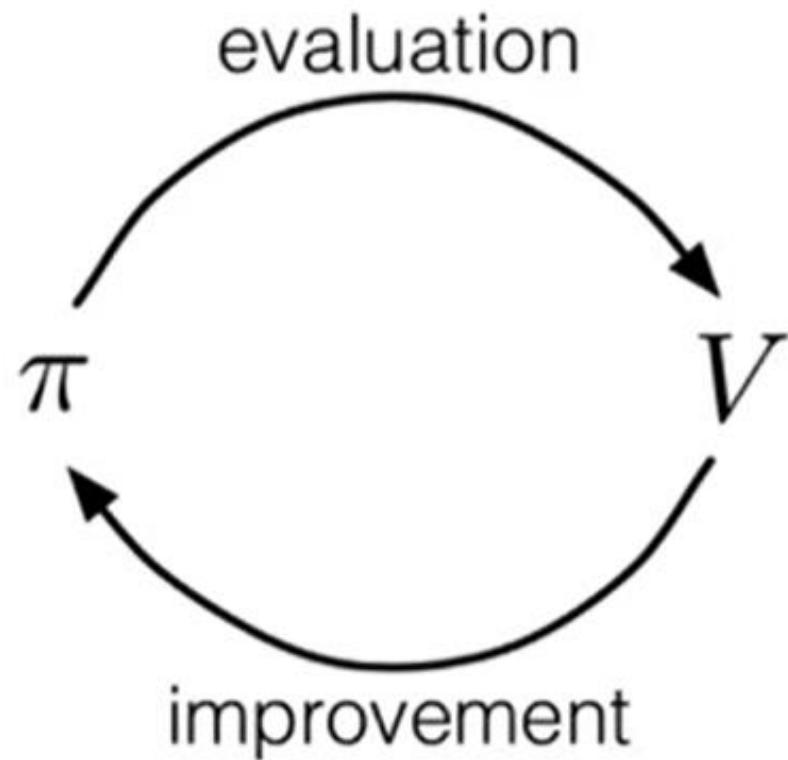


Advanced Machine Learning

(AIE 322)

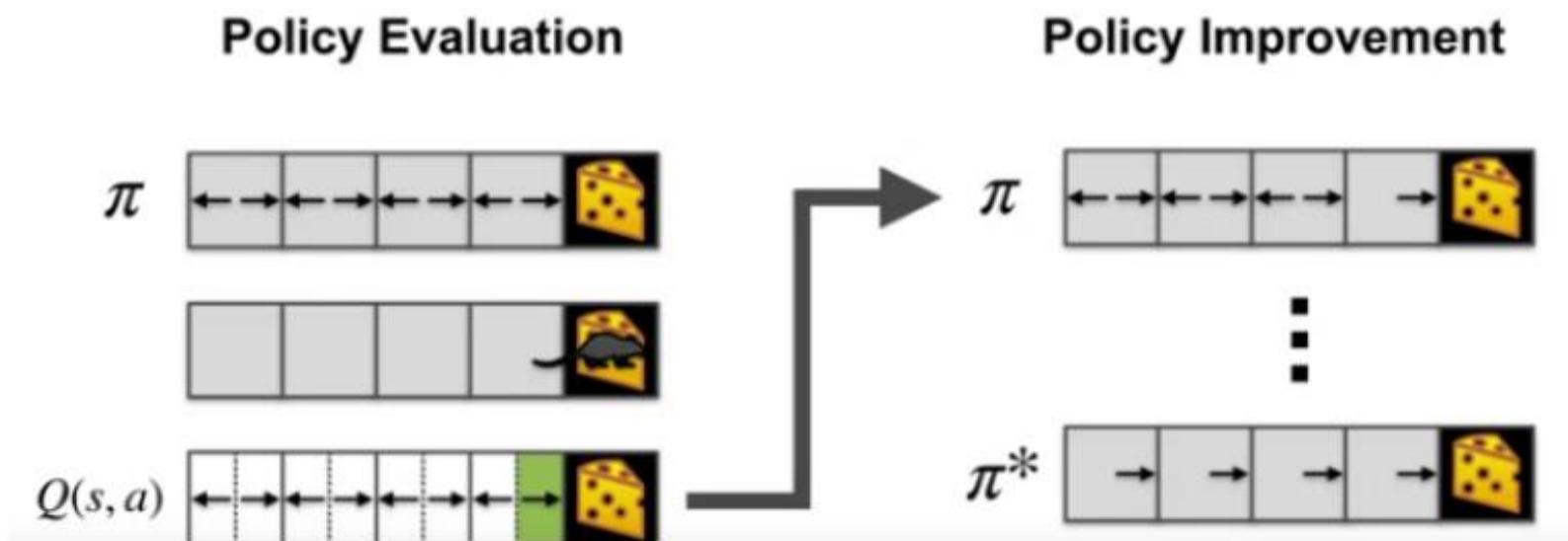
**Lecture 9: Temporal Difference Learning
Methods for Control**

Recall: Generalized Policy Iteration (GPI)



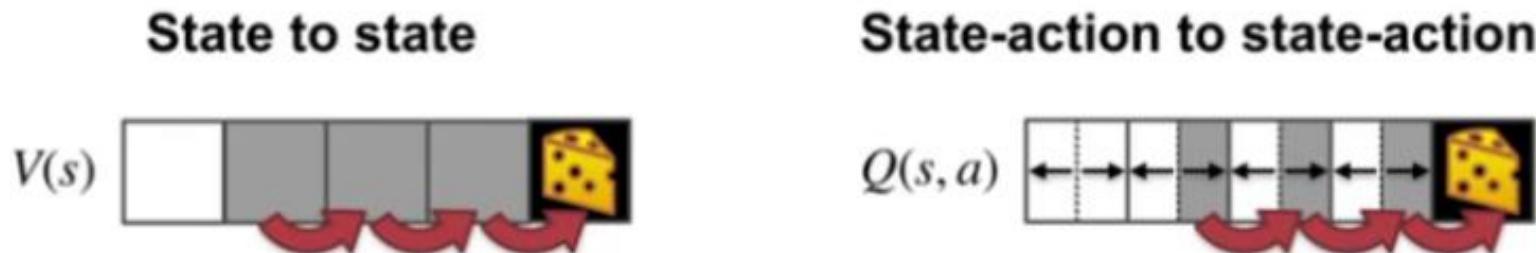
Recall: GPI with MC

- To better remember GPI with Monte Carlo, imagine a mouse in a four-state corridor with cheese at the end.
- The mouse starts out knowing nothing and follows a random policy. Eventually, the mouse will stumble into the cheese just by moving randomly.
- At that point, the mouse updates its action values. Then it improves its policy by greedy with respect to its action values.



Sarsa: GPI with TD

- To use TD within (Generalized policy iteration) GPI, we need to learn an action value function.
- So we'll need to look at slightly different version of TD than we've seen in the past.
- Instead of looking at transitions **from state to state** and learn the value of each state, let's look at transitions from **state action pair to state action pair** and learn the value of each pair → **SARSA Prediction**



SARSA Algorithm

The Sarsa algorithm

$S_t \ A_t \ R_{t+1} \ S_{t+1} \ A_{t+1}$

$\sim \epsilon$ -greedy

$S_t \quad A_t \quad R_{t+1} \quad S_{t+1} \quad A_{t+1} \sim \pi$



$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t))$$

Recall: $V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$

On-Policy vs Off-Policy

- **On-Policy:** improve and evaluate the policy being used to select actions.
 - In on-policy learning, the $Q(s,a)$ function is learned from actions that we took using our current policy $\pi(a|s)$
 - It updates the policy being used to take actions as the agent interacts with the environment.
 - The agent learns by following the current policy and then updates the policy based on the rewards received from those actions.
- **Off-Policy:** improve and evaluate a *different* policy from the one used to select actions
 - In off-policy learning, the $Q(s,a)$ function is learned from taking different actions.
 - It updates a different policy than the one being used to take actions.
 - The main advantage of off-policy learning is that it is more likely to find optimal policy

SARSA Algorithm

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$

Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+$, $a \in \mathcal{A}(s)$, arbitrarily except that $Q(\text{terminal}, \cdot) = 0$

Loop for each episode:

 Initialize S

 Choose A from S using policy derived from Q (e.g., ε -greedy)

 Loop for each step of episode:

 Take action A , observe R, S'

 Choose A' from S' using policy derived from Q (e.g., ε -greedy)

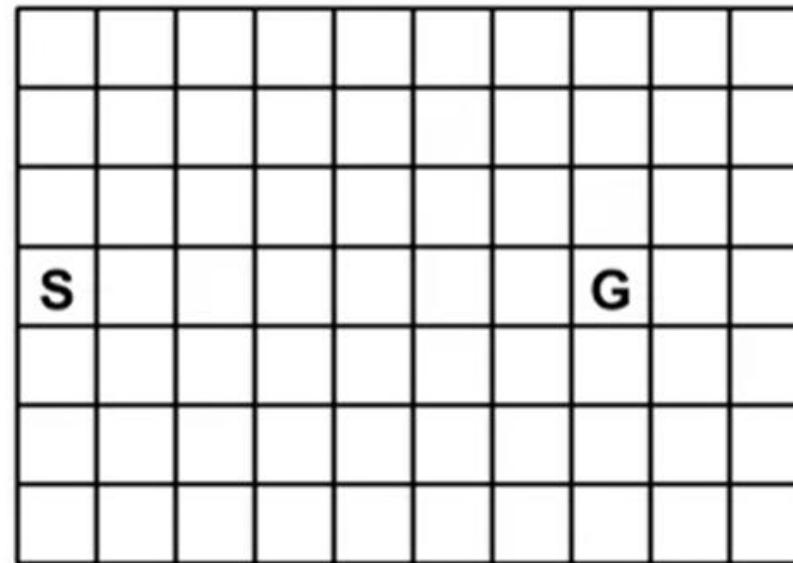
$$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma Q(S', A') - Q(S, A)]$$

