experiments

November 8, 2023

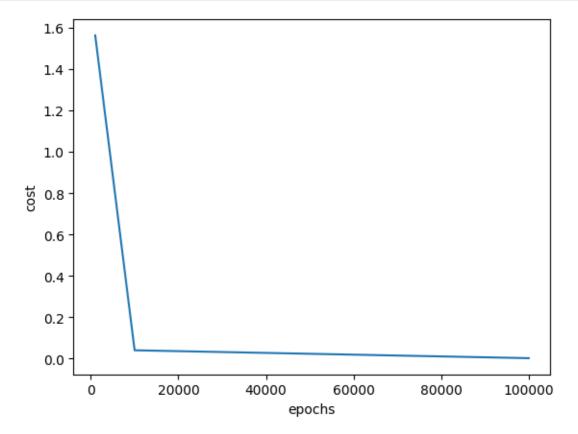
```
[186]: import numpy as np
      import ann
      from activation functions import sigmoid
      from cost_functions import squared_error, mean_squared_error
      from number generation import zeros, normal
      import matplotlib.pyplot as plt
[187]: training_examples = np.identity(8)
      number_generation_kwargs = {'mu': 0, 'sigma': 0.1}
      costs = []
      Model 1 (1000 epochs)
[188]: ann_model_1 = ann.ANN(8, [3], 8, sigmoid, squared_error, normal, zeros,
        →add_bias=True, **number_generation_kwargs)
      ann_model_1.train(training_examples, training_examples, 1000, 0.1)
[189]: predictions = ann_model_1.feed_forward(training_examples)
      for i, prediction in enumerate(predictions):
          print(f"input: {training_examples[i]}, output: {np.around(prediction, 2)}")
      cost = round(mean_squared_error(predictions, training_examples), 5)
      costs.append(cost)
      print(f"cost: {cost}")
      input: [1. 0. 0. 0. 0. 0. 0. 0.], output: [0.15 0.12 0.12 0.1 0.14 0.1 0.2
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0.14 0.13 0.11 0.12 0.15 0.13 0.15
      0.11]
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0.14 0.13 0.2 0.17 0.13 0.17 0.19
      0.15]
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0.11 0.13 0.15 0.21 0.13 0.26 0.1
      input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0.15 0.14 0.12 0.12 0.18 0.13 0.19
      0.11]
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0.11 0.13 0.16 0.25 0.14 0.31 0.09
      0.14]
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0.18 0.14 0.15 0.09 0.17 0.08 0.31
```

```
0.13]
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0.13 0.12 0.14 0.14 0.12 0.15 0.14
      0.137
      cost: 1.56273
[190]: for i, layer in enumerate(ann_model_1.network):
          print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
          print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
          print()
      layer 1 weights:
      [[ 1.
              0.61 -0.69 -1.07 0.84 -1.41 1.21 -0.13]
       [ 0.43 -0.04  0.45 -0.1  -0.63 -0.58  0.2
       [-0.08 0.44 -0.64 0.8 0.1 1.08 -1.42 0.18]]
      layer 1 biases:
      layer 2 weights:
      [[ 0.43 -0.22 -0.78]
       [0.03 - 0.74 - 0.25]
       [-1.15 -0.08 -1.19]
       [-1.6 -0.65 0.28]
       [ 0.36 -1.26 -0.52]
       [-1.89 -1.05 0.66]
       [ 0.85 -0.29 -2.03]
       [-0.73 0.02 -0.5]]
      layer 2 biases:
      [[-1.55]
       [-1.41]
       [-0.55]
       [-0.78]
       [-1.06]
       [-0.53]
       [-0.88]
       [-1.3]]
      Model 2 (10 000 epochs)
[191]: ann model_2 = ann.ANN(8, [3], 8, sigmoid, squared_error, normal, zeros,
       →add_bias=True, **number_generation_kwargs)
      ann model 2.train(training examples, training examples, 10000, 0.1)
[192]: predictions = ann_model_2.feed_forward(training_examples)
      for i, prediction in enumerate(predictions):
          print(f"input: {training_examples[i]}, output: {np.around(prediction, 2)}")
      cost = round(mean_squared_error(predictions, training_examples), 5)
      costs.append(cost)
      print(f"cost: {cost}")
```

```
input: [1. 0. 0. 0. 0. 0. 0. 0.], output: [0.93 0. 0. 0.05 0.05 0.
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0. 0.93 0.05 0.05 0.
                                                                            0.06 0.
                                                                       0.
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0. 0.04 0.91 0. 0.
                                                                       0.11 0.
      0.031
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0.05 0.05 0.
      0.061
      input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0.04 0.
                                                         0. 0.
                                                                 0.91 0.11 0.
      0.047
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0. 0. 0.05 0. 0.06 0.83 0.05 0.
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0.04 0.03 0. 0. 0. 0.11 0.91 0.
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0. 0. 0.06 0.05 0.06 0.
      0.931
      cost: 0.04051
[193]: for i, layer in enumerate(ann_model_2.network):
          print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
          print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
          print()
      layer 1 weights:
      [[ 3.06 -3.85 -4.
                         -3.29 3.82 3.63 3.39 -3.55]
      [-3.64 -3.39 3.87 -3.87 3.63 3.64 -3.98 2.48]
       [-3.53 3.01 2.95 -3.36 -3.73 4.06 3.62 -4.11]]
      layer 1 biases:
      0
      layer 2 weights:
      [[ 5.95 -6.33 -6.31]
      [-6.55 -6.28 5.94]
       [-5.64 5.52 5.06]
       [-5.77 -6.08 -5.8]
       [ 5.18 5.32 -5.31]
       [ 3.73 3.77 3.87]
       [ 5.22 -5.57 5.28]
       [-6.36 5.76 -6.88]]
      layer 2 biases:
      [[-2.74]
      [-2.68]
       [-7.76]
       [ 2.84]
       [-7.8]
       [-9.53]
       [-7.76]
       [-2.44]]
```

```
Model 3 (100 000 epochs)
[194]: ann model_3 = ann.ANN(8, [3], 8, sigmoid, squared_error, normal, zeros,
       →add_bias=True, **number_generation_kwargs)
      ann model 3.train(training examples, training examples, 100000, 0.1)
[195]: predictions = ann_model_3.feed_forward(training_examples)
      for i, prediction in enumerate(predictions):
          print(f"input: {training_examples[i]}, output: {np.around(prediction, 2)}")
      cost = round(mean squared error(predictions, training examples), 5)
      costs.append(cost)
      print(f"cost: {cost}")
      input: [1. 0. 0. 0. 0. 0. 0. 0.], output: [0.98 0.02 0.
                                                              0.
                                                                   0.01 0.
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0.01 0.98 0. 0.01 0.
                                                                        0.
                                                                             0.02 0.
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0. 0. 0.98 0.01 0.
                                                                        0.01 0.02 0.
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0. 0.01 0.01 0.98 0.02 0.
      input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0.02 0. 0. 0.01 0.97 0.01 0.
                                                                                  0.
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0. 0.
                                                         0.01 0. 0.01 0.98 0.
      0.017
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0. 0.01 0.01 0.
                                                                   0.
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0.01 0. 0. 0.
                                                                        0.01 0.02
      cost: 0.00228
[196]: for i, layer in enumerate(ann model 3.network):
          print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
          print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
          print()
      layer 1 weights:
      [[-5.33 1.09 2.86 4.89 -1.48 -4.27 5.21 -4.83]
       [-4.77 -5.02  4.21 -4.02 -1.77  4.72  4.64  2.11]
       [ 0.44 4.95 -4.91 -1.24 -6.07 -1.57 4.88 4.93]]
      layer 1 biases:
      layer 2 weights:
      [[-13.86 -11.82
                      5.73]
       [ 4.31 -9.47 11.55]
       [7.14 6.73 -9.48]
       [ 10.43 -9.92 -7.37]
```

```
[ -6.19 -7.14 -16.3 ]
 [-10.64
          9.44 -8.94]
 [ 7.28
                7.32]
          5.96
 [ -8.92
          4.54 10.26]]
layer 2 biases:
[[ 0.36]
 [-10.9]
 [-9.34]
 [-4.67]
 [ 5.78]
 [-3.76]
 [-16.78]
 [-10.34]]
```



