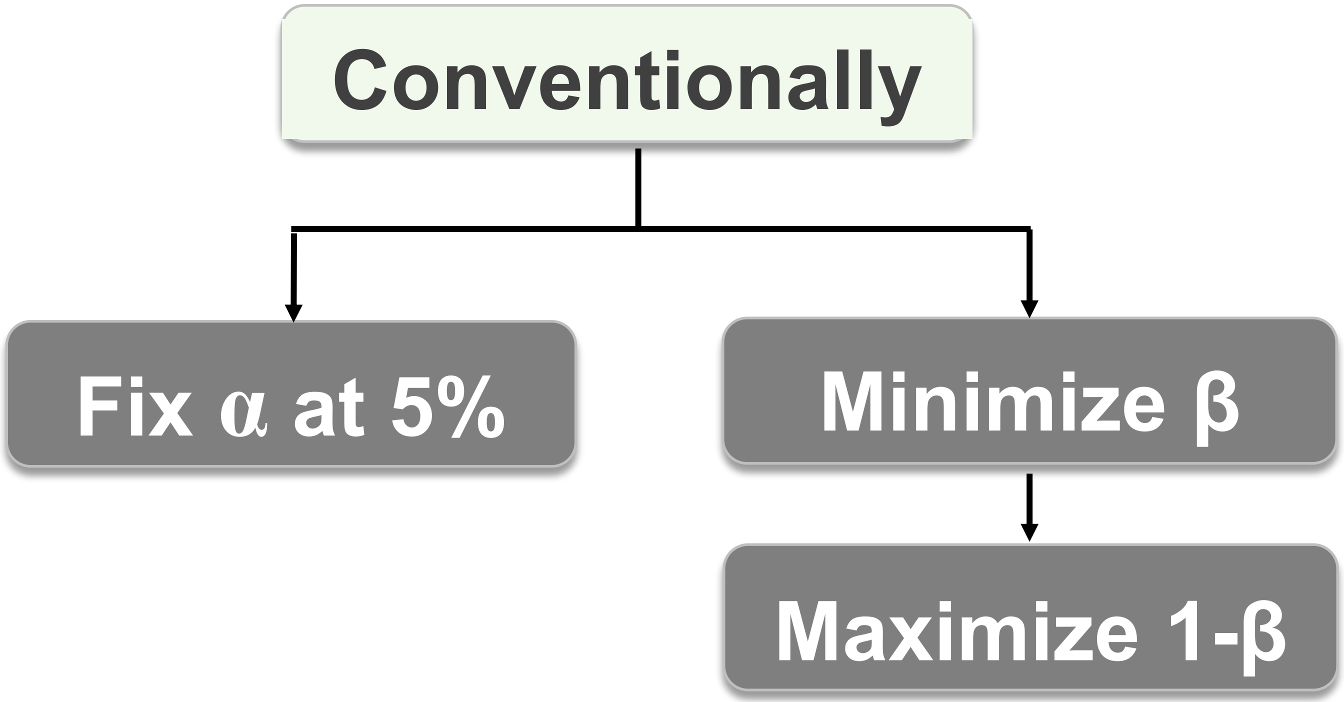
Decision	Actual Situation (H_0)	
	True	False
Do not Reject		Type II-error
Reject	Type I-error	

Testing of Hypothesis → Errors

Jury Example (Not based on data)		
Non - Statistical Example		
Verdict	**Actual Situation (H_0)**	
	Innocent (True)	**Guilty (False)**
Innocent	**Confidence Level (1-α)**	**β-error**
Guilty	**α-error**	**Power 1-β**

Any example based on data		
Statistical Example		
Decision	**Actual Situation (H_0)**	
	True	**False**
Do not Reject	**Confidence Level (1-α)**	**β-error**
Reject	**α-error**	**Power 1-β**

Testing of Hypothesis → Decision on α and β values



Example: Formulate hypothesis

It is claimed that sports-car owners drive on the average 17525 kms per year. A consumer firm believes that the average milage is probably higher. To check, the consumer firm obtained information from randomly selected 40 sports-car owners that resulted in a sample mean of 17897 kms with a sample standard deviation of 1348 kms. At what can be concluded about this claim at

(a) 5% level of significance (Critical value is 1.645)

(b) 1% level of significance (Critical value is 2.331)

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The average milage of sports-car as claimed and the sample average milage may be same

$$H_0 : \mu = 17525$$

H_1 → The average milage of sports-car as claimed may be **higher than** the sample average milage

$$H_1 : \mu > 17525$$

Testing of Hypothesis ⇒ Computation of a test statistic - example

(a) At 5% level of significance with critical value 1.645

$$Z = \frac{17897 - 17525}{\frac{1348}{\sqrt{40}}} = 1.744$$

Critical value for α = 0.05 is 1.645

Since Z =1.744 > 1.645, Reject H_0

Accept H_1

(b) At 1% level of significance with critical value 2.331

$$Z = \frac{17897 - 17525}{\frac{1348}{\sqrt{40}}} = 1.744$$

Critical value for α = 0.01 is 2.331

Since Z =1.744 < 2.331, Accept H_0

Reject H_1

P - value

In hypothesis testing, the choice of the value of α is somewhat arbitrary. For the same data, if the test is based on two different values of α, the conclusion could be different. Many Statisticians prefer to compute the so called P-value, which is calculated based on the observed test statistic. For computing the P-value, it is not necessary to specify a value of α. We can use the given value data to obtain the P-value.

Testing of Hypothesis → P - value

P – value: The strength of the evidence against the null hypothesis that the true difference in the population is zero

In other words

Corresponding to an observed value of a test statistic, the P-value (or attained level of significance) is the lowest level of significance at which the null hypothesis would have been rejected.

Testing of Hypothesis → P - value

P - value

01 Possibility that the observed differences were a chance event

02 Entire population need to be studied to know that a difference is really present with certainty

03 Research community and statisticians had to pick a level of uncertainty at which they could live

Testing of Hypothesis → P - value

If the P-value is less than **1% (< 0.01)**,

Overwhelming evidence that supports the alternative hypothesis

If the P-value is between **5% and 10%**,

Weak evidence that supports the alternative hypothesis

If the P-value is between **1% and 5%**,

Strong evidence that supports the alternative hypothesis

If the P-value exceeds **10%**,

No evidence that supports the alternative hypothesis.

Example: Formulate hypothesis

Aircrew escape system are powered by a solid propellant. The burning rate of this propellant is an important product characteristic. Specification require that the mean burning rate must be 50 cm/s. From past experience it is known that the population SD is 2 cm/s. A sample of 25 solid propellant were selected randomly to reconfirm the specification stated. The sample mean found was 51.3 cm/s. At 5% level of significance what conclusion should be drawn?

Testing of Hypothesis → Relation between P-value and CI

1. H_0: $\mu = 50$ cm/s versus H_1: $\mu \neq 50$ cm/s
2. Specify the level of significance 'α 0.05'
3. Standard normal distribution
4. Test statistic Z = 3.25 with P-value =???
5. 95% CI for μ is [50.52, 52.08]
6. The interval do not include hypothesized mean of 50 cm/s

Observation

There is a close relation between the test of a hypothesis about the parameter and the confidence interval for the parameter.

If 100 (1-α)% CI for μ is

$$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

Hypothesis to be tested is

$$H_0 : \mu = 50 \text{ vs } H_1 : \mu \neq 50$$

In the above example, H_0 is rejected using α = 0.05. The 95% CI of μ is [50.52, 52.08] in which μ = 50 is not included

Conclusion

Thus, if the null hypothesis H_0 is rejected, then the CI will not include the value $\mu = 50$.

On the other hand, if the null hypothesis is accepted then $\mu = 50$ will be included in the 95% CI.

It is claimed that sports-car owners drive on the average 17525 kms per year. A consumer firm believes that the average milage is probably higher. To check, the consumer firm obtained information from randomly selected 40 sports-car owners that resulted in a sample mean of 17897 kms with a sample standard deviation of 1348 kms. At what can be concluded about this claim at

(a) 5% level of significance (Critical value is 1.645)

(b) 1% level of significance (Critical value is 2.331)

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The average milage of sports-car as claimed and the sample average milage may be same

$$H_0 : \mu = 17525$$

H_1 → The average milage of sports-car as claimed may be **higher than** the sample average milage

$$H_1 : \mu > 17525$$

Testing of Hypothesis → Computation of a test statistic - example

(a) At 5% level of significance with critical value 1.645

$$Z = \frac{17897 - 17525}{\frac{1348}{\sqrt{40}}} = 1.744$$

μ = 17525 not included in

Critical value for α = 0.05 is 1.645

Since Z =1.744 > 1.645, Reject H_0

[17478.95, 18315.05]

(b) At 1% level of significance with critical value 2.331

$$Z = \frac{17897 - 17525}{\frac{1348}{\sqrt{40}}} = 1.744$$

μ = 17525 included in

Critical value for α = 0.01 is 2.331

Since Z =1.744 < 2.330, Accept H_0

[17347.71, 18447.29]

Parametric tests

Testing of Hypothesis → Parametric statistical test

It is assumed that the data do follow some probability distribution which is characterized by any parameters.

- **Large Sample Test** → $n \geq 30$
  - **Standard Normal Test** → Z-Test
- **Small Sample Test** → $n < 30$
  - **Student's t-test** → Unpaired t-Test, Paired t-Test
  - **Analysis of Variance** → ANOVA, Rm ANOVA

manipal PROlearn

Testing of Hypothesis → Standard Normal Variate (Z) test

Z-test → This is a test based on Standard Normal Distribution

Used for testing the

- **Mean of a single population (μ)**
- **Difference between means of two populations ($\mu_1 - \mu_2$)**
- **Proportion of a single population (P)**
- **Difference between proportions of two populations ($P_1 - P_2$)**

Testing of Hypothesis → **Assumptions of Z-test**

Z-test

- Samples are drawn from normal distribution
- The population variances should be known
- Two groups should be independent
- Subjects should be allocated randomly to both groups
- The sample size should be more than 30 (i.e., $n \geq 30$)

Standard Normal Variate (Z) test

Testing the mean of a single population (μ)

Testing of Hypothesis → Standard Normal Variate (Z) test

Z-test → Mean of a single population (μ)

Assumptions

- Assume that the samples are drawn from normal distribution
- The population variance should be known
- The sample size should be more than or equal to 30
- Subjects should be selected randomly

Testing of Hypothesis → Steps involved in testing of hypothesis

1. State null and alternative hypothesis
2. Specify the level of significance 'α'
3. Standard Normal Distribution
4. Compute the test statistic
5. Define the critical region/ rejection criteria
6. Conclusion

$$H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu < \mu_0$$
$$\text{or } H_1 : \mu > \mu_0$$
$$\text{or } H_1 : \mu \neq \mu_0$$

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \cong N(0,1)$$

Example: Formulate hypothesis

It is claimed that sports-car owners drive on the average 17525 kms per year. A consumer firm believes that the average milage is probably higher. To check, the consumer firm obtained information from randomly selected 40 sports-car owners that resulted in a sample mean of 17897 kms with a sample standard deviation of 1348 kms. At what can be concluded about this claim at

(a) 5% level of significance (Critical value is 1.645)

(b) 1% level of significance (Critical value is 2.331)

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The average milage of sports-car as claimed and the sample average milage may be same

$$H_0 : \mu = \mu_0 = 17525$$

H_1 → The average milage of sports-car as claimed may be **higher than** the sample average milage

$$H_1 : \mu > \mu_0 = 17525$$

Testing of Hypothesis → Selection of rejection criteria when α = 0.05

(a) At 5% level of significance with critical value 1.645

$$Z = \frac{17897 - 17525}{\frac{1348}{\sqrt{40}}} = 1.744$$

Critical value for

$\alpha = 0.05$ is 1.645

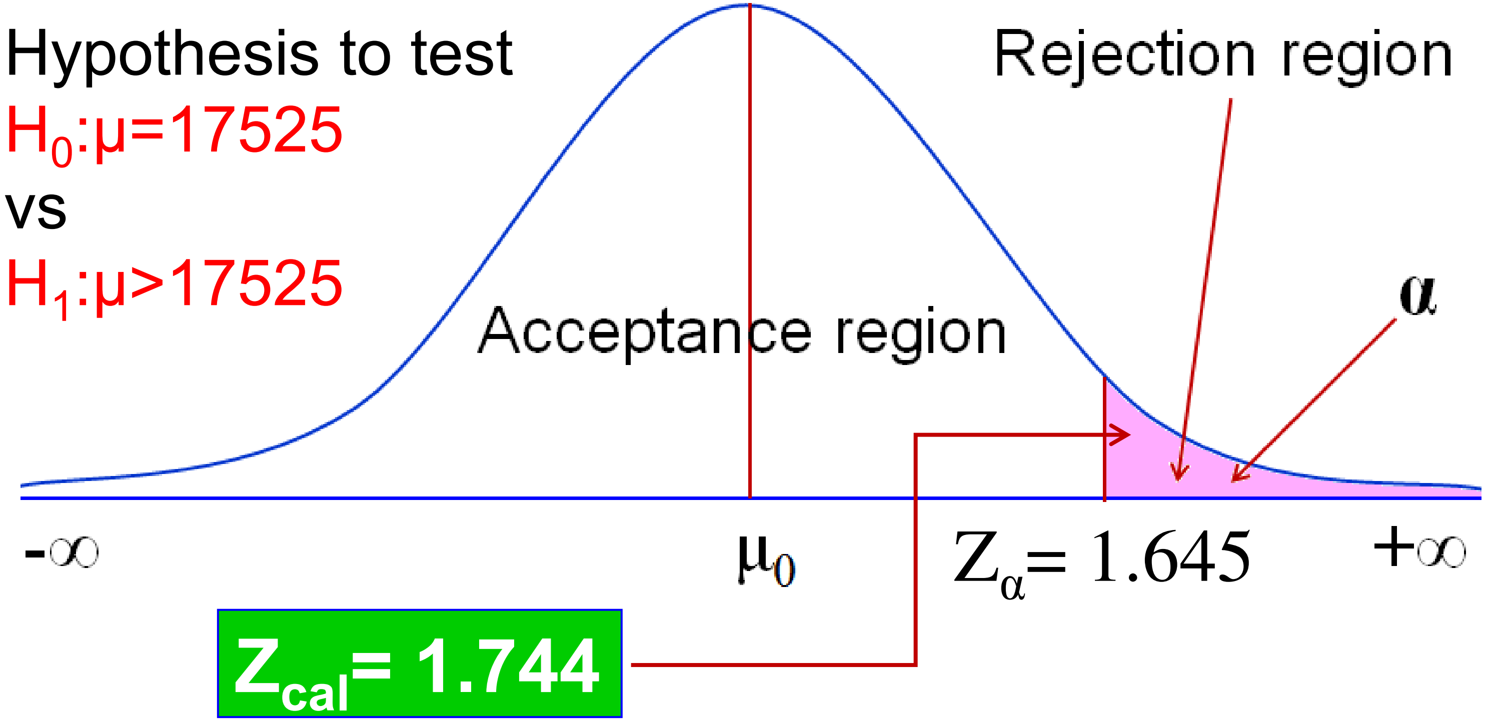
Since Z =1.744 > 1.645, Reject H_0 and Accept H_1

Hypothesis to test

$H_0: \mu = 17525$

vs

$H_1: \mu > 17525$



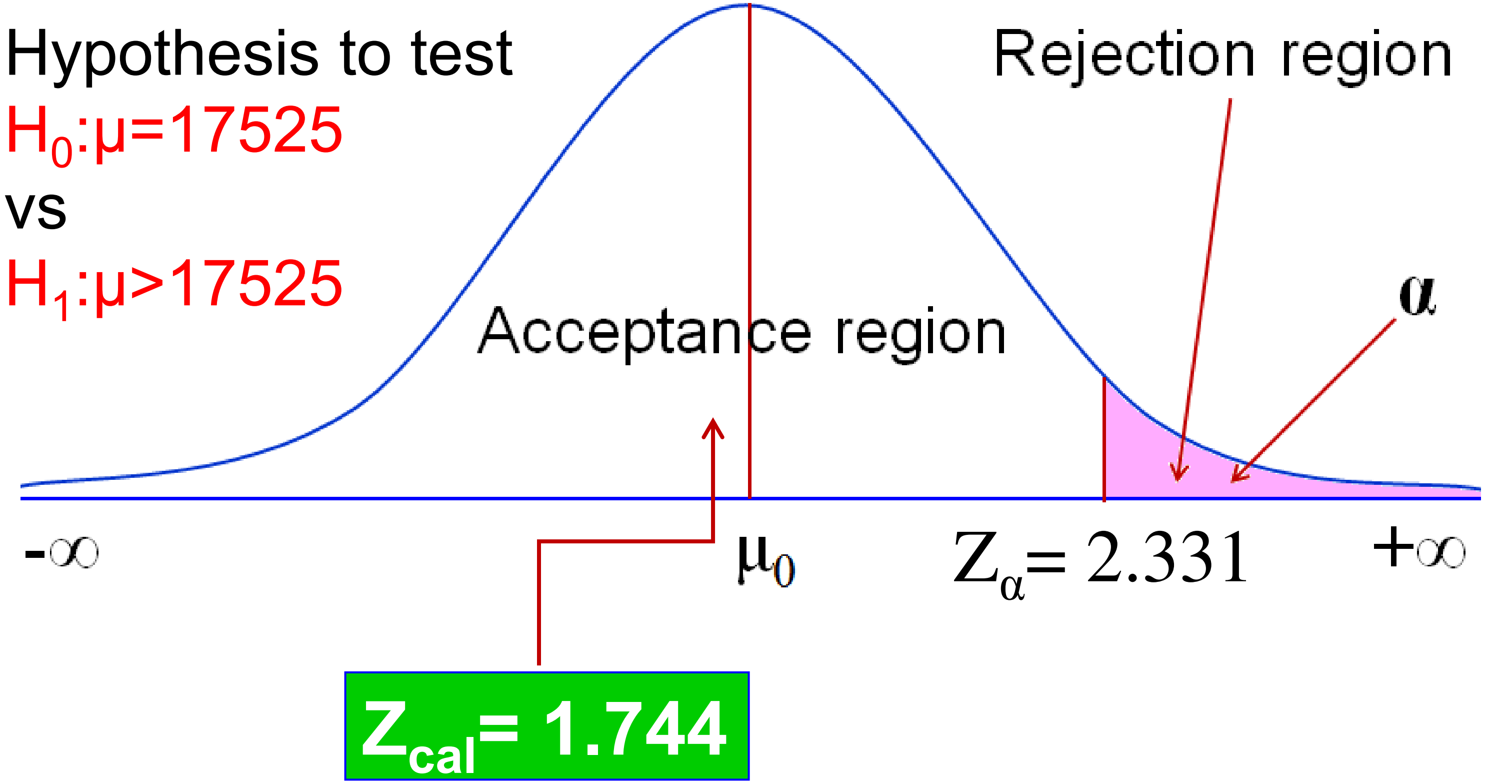
Testing of Hypothesis → Selection of rejection criteria when α = 0.05

(b) At 1% level of significance with critical value 2.331

$$Z = \frac{17897 - 17525}{\frac{1348}{\sqrt{40}}} = 1.744$$

Critical value for
$\alpha = 0.01$ is 2.331
Since Z =1.744
< 2.330, Accept H_0
Reject H_1

Hypothesis to test
$H_0: \mu = 17525$
vs
$H_1: \mu > 17525$



Standard Normal Variate (Z) test

Testing difference between means of a two populations (μ_1-μ_2)

Testing of Hypothesis → Z – test: Assumptions

Z-test → **Difference between means of two populations (μ_1 - μ_2)**

Z-test

- **Samples are drawn from normal distribution**
- **The population variances should be known**
- **Two groups should be independent**
- **Subjects should be allocated randomly to both groups**
- **The sample size should be more than 30 (i.e., n ≥ 30)**

Testing of Hypothesis → Steps involved in testing of hypothesis

1. State null and alternative hypothesis
2. Specify the level of significance 'α'
3. Standard Normal Distribution
4. Compute the test statistic
5. Define the critical region/ rejection criteria
6. Conclusion

$$H_0 : \mu_1 = \mu_2 \text{ vs } H_1 : \mu_1 < \mu_2$$
$$\text{or } H_1 : \mu_1 > \mu_2$$
$$\text{or } H_1 : \mu_1 \neq \mu_2$$

$$Z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \simeq N(0,1)$$

Example: Formulate hypothesis

The management of a local health club claims that its members lose on the average 7 kgs or more within 3 months after joining the club. To check this claim, a consumer agency took a random sample of 45 members of this health club and found that they lost an average of 6.26 kgs within the first three months of membership. From the past experience, it is known that the population standard deviation 1.91 kgs. Test at 1% level of significance whether the claim made by management of a local health club is acceptable or not? Also find the P-value of this test.

