Bornholdt's Model and Extensions

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Complex System Simulation

Motivation & Research Questions

“Can we reproduce key stylized facts of financial markets such as herding, volatility clustering, and regime shifts, without rational agents, external shocks, or fine-tuned strategies?”

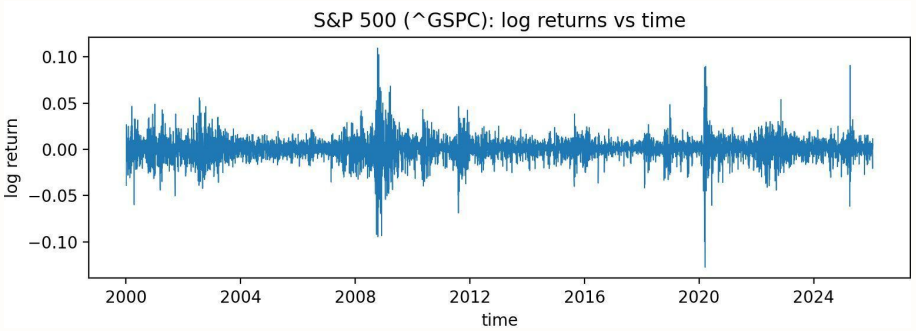
Reference models: Bornholdt (2001) and Yamano (2002)

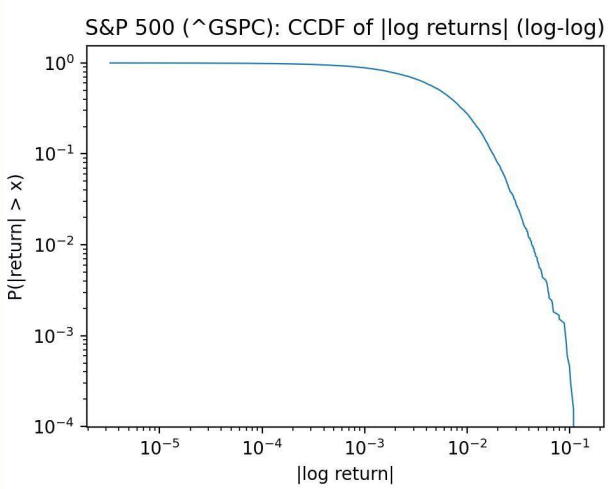
Hypothesis & Research Questions:

H1 - Criticality & Frustration

H2 - Agent heterogeneity

H3 - Topology/Network





Baseline Model: Bornholdt (Mechanics)

Agent state:

- Decision spin: (buy, sell) $S_i(t) \in \{+1, -1\}$
- Strategy spin: (fundamentalist, chartisti) $C_i(t) \in \{+1, -1\}$

Spin decision update:

$$S_i(t+1) = +1 \; with \; probability \; p = \frac{1}{1+\exp[-2\beta h_i(t)]} \; with \; \beta = \frac{1}{T}$$

Local field (two scales):

$$h_i(t) = \sum_{j=1}^{N} J_{ij} S_j(t) \; - \; \alpha \, C_i(t) \, \frac{1}{N} \sum_{j=1}^{N} S_j(t)$$

Strategy switching rule:

$$C_i(t+1) = -C_i(t) \quad if \quad C_i(t) \, S_i(t) \, M(t) < 0$$

Returns:

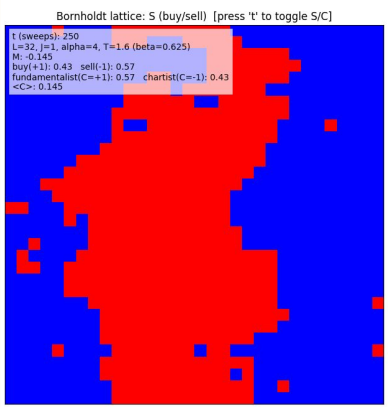
$$r(t) = \ln(|M(t)|) - \ln(|M(t-1)|)$$

Baseline Model: Frustration

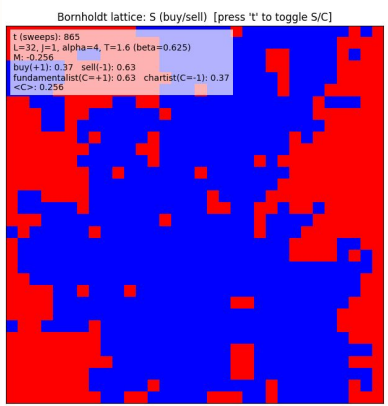
Agents are simultaneously driven by local imitation and global market pressure, introducing competing incentives across scales and generating frustration.