Since the cost only decreased by 0.02792. Somewhere around 10 000 epochs lays the optimal amount. So for the next model 10 000 epochs will be used.

```
Model 4 (No bias)
```

```
[197]: ann_model_4 = ann.ANN(8, [3], 8, sigmoid, squared_error, normal, zeros,
        →add_bias=False, **number_generation_kwargs)
      ann model 4.train(training examples, training examples, 10000, 0.1)
[198]: predictions = ann model 4.feed forward(training examples)
      for i, prediction in enumerate(predictions):
          print(f"input: {training examples[i]}, output: {np.around(prediction, 2)}")
      cost = round(mean_squared_error(predictions, training_examples), 5)
      print(f"cost: {cost}")
      input: [1. 0. 0. 0. 0. 0. 0.], output: [0.88 0.
                                                          0.
                                                                              0.09
                                                               0.06 0.06 0.
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0. 0.88 0.06 0.06 0.06 0.
      0.071
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0. 0.09 0.88 0.09 0.09 0.09 0.
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0.07 0.07 0.08 0.28 0.28 0.06 0.09
      input: [0. 0. 0. 0. 1. 0. 0.], output: [0.06 0.07 0.08 0.28 0.28 0.06 0.09
      0.081
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0. 0. 0.08 0.06 0.06 0.88 0.07 0.
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0.1 0. 0. 0.09 0.09 0.1 0.86 0.
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0.08 0.09 0. 0.09 0.09 0.
      0.881
      cost: 0.37656
[199]: for i, layer in enumerate(ann_model_4.network):
          print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
          print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
          print()
      layer 1 weights:
      [[-4.47 4.55 4.82 -1.36 -1.35 1.48 -1.83 -0.53]
       [ 1.69 -4.79 -0.55 -1.37 -1.38 4.83 4.47 -1.71]
       [ 4.66  1.22  -2.04  -1.4  -1.4  -4.82  -0.58  4.88]]
      layer 1 biases:
      0
      layer 2 weights:
      [[-14.91 -1.36 3.34]
       [ 3.15 -14.53 -1.29]
```

```
[ 5.91 -7.21 -11.07]
       [ -1.56 -1.53 -1.48]
       [-1.54 -1.54 -1.49]
       [-1.82]
               3.58 - 15.34
       [-10.23 5.92 -7.3]
       [ -7.51 -10.52
                      6.42]]
      layer 2 biases:
      0
      The cost is 0.52 for the model without bias with the same amount of epochs.
      Model 5 (learning rate 100)
[200]: ann_model_5 = ann.ANN(8, [3], 8, sigmoid, squared_error, normal, zeros,
       →add bias=True, **number generation kwargs)
      ann model_5.train(training_examples, training_examples, 10000, 100)
[201]: predictions = ann_model_5.feed_forward(training_examples)
      for i, prediction in enumerate(predictions):
          print(f"input: {training_examples[i]}, output: {np.around(prediction, 2)}")
      cost = round(mean_squared_error(predictions, training_examples), 5)
      print(f"cost: {cost}")
      input: [1. 0. 0. 0. 0. 0. 0. 0.], output: [0. 0. 0. 0. 0. 1. 0. 0.]
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0. 0. 0. 0. 0. 1. 0. 0.]
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0. 0. 0. 0. 0. 1. 0. 0.]
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0. 0. 0. 0. 0. 1. 0. 0.]
      input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0. 0. 0. 0. 0. 1. 0. 0.]
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0. 0. 0. 0. 0. 1. 0. 0.]
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0. 0. 0. 0. 0. 1. 0. 0.]
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0. 0. 0. 0. 0. 1. 0. 0.]
      cost: 3.5
[202]: for i, layer in enumerate(ann_model_5.network):
           print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
          print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
          print()
      layer 1 weights:
      [[ 0.07  0.01  0.21  0.14 -0.15 -1.24 -0.03  0.04]
       [-0.1 -0.01 -0.12 -0.09 0.02 0.11 -0.02 -0.03]
       [ 0.19  0.07  0.05 -0.08 -0.09 -0.26  0.04 -0.01]]
      layer 1 biases:
      layer 2 weights:
      [[-6.12 -6.97 -6.75]
       [-6.11 -6.63 -6.36]
```