$S \leftarrow S'; A \leftarrow A'$;

 until S is terminal

Example: Windy Gridworld

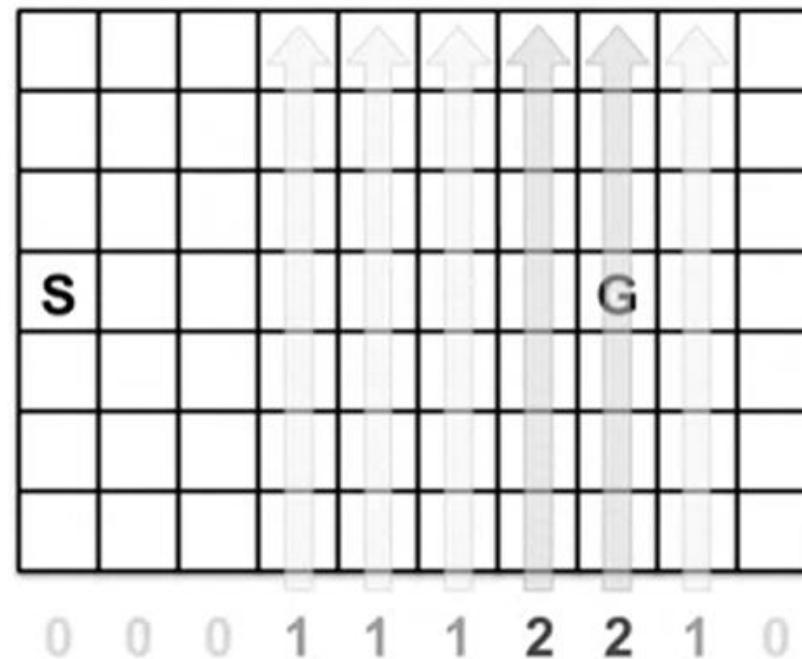
The Windy Gridworld



$$R_{step} = -1$$
$$\gamma = 1$$

Example: Windy Gridworld

The Windy Gridworld

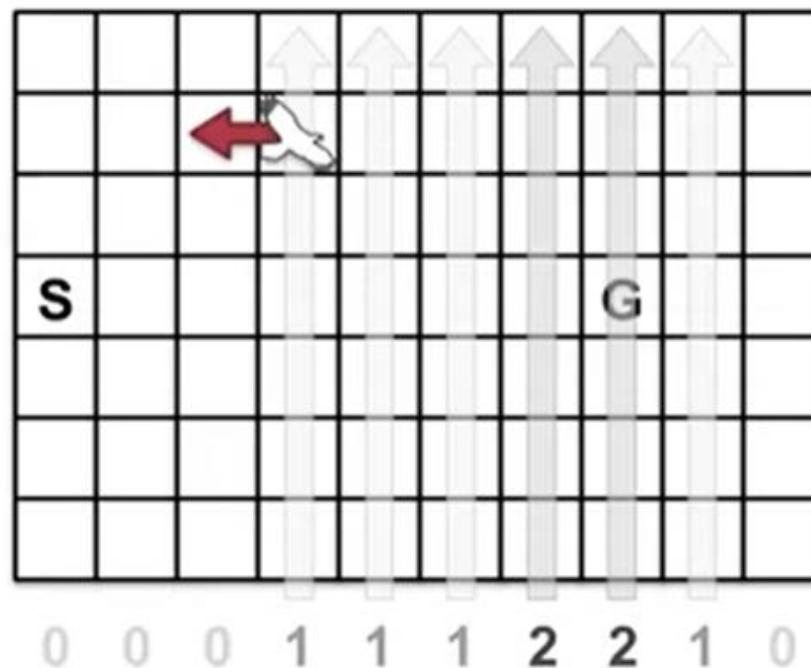


Actions

$$R_{step} = -1$$
$$\gamma = 1$$

Example: Windy Gridworld

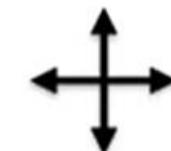
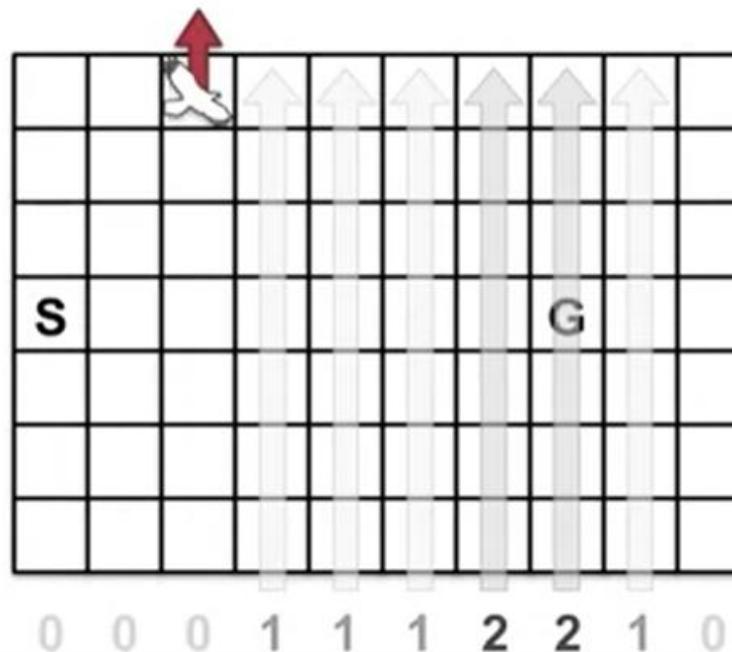
The Windy Gridworld