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The average weight loss as claimed by the health club management is 7

$$H_0 : \mu = \mu_0 = 7$$

H_1 → The average weight loss as claimed by the health club management of 7 may be **higher than** the sample average weight loss

$$H_1 : \mu < \mu_0 = 7$$

Testing of Hypothesis → Selection of rejection criteria when α = 0.05

At 1% (0.01) level of significance with critical value - 2.331

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{6.26 - 7}{\frac{1.91}{\sqrt{45}}} = -2.596$$

Hypothesis to test

$H_0: \mu = \mu_0 = 7$

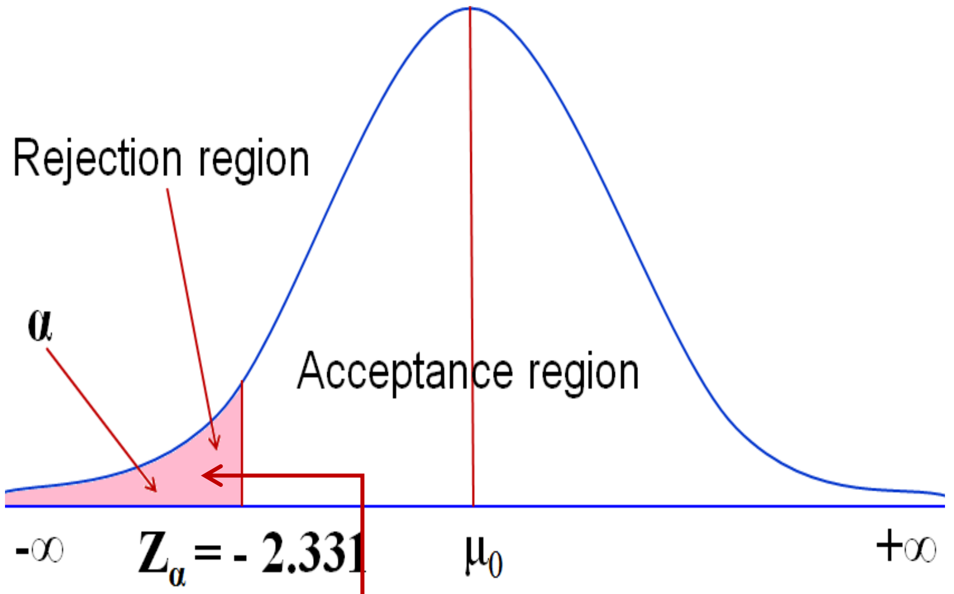
vs

$H_1: \mu = \mu_0 < 7$

???

$$P(Z < -2.60) = 1 - P(Z > 2.60)$$
$$= 1 - 0.9953$$
$$= 0.0047$$

99% CI for μ is [5.596, 6.924]



Z_{cal} = - 2.596

Critical value for α = 0.01 is - 2.331. Since Z =- 2.596 < - 2.331, Reject H_0 Accept H_1

Example: Formulate hypothesis

The manager of a courier service believes that packets delivered at the end of the month are heavier than those delivered early in the month. As an experiment, he weighed a random sample of 20 packets at the beginning of the month and found that the mean weight was 5.25 kg. A randomly selected 10 packets at the end of the month had a mean weight of 4.96 kg. It was observed from the past experience that the population variances are 1.20 kg and 1.15 kg. At 5% level of significance, can it be concluded that the packets delivered at the end of the month weigh more? Also find P-value and 95% confidence interval for the difference between the means.

Testing of Hypothesis → Formulation of Statistical hypothesis

H_0 → The mean weight of packets delivered at the early in the month and at the end of month may be same

$$H_0 : \mu_1 = \mu_2$$

H_1 → The mean weight of packets delivered at the end of the month may be higher than at the early of month

$$H_1 : \mu_2 > \mu_1$$

Testing of Hypothesis → Selection of rejection criteria when α = 0.05

At 5% (0.05) level of significance with critical value 1.645

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{5.25 - 4.96}{\sqrt{\frac{(1.20)^2}{20} + \frac{(1.15)^2}{10}}} = 0.642$$

Hypothesis to test

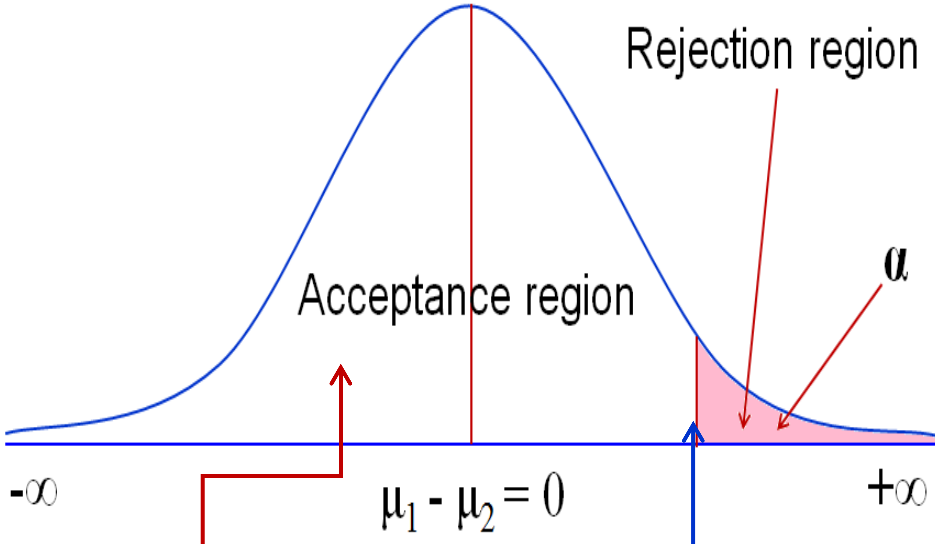
$H_0: \mu_1 - \mu_2 = 0$

vs

$H_1: \mu_1 - \mu_2 > 0$

$$P(Z < 0.642) = 0.7389$$

???



95% CI for μ is [- 454, 1.033]

$Z_{cal} = 0.642$

$Z_{\alpha} = 1.645$

Critical value for α = 0.05 is 1.645. Since Z = 0.642 < 1.645, Accept H_0 Reject H_1

A sample of 100 lung cancer patients on a new drug (Group I) are observed to have a mean survival of 27.5 months, in another sample of 100 lung cancer patients with old drug (Group II), the mean survival was observed to be 25.2 months. The Population variances are 6.25 months and 9.1 months respectively.

The investigators want to know on the basis of the data whether the new drug prolongs the survival?

Testing of Hypothesis → Example: Formulate hypothesis

Company A

Company B

Sample of 100 electric bulbs

- Mean survival of 1237.5 hours
- Population variance is 456.25 hrs.

- Mean survival of 1225.2 months
- Population variance is 629.1 hrs.

Whether the company A bulbs have higher survival time than company B?

Testing of Hypothesis → Example: Formulate hypothesis

The Edison Electric Institute has published figures on the annual number of kilowatt-hours expended by various home appliances. It is claimed that a vacuum cleaner expends an average of 46 kilowatt-hours per year. If a random sample of 45 homes included in a planned study indicates that vacuum cleaners expends an average of 42 kilowatt-hours per year with a standard deviation of 11.9 kilowatt-hours. Does this suggest at the 0.05 level of significance that vacuum cleaners expends, on an average less than 46 kilowatt-hors annually? Assume population of kilowatt-hours to be normal.

Testing of Hypothesis ⟹ Selection of rejection criteria when α = 0.05

At 1% (0.01) level of significance with critical value - 2.331

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{42 - 46}{\frac{11.9}{\sqrt{45}}} = -2.255$$

Hypothesis to test

$H_0: \mu = \mu_0 = 46$

vs

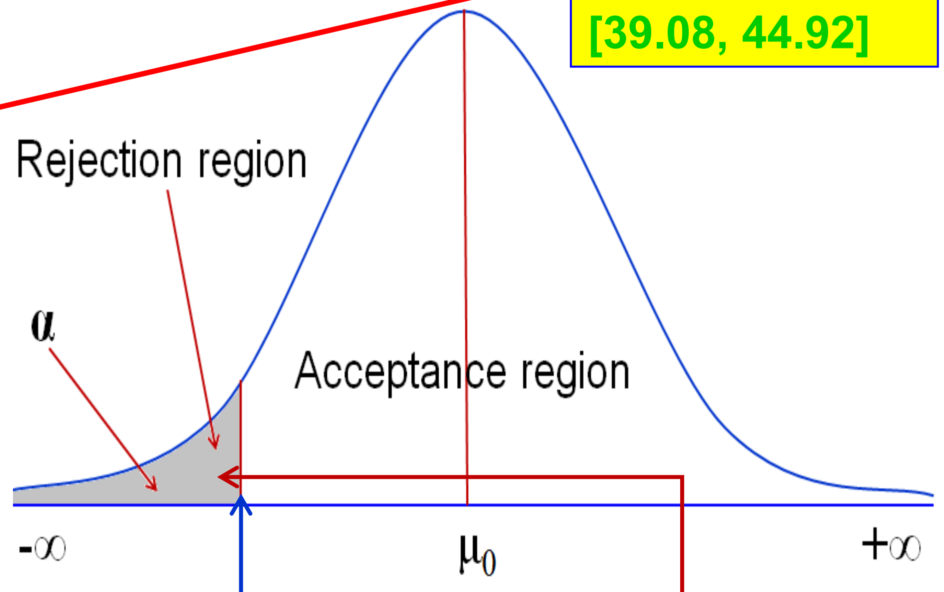
$H_1: \mu = \mu_0 < 46$

46 is not included [39.08, 44.92]

$$P(Z < -2.255) = 1 - P(Z > 2.255)$$
$$= 1 - 0.9881$$
$$= 0.0119$$

???

99% CI for μ is [39.08, 44.92]



$Z_\alpha = -2.331$

$Z_{cal} = -2.255$

Critical value for α = 0.01 is - 2.331. Since Z =- 2.255 < - 2.331, Reject H_0 Accept H_1

Random samples of 25 and 20 were selected from two thermocouples. The sample means were 315, 303 and population variances were 16, 49 respectively.

(a) Construct 95% CI for difference in means

(b) Test whether there is any significant difference in the means of two thermocouples at 5% level of significance

(c) Find the P-value

Testing of Hypothesis → **Example: Formulate hypothesis**

A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilograms. In a random sample 50 fishing lines, the mean breaking strength was found to be 7.8 kilograms. Test at 5% level of significance whether the observed value and the estimated differs. Find the P-value and also construct 95% confidence interval for population mean.

Standard Normal Variate (Z) test

Testing the proportion of a single population (P)

Testing of Hypothesis → Standard Normal Variate (Z) test

Z-test → Proportion of a single population (P)

Assumptions

- Assume that the samples are drawn from normal distribution
- The sample size should be more than or equal to 30
- Subjects should be selected randomly

Testing of Hypothesis → Steps involved in testing of hypothesis

1. State null and alternative hypothesis

$$H_0 : P = P_0 \text{ vs } H_1 : P < P_0$$
$$\text{or } H_1 : P > P_0$$
$$\text{or } H_1 : P \neq P_0$$

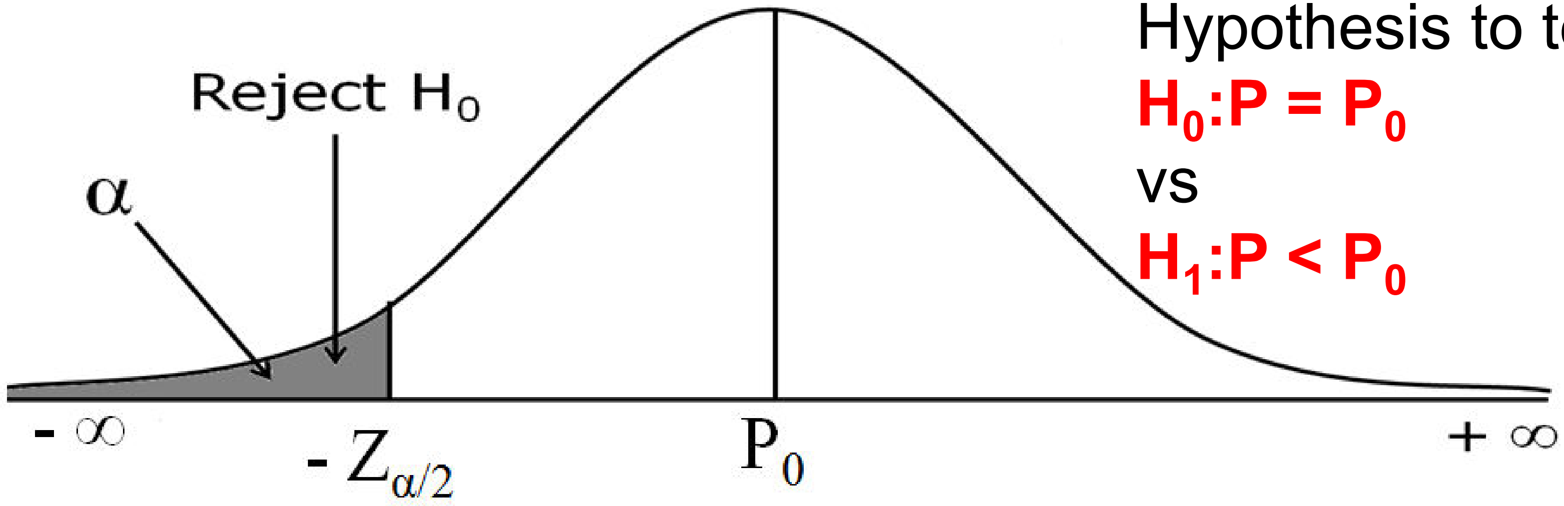
2. Specify the level of significance 'α'
3. Standard Normal Distribution
4. Compute the test statistic

$$Z = \frac{p - P_0}{\sqrt{\frac{pq}{n}}} \cong N(0, 1)$$

5. Define the critical region/ rejection criteria
6. Conclusion

5 Define the critical region/ rejection criteria

(i) **Reject H_0 if computed value of Z is less than the critical value, ie., $P(Z < -z_\alpha)$, otherwise do not reject H_0**



Hypothesis to test

$H_0: P = P_0$

vs

$H_1: P < P_0$

6 Conclusion

5 Define the critical region/ rejection criteria

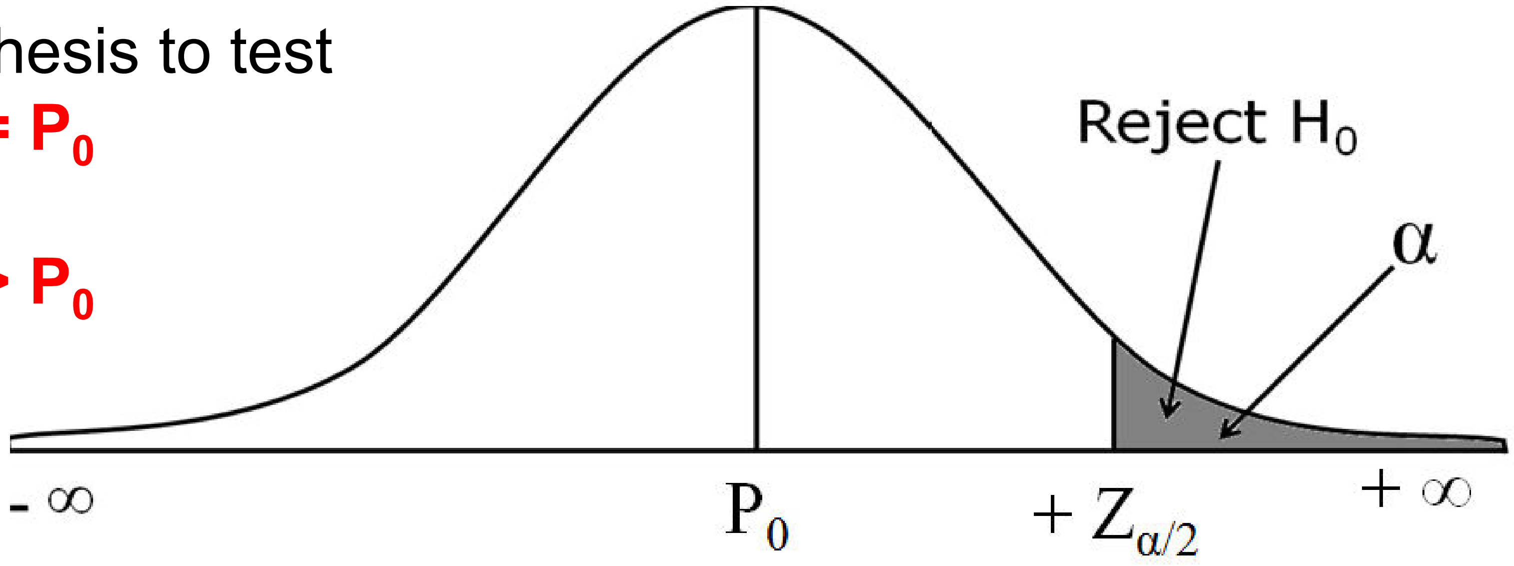
(ii) Reject H_0 if computed value of Z is greater than the critical value, ie., $P(Z > z_\alpha)$, otherwise do not reject H_0