[-6.06 -6.7 -6.38]

```
[-5.94 -6.42 -6.29]
       [5.87 6.48 6.07]
       [-6.63 -6.96 -6.71]
       [-5.82 -6.28 -6.08]]
      layer 2 biases:
      [[-13.27]
       [-12.62]
       [-12.66]
       [-12.55]
       [-12.24]
       [ 12.06]
       [-13.66]
       [-11.86]]
      Model 6 (learning rate 10)
[203]: ann_model_6 = ann.ANN(8, [3], 8, sigmoid, squared_error, normal, zeros,
        →add_bias=True, **number_generation_kwargs)
       ann_model_6.train(training_examples, training_examples, 10000, 10)
[204]: predictions = ann_model_6.feed_forward(training_examples)
       for i, prediction in enumerate(predictions):
           print(f"input: {training_examples[i]}, output: {np.around(prediction, 2)}")
       cost = round(mean_squared_error(predictions, training_examples), 5)
       print(f"cost: {cost}")
      input: [1. 0. 0. 0. 0. 0. 0. 0.], output: [0.99 0.
                                                                           0.
                                                                                0.
                                                           0.
                                                                0.
                                                                     0.
                                                                                     0.
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0. 0.99 0.01 0.01 0.
                                                                                     0.
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0.01 0.
                                                           0.99 0.
                                                                     0.
                                                                           0.
                                                                                0.
                                                                                     0.
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0.
                                                                0.99 0.
                                                      0.01 0.
                                                                                     0.
      input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0.
                                                      0.01 0.
                                                                0.
                                                                     0.99 0.01 0.
                                                                                     0.
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0.
                                                      0.
                                                           0.
                                                                0.
                                                                     0.
                                                                           0.99 0.
                                                                                     0.
      input: [0. 0. 0. 0. 0. 1. 0.], output: [0.
                                                                                0.99 0.
                                                      0.
                                                           0.
                                                                0.
                                                                     0.
                                                                           0.
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0.
                                                                                0.01
                                                      0.
                                                           0.
                                                                0.
                                                                     0.
                                                                           0.
      0.991
      cost: 0.00023
[205]: for i, layer in enumerate(ann_model_6.network):
           print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
           print()
```