$$R_{step} = -1$$
$$\gamma = 1$$

Example: Windy Gridworld

The Windy Gridworld

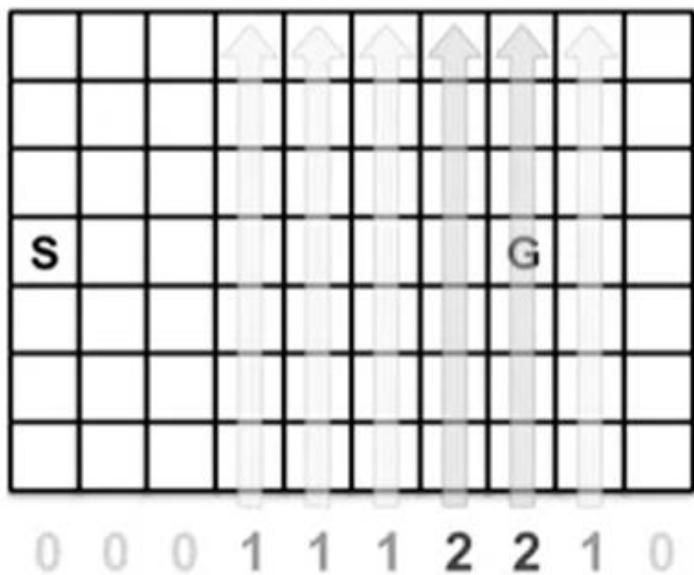


Actions

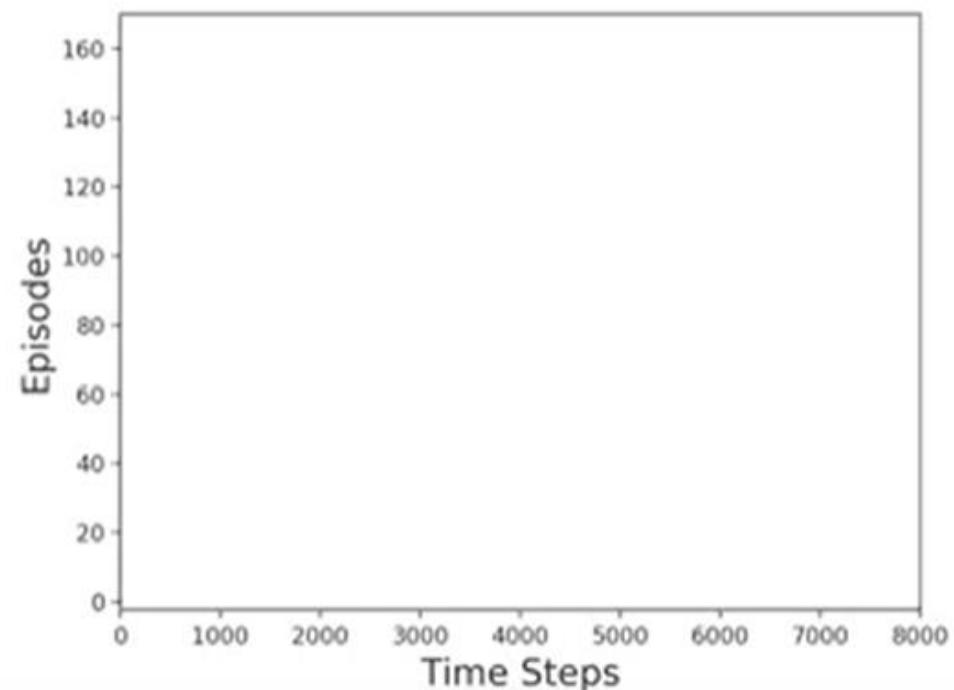
$$R_{step} = -1$$
$$\gamma = 1$$

Example: Windy Gridworld

Sarsa on the Windy Gridworld $\sim \epsilon$ -greedy



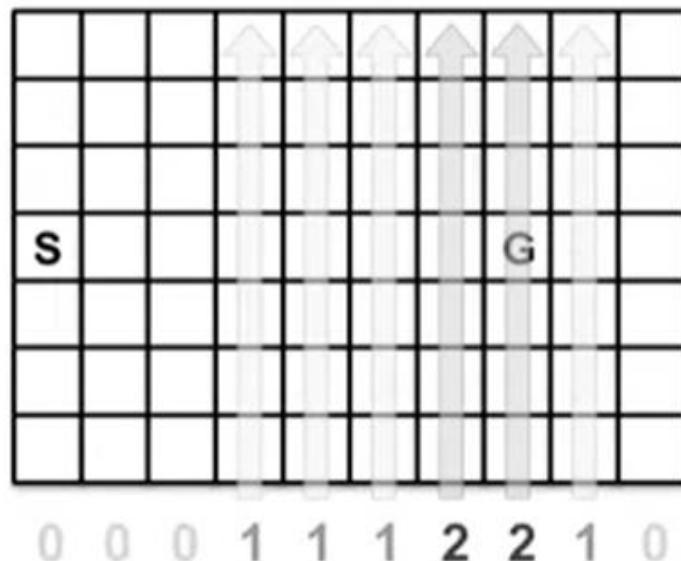
Sarsa: $\epsilon = 0.1$
 $\alpha = 0.5$



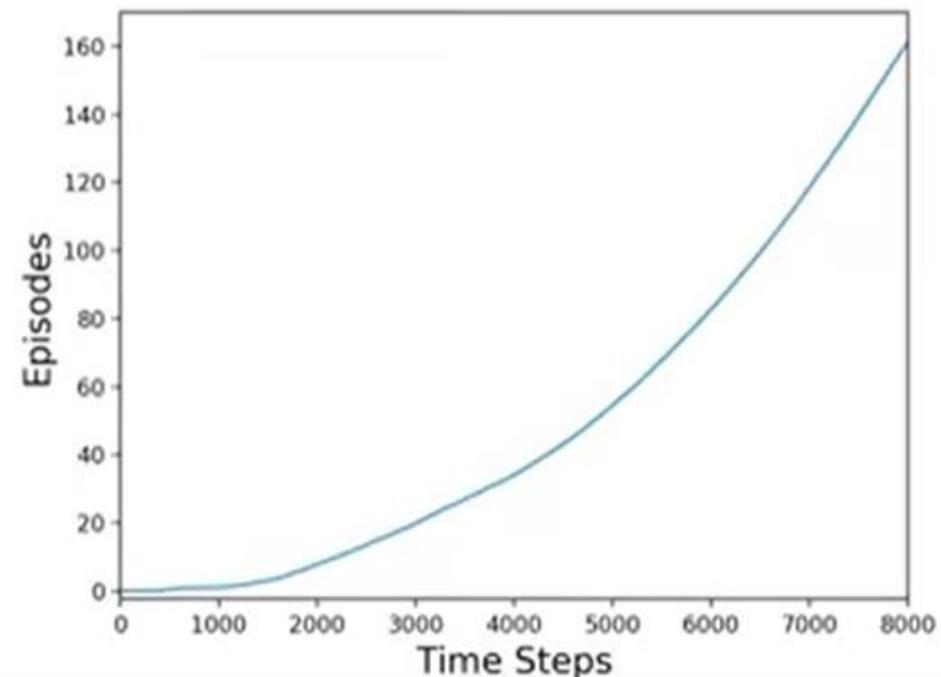
Take a random action
every 10 steps

Example: Windy Gridworld

Sarsa on the Windy Gridworld



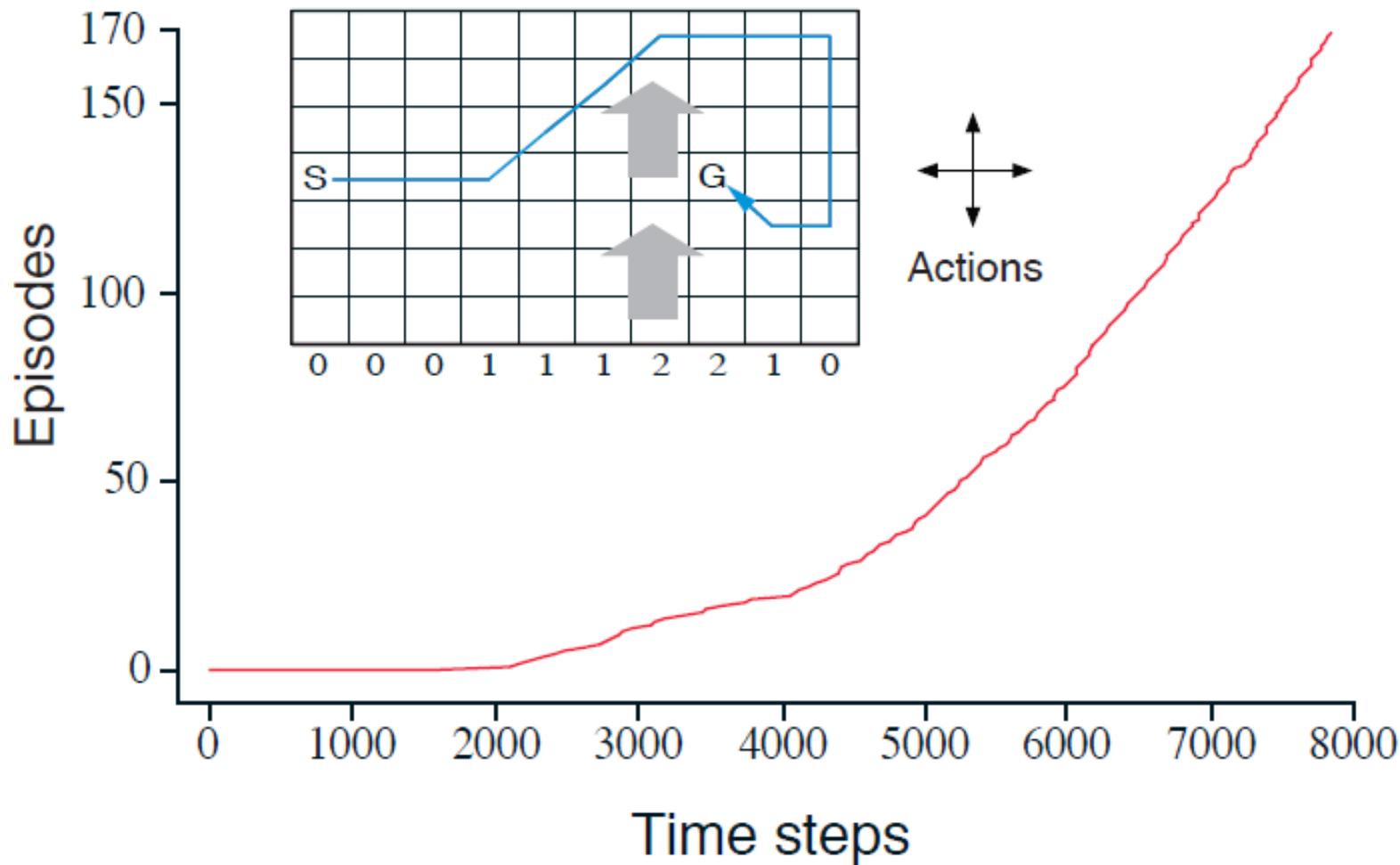
Sarsa: $\epsilon = 0.1$
 $\alpha = 0.5$



Can Monte Carlo methods be used on this task? !
No, since termination is not guaranteed for all policies.!

Example: Windy Gridworld

Sarsa on the Windy Gridworld



Unlike Monte Carlo Sarsa's Step-by-step learning methods can quickly learn during the episode that such policies are poor, and switch to something else!

What is Q-learning?

- Q-learning is a model-free, value-based, off-policy TD control algorithm that will find the best series of actions based on the agent's current state.
- The “Q” stands for quality. Quality represents how valuable the action is in maximizing future rewards.
- It is defined by:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \right]$$

- Here the learned action-value function, Q , directly approximates q^* , the optimal action-value function, independent of the policy being followed.
- This dramatically simplifies the analysis of the algorithm and enabled early convergence proofs.