Hypothesis to test

$H_0: P = P_0$

vs

$H_1: P > P_0$



6 Conclusion

5 Define the critical region/ rejection criteria

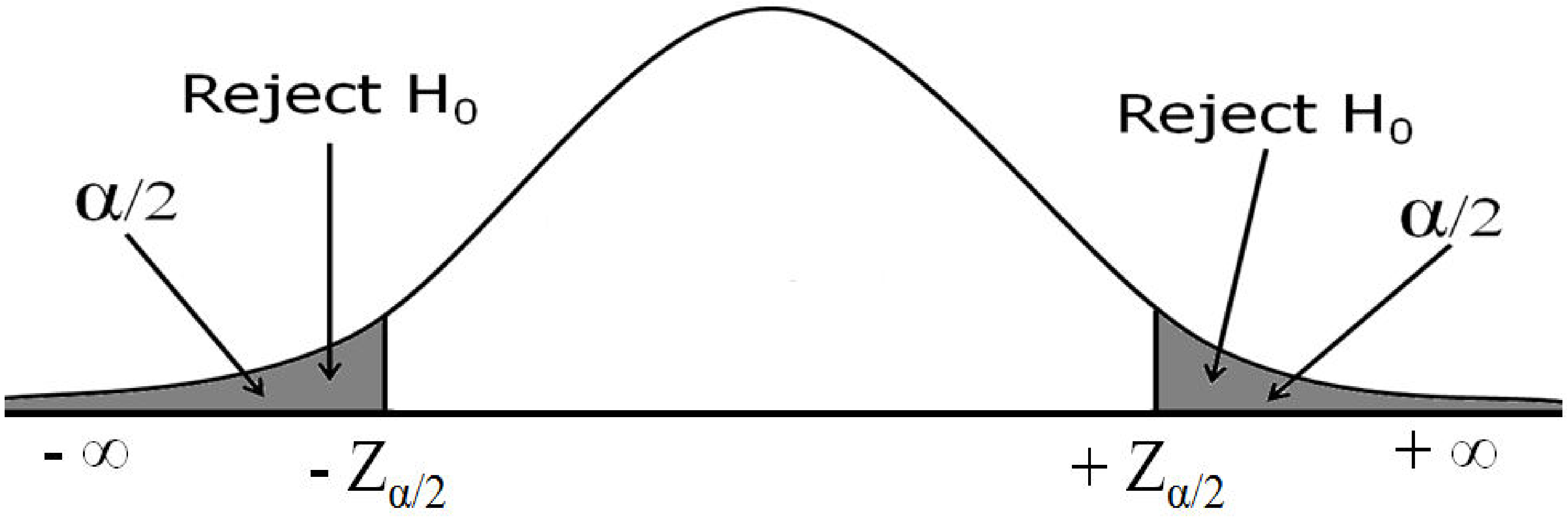
(iii) **Reject H_0 if computed value of Z is less than or greater than the critical value, ie., $P(Z < -z_{\alpha/2})$ or $P(Z > z_{\alpha/2})$, otherwise do not reject H_0**

Hypothesis to test

$H_0: P = P_0$

vs

$H_1: P \neq P_0$



6 Conclusion

Example: Formulate hypothesis

A builder claims that heat pumps are installed in 70% of all homes being constructed today in the city of Bangalore. Would you agree with this claim if a random sample of new homes in this city shows that 28 out of 55 had heat pumps installed? What P-value and confidence interval are related in this situation?

A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use 0.05 level of significance. Also find the P-value and construct (i) 95% (ii) 99% confidence interval for population proportion. Which one of the CI do you suggest for this data? Also compute P-value and comment your result on based on P-value.

Standard Normal Variate (Z) test

Testing the difference between proportions of a populations (P_1-P_2)

Testing of Hypothesis → **Standard Normal Variate (Z) test**

Z-test → Difference between proportion of two population (P_1-P_2)

Assumptions

- Assume that the samples are drawn from normal distribution
- The sample size should be more than or equal to 30
- Subjects should be selected randomly
- Two groups should be independent of each other

Testing of Hypothesis → Steps involved in testing of hypothesis

1. State null and alternative hypothesis
2. Specify the level of significance 'α'
3. Standard Normal Distribution
4. Compute the test statistic
5. Define the critical region/ rejection criteria
6. Conclusion

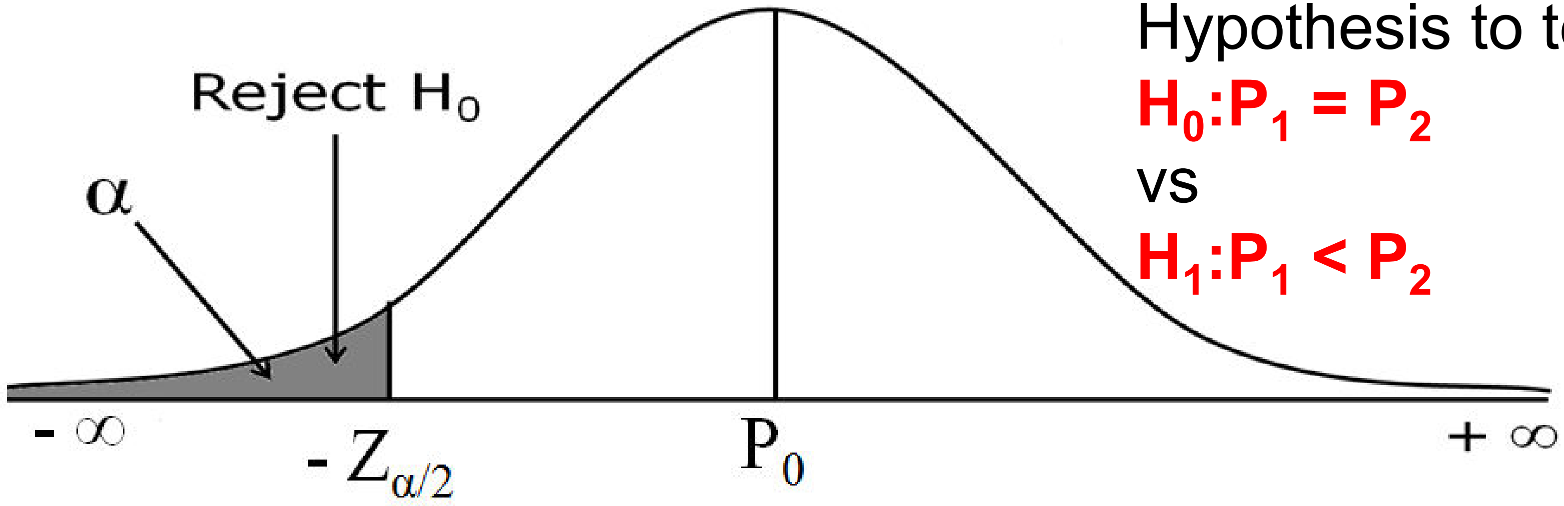
$$H_0 : P_1 = P_2 \text{ vs } H_1 : P_1 < P_2$$
$$\text{or } H_1 : P_1 > P_2$$
$$\text{or } H_1 : P_1 \neq P_2$$

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \cong N(0,1)$$

$$Z = \frac{p_1 - p_2}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} \cong N(0,1)$$

5 Define the critical region/ rejection criteria

(i) Reject H_0 if computed value of Z is less than the critical value, ie., $P(Z < -z_\alpha)$, otherwise do not reject H_0



Hypothesis to test

$H_0: P_1 = P_2$

vs

$H_1: P_1 < P_2$

6 Conclusion

Testing of Hypothesis → **Steps involved in testing of hypothesis**

5 **Define the critical region/ rejection criteria**

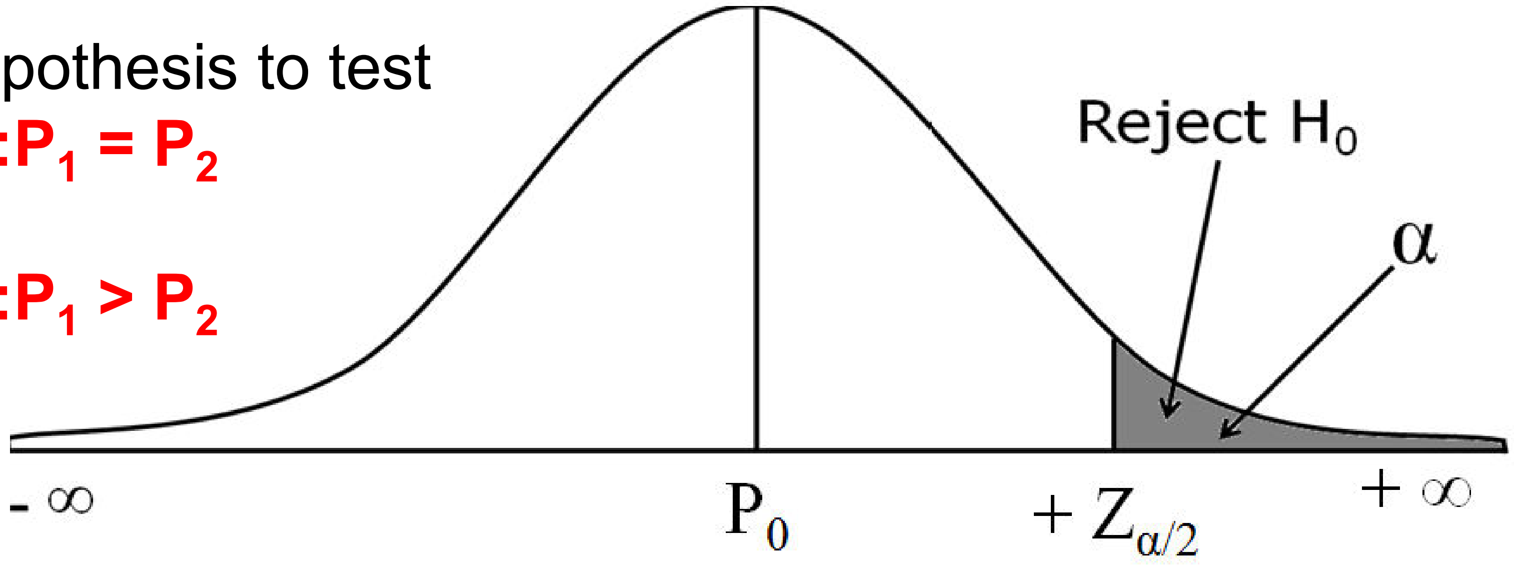
(ii) **Reject H_0 if computed value of Z is greater than the critical value, ie., $P(Z > z_\alpha)$, otherwise do not reject H_0**

Hypothesis to test

$H_0: P_1 = P_2$

vs

$H_1: P_1 > P_2$



6 Conclusion

Testing of Hypothesis → **Steps involved in testing of hypothesis**

5 **Define the critical region/ rejection criteria**

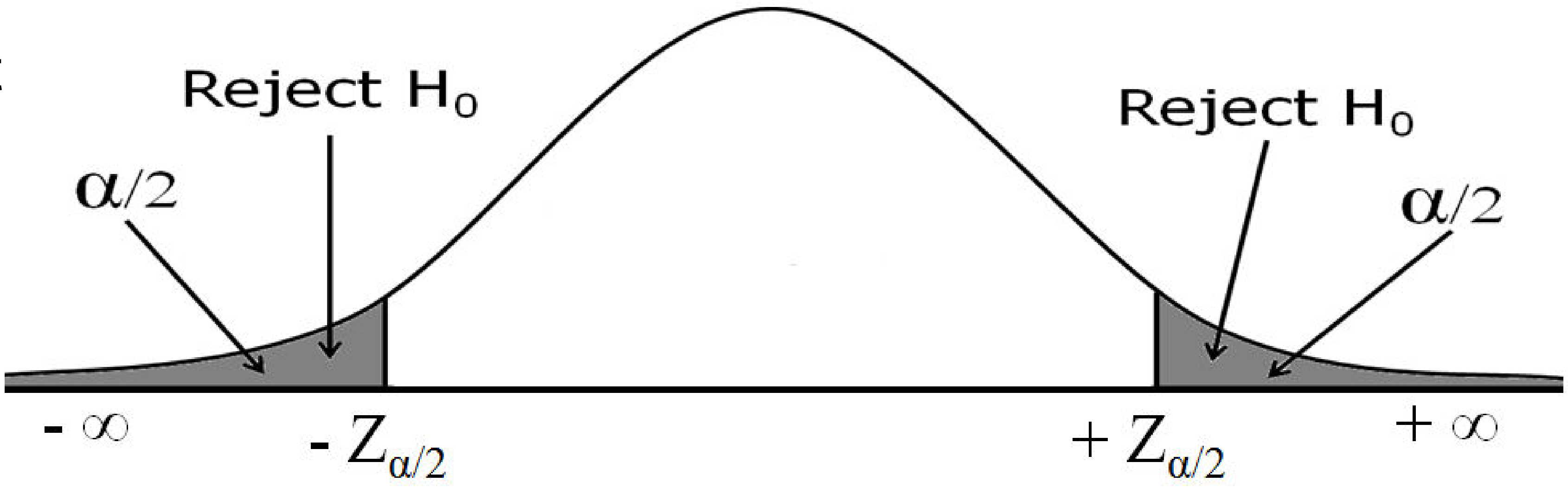
(iii) **Reject H_0 if computed value of Z is less than or greater than the critical value, ie., $P(Z < -z_{\alpha/2})$ or $P(Z > z_{\alpha/2})$, otherwise do not reject H_0**

Hypothesis to test

$H_0: P_1 = P_2$

vs

$H_1: P_1 \neq P_2$



6 Conclusion

Testing of Hypothesis → Example: Formulate hypothesis

An economic firm wants to test the hypothesis that whether both urban and rural people will equally support demonitization of India's old currency of Rs.500 and Rs.1000. The firm has selected 30 each of urban and rural people enquired about the effect of demonitization on their day-to-day life. It was and found that 68.4% of Urban population and 41.7% of Rural population have said that the demonitization has affected the day-to-day life. Does this data at 5% support the economic firm to test the hypothesis what they are intendent to test? Find 95% confidence interval for difference in population proportions. Compute P-value

Testing of Hypothesis → **Example: Formulate hypothesis**

387 of the 1500 BSNL telephones connections were disconnected in Bangalore in 2015. In the same year 310 out of 1200 were disconnected in Kochi by BSNL.

a) Construct 99% confidence interval for difference in proportions.

b) Is there a significant difference between the two proportion of telephones disconnection by BSNL? Use $\alpha = 1\%$.

c) What conclusions can you draw from the confidence interval in part (a)

Testing of Hypothesis → Example: Formulate hypothesis

Example on Z-test on difference in proportions

Bangalore 2015

387/1500 BSNL telephones connections were disconnected

Kochi 2015

310/1200 BSNL telephones connections were disconnected

- Is there a significant difference between the two proportion of telephones disconnection by BSNL?

Use α = 1%.

Construct 99% confidence interval for difference in proportions

Student’s t - test

What, if population variance unknown?

Testing of Hypothesis → Small sample test (n < 30)

Unpaired t-test

OR

Independent sample t-test

AND

Paired t-test

Independent Sample t-test

- Testing mean of a single population
- Testing difference between means of two populations

Testing of Hypothesis → Student's unpaired t-test

t-test → **Testing mean of a single population (μ)**

Assumptions

- Assume that the samples are drawn from normal distribution
- The population variance may be unknown
- The sample size should be less than 30 ($n < 30$)
- Subjects should be selected randomly

Testing of Hypothesis → Steps involved in testing of hypothesis

1. State null and alternative hypothesis
2. Specify the level of significance 'α'
3. Student's t-distribution
4. Compute the test statistic
5. Define the critical region/ rejection criteria
6. Conclusion

$$H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu < \mu_0$$
$$\text{or } H_1 : \mu > \mu_0$$
$$\text{or } H_1 : \mu \neq \mu_0$$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \cong t_{(\alpha, n-1)}$$

Testing of Hypothesis → **Steps involved in testing of hypothesis**

5 Define the critical region/ rejection criteria

(i) Reject H_0 if computed value of t is less than the critical value, ie., $P(t < -t_\alpha)$, otherwise do not reject H_0

(ii) Reject H_0 if computed value of t is greater than the critical value, ie., $P(t > t_\alpha)$, otherwise do not reject H_0

By combining both (i) and (ii), Reject H_0 if computed value of $|t|$ is greater than the critical value, ie., $P(|t| > t_\alpha)$, otherwise do not reject H_0. Besides α, the df is also important.

6 Conclusion

Testing of Hypothesis → **Steps involved in testing of hypothesis**

5 **Define the critical region/ rejection criteria**

(iii) **Reject H_0 if computed value of t is less than or greater than the critical value, ie., $P(t < -t_{\alpha/2})$ or $P(t > t_{\alpha/2})$, otherwise do not reject H_0**

Alternatively, reject H_0 if computed value of $|t|$ is greater than the critical value, ie., $P(|t| > t_{\alpha/2})$, otherwise do not reject H_0. Besides α, the degrees of freedom is also important.

6 Conclusion

Testing of Hypothesis → **Example on one sample t-test**

It is claimed that sports-car owners drive on the average 18580 kms per year. A consumer firm believes that the average milage is probably higher. To check, the consumer firm obtained information from randomly selected 10 sports-car owners that resulted in a sample mean of 17352 kms with a sample standard deviation of 2012 kms. What can be concluded about this claim at

(a) 5% level of significance

(b) 1% level of significance

Testing of Hypothesis → Example on one sample t-test

H_0 → The average milage of sports-car as claimed and the sample average milage may be same

$$H_0 : \mu = \mu_0 = 18580$$

H_1 → The average milage of sports-car as claimed may be **higher than** the sample average milage

$$H_1 : \mu > \mu_0 = 18580$$

(a) At 5% level of significance with critical value 1.645

$$|t| = \frac{|17352 - 18580|}{\frac{2012}{\sqrt{10}}} = 1.929$$

95% CI for μ is

$$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = [16184.91, 18519.09]$$

$$P - \text{value} = 0.0428$$

Hypothesis to test

$H_0: \mu = 18580$ vs $H_1: \mu > 18580$

Critical value for α = 0.05 is 1.833 for 9 degree of freedom

Since | t | =1.929 > 1.833, Reject H_0 and Accept H_1

Example on single population t-test

The management of a local health club claims that its members lose on the average 7 kgs or more within 3 months after joining the club. To check this claim, a consumer agency took a random sample of 15 members of this health club and found that they lost an average of 6.26 kgs within the first three months of membership. From the past experience, it is known that the population standard deviation 1.91 kgs. Test at 1% level of significance whether the claim made by management of a local health club is acceptable or not? Also find the P-value of this test.