[-6.03 -6.58 -6.41]

```
layer 1 weights:
      [[ 6.34 -7.33 -0.21 -0.84 -0.94 5.22 5.24 -1.71]
       [ 0.39 -0.72 -5.71 5.63 -5.54 -4.84 5.14 2.55]
       [ 5.61 0.31 3.38 4.63 -6. -1.56 -4.53 -5.46]]
      layer 2 weights:
      [[ 15.28
                 1.74 10.99]
       [-29.8 -4.47]
                       5.017
       [ -1.8 -17.27 13.52]
       [ -7.91 11.95 10.94]
       [-10.65 -13.41 -15.84]
       [ 15.66 -11.94 -5.05]
       [ 11.64 9.59 -7.32]
       [-13.6
                10.76 -11.49]]
      Model 7 (learning rate 0.01)
[206]: ann_model_7 = ann.ANN(8, [3], 8, sigmoid, squared_error, normal, zeros,
        →add_bias=True, **number_generation_kwargs)
      ann_model_7.train(training_examples, training_examples, 10000, 0.01)
[207]: predictions = ann_model_7.feed_forward(training_examples)
      for i, prediction in enumerate(predictions):
          print(f"input: {training_examples[i]}, output: {np.around(prediction, 2)}")
      cost = round(mean_squared_error(predictions, training_examples), 5)
      print(f"cost: {cost}")
      input: [1. 0. 0. 0. 0. 0. 0. 0.], output: [0.18 0.09 0.14 0.17 0.09 0.19 0.14
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0.1 0.43 0.11 0.16 0.36 0.06 0.16
      0.12]
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0.14 0.09 0.17 0.12 0.14 0.27 0.11
      0.13]
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0.18 0.15 0.14 0.21 0.12 0.14 0.17
      input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0.07 0.32 0.11 0.09 0.42 0.08 0.11
      0.09]
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0.17 0.05 0.22 0.11 0.1 0.47 0.1
      0.13]
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0.14 0.14 0.11 0.16 0.14 0.11 0.15
      0.14]
      input: [0. 0. 0. 0. 0. 0. 0. 1.], output: [0.16 0.1 0.13 0.16 0.11 0.14 0.14
      0.15]
      cost: 1.40664
[208]: for i, layer in enumerate(ann_model_7.network):
          print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
          print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
```

```
print()
layer 1 weights:
[[-0.54 -0.43  0.67 -1.28  1.26  1.21 -0.57 -0.5]
[ 1.1 -1.96 0.49 0.26 -1.3
                             1.06 0.56 0.92]]
layer 1 biases:
0
layer 2 weights:
[[-0.96 -0.95 0.45]
[-0.69 0.97 -2.83]
[ 0.29 -1.36 -0.22]
[-1.67 -0.64 -0.36]
[ 0.96 1.06 -2.1 ]
[ 0.96 -2.89 0.46]
[-1.05 -0.03 -0.24]
[-0.97 -0.47 0.19]]
layer 2 biases:
[[-1.11]
[-0.42]
[-1.14]
Γ-0.431
[-1.51]
[-0.89]
[-1.21]
[-1.25]
```