Q-learning Algorithm

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$

Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\text{terminal}, \cdot) = 0$

Loop for each episode:

 Initialize S

 Loop for each step of episode:

 Choose A from S using policy derived from Q (e.g., ε -greedy)

 Take action A , observe R, S'

→
$$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$$

$S \leftarrow S'$

 until S is terminal

SARSA and Bellman Equation

Revisiting Bellman equations

Sarsa:
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t))$$

$$q_\pi(s, a) = \sum_{s', r} p(s', r | s, a) \left(r + \gamma \sum_{a'} \pi(a' | s') q_\pi(s', a') \right)$$


Q-learning and Bellman Equation

Revisiting Bellman equations

Q-learning:
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a') - Q(S_t, A_t))$$

$$q_*(s, a) = \sum_{s', r} p(s', r | s, a) \left(r + \gamma \max_{a'} q_\pi(s', a') \right)$$

Q-learning and Bellman Equation

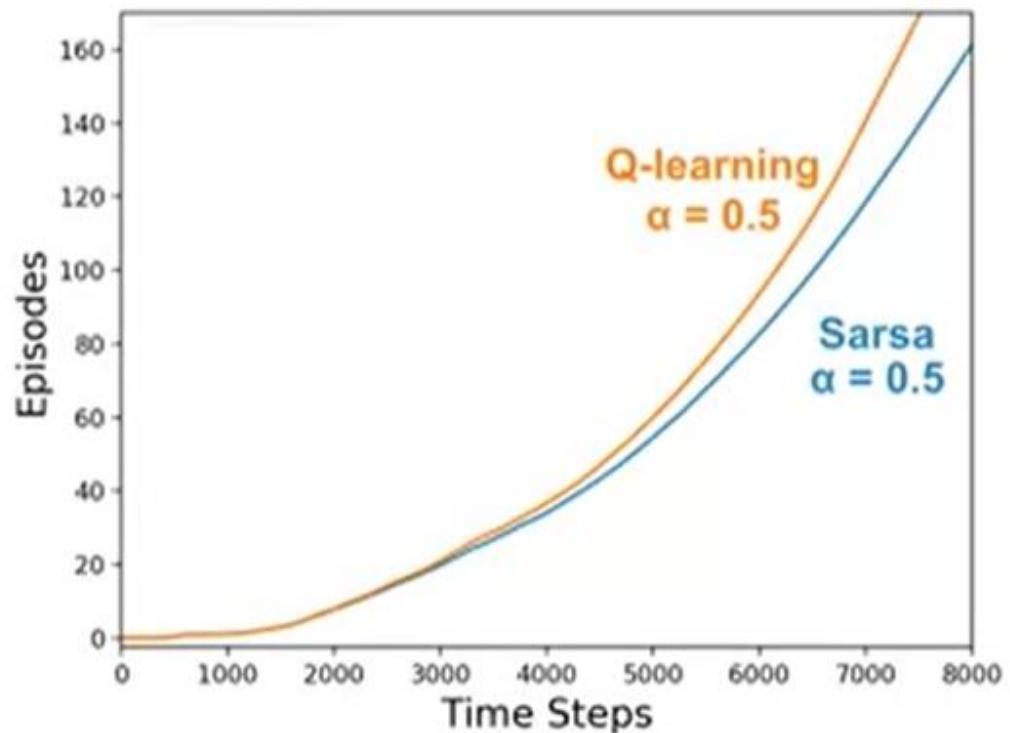
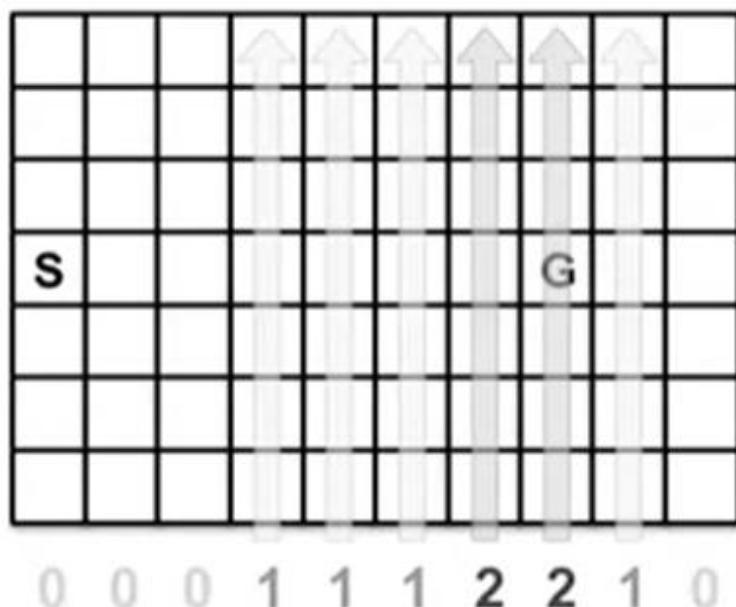
Revisiting Bellman equations

Q-learning:
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a') - Q(S_t, A_t))$$

$$q_*(s, a) = \sum_{s', r} p(s', r | s, a) \left(r + \gamma \max_{a'} q_\pi(s', a') \right)$$

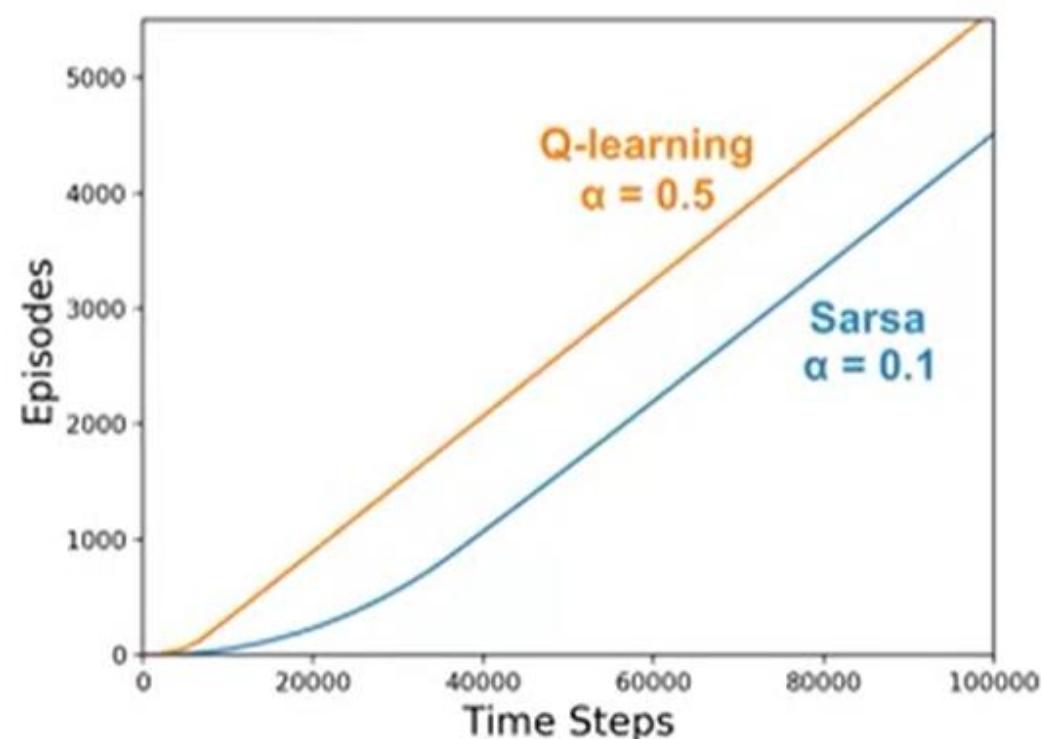
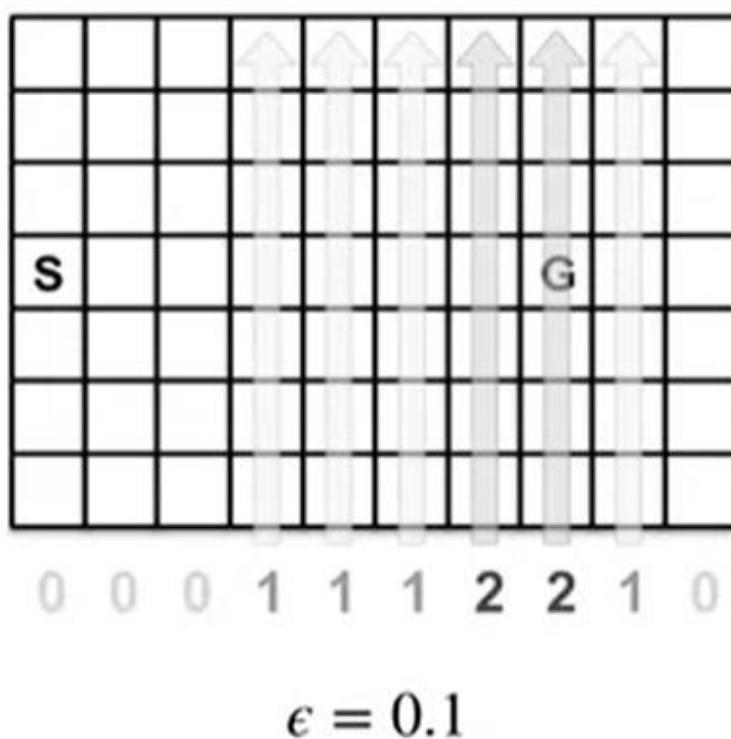
Example: Windy Gridworld (Q-learning vs SARSA)

The Windy Gridworld



Example: Windy Gridworld (Q-learning vs SARSA)

The Windy Gridworld



On-Policy vs Off-Policy

Comparison with Sarsa

Sarsa:
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma Q(S_{t+1}, \overset{\sim}{\pi}_{t+1}) - Q(S_t, A_t))$$