Testing of Hypothesis → Example on single population t-test

H_0

The average weight loss as claimed by the health club management is 7

$$H_0 : \mu = \mu_0 = 7$$

H_1

The average weight loss as claimed by the health club management of 7 may be **higher than** the sample average weight loss

$$H_1 : \mu < \mu_0 = 7$$

Testing of Hypothesis → Example on single population t-test

At 1% (0.01) level of significance with critical value - 2.331

$$|t| = \frac{|\bar{x} - \mu_0|}{\frac{\sigma}{\sqrt{n}}} = \frac{|6.26 - 7|}{\frac{1.91}{\sqrt{15}}} = 1.51$$

Hypothesis to test

$H_0: \mu = \mu_0 = 7$

vs

$H_1: \mu = \mu_0 < 7$

95% CI for μ

[5.40, 7.12]

includes $\mu_0 = 7$

99% CI for μ is [5.40, 7.12]

Critical value for α = 0.05 is 1.761 for df=14. | t | = 1.51 < 1.761, accept H_0 & Reject H_1

Example on single population t-test

The Edison Electric Institute has published figures on the annual number of kilowatt-hours expended by various home appliances. It is claimed that a vacuum cleaner expends an average of 46 kilowatt-hours per year. If a random sample of 20 homes included in a planned study indicates that vacuum cleaners expends an average of 42 kilowatt-hours per year with a sample standard deviation of 11.9 kilowatt-hours. Does this suggest at the 0.05 level of significance that vacuum cleaners expends, on an average less than 46 kilowatt-hors annually? Assume population of kilowatt-hours to be normal.

Testing of Hypothesis → Example on single population t-test

At 1% (0.01) level of significance with critical value 2.539

$$|t| = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{|42 - 46|}{\frac{11.9}{\sqrt{20}}} = 1.503$$

Hypothesis to test

$H_0 : \mu = \mu_0 = 46$

vs

$H_1 : \mu = \mu_0 < 46$

46 is included [39.08, 44.92]

???

99% CI for μ is [-2.759, 10.759]

Critical value for α = 0.01 is - 2.331. Since | t | = 1.503 < 2.539, Accept H_0 & Reject H_1

A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilograms. In a random sample 50 fishing lines, the mean breaking strength was found to be 7.8 kilograms. Test at 5% level of significance whether the observed value and the estimated differs. Find the P-value and also construct 95% confidence interval for population mean.

Student's t -test

Testing difference between means of a two populations (μ_1-μ_2)

Testing of Hypothesis → t – test: Assumptions

t-test → Difference between means of two populations ($\mu_1 - \mu_2$)

t-test

- Samples are drawn from normal distribution
- The population variances should be unknown
- The sample size should be less than 30 (i.e., $n < 30$)
- The population variances should be equal
- Two groups should be independent
- Subjects should be allocated randomly to both groups

- However even if sample size more than 30 (i.e., $n > 30$) and population variances unknown, t-test should be continue to apply, because of central limit theorem it approaches normal.

Testing of Hypothesis → Steps involved in testing of hypothesis

1. State null and alternative hypothesis
2. Specify the level of significance ‘α’
3. Standard Normal Distribution
4. Compute the test statistic
5. Define the critical region/ rejection criteria
6. Conclusion

$$H_0 : \mu_1 = \mu_2 \text{ vs } H_1 : \mu_1 < \mu_2$$
$$\text{or } H_1 : \mu_1 > \mu_2$$
$$\text{or } H_1 : \mu_1 \neq \mu_2$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \cong t_{(\alpha, n_1+n_2-2)}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}\right)^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

5 Define the critical region/ rejection criteria

(i) Reject H_0 if computed value of t is less than the critical value, ie., $P(t < -t_\alpha)$, otherwise do not reject H_0

(ii) Reject H_0 if computed value of t is greater than the critical value, ie., $P(t > t_\alpha)$, otherwise do not reject H_0

By combining both (i) and (ii), Reject H_0 if computed value of $|t|$ is greater than the critical value, ie., $P(|t| > t_\alpha)$, otherwise do not reject H_0. Besides α, the df is also important.

6 Conclusion

Testing of Hypothesis → **Steps involved in testing of hypothesis**

5 **Define the critical region/ rejection criteria**

(iii) **Reject H_0 if computed value of t is less than or greater than the critical value, ie., $P(t < -t_{\alpha/2})$ or $P(t > t_{\alpha/2})$, otherwise do not reject H_0**

Alternatively, reject H_0 if computed value of $|t|$ is greater than the critical value, ie., $P(|t| > t_{\alpha/2})$, otherwise do not reject H_0. Besides α, the degrees of freedom is also important.

6 Conclusion

The manager of a courier service believes that packets delivered at the end of the month are heavier than those delivered early in the month. As an experiment, he weighed a random sample of 20 packets at the beginning of the month and found that the mean weight was 5.25 kg. A randomly selected 10 packets at the end of the month had a mean weight of 4.96 kg. The respective sample standard deviations are 1.20 kg and 1.15 kg. At 5% level of significance, can it be concluded that the packets delivered at the end of the month weigh more? Also find P-value and 95% confidence interval for the difference between the means.

Testing of Hypothesis → Example on two population t-test

H_0

The mean weight of packets delivered at the early in the month and at the end of month may be same

$$H_0 : \mu_1 = \mu_2$$

H_1

The mean weight of packets delivered at the end of the month may be higher than at the early of month

$$H_1 : \mu_1 < \mu_2$$

Testing of Hypothesis → Example on two population t-test

At 5% (0.05) level of significance with critical value

$$|t| = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{5.25 - 4.96}{\sqrt{\frac{(1.20)^2}{20} + \frac{(1.15)^2}{10}}} = 0.642$$

Hypothesis to test

$H_0: \mu_1 - \mu_2 = 0$

vs

$H_1: \mu_1 - \mu_2 > 0$

???

95% CI for μ is [- 479, 1.059] includes 0

95% CI for μ is [- 479, 1.059]

Critical value for α = 0.05 is 1.645. Since | t | = 0.642 < 1.701, Accept H_0 & Reject H_1

Random samples of 15 and 10 were selected from two thermocouples. The sample means were 315, 303 and sample standard deviations were 3.8, 4.9 respectively.

(a) Construct 95% CI for difference in means

(b) Test whether there is any significant difference in the means of two thermocouples at 5% level of significance

(c) Find the P-value

Testing of Hypothesis → Example on two population t-test

H_0 → The mean of two thermocouples may be same

$$H_0 : \mu_1 = \mu_2$$

H_1 → The mean of two thermocouples may be different

$$H_1 : \mu_1 \neq \mu_2$$

Testing of Hypothesis → Example on two population t-test

At 5% (0.05) level of significance with critical value

$$|t| = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{315 - 303}{\sqrt{\frac{(3.8)^2}{15} + \frac{(4.9)^2}{10}}} = 6.543$$

Hypothesis to test

$H_0: \mu_1 - \mu_2 = 0$

vs

$H_1: \mu_1 - \mu_2 > 0$

???

95% CI for μ is [6.24, 17.76] not includes 0

95% CI for μ is [6.24, 17.76]

Critical value for α = 0.05 is 1.714. Since | t | = 3.571 > 1.714, Reject H_0 & Accept H_1

Exercise on two sample t-test

Suppose 10 regular users of oral contraceptives were studied to determine, if cholesterol levels in such women are significantly different from those of non-users of contraceptives. The mean cholesterol was 8.96 mmol/L and 5.74 mmol/L for contraceptive users and non-users respectively with respective standard deviations 1.02 mmol/L and 0.99 mmol/L. Test at 5% level pf significance whether the cholesterol between oral Contraceptives users and non-users differs significantly. Find P-value and 95% confidence interval for the difference in population means.

Testing of Hypothesis ➡ Exercise on two population t-test

Test 1

Test 2

➢ Is there sufficient evidence to conclude that both tests give the same mean impurity level

Using α = 0.01

Specimen	1	2	3	4	5	6	7	8
Test 1	1.2	1.3	1.5	1.4	1.7	1.8	1.4	1.3
Test 2	1.4	1.7	1.5	1.3	2.0	2.1	1.7	1.6

Construct 99% confidence interval for mean difference

Student's t - test

Testing the paired mean of a single population (μ_d)

t-test → Testing mean before and after observations of a single population (μ_d)

Assumptions

- Assume that the difference between before and after observations follow normal distribution
- The sample size should be less than 30 ($n < 30$)
- Subjects should be selected randomly

Testing of Hypothesis → Steps involved in testing of hypothesis

1. State null and alternative hypothesis
2. Specify the level of significance 'α'
3. Student's t-distribution
4. Compute the test statistic
5. Define the critical region/ rejection criteria
6. Conclusion

$$H_0 : \mu_d = 0 \text{ vs } H_1 : \mu_d < 0$$
$$\text{or } H_1 : \mu_d > 0$$
$$\text{or } H_1 : \mu_d \neq 0$$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} \cong t_{(\alpha, n-1)}$$

But μ_d under H_0 will be 0

Testing of Hypothesis → **Steps involved in testing of hypothesis**

5 Define the critical region/ rejection criteria

(i) Reject H_0 if computed value of t is less than the critical value, ie., $P(t < -t_\alpha)$, otherwise do not reject H_0

(ii) Reject H_0 if computed value of t is greater than the critical value, ie., $P(t > t_\alpha)$, otherwise do not reject H_0

By combining both (i) and (ii), Reject H_0 if computed value of $|t|$ is greater than the critical value, ie., $P(|t| > t_\alpha)$, otherwise do not reject H_0. Besides α, the df is also important.

6 Conclusion

Testing of Hypothesis → **Steps involved in testing of hypothesis**

5 Define the critical region/ rejection criteria

(iii) **Reject H_0 if computed value of t is less than or greater than the critical value, ie., $P(t < -t_{\alpha/2})$ or $P(t > t_{\alpha/2})$, otherwise do not reject H_0**

Alternatively, reject H_0 if computed value of $| t |$ is greater than the critical value, ie., $P(| t | > t_{\alpha/2})$, otherwise do not reject H_0. Besides α, the degrees of freedom is also important.

6 Conclusion

Example on paired t-test

The pulse rate of 8 persons were measured before and after administration of a drug. The results are as follows: Test at 5% level of significance whether there is any increase in pulse rate?

Subjects	Before (x)	After (y)
1	58	66
2	65	69
3	68	75
4	70	68
5	66	73
6	75	75
7	62	68
8	72	69

Testing of Hypothesis → **Example on paired t-test**

Subjects	Before (x)	After (y)	d = y-x	(d-mean)^2
1	58	66	8	64
2	65	69	4	16
3	68	75	7	49
4	70	68	- 2	4
5	66	73	7	49
6	75	75	0	0
7	62	68	6	36
8	72	69	- 3	9
Total	67.0	70.375	27	

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n} = \frac{27}{8} = 3.38$$

$$S_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$$

$$S_d = \sqrt{\frac{135.88}{7}} = 4.41$$

Testing of Hypothesis → Example on paired t-test

At 5% (0.05) level of significance with critical value is 1.895

$$|t| = \frac{\bar{d} - \mu_d}{\frac{S_d}{\sqrt{n}}} = \frac{3.38 - 0}{\frac{4.41}{\sqrt{8}}} = 2.168$$

Hypothesis to test

$H_0: \mu_d = 0$

vs

$H_1: \mu_d > 0$

???

95% CI for μ is [0.426, 6.334] includes 0

95% CI for μ is [0.426, 6.334]

Critical value for α = 0.05 is 1.895. Since | t | = 3.571 > 1.895, Reject H_0 & Accept H_1

Testing of Hypothesis → Example on paired t-test

The HRD manager wishes to see if there has been any change in the ability of trainees after a specific training programme. The trainees take a aptitude test before and after training programme.

Subjects	Before (x)	After (y)
1	75	70
2	70	77
3	46	57
4	68	60
5	68	79
6	43	64
7	55	55
8	68	77
9	77	76

Testing of Hypothesis → **Example on paired t-test**

Subjects	Before (x)	After (y)	d = y-x	(d-mean)^2
1	75	70	-5	100
2	70	77	7	4
3	46	57	11	36
4	68	60	-8	169
5	68	79	11	36
6	43	64	21	256
7	55	55	0	25
8	68	77	9	16
9	77	76	-1	36
Total			45	678

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n} = \frac{45}{9} = 5$$

$$S_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$$

$$S_d = \sqrt{\frac{678}{8}} = 9.21$$

Example on paired t-test

At 5% (0.05) level of significance with critical value is 3.31

$$|t| = \frac{\bar{d} - \mu_d}{\frac{S_d}{\sqrt{n}}} = \frac{5-0}{\frac{9.21}{\sqrt{9}}} = 3.07$$

Hypothesis to test

$H_0 : \mu_d = 0$

vs

$H_1 : \mu_d > 0$

???

95% CI for μ is [[-2.52, 12.52] not includes 0

95% CI for μ is [-2.52, 12.52]

Critical value for α = 0.05 is 1.895. Since | t | = 1.63 < 2.31, Accept H_0 & Reject H_1

Testing of Hypothesis → Example on paired t-test

Diet-modification Program

Ten individuals have participated

Subject	1	2	3	4	5	6	7	8	9	10
Weight Before	195	213	247	201	187	210	215	246	294	310
Weight After	187	195	221	190	175	197	199	221	278	285

Is there sufficient evidence to support claim that this program is effective in reducing weight?

Use **α = 0.05**.
Construct 95% confidence interval for mean difference.

Analysis of Variance (ANOVA)

Introduction:

- When there are more than two groups to be compared, it is not correct to compare the groups in pairs, as this type of comparison will not take the within variability into consideration

- The Analysis procedure used in such comparisons is known as **ANALYSIS OF VARIANCE**

Testing of Hypothesis → Analysis of variance (ANOVA)

Introduction:

- In this analysis the variability of observations between and within groups are taken into consideration
- The total variability is split in these components and test is applied
- The test used to compare these variability is F test

Testing of Hypothesis → Analysis of Variance (ANOVA)

Types of ANOVA

- One-way
- Two-way
- Three-way (LSD)
- Repeated measures of ANOVA (Rm ANOVA)

Note: In this unit we discuss only One-way ANOVA and Two ANOVA in detail

Testing of Hypothesis → ANOVA: Assumptions

Student's t-test cannot be applied

↓

No. of groups are more than two (say k) and are independent

↓

If t-test is applied, the type-I error will increase

Used to test equality of more than two population means against not equal

Testing of Hypothesis → **ANOVA: Assumptions**

ANOVA → Testing equality of k group means against not equal

- Samples are drawn from normal distribution
- The population variances should be equal
- The sample size should be less than 30 (i.e., n < 30)

ANOVA

- Groups should be independent
- Subjects should be allocated randomly to both groups
- However even if sample size more than 30 (i.e., n > 30) and population variances unknown, t-test should be continue to apply, because of central limit theorem it approaches normal.

Testing of Hypothesis → **Exercise on single population t-test**

G_1	G_2	G_3	. . .	G_k
X_{11}	X_{21}	X_{31}	. . .	X_{k1}
X_{12}	X_{22}	X_{32}	. . .	X_{k2}
X_{13}	X_{23}	X_{33}	. . .	X_{k3}
. . .	. . .	. . .	. . .	. . .
X_{1n1}	X_{2n2}	X_{3n1}	. . .	X_{kn1}
C_1	C_2	C_3	. . .	C_k

$$n_1 + n_2 + n_3 + \ldots + n_k = n$$

$$C_1 + C_2 + C_3 + \ldots + C_k = G$$

Calculation of sum of squares

1. Correction factor (CF) : $\dfrac{G^2}{n}$

2. Total sum of squares (TSS) : $\sum_i \sum_i x_{ij}^2 - CF$

3. Between group sum of squares (GSS) : $\sum_{i=1}^{k} \dfrac{C_i^2}{n_i} - CF$

4. Within group sum of squares (WSS) : TSS - GSS

One-way ANOVA table

Source of variation	df	Sum of squares	Mean sum of squares	F-ratio
Between groups	k - 1	GSS	$\text{MGSS} = \frac{\text{GSS}}{\text{k - 1}}$	$F = \frac{\text{MGSS}}{\text{MWSS}}$
Within groups	n - k	WSS	$\text{MWSS} = \frac{\text{WSS}}{\text{n - k}}$	
Total	n - 1	TSS	$F \approx$ F - distribution with k - 1 and n - k df	

Testing of Hypothesis → Analysis of variance

1	State null and alternative hypothesis
2	Specify the level of significance ‘α’
3	Student’s t-distribution
4	Compute the test statistic
5	Define the critical region/ rejection criteria
6	Conclusion

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

vs

$$H_1 : \mu_1 \neq \mu_2 \neq \ldots \neq \mu_k$$

$$F = \frac{\text{MGSS}}{\text{MWSS}} \approx F_{(k-1, n-k)}$$

MGSS- Mean group sum of squares

MWSS- Mean within group sum of squares

Testing of Hypothesis → Analysis of Variance (ANOVA)

- Reject H_0, if $F > F_{(\alpha, k-1, n-k)}$
- If H_0 is rejected, then further mean differences between any two groups should be tested using Post-hoc tests. Following are different Post-hoc tests.
  - Bonferroni's test
  - Tucky's HSD test
  - Schaeffe's test
  - Duncan's test
  - Least Significant Difference (LSD) test

Testing of Hypothesis → Analysis of Variance (ANOVA)

Test at 5% level of significance is there any significant difference in mean iron intake among four groups of patients? Also test if significant, which group means have contributed to the difference in means using LSD test

A clinical trial Iron intake of four groups of patients (mg)

Group 1	Group 2	Group 3	Group 4
11.5	19.5	18.5	30.0
12.5	18.5	16.5	26.5
18.5	16.0	24.5	27.0
21.0	22.0	30.0	34.0
28.0	30.0	28.5	20.0
26.0	24.5	14.0	22.5
14.0	19.0	19.0	28.0
22.0	24.0	17.0	32.0
20.0	19.5	18.0	27.0
22.0	15.0	29.0	25.5

Calculation of sum of squares

1. Correction factor (CF) : $\frac{G^2}{n} = \frac{981^2}{40} = 19847.03$

2. $TSS = \sum_i \sum_j x_{ij}^2 - CF = 21119.5 - 19847.03 = 1272.47$

3. $GSS = \sum_{i=1}^{k} \frac{C_i^2}{n_i} - CF = 20196.6 - 19847.03 = 349.52$

4. $WSS = TSS - GSS = 1272.47 - 349.52 = 922.95$

One-way ANOVA table

Source of variation	df	Sum of squares	Mean sum of squares	F-ratio
Between groups	3	349.52	116.51	F=4.455
Within groups	36	922.95	25.64	
Total	39	1272.47	$F(4.46) > F_{(0.05;\,3,\,36)} = 4.38$	

H_0 may be rejected and **H_1 may be accepted**.