With a learning rate of 100 the gradient overshoots and does not learn correctly, with a rate of 10 it learns almost perfectly within 10 000 epochs, and with a learning rate of 0.01 it didn't converge within 10 000 epochs.

Model 8 (weights initialized as 0, 1000 epochs)

```
input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
     0.12]
     input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
     0.12]
     input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
     0.127
     input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
     0.127
     input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
     0.12]
     input: [0. 0. 0. 0. 0. 0. 0. 1.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
     0.12
     cost: 1.75001
[211]: for i, layer in enumerate(ann_model_8.network):
         print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
         print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
         print()
     layer 1 weights:
     layer 1 biases:
     0
     layer 2 weights:
     [[-0.55 - 0.55 - 0.55]
      [-0.55 - 0.55 - 0.55]
      [-0.55 - 0.55 - 0.55]
      [-0.55 -0.55 -0.55]
      [-0.55 -0.55 -0.55]
      [-0.55 -0.55 -0.55]
      [-0.55 - 0.55 - 0.55]
      [-0.55 -0.55 -0.55]]
     layer 2 biases:
     [[-1.09]
      [-1.09]
      [-1.09]
      [-1.1]
      [-1.09]
      [-1.1]
      [-1.09]
      [-1.09]]
```

Model 9 (weights initialized as 0, 10000 epochs)

```
[212]: ann_model_9 = ann.ANN(8, [3], 8, sigmoid, squared_error, zeros, zeros,
        →add_bias=True, **number_generation_kwargs)
      ann model 9.train(training examples, training examples, 10000, 0.1)
[213]: predictions = ann model 9.feed forward(training examples)
      for i, prediction in enumerate(predictions):
          print(f"input: {training_examples[i]}, output: {np.around(prediction, 2)}")
      cost = round(mean_squared_error(predictions, training_examples), 5)
      print(f"cost: {cost}")
      input: [1. 0. 0. 0. 0. 0. 0. 0.], output: [0.18 0.1 0.18 0.18 0.09 0.
                                                                               0.09
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0.09 0.15 0.09 0.09 0.
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0.18 0.1 0.18 0.18 0.09 0.
                                                                               0.09
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0.18 0.1 0.18 0.18 0.09 0.
                                                                               0.09
      0.071
      input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0.25 0.08 0.25 0.25 0.83 0.
                                                                               0.07
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0.05 0.2 0.05 0.05 0.
                                                                          0.79 0.23
      0.27
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0.08 0.15 0.08 0.08 0.
                                                                         0.12 0.16
      0.16]
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0.08 0.16 0.08 0.08 0.
                                                                         0.17 0.16
      0.17]
      cost: 1.28603
[214]: for i, layer in enumerate(ann_model_9.network):
          print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
          print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
          print()
      layer 1 weights:
      [[-1.22 0.47 -1.22 -1.22 -3.65 3.37 0.62 0.78]
       [-1.22 0.47 -1.22 -1.22 -3.65 3.37 0.62 0.78]
       [-1.22 0.47 -1.22 -1.22 -3.65 3.37 0.62 0.78]]
      layer 1 biases:
      0
      layer 2 weights:
      [[-0.68 -0.68 -0.68]
       [ 0.36  0.36  0.36]
       [-0.67 - 0.67 - 0.67]
       [-0.68 - 0.68 - 0.68]
       [-6.39 -6.39 -6.39]
       [ 3.46 3.46 3.46]
```

```
[ 0.51 0.51 0.51]
       [ 0.73 0.73 0.73]]
      layer 2 biases:
      [[-1.07]
       [-2.43]
       [-1.07]
       [-1.06]
       [ 2.09]
       [-8.72]
       [-2.67]
       [-3.09]]
      Model 10 (weights initialized as 0, 10000 epochs, no bias)
[215]: ann_model_10 = ann.ANN(8, [3], 8, sigmoid, squared_error, zeros, zeros,
       →add bias=False, **number generation kwargs)
       ann_model_10.train(training_examples, training_examples, 10000, 0.1)
[216]: predictions = ann_model_10.feed_forward(training_examples)
       for i, prediction in enumerate(predictions):
          print(f"input: {training_examples[i]}, output: {np.around(prediction, 2)}")
       cost = round(mean_squared_error(predictions, training_examples), 5)
       print(f"cost: {cost}")
      input: [1. 0. 0. 0. 0. 0. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
      input: [0. 1. 0. 0. 0. 0. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
      0.12]
      input: [0. 0. 1. 0. 0. 0. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
      0.12]
      input: [0. 0. 0. 1. 0. 0. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
      input: [0. 0. 0. 0. 1. 0. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
      input: [0. 0. 0. 0. 0. 1. 0. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
      0.127
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0.12 0.12 0.12 0.12 0.12 0.12 0.12
      0.12]
      cost: 1.75
[217]: for i, layer in enumerate(ann_model_10.network):
           print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
          print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
          print()
```