This frustration leads to metastable ordered phases punctuated by sudden global reorganizations, producing intermittent dynamics.

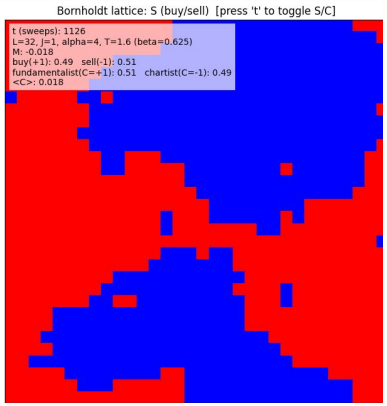
Temperature is crucial: emergence is observed only below the critical temperature.



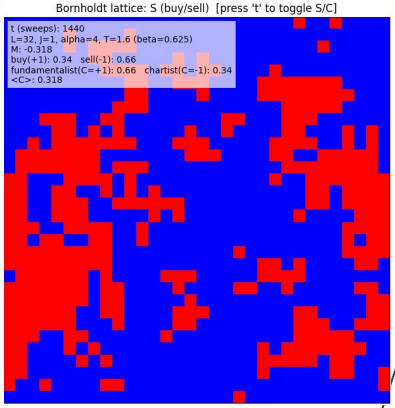
Sweep n. 250



Sweep n. 865

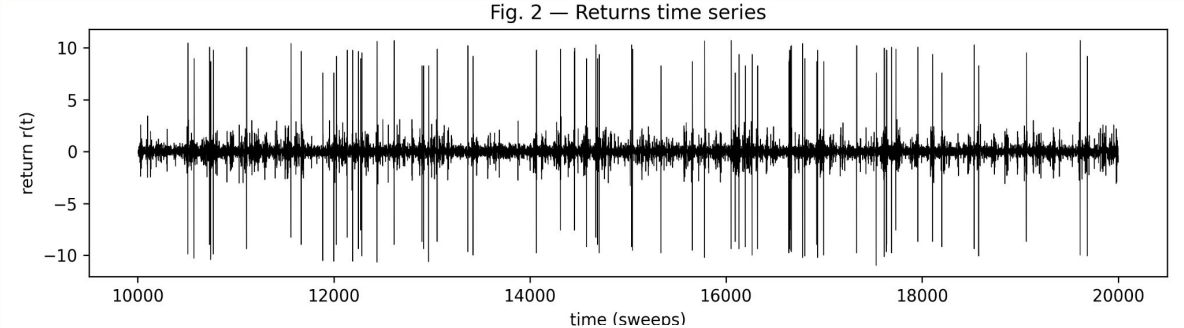


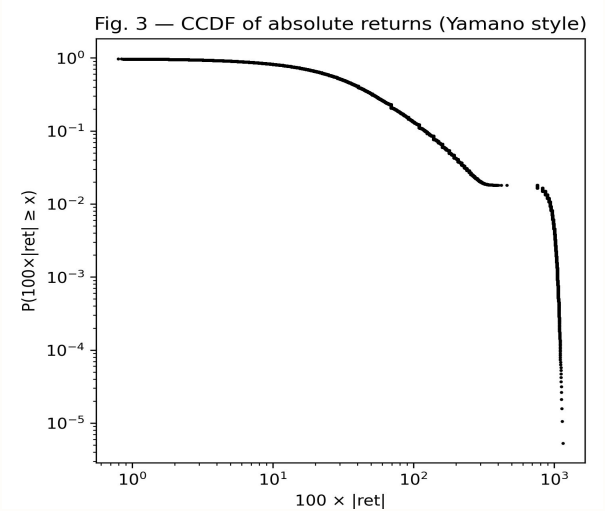
Sweep n. 1126

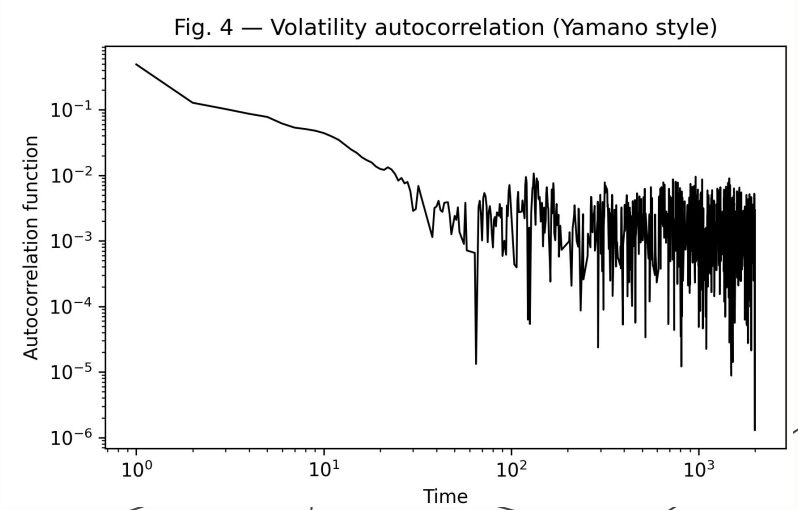


Sweep n. 1140

Baseline Model: Results







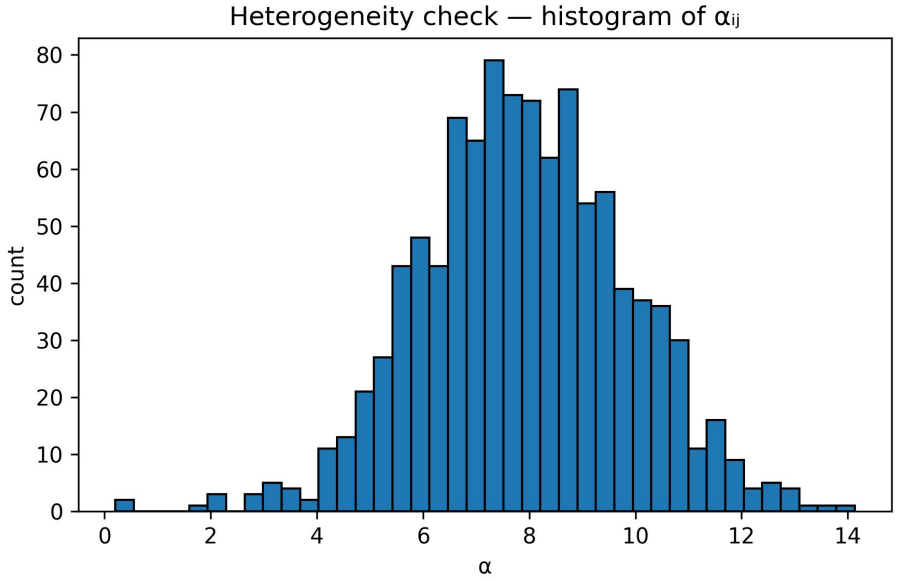
Agent Heterogeneity: Motivation and Mechanics