Testing of Hypothesis → Analysis of Variance (ANOVA)

- Least significant difference (LSD) test
- Analysis of Variance provides estimate of Standard error for testing which of the differences between the villages is significant. An estimate of the standard error of the differences between the group means is equal to
  - $$\sqrt{\frac{2S^2}{k}}$$
  - Where S^2 is the 'Within groups mean sum of squares and k is the number of observations in each of the group under comparison

Testing of Hypothesis → Analysis of Variance (ANOVA)

- S^2 = Within Groups Mean sum of squares = 25.6375
- k = Number of observations in each Group = 10 df of within villages = 36
- t Statistics value corresponding to P = 0.05 for 36 df is 2.03
- t Statistics value corresponding to P = 0.05 for 36 df is 2.03

$$\left\{ t_{0.05} \sqrt{\frac{2S^2}{k}} \right\}$$

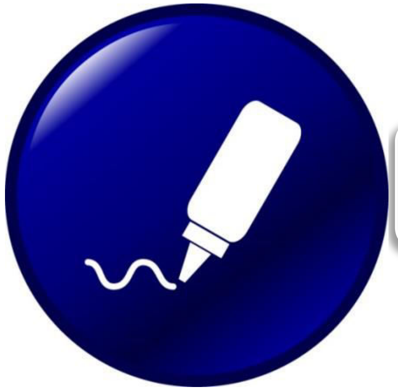
Testing of Hypothesis → Analysis of Variance (ANOVA)

$$\left\{2.03\sqrt{\frac{225.64}{10}}\right\} = 4.597$$

Group of patients			
1	2	3	4
Mean iron intake (mg)			
20	21	22	28

It can be seen that only the difference between Group 4 is different from other groups as only this difference is more than 4.597. Hence, Group 4 makes the difference in significance.

Testing of Hypothesis → Analysis of Variance (ANOVA)



Three drying formulas for curing glue are studied

Formula A	13	10	8	11	8	
Formula B	13	11	14	14		
Formula C	4	1	3	4	2	4

Test at 5% level of significance whether is any difference in the mean curing time of glue?

Find between which two formulas the mean difference has contributed significantly using least significant difference post-hoc test?

Testing of Hypothesis → Analysis of Variance (2-way ANOVA)

A clinical trial Iron intake of four groups of patients (mg). Test at 5% level of significance is there any significant difference in mean iron intake among ten groups of patients as well as between three trimesters? Also test if significant, which group means have contributed to the difference in means using LSD test

Trimester	Group means for iron intake										Row Total
	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}	
I	11.5	19.5	18.5	12.5	18.5	16.5	26.5	18.5	16.0	24.5	182.5
II	27.0	28.0	22.0	21.0	15.0	19.5	20.0	26.0	30.0	28.5	237.0
III	28.0	30.0	26.0	30.0	24.5	28.5	26.0	30.0	27.0	25.5	275.5
Column Total	66.5	77.5	66.5	63.5	58.0	64.5	72.5	74.5	73.0	78.5	695

Testing of Hypothesis → Analysis of Variance (2-way ANOVA)

SUMMARY	Average	SD	Variance
Trimester 1	18.25	4.66	21.74
Trimester 2	23.70	4.88	23.84
Trimester 3	27.55	2.05	4.19
Group 1	22.17	9.25	85.58
Group 2	25.83	5.58	31.08
Group 3	22.17	3.75	14.08
Group 4	21.17	8.75	76.58
Group 5	19.33	4.80	23.08
Group 6	21.50	6.24	39.00
Group 7	24.17	3.62	13.08
Group 8	24.83	5.84	34.08
Group 9	24.33	7.37	54.33
Group 10	26.17	2.08	4.33

Source of Variation	df	SS	MS	F	P-value	F crit
Trimester	2	436.72	218.36	12.53	0.0003	3.55
Group	9	134.17	14.91	0.86	0.58	2.46
Error	18	313.78	17.43			
Total	29	884.67				

ф **The mean difference of trimester varies significantly ($F=12.53 > F_{(0.05;\ 2,\ 18)} = 3.55$) and the mean group difference is not significant ($F=0.86 > F_{(0.05;\ 9,\ 18)} = 2.46$) . Carry out Post-hoc test for trimester**

Testing of Hypothesis → Analysis of Variance (ANOVA)

Three-way ANOVA (Latin Square Design (LSD))

- Three-way ANOVA: deals with measurement of variation in three different areas viz., Plots, Blocks, and treatments.

Repeated measures of ANOVA (Rm ANOVA)

- Repeated measures of ANOVA: deals with measurements made repeatedly on a single group (Plot) more the two times which forms depended observations. Even though there are more than two sets of observations because of dependence, one-way ANOVA should not be applied. In this case repeated measures of ANOVA has to be applied to see the variation at different levels of time point.

Non-Parametric tests

Testing of Hypothesis → Non-parametric Statistical test

It is assumed that the data do not follow any probability distribution which is not characterized by any parameters.

→ **Distribution - free tests**

- Chi - Square Test
- Fisher’s exact probabilities
- Mann – Whitney U test
- Wilcoxon Signed Rank Test
- Kruskal - WallisTest
- Friedman’sTest

Testing of Hypothesis → Non-parametric Statistical test

Chi-square test

Chi-square Test

- Goodness-of-fit
- Independence

Should be applied ONLY for Frequencies

Not for percentages, ratios, mean etc.

Testing of Hypothesis → Non-parametric Statistical test

2 x 2 Contingency Table

Categorical Variable 1	Categorical variable 2		Total
	Present	Absent	
Present	O_1 $\boxed{E_1}$	O_2 $\boxed{E_2}$	r_1
Absent	O_3 $\boxed{E_3}$	O_4 $\boxed{E_4}$	r_2
Total	c_1	c_2	n

Calculation of expected frequencies

$$E_1 = \frac{r_1 c_1}{n}$$

$$E_2 = \frac{r_1 c_2}{n}$$

$$E_3 = \frac{r_2 c_1}{n}$$

$$E_4 = \frac{r_2 c_2}{n}$$

Testing of Hypothesis → Non-parametric Statistical test

If the expected cell frequencies is **< 5**

Yate's correction should be applied for continuity

In a **2 x 2 contingency table**, if one or more of the cell has the expected cell frequencies is **< 5**,

Fisher's exact probabilities should be computed

For the use of **Chi-square test**

The sample size should not be **less than 20**.

The Fisher's exact Probability

$$P = \frac{1}{n!}\frac{r_1!}{a!}\frac{r_2!}{b!}\frac{c_1!}{c!}\frac{c_2!}{d!}$$

$$E_1 = \frac{r_1 c_1}{n}$$

$$E_2 = \frac{r_1 c_2}{n}$$

$$E_3 = \frac{r_2 c_1}{n}$$

$$E_4 = \frac{r_2 c_2}{n}$$

Chi-square is calculated by

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \approx \chi^2_{(\alpha, r-1, c-1)}$$

where k = r x c is the total number of cells in the r x c contingency table, r = total no. of rows and c is total no. of columns.

Testing of Hypothesis → Non-parametric Statistical test

Categorical variable 1	Categorical variable 2		Total
	Present	Absent	
Present	a	b	r_1
Absent	c	d	r_2
Total	c_1	c_2	n

If the expected count is more than 5, the alternative formula for calculation of Chi-square is

$$\chi^2 = n\frac{(ad-bc)^2}{r_1 r_2 c_1 c_2}$$

Testing of Hypothesis → Non-parametric Statistical test

Three pension plans

Independent of job classification

Use **α = 0.05**

The opinion of a random sample of 500 employees are shown below

Job Classification	Pension Plan 1	2	3	Total
Salaried workers	166	86	68	320
Hourly workers	84	64	32	180
Total	250	150	100	500

$$E_1 = \frac{r_1 c_1}{n} = \frac{87 \text{ x } 69}{180} = 33.35$$

$$E_2 = \frac{r_1 c_2}{n} = \frac{87 \text{ x } 62}{180} = 29.97$$

$$E_3 = \frac{r_1 c_3}{n} = \frac{87 \text{ x } 49}{180} = 23.68$$

$$E_4 = \frac{r_2 c_1}{n} = \frac{93 \text{ x } 69}{180} = 35.65$$

$$E_5 = \frac{r_2 c_2}{n} = \frac{93 \text{ x } 62}{180} = 32.03$$

$$E_6 = \frac{r_2 c_3}{n} = \frac{93 \text{ x } 49}{180} = 25.32$$

Testing of Hypothesis → Non-parametric Statistical test

Sl No	(O_i)	(E_i)	(O_i-E_i)	$(O_i-E_i)^2$	$(O_i-E_i)^2/E_i$
1	21	33.35	- 12.35	152.52	4.57
2	36	29.97	6.03	36.36	1.21
3	30	23.68	6.32	39.94	1.69
4	48	35.65	12.35	152.52	4.28
5	26	32.03	- 6.03	36.36	1.14
6	19	25.32	- 6.32	39.94	1.58
Total	180	180	Chi-square value		14.46

Testing of Hypothesis → Non-parametric Statistical test

- H_0: Job satisfaction and pension plan are independently distributed
- H_1: Job satisfaction and pension plan are not independently distributed (Associated)
- $\chi^2 = 14.46$
- DF=2
- $P<0.001$
- Inference: Reject H0, which shows Job satisfaction and pension plan are associated

Testing of Hypothesis → Non-parametric Statistical test

To assess the length of stay and the type of insurance, data were taken on 70 individuals

Type of Insurance	Length of Hospital Stay (days)		Total
	≤10	>10	
Type 1	39	3	42
Type 2	21	7	28
Total	60	10	70

Examine whether Chi-square test can be applied to this data to test the independence between type of insurance and length of hospital stay?

Calculation of expected frequencies

- $E_1 = (42 \times 60)/70 = 36.00$
- $E_2 = (42 \times 10)/70 = 6.00$
- $E_3 = (28 \times 60)/70 = 24.00$
- $E_4 = (10 \times 28)/70 = 4.00$

Testing of Hypothesis → Non-parametric Statistical test

Sl No	(O_i)	(E_i)	(O_i-E_i)	$(O_i-E_i)^2$	$(O_i-E_i)^2/E_i$
1	39	36	3	9	0.25
2	3	6	- 3	9	1.50
3	21	24	- 3	9	0.38
4	7	4	3	9	2.25
Total	70	70	Chi-square value		4.38

Since the expected frequency in the 4 is less than 5 Chi-square cannot be applied and hence the Fisher's exact probabilities has to be calculated.

Testing of Hypothesis → Non-parametric Statistical test

Computed Probability

$$P = \frac{1}{70!}\frac{42!}{39!}\frac{28!}{3!}\frac{60!}{21!}\frac{10!}{7!}$$

Two tailed: P=0.077

H_0: Type of insurance plan and length of hospital stay may be independent

H_1: Type of insurance plan and length of hospital stay may be associated

Conclusion: H_0 is accepted and hence the type of insurance plan and length at may be independent

Testing of Hypothesis → Non-parametric Statistical test

Mann-Whitney U test

- Denote the samples $\mathbf{G_1}$ and $\mathbf{G_2}$
- Denote the size of each of the samples $\mathbf{n_1}$ and $\mathbf{n_2}$

$$n = n_1 + n_2$$

- Combine the data, keeping track of the sample from which each datum arose

Rank the Data

Lowest value with rank 1

Next value with rank 2

Highest value with rank n

- Add the ranks of each sample separately, naming the sums $\mathbf{R_1}$ and $\mathbf{R_2}$

Testing of Hypothesis → Non-parametric Statistical test

Sum the rank of the smaller group and denote it as U_1 and sum the other group and denote it as U_2

$U_1 = n_1 n_2 + (n_1(n_1+1)/2) - R_1$ **and** $U_2 = n_1 n_2 + (n_2(n_2+1)/2) - R_2$

n_1 is the sample size of the **smaller** group

n_2 is the sample size of the **bigger** group

With Mean $U = n_1 n_2/2$

and $S_U = SQRT(n_1 n_2 (n_1+n_2+1)/12)$

If $n_1 + n_2 = n > 20$ approximate $Z \approx N(0,1)$

Find the U = Minimum (U_1, U_2)

The calculated value more than The critical value

The null hypothesis in not rejected, otherwise it is rejected

Testing of Hypothesis → Non-parametric Statistical test

- The results serum fibronogen degradation product FDP values are follows:

Control group	10	180	80	5	40	15	30	160		
Treated group	7.8	40	10	80	180	5	80	10	5	7.5

- Do the data suggest that the data suggest the significant difference between FDP value of control and treated group

Testing of Hypothesis → Non-parametric Statistical test

Sl No	Group	FDP	Ranks
4	Control group	5.0	2
14	Treated group	5.0	2
17	Treated group	5.0	2
18	Treated group	7.5	4
9	Treated group	7.8	5
1	Control group	10.0	7
11	Treated group	10.0	7
16	Treated group	10.0	7
6	Control group	15.0	9

Testing of Hypothesis → Non-parametric Statistical test

Sl No	Group	FDP	Ranks
7	Control group	30.0	10
5	Control group	40.0	11.5
10	Treated group	40.0	11.5
3	Control group	80.0	14
12	Treated group	80.0	14
15	Treated group	80.0	14
8	Control group	160.0	16
2	Control group	180.0	17.5
13	Treated group	180.0	17.5

Testing of Hypothesis → Non-parametric Statistical test

Sl. No.	Control group		Treated group	
	FDP value	Rank of FDP value	FDP value	Rank of FDP value
1	10.0	7.0	7.8	5.0
2	180.0	17.5	40.0	11.5
3	80.0	14.0	10.0	7.0
4	5.0	2.0	80.0	14.0
5	40.0	11.5	180.0	17.5
6	15.0	9.0	5.0	2.0
7	30.0	10.0	80.0	14.0
8	160.0	6.0	10.0	7.0
9			5.0	2.0
10			7.5	4.0
Total		$R_1 = 77.0$		$R_2 = 84.0$

Testing of Hypothesis → Non-parametric Statistical test

- Sum of the ranks of smaller sample is calculated
- $U_1 = n_1 n_2 + ((n_1(n_1+1)/2) – R_1 = [8*9 + ((8(8+1)/2) – 77 = 39$
- $U_2 = n_1 n_2 + ((n_2(n_2+1)/2) – R_2 = [8*9 + ((10(10+1)/2) – 84 = 51$
- where n_1 is the smaller sample size n_2 is of bigger sample
- $U = \text{Min}\ (U_1, U_2) = \text{Min}\ (39, 51) = 39$
- Since 39 is smaller than 53 we refer to this value in the table to obtain P value.
- Since $U = 39 < 53$, the critical value H_0 is rejected and H_1 is accepted

Testing of Hypothesis → Non-parametric Statistical test

The results tensile adhesion tests on two different alloy specimen load at failure (in MPa) are follows:

Alloy 1	19.8	18.5	17.6	16.7	15.8	15.4	14.1	13.6	11.9	11.4	11.4
Alloy 2	8.8	7.5	15.4	15.4	19.5	14.9	12.7	11.9	11.4	10.1	7.9

Do the data suggest that mean load at failure exceeds 10MPa?

Assume that load at failure is not normally distributed and use α=0.05.

Testing of Hypothesis → Non-parametric Statistical test

Wilcoxon Signed Rank test

Denote the before and after observation by **X** and **Y**

Find the difference between **X** and **Y**

Ignore the sign of the difference

Rank the Difference

Rank 1 for smaller difference

Rank 2 for next smaller difference

Rank n for the larger difference

Assign average rank for the tied values in the difference

Testing of Hypothesis → Non-parametric Statistical test

Attach the original sign to the ranks assigned to the difference

↓

Find the sum of the positive ranks and negative ranks separately.

↓

Choose the minimum of sum of positive and negative ranks.

The calculated value | more than | The critical value

↳ The null hypothesis in not rejected, otherwise it is rejected

Testing of Hypothesis → Non-parametric Statistical test

Ten workers were given on the job training with a view to shorten their assembly time for a certain mechanism. The results of the time (min) before and after are as follows: Test at 5% whether the training program has shortened the assembly time?

Sl No	Before	After	Difference	Ranks for magnitude	Ranks
1	61	59	2	3.5	3.5
2	62	63	-1	1.5	-1.5
3	55	57	-2	3.5	- 3.5
4	62	54	8	8	8
5	59	87	-28	9	- 9
6	74	70	4	5	5
7	62	67	-5	6.5	- 6.5
8	97	96	1	1.5	1.5
9	10	59	-49	10	- 10
10	2	7	-5	6.5	- 6.5

Testing of Hypothesis → Non-parametric Statistical test

- Sum of (+) ranks = 18
- Sum of (-) ranks = 37
- For 10 pairs, a minimum rank sum of less than or equal to 10 is required for rejection of the null hypothesis at 5% level.
- Since the calculated rank sum 18 is more than 10, the null hypothesis is accepted,

Testing of Hypothesis → Non-parametric Statistical test

Serum fibronogen degradation product values (µgm/ml) of a group of 12 persons

Sl No	Before	After	Difference	Ranks for magnitude	Ranks
1	5.0	7.8	- 2.8	2	- 2
2	10.0	180.0	- 170.0	11	- 11
3	18.0	10.0	8.0	5	+ 5
4	5.0	80.0	- 75.0	10	- 10
5	10.0	15.0	- 5.0	3.5	- 3.5
6	20.0	10.0	10.0	6	+ 6
7	5.0	180.0	- 175.0	12	- 12
8	2.5	40.0	- 37.5	8	- 8
9	15.0	10.0	5.0	3.5	+ 3.5
10	10.0	7.5	2.5	1	+ 1
11	80.0	10.0	70.0	9	+ 9
12	5.0	20.0	- 15.0	7	- 7

Testing of Hypothesis → Non-parametric Statistical test

- Sum of (+) ranks = 24.5
- Sum of (-) ranks = 53.5
- For 12 pairs, a minimum rank sum of less than or equal to 14 is required for rejection of the null hypothesis at 5% level.
- Since the calculated rank sum 24.5 is more than 14, the null hypothesis, that the pre and post operative values of F.D.P. is not significantly different is accepted at $P>0.05$.

Testing of Hypothesis → Non-parametric Statistical test

Diet-modification Program

Ten individuals have participated

Subject	1	2	3	4	5	6	7	8	9	10
Weight Before	195	213	247	201	187	210	215	246	294	310
Weight After	187	195	221	190	175	197	199	221	278	285

Is there sufficient evidence to support claim that this program is effective in reducing weight?

Assume that the difference between before and after do not follow normal distribution. Use **α = 0.05**.