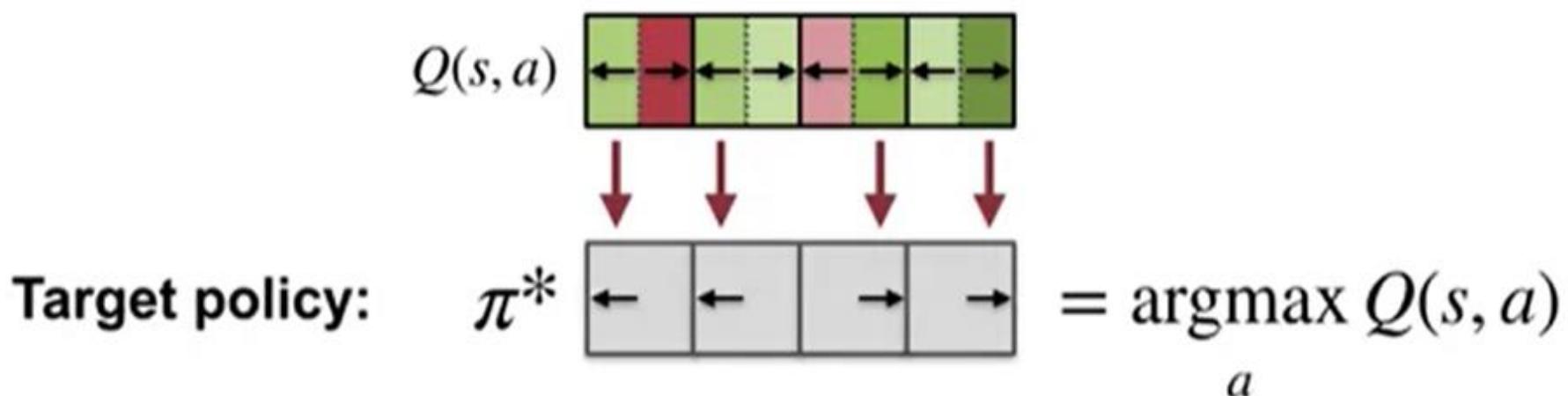
$\overset{\sim}{\pi}$ **On-Policy**

Q-learning:
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, \overset{\sim}{\pi}_*) - Q(S_t, A_t))$$

$\overset{\sim}{\pi}_*$ $\not\propto \pi$ **Off-Policy**

Behavior vs Target Policies

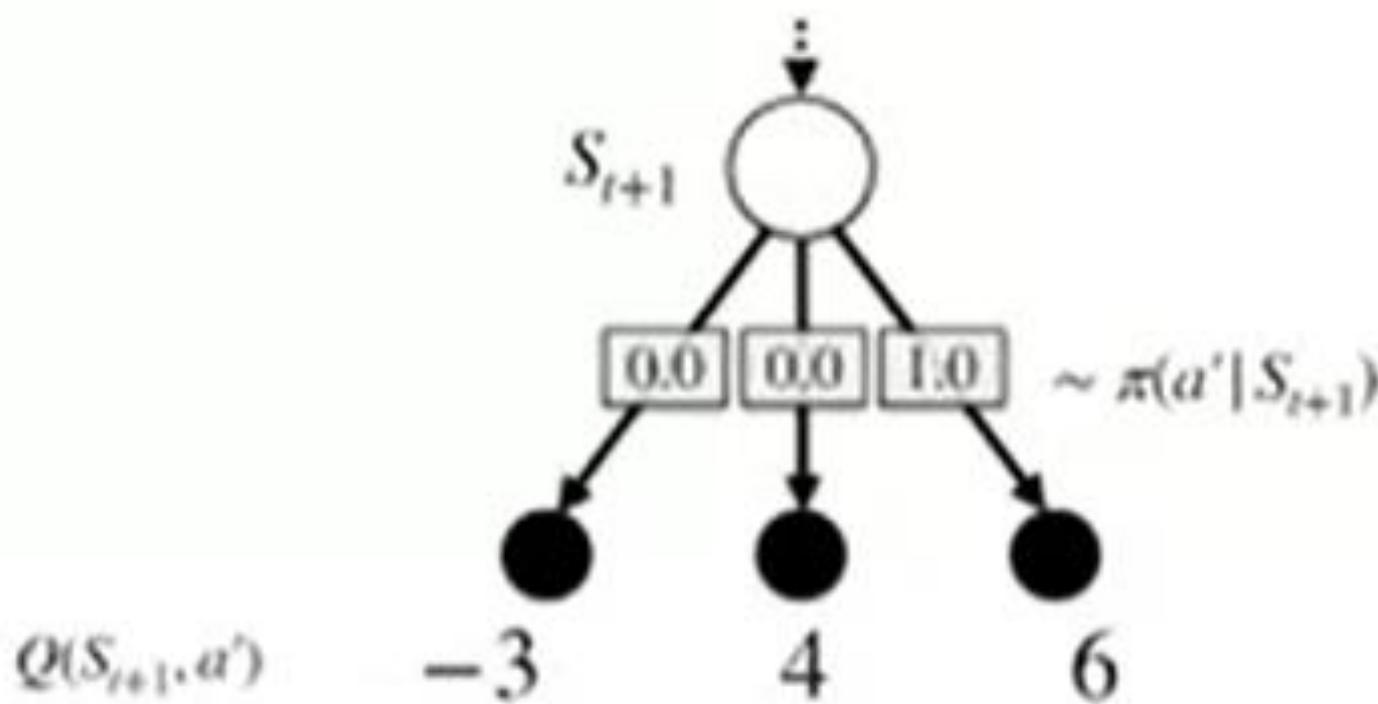
Behavior and target policies



Behavior policy: π

Can be ε -greedy

Q-learning Off-Policy Learning Technique

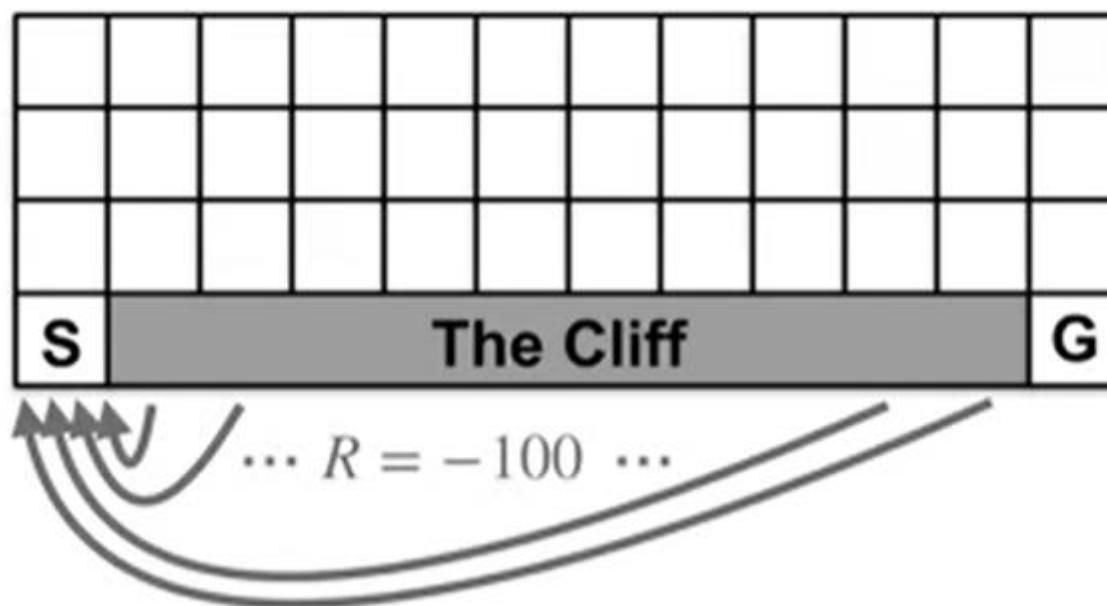


$$\sum_{a'} \pi(a'|S_{t+1}) Q(S_{t+1}, a') = \mathbb{E}_\pi[G_{t+1}|S_{t+1}] = \max_a Q(S_{t+1}, a) = 6$$

Since the agents target policies greedy, with respect to its action values, all non-maximum actions have probability 0.

Example: Cliff Walking

The cliff walking environment

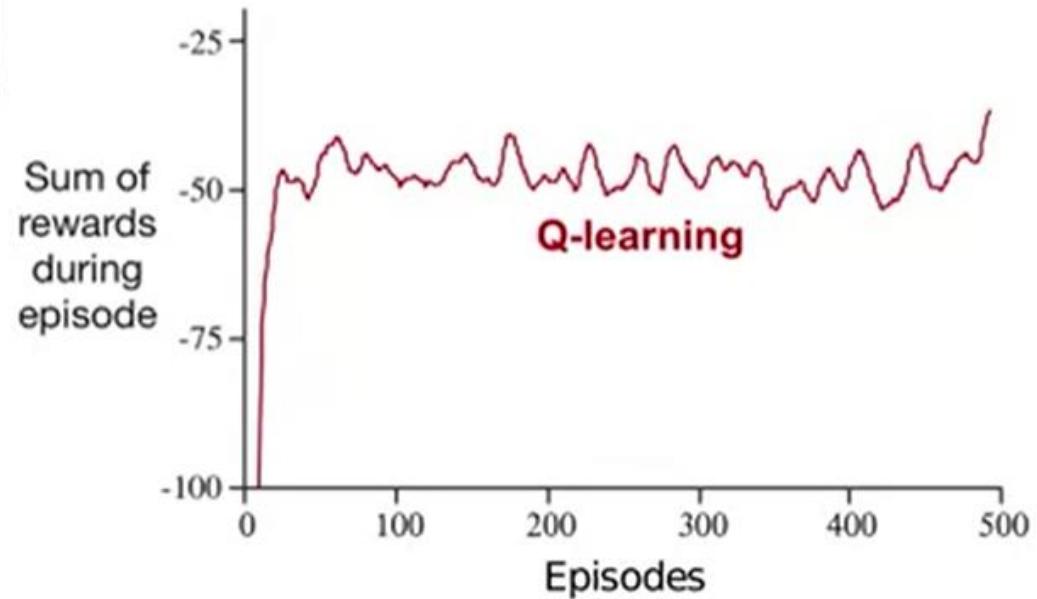
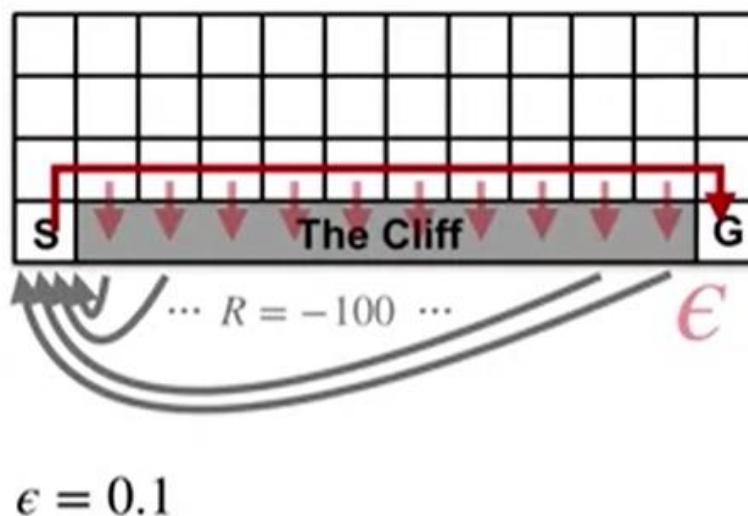