Real markets consist of non-identical agents.

Agents differ in risk sensitivity and in how strongly they react to global market signals.

Each agent is assigned an individual coupling α randomly drawn from a fixed distribution at initialization.

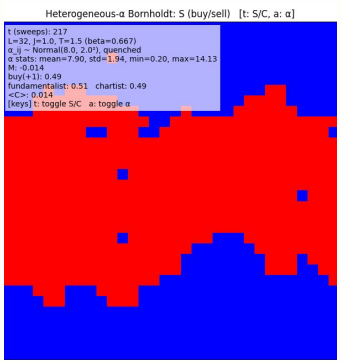
Large alpha: strongly contrarian agents.
Small alpha: weakly sensitive agents.



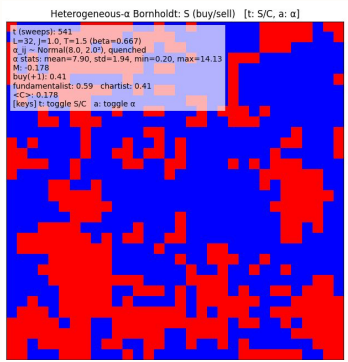
Agent Heterogeneity: Frustration

As in the baseline model, agents face competing forces across scales, but this frustration becomes asymmetric across agents: some agents react strongly and flip early, while others resist longer.