layer 1 weights:

```
[0.24 0.24 0.24 0.24 0.24 0.24 0.24 0.24]]
      layer 1 biases:
      0
      layer 2 weights:
      [[-1.16 -1.16 -1.16]
       [-1.16 -1.16 -1.16]
       [-1.16 - 1.16 - 1.16]
       [-1.16 -1.16 -1.16]
       [-1.16 -1.16 -1.16]
       [-1.16 -1.16 -1.16]
       [-1.16 -1.16 -1.16]
       [-1.16 -1.16 -1.16]]
      layer 2 biases:
      With the weights initialized as 0 the model learns minimally, this has probably to do with the fact
      that almost all backpropagation formula's use multiplication or the dot product only the bias can
      have non zero values.
      Model 11 (regularization, train with first 6 examples)
[218]: ann_model_11 = ann.ANN(8, [3], 8, sigmoid, squared_error, normal, zeros,
        add_bias=True, **number_generation_kwargs)
       ann model 11.train(training examples[0:6], training examples[0:6], 10000, 0.1)
[219]: predictions = ann_model_11.feed_forward(training_examples[6:8])
       for i, prediction in enumerate(predictions):
           print(f"input: {training_examples[6:8][i]}, output: {np.around(prediction,__
        →2)}")
       cost = round(mean_squared_error(predictions, training_examples[6:8]), 5)
       print(f"cost: {cost}")
      input: [0. 0. 0. 0. 0. 0. 1. 0.], output: [0.03 0.03 0.02 0.04 0.03 0.02 0.
                                                                                        0.
      input: [0. 0. 0. 0. 0. 0. 1.], output: [0.01 0.02 0.03 0.04 0.03 0.03 0.
                                                                                       0.
      cost: 1.99211
[220]: for i, layer in enumerate(ann_model_11.network):
           print(f"layer {i + 1} weights: \n{np.around(layer.weights, 2)}")
           print(f"layer {i + 1} biases: \n{np.around(layer.bias, 2)}")
```

[[0.24 0.24 0.24 0.24 0.24 0.24 0.24 0.24] [0.24 0.24 0.24 0.24 0.24 0.24 0.24 0.24]

print()

layer 1 weights:

[[-3.53 -4.15 3.97 2.33 -1.46 3.27 -0.2

[-3.03 3.2 -3.85 3.72 -3.34 2.78 -0.07 -0.08]

```
[ 3.22 -0.88  0.64  4.17 -4.  -3.99  0.01 -0.09]]
layer 1 biases:
0
layer 2 weights:
[[-5.71 -4.02 5.62]
 [-7.36 5.85 -1.95]
 [ 6.79 -6.59 0.87]
 [ 1.99 4.26 5.88]
 [-2.36 -5.14 -6.88]
 [ 4.79 2.98 -6.98]
 [-1.26 -1.38 -1.44]
 [-1.37 -1.42 -1.35]]
layer 2 biases:
[[-1.94]
 [-1.94]
 [-4.22]
 [-9.12]
 [3.44]
 [-4.25]
 [-3.44]
 [-3.43]
```

The model is not able to generalize to the last 2 examples, this is probably because the model is overfitted to the first 6 examples.