$$\gamma = 1$$

$$R_{step} = -1$$

Example: Cliff Walking

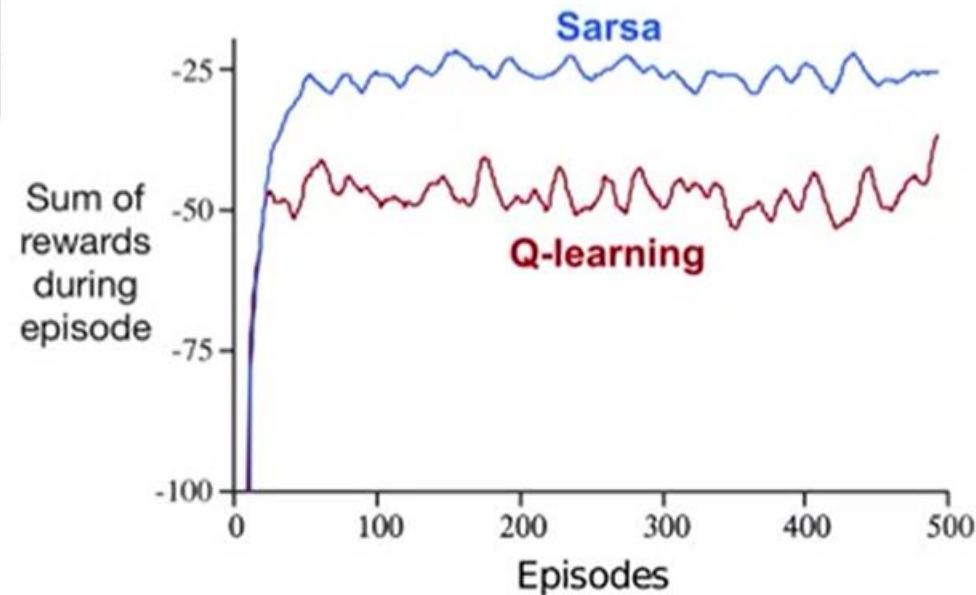
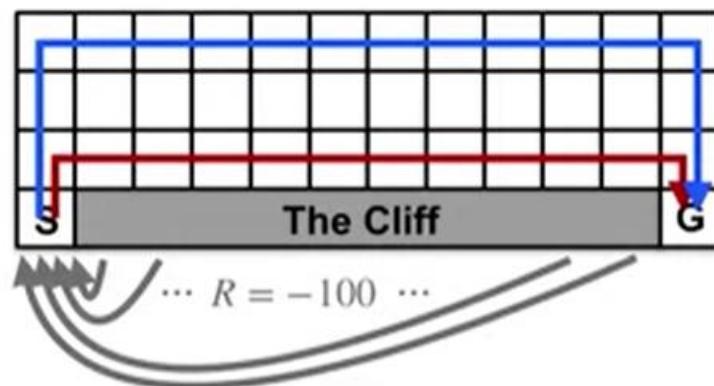
The cliff walking environment



Q-Learning is less desirable in specific situations

Example: Cliff Walking

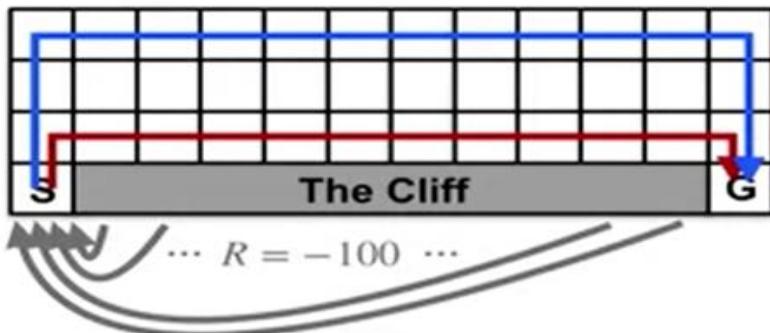
The cliff walking environment



Q-Learning is less desirable in specific situations

Example: Cliff Walking

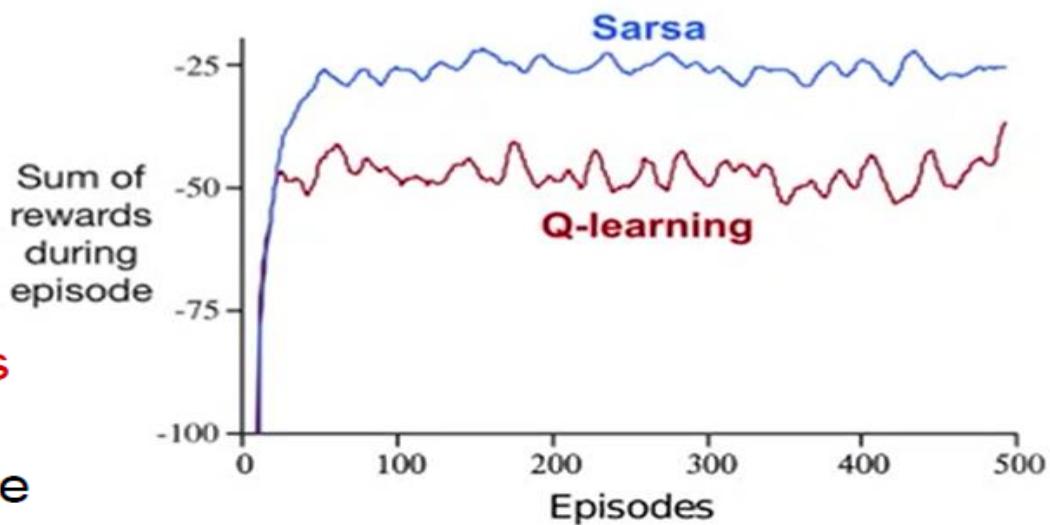
The cliff walking environment



$$\epsilon = 0.1$$

Q-learning learns quickly **values for the optimal policy**, that which travels right along the edge of the cliff. Unfortunately, this results in its occasionally falling off the cliff because of the ϵ -greedy action selection.

Sarsa takes the action selection **into account** and learns the longer but safer path through the upper part of the grid.



Q-learning Solved Example 1

d	e	f
a	b	c

$$\alpha = 0.9 \quad \gamma = 1$$

Initialize

$$Q(s, a) = 0$$

d	e	f
a	b	c

$$c \rightarrow f$$

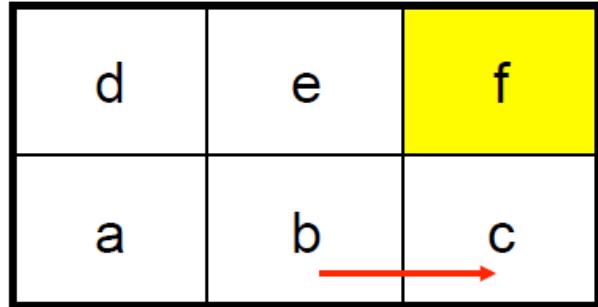
$$Q(c, \uparrow) \leftarrow 0 + 0.9[100 + 0 - 0] = 90$$

d	e	f
a	b	c

$$e \rightarrow f$$

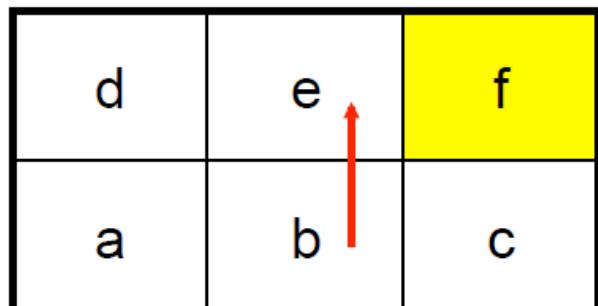
$$Q(e, \rightarrow) \leftarrow 0 + 0.9[100 + 0 - 0] = 90$$