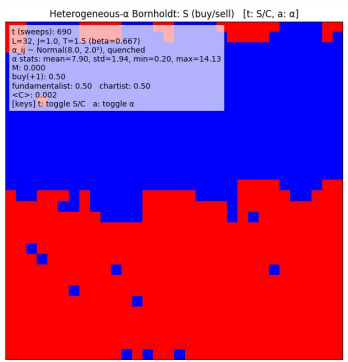
What we observed: Frustration still drives emergence, but heterogeneity accelerates the dynamics.



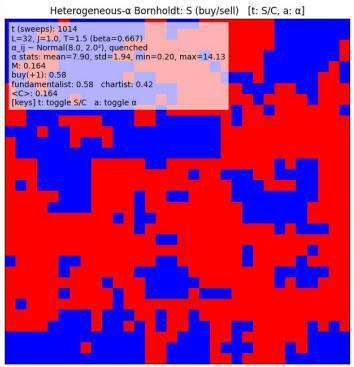
Sweep n. 217



Sweep n. 541

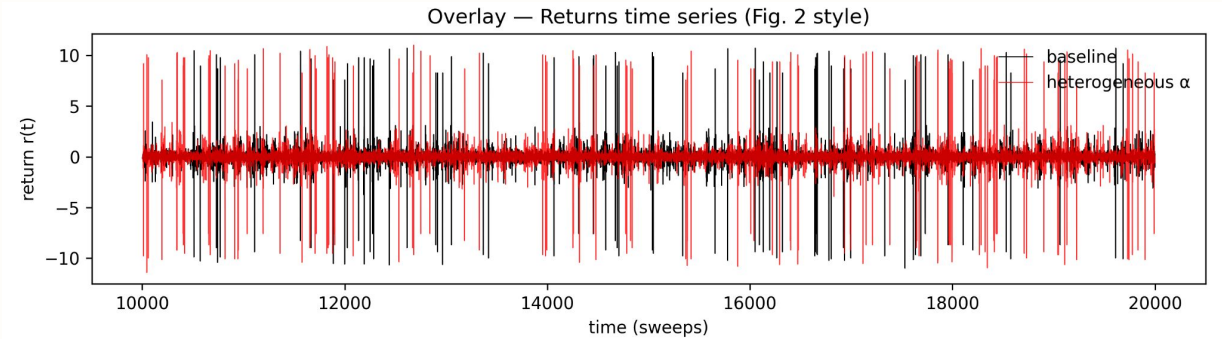


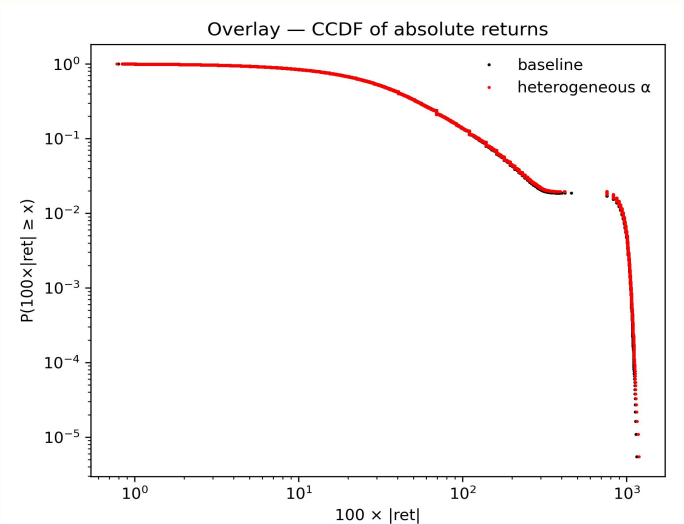
Sweep n. 690

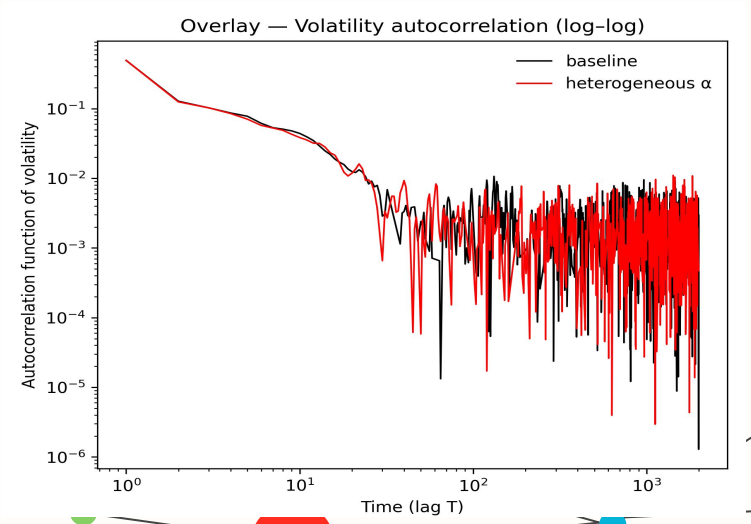


Sweep n. 1014

Agent Heterogeneity: Results and Comparison







Network Topology Motivation

- 2D lattice puts us at a disadvantageous communication structure (every trader interacts with the same small set of nearest neighbors)
- Markets are relationally dependent, not just spatial.
- Networks give realistic structural knobs like clustering (communities), shortcuts (fast information spread), hubs (dominant players).