Kruskal-Wallis test

- Denote the k samples as $G_1, G_2, G_3, \ldots, G_k$
- Denote the size of each of the samples as $n_1, n_2, n_3, \ldots, n_k$

$$n = n_1 + n_2 + n_3 + \ldots + n_k$$

- Combine the data, keeping track of the sample from which each datum arose

Rank the Data

Lowest value with rank 1

Next value with rank 2

Highest value with rank n

- Add the ranks of each sample separately, naming the sums $R_1, R_2, R_3, \ldots, R_k$

Testing of Hypothesis → Non-parametric Statistical test

Calculate the test-statistic given by

$$H = \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(n+1)$$

→ Chi-square with k-1 degrees of freedom

- $\mathbf{R_i}$ is the sum of the rank of the i^{th} group
- $\mathbf{n_i}$ is the number of observation in the i^{th} group

The calculated value → more than → The critical value

→ The null hypothesis in not rejected, otherwise it is rejected

Testing of Hypothesis → Non-parametric Statistical test

BPH	Positive biopsy	Negative biopsy
5.3	7.1	11.4
7.9	6.6	0.5
8.7	6.5	1.6
4.3	14.8	2.3
6.6	17.3	3.1
6.4	3.4	1.4
	13.4	4.4
	7.6	5.1

Testing of Hypothesis → Non-parametric Statistical test

Sl No	Group	PSA (ng/ml)	Ranks
16	Negative biopsy	0.5	1
20	Negative biopsy	1.4	2
17	Negative biopsy	1.6	3
18	Negative biopsy	2.3	4
19	Negative biopsy	3.1	5
12	Positive biopsy	3.4	6
4	BPH	4.3	7
21	Negative biopsy	4.4	8
22	Negative biopsy	5.1	9

Testing of Hypothesis → Non-parametric Statistical test

Sl No	Group	PSA (ng/ml)	Ranks
1	BPH	5.3	10
6	BPH	6.4	11
9	Positive biopsy	6.5	12
5	BPH	6.6	13.5
8	Positive biopsy	6.6	13.5
7	Positive biopsy	7.1	15
14	Positive biopsy	7.6	16
2	BPH	7.9	17
3	BPH	8.7	18

Testing of Hypothesis → Non-parametric Statistical test

Sl No	Group	PSA (ng/ml)	Ranks
16	Negative biopsy	0.5	1
20	Negative biopsy	1.4	2
17	Negative biopsy	1.6	3
18	Negative biopsy	2.3	4
19	Negative biopsy	3.1	5
12	Positive biopsy	3.4	6
4	BPH	4.3	7
21	Negative biopsy	4.4	8
22	Negative biopsy	5.1	9

Testing of Hypothesis → Non-parametric Statistical test

Sl No	Group	PSA (ng/ml)	Ranks
1	BPH	5.3	10
6	BPH	6.4	11
9	Positive biopsy	6.5	12
5	BPH	6.6	13.5
8	Positive biopsy	6.6	13.5
7	Positive biopsy	7.1	15
14	Positive biopsy	7.6	16
2	BPH	7.9	17
3	BPH	8.7	18

Testing of Hypothesis → Non-parametric Statistical test

Sl No	Group	PSA (ng/ml)	Ranks
15	Negative biopsy	11.4	19
13	Positive biopsy	13.4	20
10	Positive biopsy	14.8	21
11	Positive biopsy	17.3	22

Testing of Hypothesis → Non-parametric Statistical test

BPH		Positive biopsy		Negative biopsy	
PSA (ng/ml)	Rank	PSA (ng/ml)	Rank	PSA (ng/ml)	Rank
5.3	10	7.1	15	11.4	19
7.9	17	6.6	13.5	0.5	1
8.7	18	6.5	12	1.6	3
4.3	7	14.8	21	2.3	4
6.6	13.5	17.3	22	3.1	5
6.4	11	3.4	6	1.4	2
		13.4	20	4.4	8
		7.6	16	5.1	9
Total	76.5		125.5		51

Testing of Hypothesis → Non-parametric Statistical test

- n = total no. of observations = 24
- = {12 / (22 x23) } { $(76.5)^2$/6) +$(125.5)^2$ /8)+$(51)^2$ /8) } – 3(22+1)
- = (0.02) [(5852.25 /6) + (15750.25 /8) + (2601 /8))- 75
- = 8.53
- Here the degrees of freedom = 2

- n = total no. of observations = 6+8+8=22

$$H = \frac{12}{22(22+1)}\left[\frac{76.5^2}{6} + \frac{125.5^2}{8} + \frac{51^2}{8}\right] - 3(22+1)$$

$$= 8.53$$

- H has Chi-square distribution with 2 degrees of freedom. The critical values for 2 degrees of freedom is 5.99 and the null hypothesis of equality of medians is rejected and alternative hypothesis is accepted.

Testing of Hypothesis → Non-parametric Statistical test

Three preventive methods against corrosion yielded in pieces of wire (in thousands of an inch)

Method A	77	54	67	74	71	66	
Method B	60	41	59	65	62	64	52
Method C	49	52	69	47	56		

Use α = 0.05 level of significance to test the null hypothesis

Testing of Hypothesis

Choosing a test for means of two or more samples of scores of experiments with one treatment factor

Data	Between (Independent samples)	Within (Related samples)
	2 samples	
Interval	Independent t-test	Paired t-test
Ordinal	Mann-Whitney U-test	Wilcoxon signed test
Nominal	Chi-square test	Mc Nemar test
	More than 2 samples	
Interval	One-way ANOVA	Repeated Measures ANOVA
Ordinal	Kruska-Wallis test	Friedman test
Nominal	Chi-square test	Cochran Q test
		(Dichotomous data only)

Design	Data Summary	Statistics and Tests
2 Independent groups	Proportions Mean Rank ordered Survival	Chi-square test, Fisher's exact test, Z-test Unpaired t-test Mann-Whitney test Mantel Haenzel, Long Rank test
2 related groups	Proportions Rank ordered Mean	McNemar Chi-square test Sign-test Wilcoxon Signed Rank test Paired t-test
More than 2 Independent groups	Proportions Rank ordered Mean Survival	Chi-square test Kruskal-Wallis test ANOVA Long Rank test
More than 2 related groups	Proportions Rank ordered Mean	Cochran Q Friedman Repeated Measures ANOVA
Study of causation; one independent variable (Univariate)	Proportions Mean	Relative Risk Odds Ratio Correlation Coefficient
Study of causation; More than one independent variable (Multivariate)	Proportions Mean	Discriminant Analysis Multiple Logistic Regression Log-linear Model; Regression Analysis Multiple Classification Analysis

Summary

In this lesson you learnt:

01 Concepts of hypothesis, null and alternative hypothesis

02 Concepts of Type-I and Type-II errors

03 Concepts of α-errors, β-errors, confidence level

04 Concepts of power of the test and P-value

05 Example to each of the concepts

In this lesson you learnt:

01 Concepts of parametric test

02 Different types of parametric test

03 Examples for each type of parametric test

Summary on Non-Parametric tests

In this lesson you learnt:

01 Concepts of non-parametric test

02 Different types of non-parametric test

03 Examples for each type of non-parametric test

Choosing a test for means of two or more samples of scores of experiments with one treatment factor

Data	Between (Independent samples)	Within (Related samples)
	2 samples	
Interval	Independent t-test	Paired t-test
Ordinal	Mann-Whitney U-test	Wilcoxon signed test
Nominal	Chi-square test	Mc Nemar test
	More than 2 samples	
Interval	One-way ANOVA	Repeated Measures ANOVA
Ordinal	Kruska-Wallis test	Friedman test
Nominal	Chi-square test	Cochran Q test

(Dichotomous data only)

Choosing appropriate Statistical method

Descriptive Statistics

Type of statistical analysis:

It is based the**Study design**

and the **Type of variable**

Measured.

If the study design is **Descriptive** and the variable measured is **QUALITATIVE** then construct a Frequency table

and compute **percentages** and represent data graphically where ever necessary.

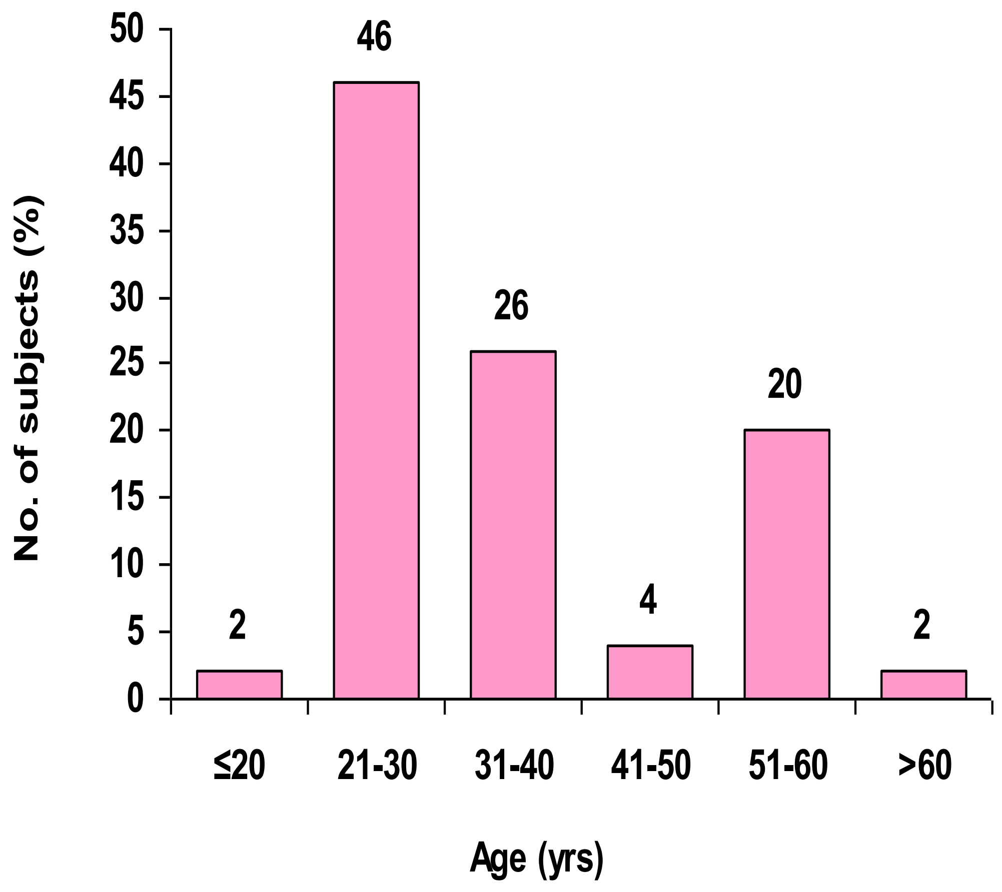
If the study design is
Descriptive
and the variable measured is
QUANTITATIVE
then construct a Frequency table
and compute **Descriptive statistics**
and represent data graphically where ever necessary.

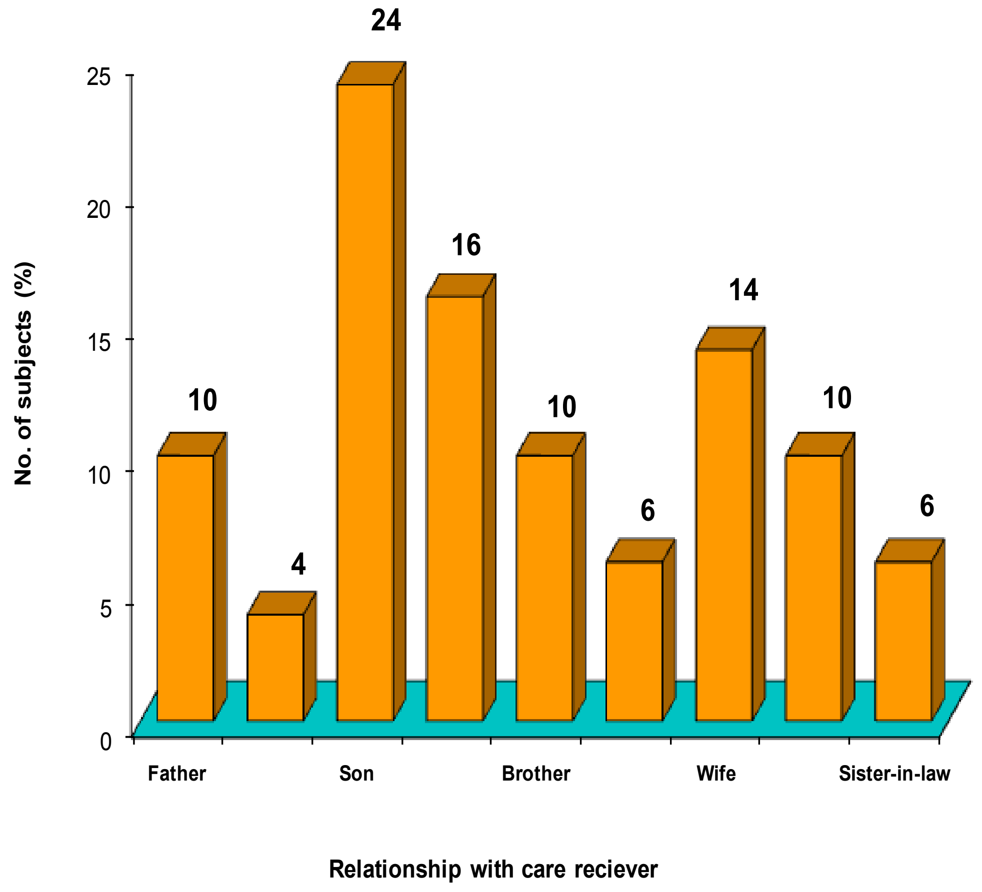
Categorical data

Bar Chart

Graphical Illustrations

Exploring data


No. of subjects (%)
50
45
40
35
30
25
20
15
10
5
0
2
46
26
4
20
2
≤20
21-30
31-40
41-50
51-60
>60
Age (yrs)


No. of subjects (%)
25
20
15
10
5
0
10
4
24
16
10
6
14
10
6
Father
Son
Brother
Wife
Sister-in-law
Relationship with care reciever

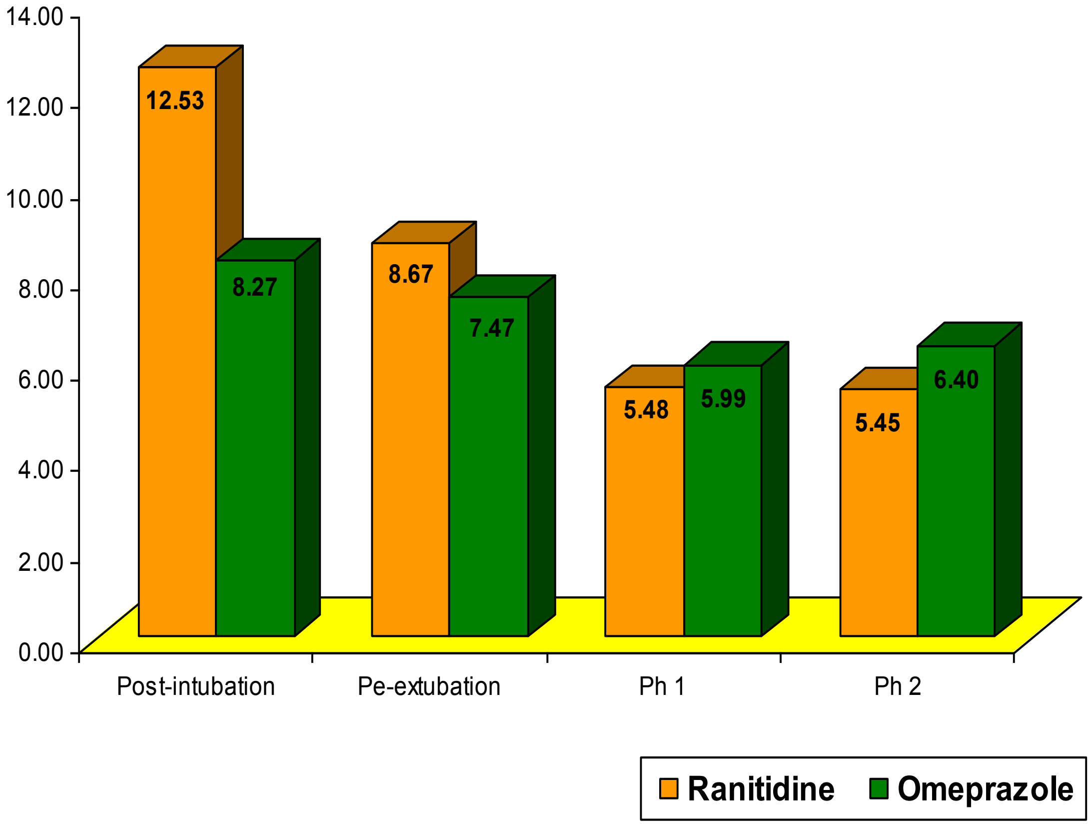
Categorical data

Bar Chart

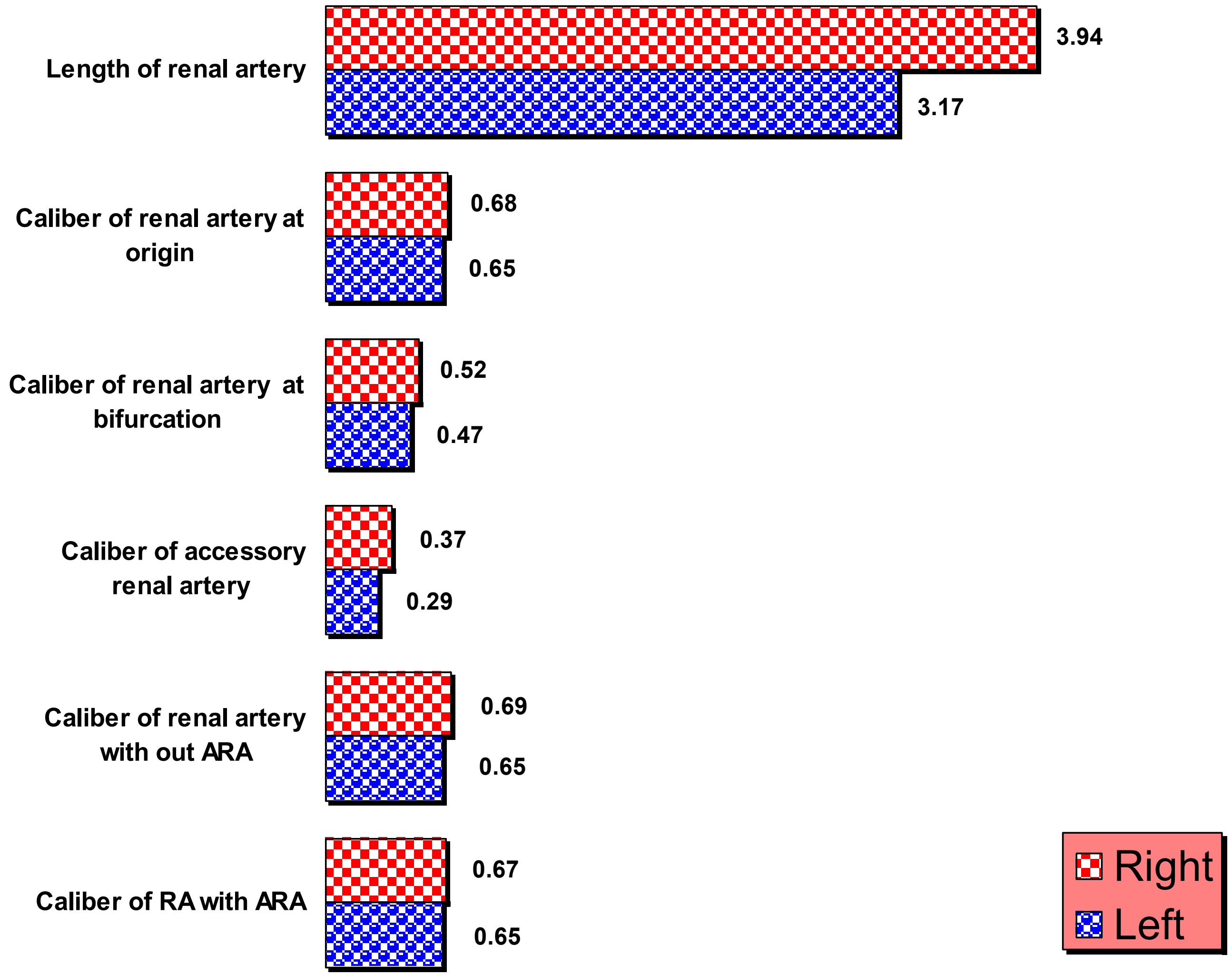
Multiple Bar Chart

Graphical Illustrations

Exploring data



14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
12.53
8.27
8.67
7.47
5.48
5.99
5.45
6.40
Post-intubation
Pe-extubation
Ph 1
Ph 2
Ranitidine
Omeprazole