Q-learning Solved Example 1



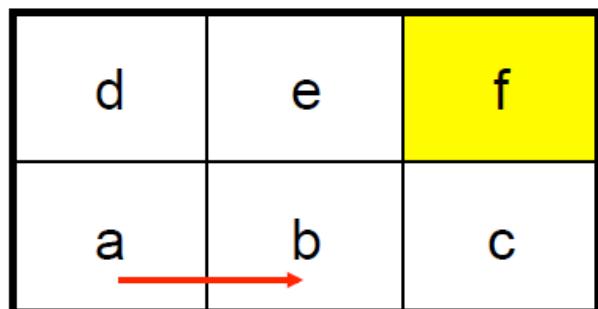
$b \rightarrow c$

$$Q(b, \rightarrow) \leftarrow 0 + 0.9[0 + 90 - 0] = 81$$



$b \rightarrow e$

$$Q(b, \uparrow) \leftarrow 0 + 0.9[0 + 90 - 0] = 81$$



$a \rightarrow b$

$$Q(a, \rightarrow) \leftarrow 0 + 0.9[0 + 81 - 0] \approx 73$$

Q-learning Solved Example 1

d	e	f
a	b	c

$b \rightarrow a$

$$Q(b, \leftarrow) \leftarrow 0 + 0.9[0 + 73 - 0] \approx 66$$

d	e	f
a	b	c

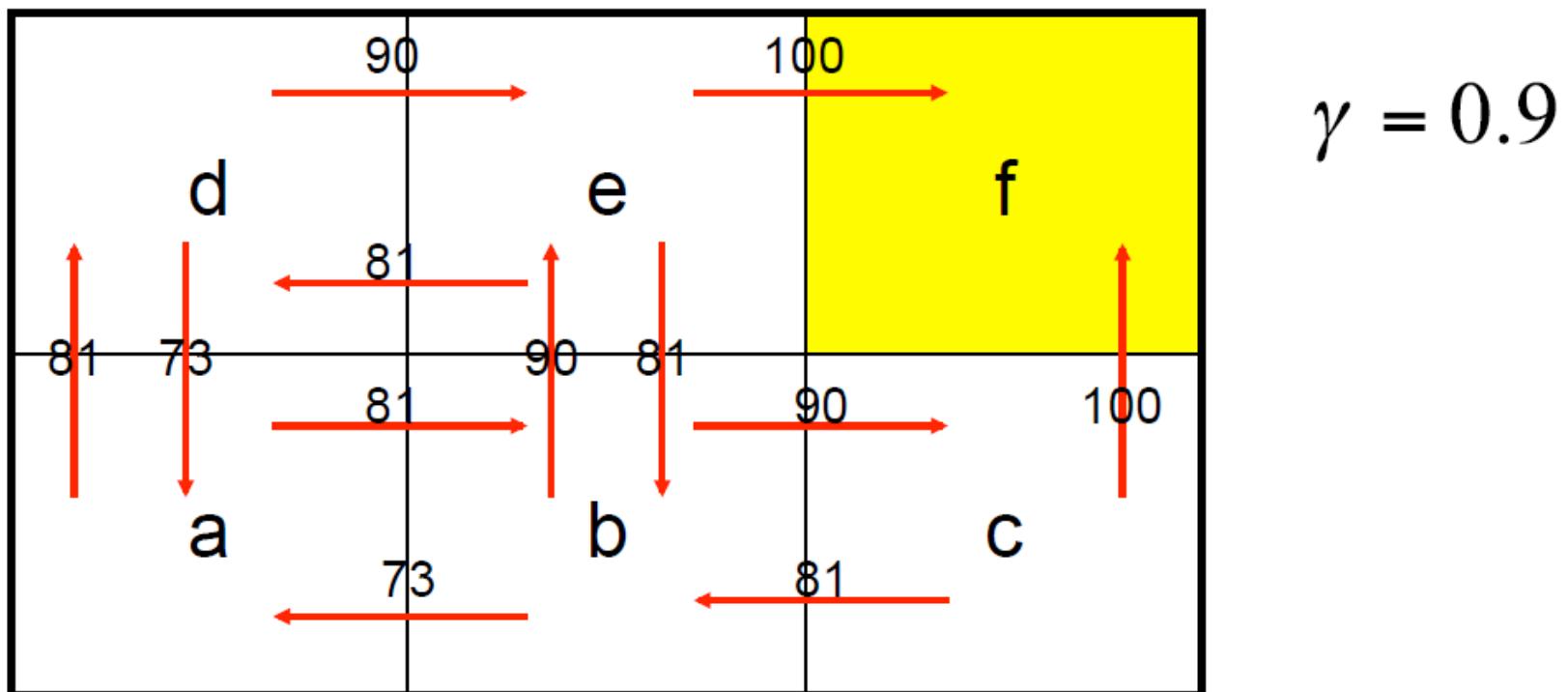
81

66

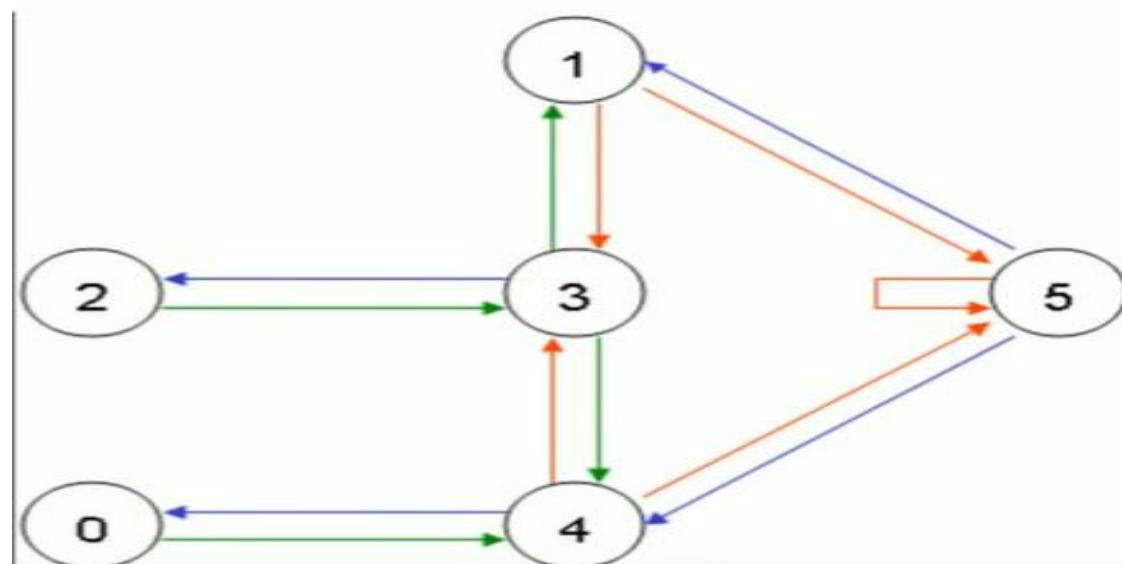
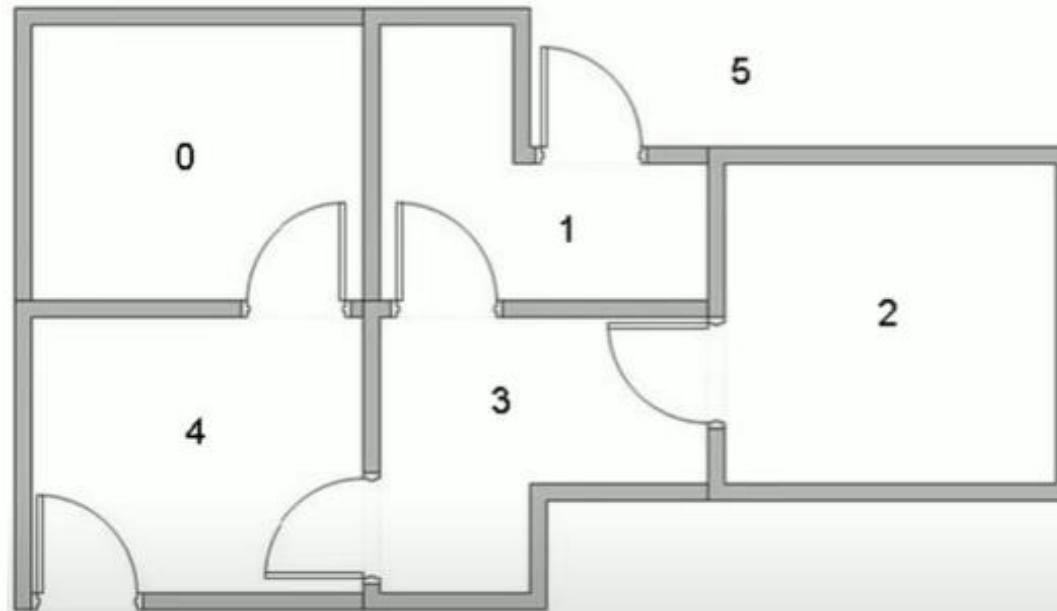
81