- Easy to extend
  - Isolate the effect of topology while keeping the trading rules and observables identical.
  - Spin dynamics (heat-bath update), two-scale local field, and returns from magnetization all stay the same
  - What changes:
    - Lattice: $J_{ij} = J$ only for nearest neighbors.
    - Network: $J_{ij} = J$ only if there is an edge (i, j), i.e. $J_{ij} = J\,A_{ij}$.

Results WS

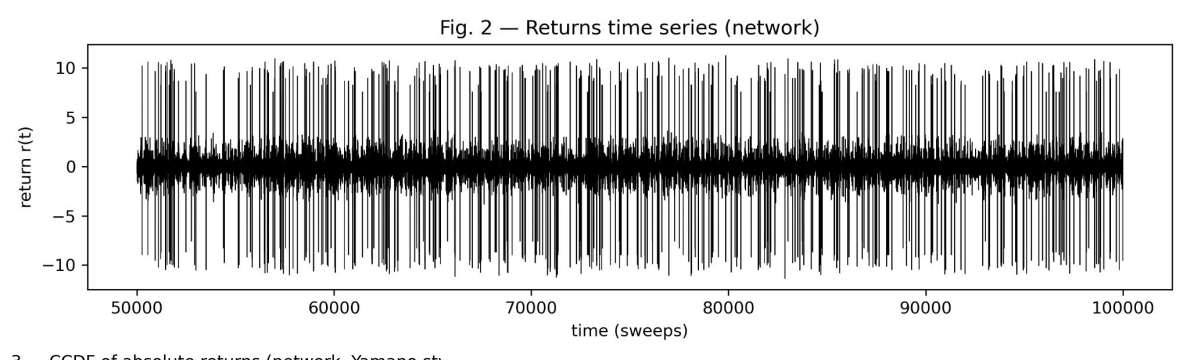


Fig. 2 — Returns time series (network)
return r(t)
time (sweeps)
10
5
0
−5
−10
50000
60000
70000
80000
90000
100000

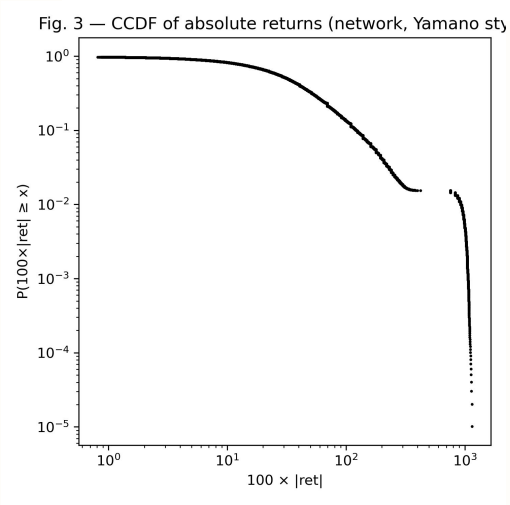


Fig. 3 — CCDF of absolute returns (network, Yamano sty
P(100×|ret| ≥ x)
100 × |ret|