Length of renal artery
3.94
3.17
Caliber of renal artery at origin
0.68
0.65
Caliber of renal artery at bifurcation
0.52
0.47
Caliber of accessory renal artery
0.37
0.29
Caliber of renal artery with out ARA
0.69
0.65
Caliber of RA with ARA
0.67
0.65
Right
Left

Exploring data

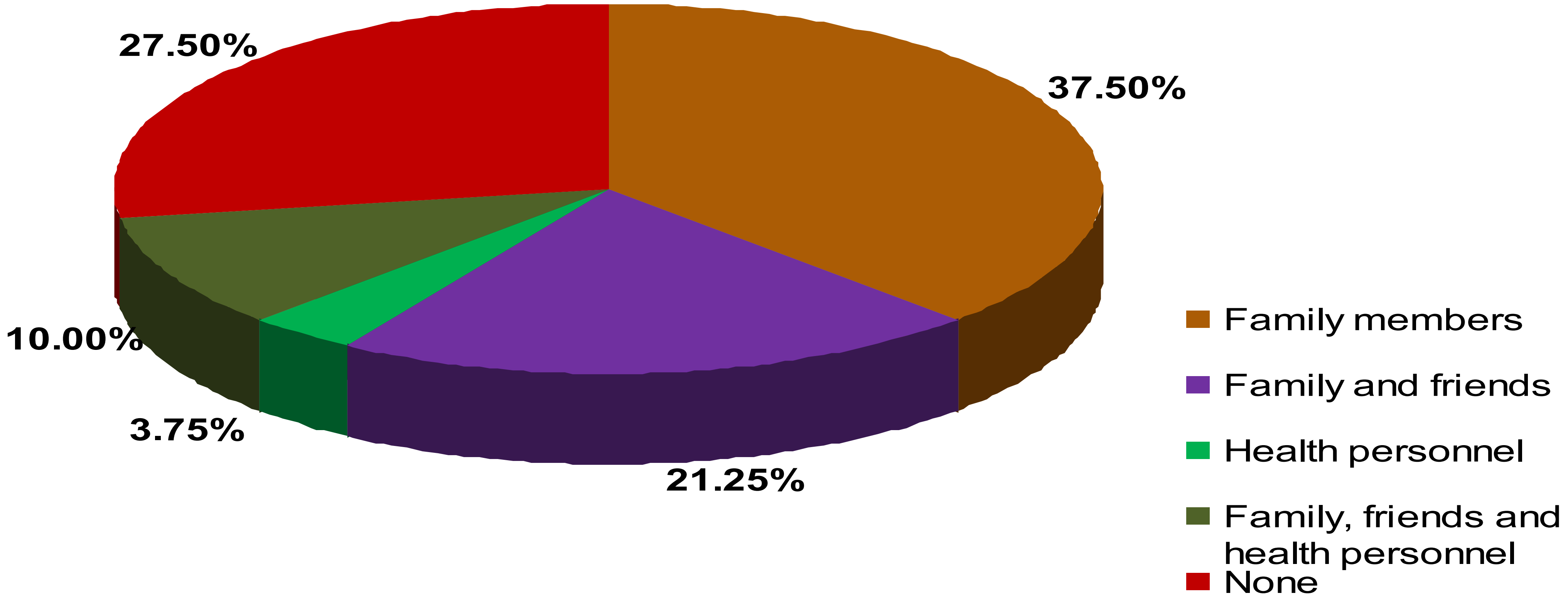
Graphical Illustrations

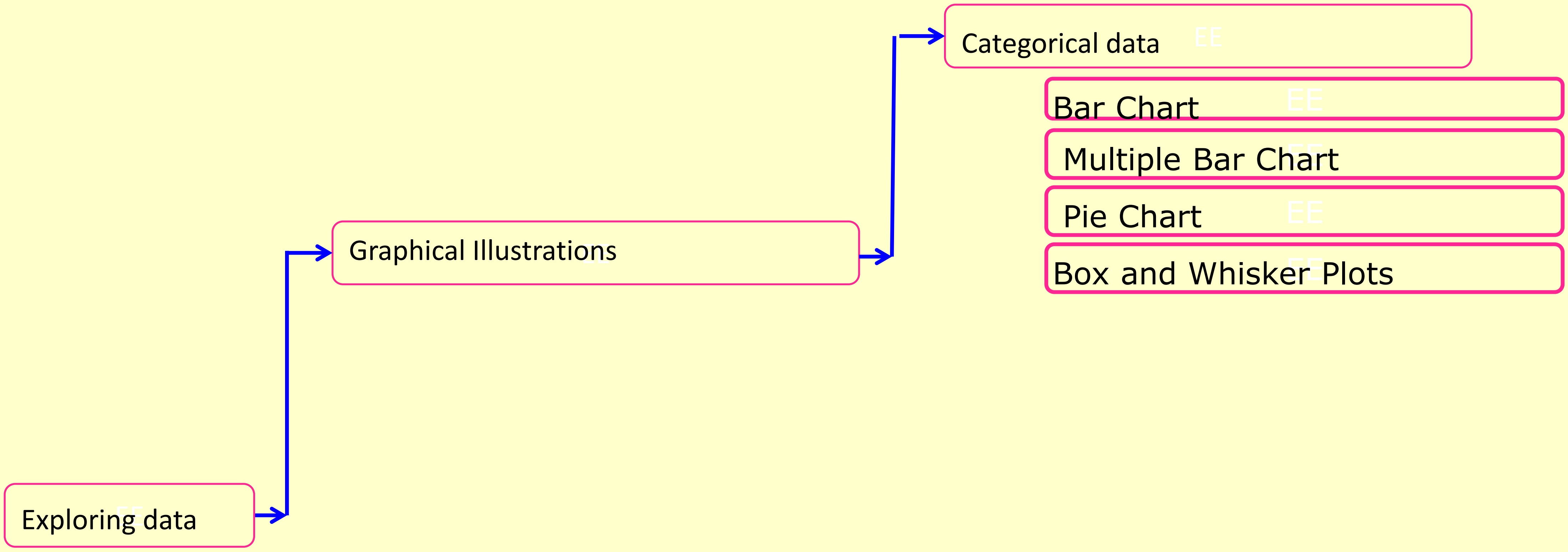
Categorical data

Bar Chart

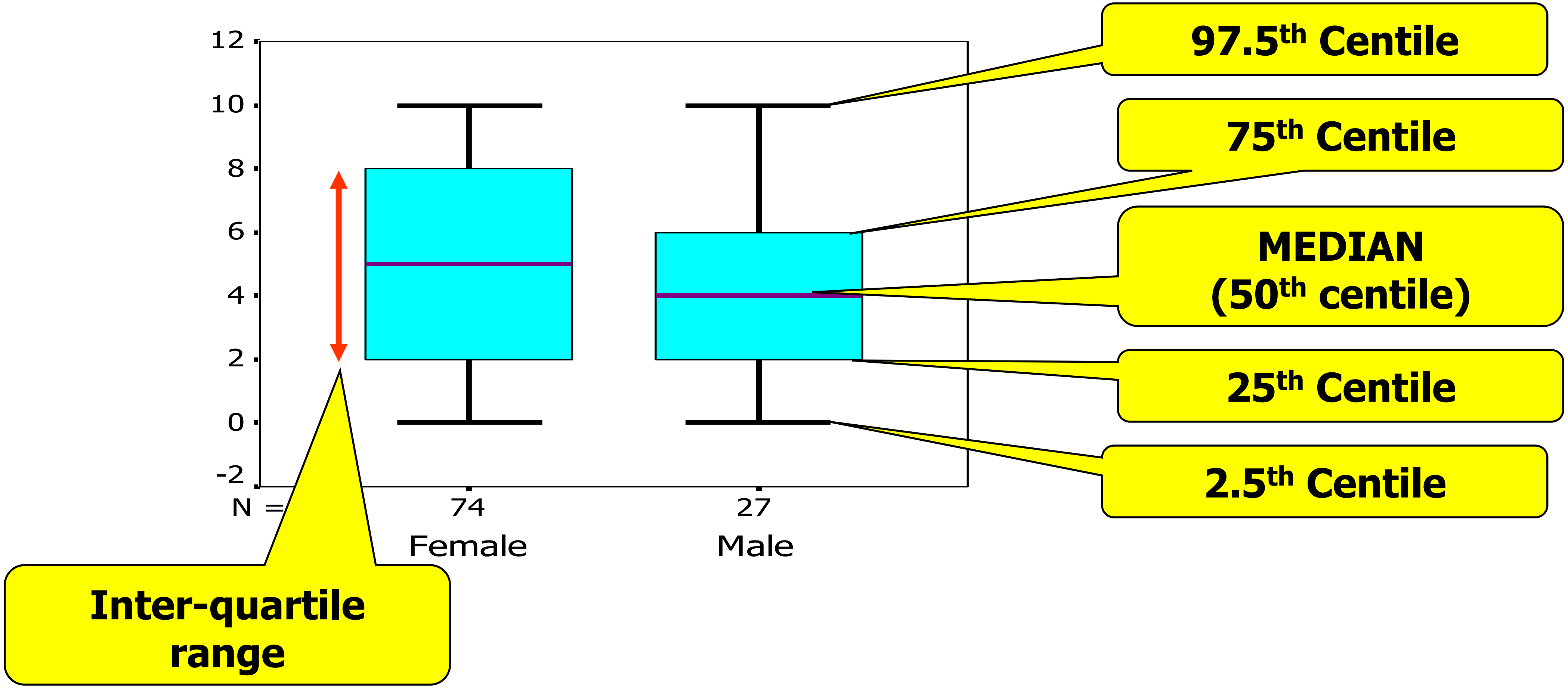
Multiple Bar Chart

Pie Chart

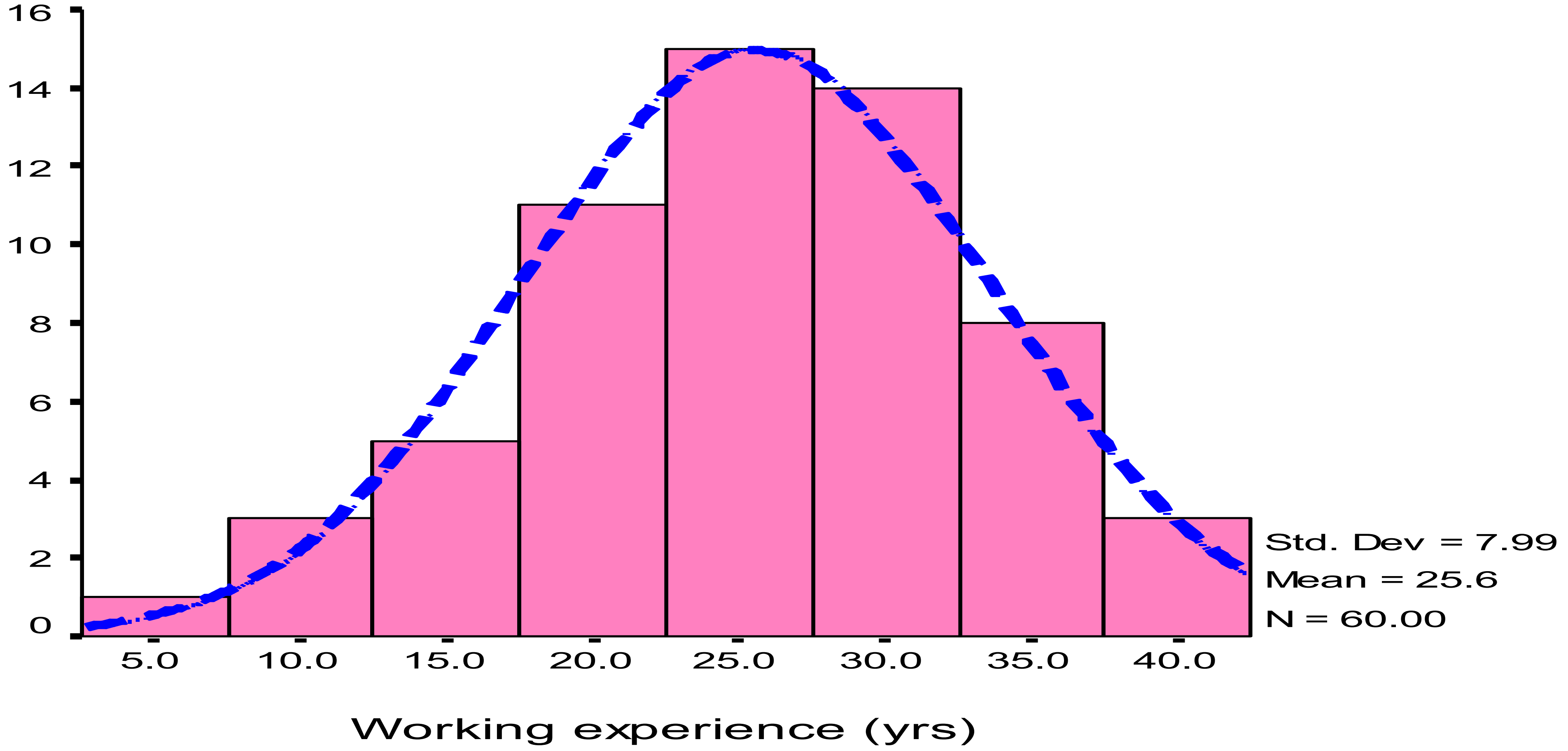

27.50%
37.50%
10.00%
3.75%
21.25%
Family members
Family and friends
Health personnel
Family, friends and health personnel
None


Exploring data
Graphical Illustrations
Categorical data
Bar Chart
Multiple Bar Chart
Pie Chart
Box and Whisker Plots

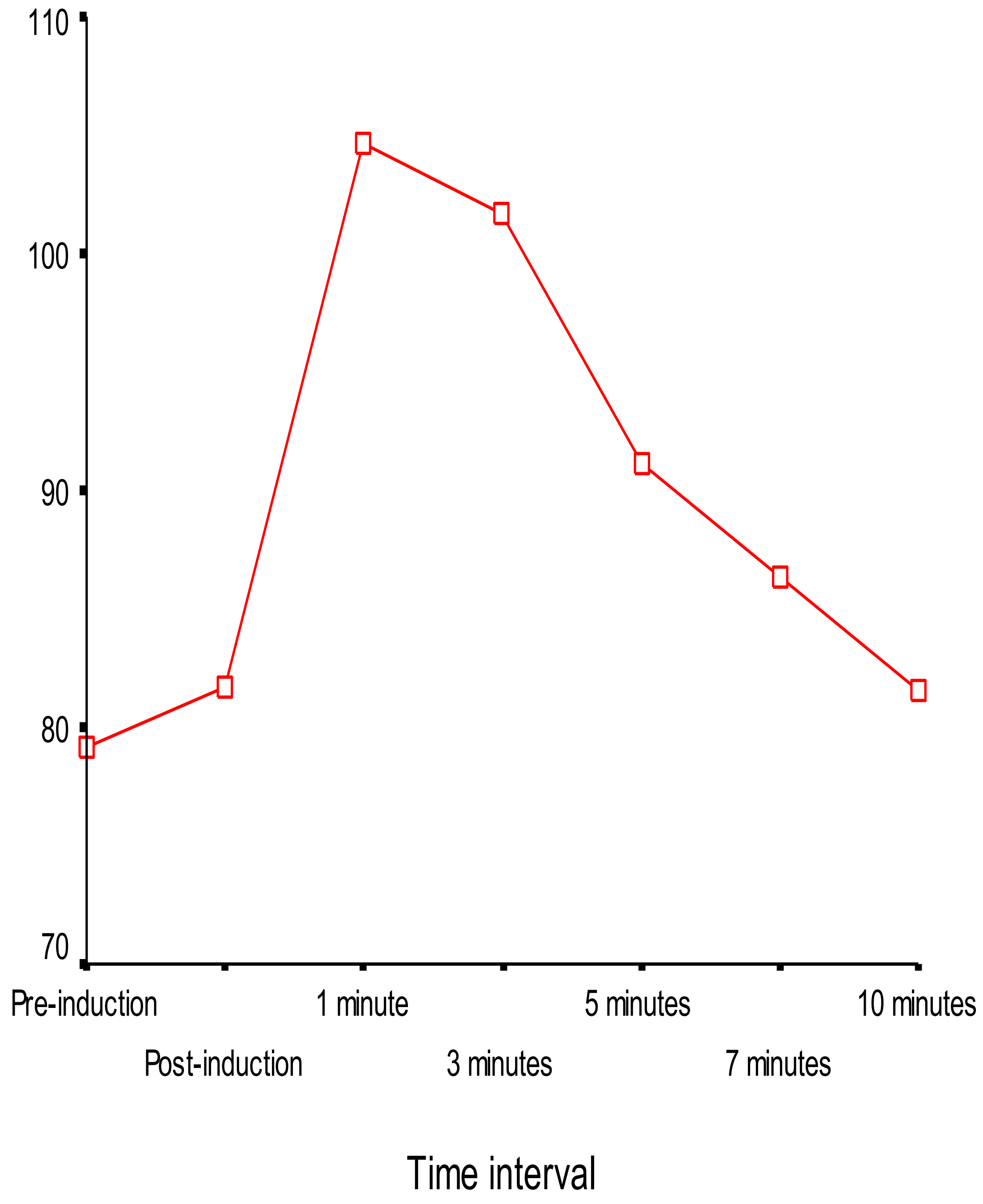
Boxplot (Box and Whisker plot)