81

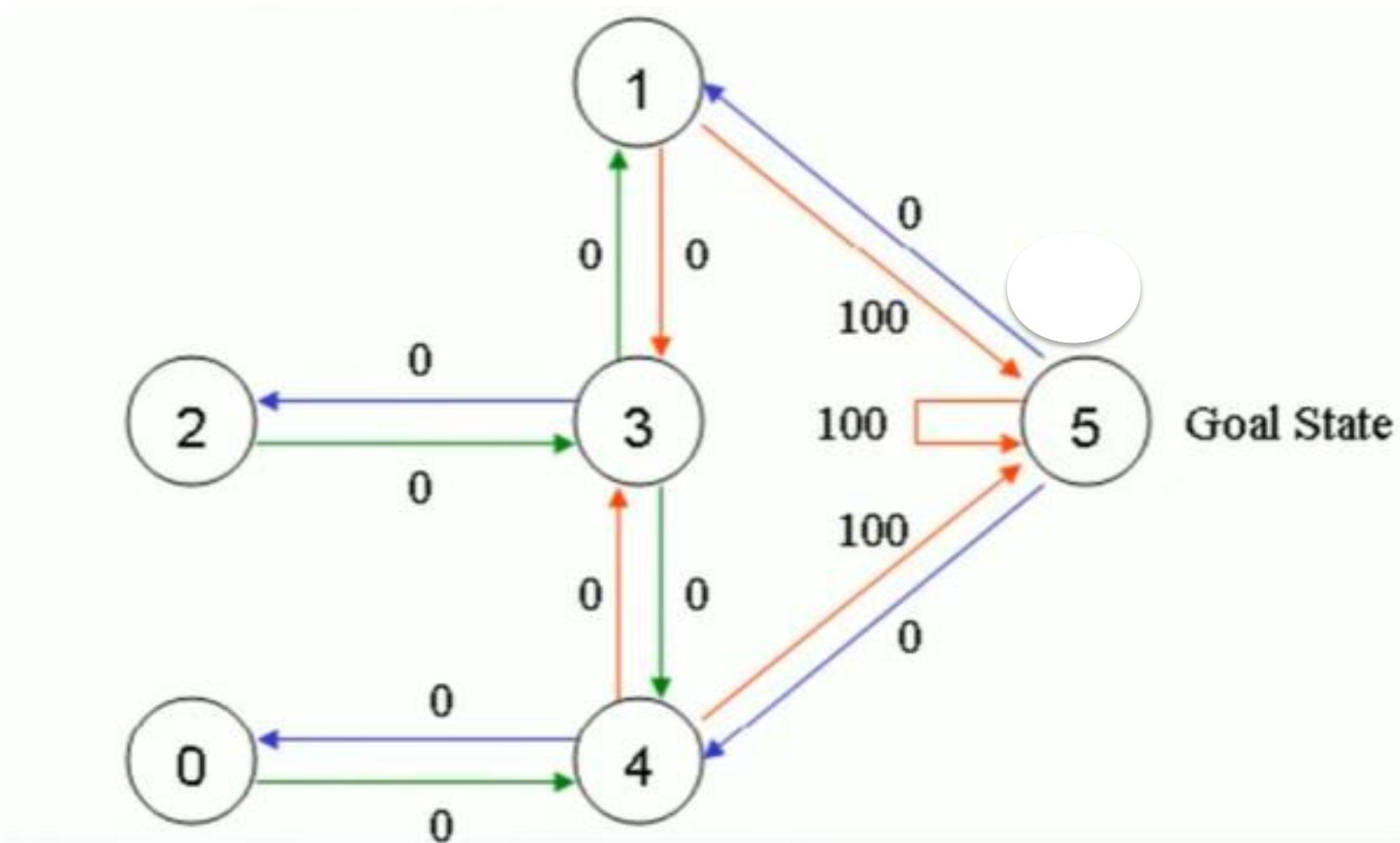
Q-learning Solved Example 1



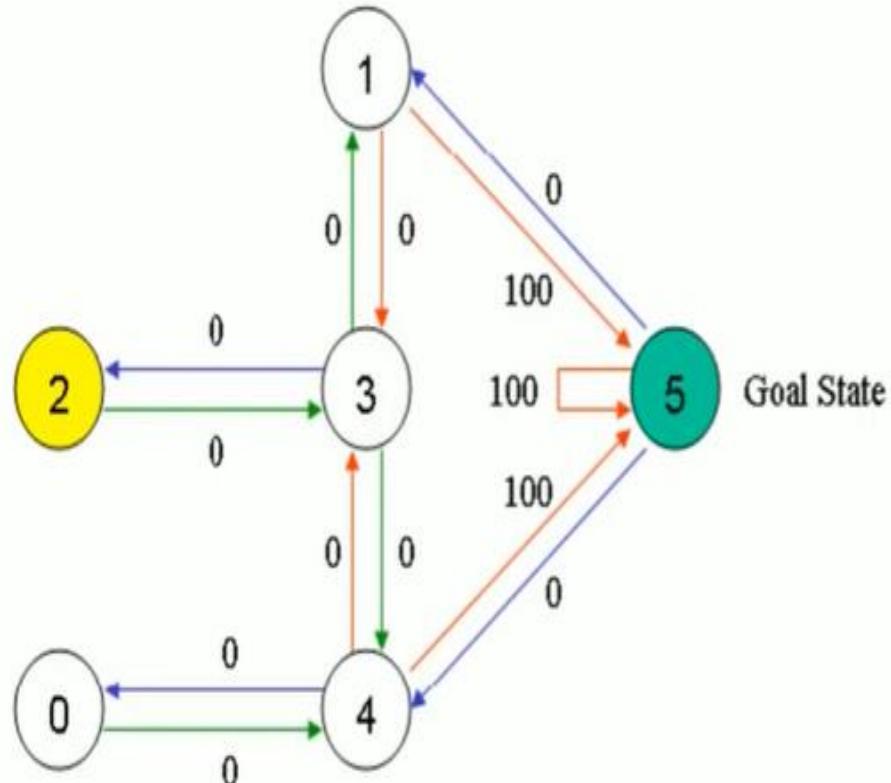
Q-learning Solved Example 2



Q-learning Solved Example



Q-learning Solved Example



Rewards Matrix

State	Action					
	0	1	2	3	4	5
0	-1	-1	-1	-1	0	-1
1	-1	-1	-1	0	-1	100
2	-1	-1	-1	0	-1	-1
3	-1	0	0	-1	0	-1
4	0	-1	-1	0	-1	100
5	-1	0	-1	-1	0	100

The -1 in the table represent null values (where there isn't a link between nodes)

Q-learning Solved Example

- $\alpha = 0.8$ and $\gamma=1$
 - Initial state at room 0
 - Initial Q-Matrix is 0

Q-Matrix (Action-Value Function)

	Action					
State	0	1	2	3	4	5
0	-1	-1	-1	-1	0	-1
1	-1	-1	-1	0	-1	100
2	-1	-1	-1	0	-1	-1
3	-1	0	0	-1	0	-1
4	0	-1	-1	0	-1	100
5	-1	0	-1	-1	0	100

$$Q = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Q-learning Solved Example

- Look at the second row (Initial state 1) in matrix R
- There are two possible actions for the current state 1: go to state 3 or go to state 5
- By random selection, we select to 5 as our action
- What would happen if our agent were in state 5 (next state)? It can go to state 1, 4 or 5 (the new action)

Q-Matrix (Action-Value Function)

State	Action					
	0	1	2	3	4	
0	-1	-1	-1	-1	0	-1
1	-1	-1	-1	0	-1	100
2	-1	-1	-1	0	-1	-1
3	-1	0	0	-1	0	-1
4	0	-1	-1	0	-1	100
5	-1	0	-1	-1	0	100

$R =$

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Q-learning Solved Example

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \right]$$

$$\begin{aligned} Q(1, 5) &= 0 + 1 [100 + 0.8 (\max (Q(5, \text{goto } 1), Q(5, \text{goto } 4), Q(5, \text{goto } 5)) - 0] \\ &= 0 + 1 [100 + 0.8 \times 0 - 0] = 100 \end{aligned}$$

End of the 1st episode

Q-Matrix (Action-Value Function)

$$R = \begin{array}{c|cccccc} & & \text{Action} & & & & \\ \hline \text{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array} \quad Q = \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Q-learning Solved Example

- Let the initial state be state 3 for the next episode
- There are three possible actions for the current state: go to state 1 or 2 or 4
- Assume using current policy we will go to state 1
- In this case we can either goto state 3 or state 5 from state 1

Q-Matrix (Action-Value Function)

$$R = \begin{array}{c|cccccc} & & \text{Action} & & & & \\ \hline \text{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \end{bmatrix} & & & & & \\ 1 & \begin{bmatrix} -1 & -1 & -1 & 0 & -1 & 100 \end{bmatrix} & & & & & \\ 2 & \begin{bmatrix} -1 & -1 & -1 & 0 & -1 & -1 \end{bmatrix} & & & & & \\ 3 & \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & -1 \end{bmatrix} & & & & & \\ 4 & \begin{bmatrix} 0 & -1 & -1 & 0 & -1 & 100 \end{bmatrix} & & & & & \\ 5 & \begin{bmatrix} -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} & & & & & \end{array}$$
$$Q = \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 100 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Q-learning Solved Example

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \right]$$

$$\begin{aligned} Q(3, 1) &= 0 + 1 [0 + 0.8 (\max (Q(1, \text{goto } 3), Q(1, \text{goto } 5)) - 0] \\ &= 0 + 1 [0 + 0.8(\max(0, 100) - 0] = 80 \end{aligned}$$

Q-Matrix (Action-Value Function)

State	Action					
	0	1	2	3	4	5
0	-1	-1	-1	-1	0	-1
1	-1	-1	-1	0	-1	100
2	-1	-1	-1	0	-1	-1
3	-1	0	0	-1	0	-1
4	0	-1	-1	0	-1	100
5	-1	0	-1	-1	0	100

$$R = \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix}$$
$$Q = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 100 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 80 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$