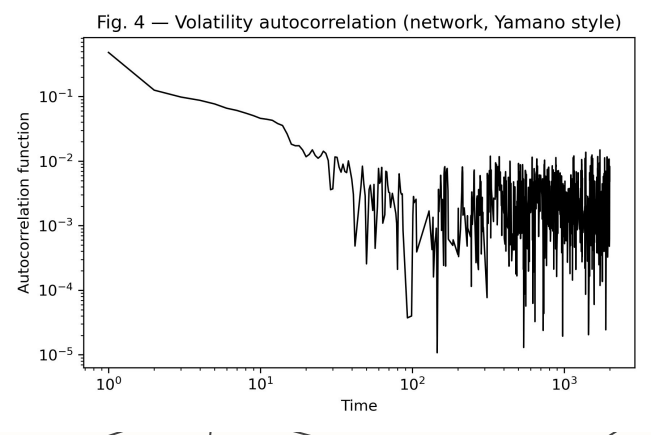
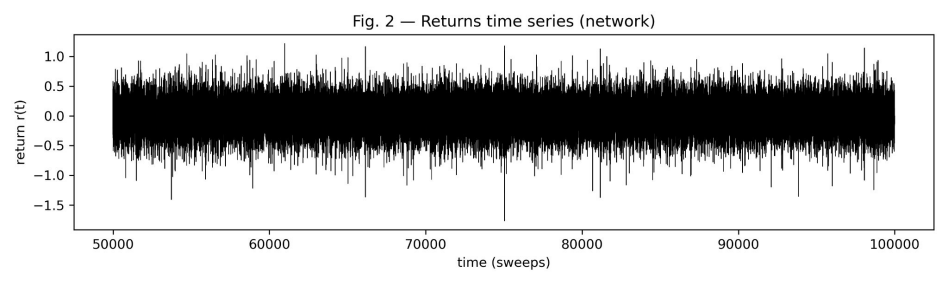


Fig. 4 — Volatility autocorrelation (network, Yamano style)
Autocorrelation function
Time

Results ER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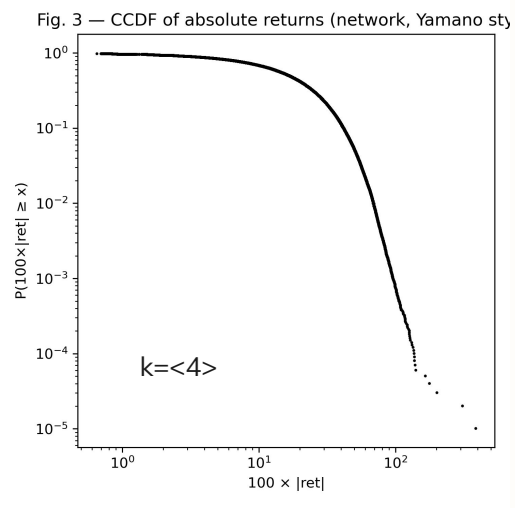


Fig. 3 — CCDF of absolute returns (network, Yamano sty
P(100×|ret| ≥ x)
100 × |ret|
k=<4>

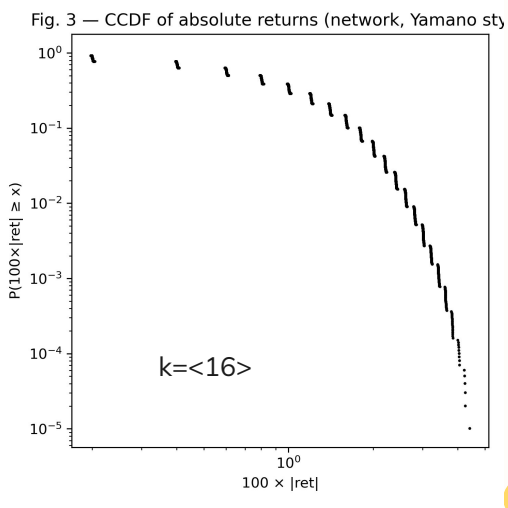
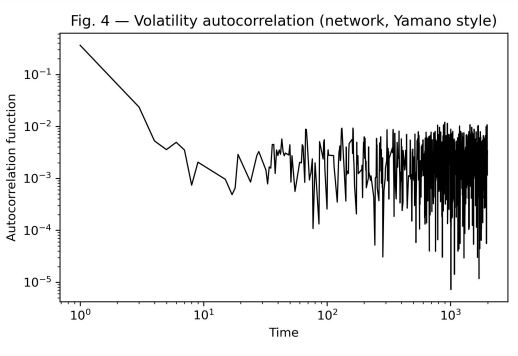