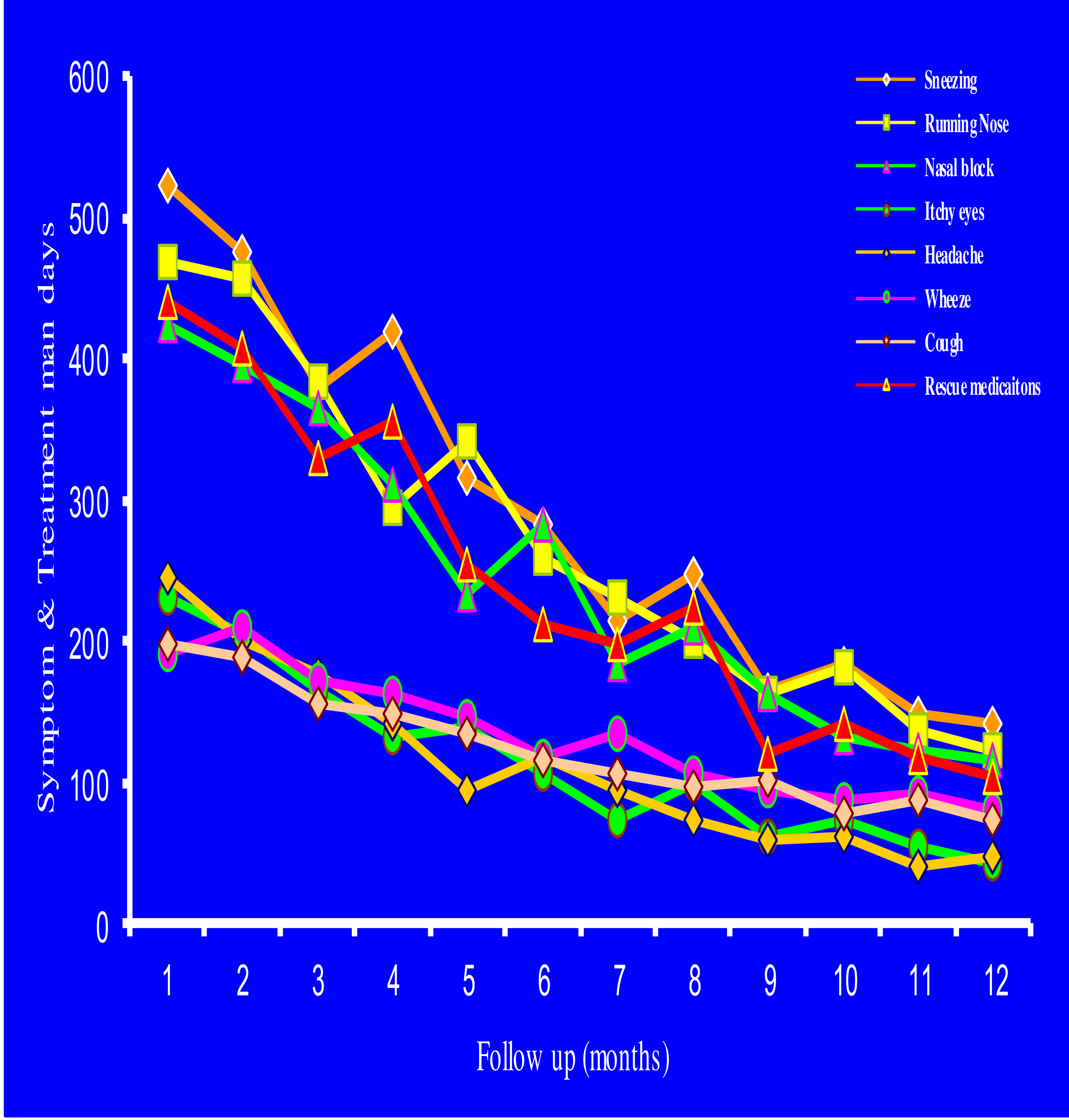


- Exploring data
  - Graphical Illustrations
    - Categorical data
      - Bar Chart
      - Multiple Bar Chart
      - Pie Chart
      - Box and Whisker plots
    - Continuous data
      - Histogram


16
14
12
10
8
6
4
2
0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
Std. Dev = 7.99
Mean = 25.6
N = 60.00
Working experience (yrs)

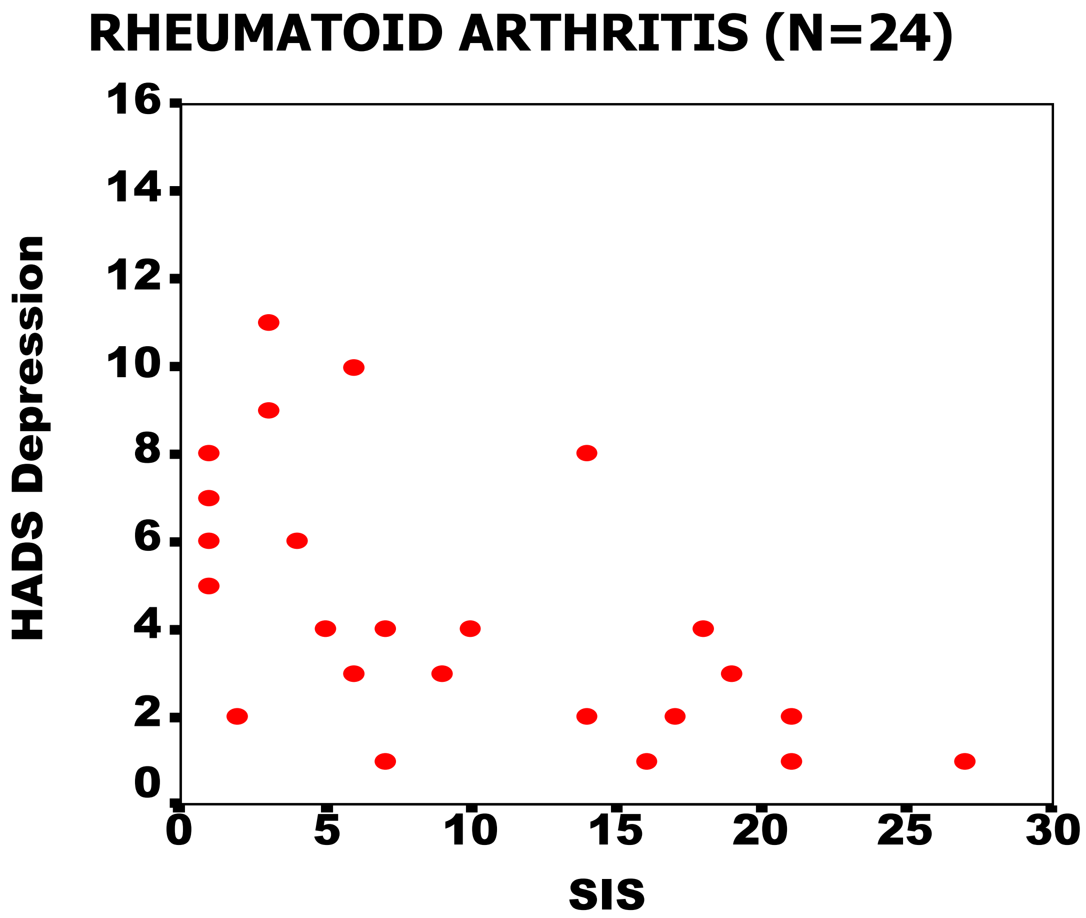
- Exploring data
  - Graphical Illustrations
    - Categorical data
      - Bar Chart
      - Multiple Bar Chart
      - Pie Chart
      - Box & Whisker plot
    - Continuous data
      - Histogram
      - Line graphs


110
100
90
80
70
Pre-induction
Post-induction
1 minute
3 minutes
5 minutes
7 minutes
10 minutes
Time interval


600
500
400
300
200
100
0
Symptom & Treatment man days
1
2
3
4
5
6
7
8
9
10
11
12
Follow up (months)
Sneezing
Running Nose
Nasal block
Itchy eyes
Headache
Wheeze
Cough
Rescue medicaitons

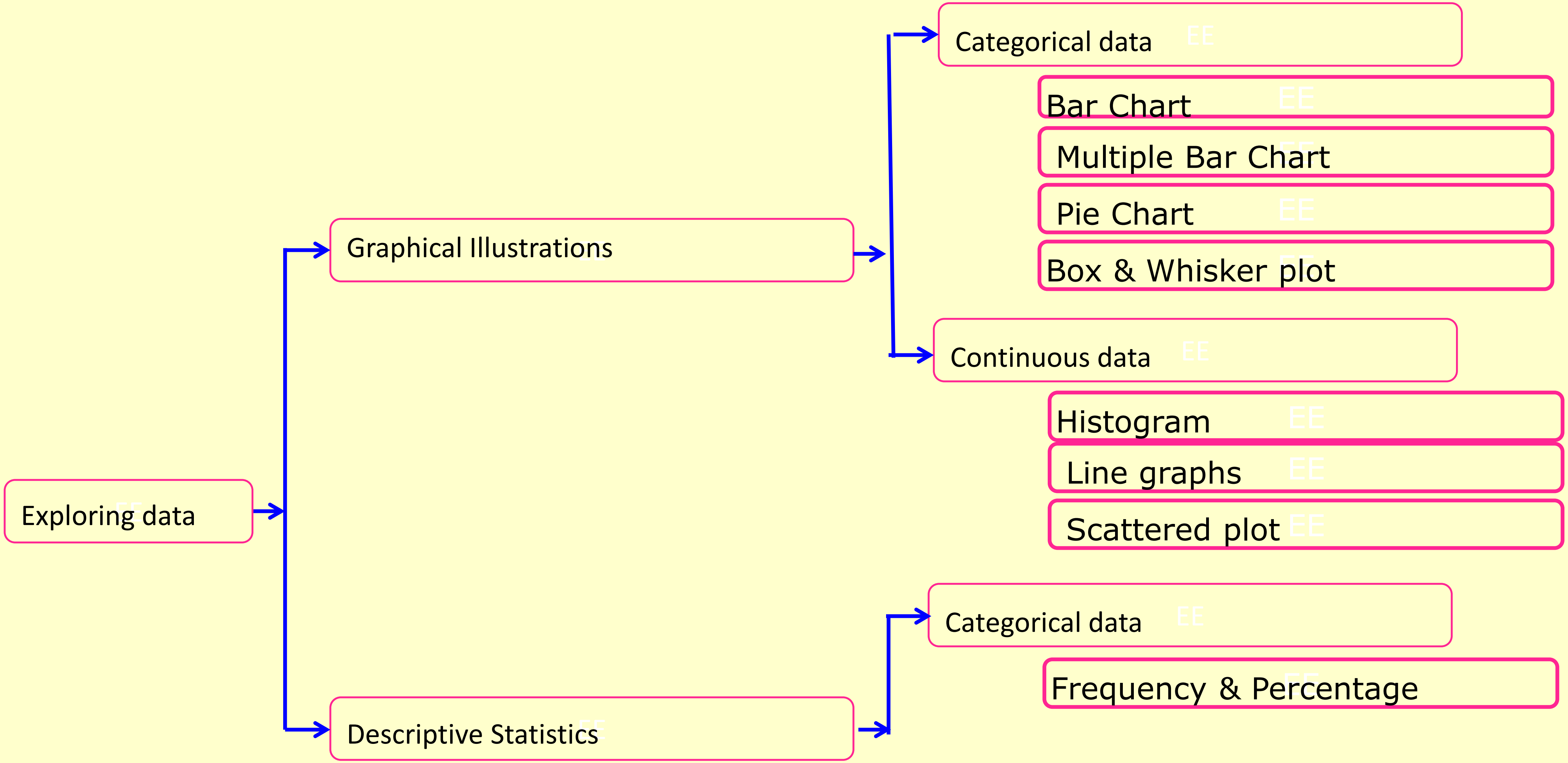
- Exploring data
  - Graphical Illustrations
    - Categorical data
      - Bar Chart
      - Multiple Bar Chart
      - Pie Chart
      - Box & Whisker plot
    - Continuous data
      - Histogram
      - Line graphs
      - Scattered plot

Correlation

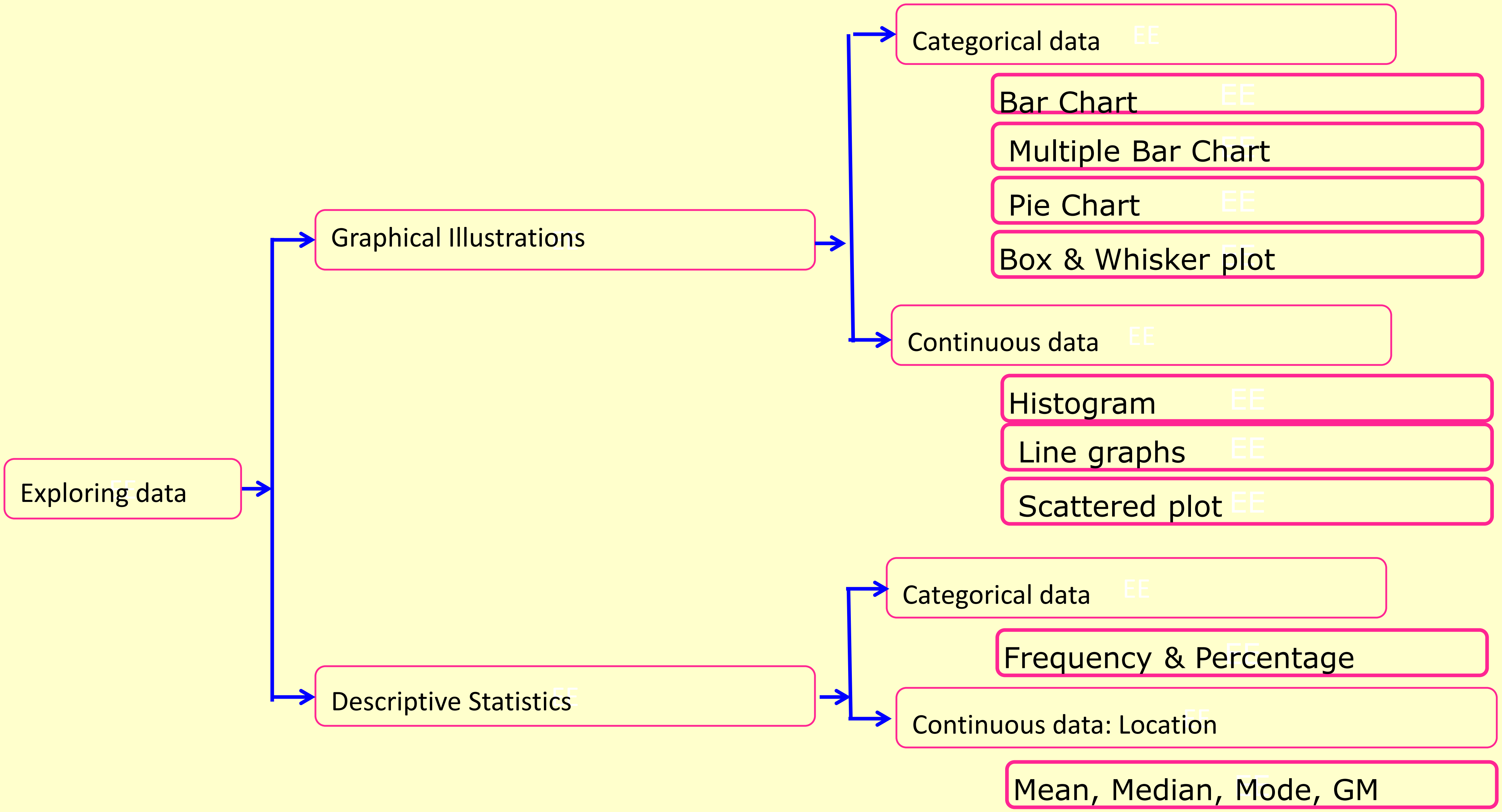


Here, there are two variables (HADS depression score and SIS) plotted against each other

The question is –
do HADS scores correlate with SIS ratings?


Exploring data
Graphical Illustrations
Categorical data
Bar Chart
Multiple Bar Chart
Pie Chart
Box & Whisker plot
Continuous data
Histogram
Line graphs
Scattered plot
Descriptive Statistics
Categorical data
Frequency & Percentage

Religion	Group A	Group B	Total
Hindu	26	22	48
Christian	11	5	16
Muslim	9	9	18
Others	4	4	8
Total	50	40	90


Exploring data
Graphical Illustrations
Categorical data
Bar Chart
Multiple Bar Chart
Pie Chart
Box & Whisker plot
Continuous data
Histogram
Line graphs
Scattered plot
Descriptive Statistics
Categorical data
Frequency & Percentage
Continuous data: Location
Mean, Median, Mode, GM

Measures of central tendency

- Mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

- Median

$$\text{Median} = \frac{(n+1)}{2}, \text{ if } n \text{ is odd}$$

$$\text{Median} = \frac{\left(\frac{n}{2}\right) + \left(\frac{n}{2} + 1\right)}{2}, \text{ if } n \text{ is even}$$

- Mode: Highest frequency

$$GM = \text{Antilog}\left(\frac{\sum_{i=1}^{n} \log x_i}{n}\right)$$

- Geometric Mean

- Exploring data
  - Graphical Illustrations
    - Categorical data
      - Bar Chart
      - Multiple Bar Chart
      - Pie Chart
      - Box & Whisker plot
    - Continuous data
      - Histogram
      - Line graphs
      - Scattered plot
  - Descriptive Statistics
    - Categorical data
      - Frequency & Percentage
    - Continuous data: Location
      - Mean, Median, Mode, GM
    - Continuous data: Variation
      - Range, SD, Variance, IQR

Measures of variation

- Range: $$R = X_{min} - X_{max}$$

- Standard Deviation: $$s = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n-1}}$$

- Variance: $$s^2 = \frac{\sum (x_i - \overline{x})^2}{n-1}$$

- Inter Quartile Range: $$IQR = Q_3 - Q_1$$

Choosing appropriate Statistical Test

Type of statistical analysis:

It is based the

Study design

and the

Type of variable

measured

If the study design is **COMPARATIVE** and the variables measured are **INDEPENDENT & QUALITATIVE** apply **Z-test** For testing the difference between two proportions

If the study design is

COMPARATIVE

and the variables measured are

INDEPENDENT & QUANTITATIVE

Then there are various

Statistical test

to apply

THANK YOU

Manipal ProLearn
#7, Service Road, Pragathi Nagar, Electronic City,
Bengaluru 560100
contact@manipalprolearn.com | manipalprolearn.com

Highlights of the program

- **First-of-its-kind program in the Data Science space**, equipping learners with competencies and skills boosting their visibility and credibility for future employment prospects.
- **Industry-relevant course curriculum**, with applications in multiple domains, where such talent is in demand
- **Highly experienced Subject Matter Experts (SMEs)** from academia, IT and Data Science industry
- **Enhanced learning experience** through the digital LMS - **EduNxt**
- State-of-the-art infrastructure, latest technology and a well-equipped, 77,000 square feet residential campus.

Highlights of the program

- Delivery Models: Online, blended, face to face
- Domain Expertise in Banking, Retail, Healthcare
- Industry Partnerships with Genpact, IBM-BDU, Coursera
- Centre Of Excellence with Deakin University
- Academic Partnership with Manipal University