Fig. 3 — CCDF of absolute returns (network, Yamano sty
P(100×|ret| ≥ x)
100 × |ret|
k=<16>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Autocorrelation function
Time

Results BA

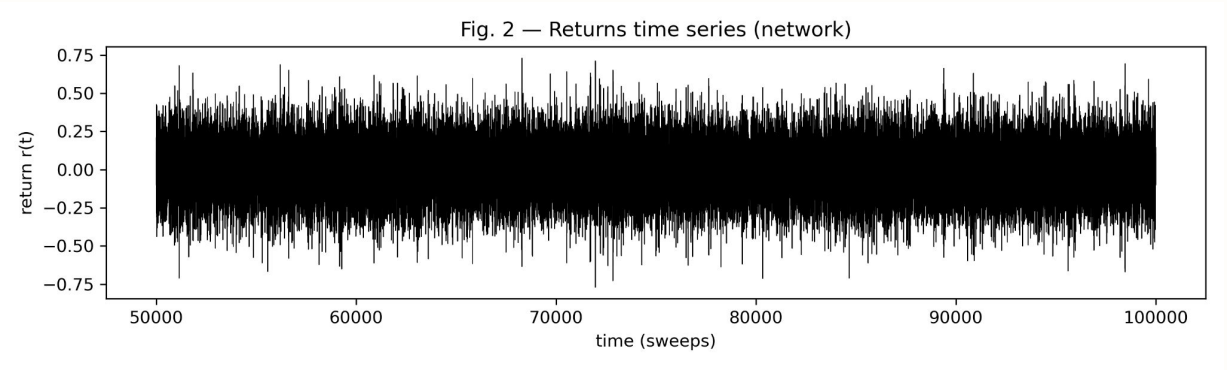
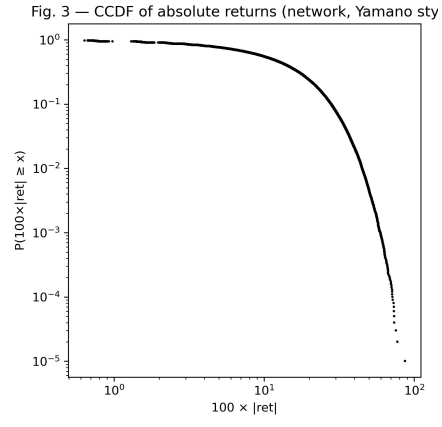


Fig. 2 — Returns time series (network)
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
return r(t)
50000
60000
70000
80000
90000
100000
time (sweeps)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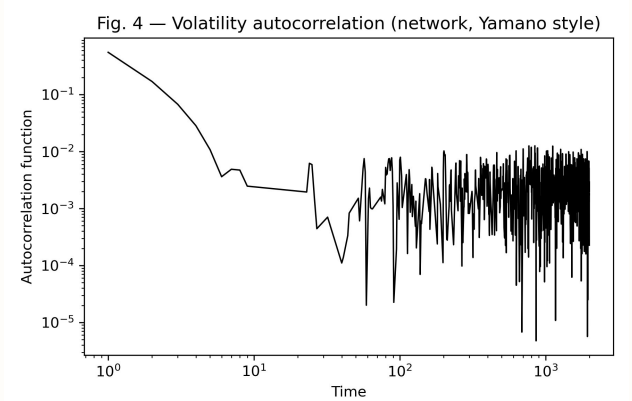
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Autocorrelation function
Time

Conclusion

- Agent heterogeneity does not change the qualitative behavior of the system but accelerates regime switching
- Confirmation that networks can correctly capture dynamics of market from Ising model
- Overall, our results support the view that market-like complexity can emerge from minimal microscopic rules, and that both heterogeneity and network structure act as modulators rather than primary drivers of critical dynamics.
- Limitations & outlook: parameter exploration was limited; future work should include systematic temperature and network sweeps, finite-size scaling, to better locate phase transitions and universality classes.