basanta-ws02

March 13, 2025

0.1 Some Helper Function:

0.1.1 Softmax Function:

[ ]:
```
import numpy as np

def softmax(z):
    """
    Compute the softmax probabilities for a given input matrix.

    Parameters:
    z (numpy.ndarray): Logits (raw scores) of shape (m, n), where
                       - m is the number of samples.
                       - n is the number of classes.

    Returns:
    numpy.ndarray: Softmax probability matrix of shape (m, n), where
                   each row sums to 1 and represents the probability
                   distribution over classes.

    Notes:
    - The input to softmax is typically computed as: z = XW + b.
    - Uses numerical stabilization by subtracting the max value per row.
    """

    # Prevent numerical instability by normalizing input
    z_shifted = z - np.max(z, axis=1, keepdims=True)
    exp_z = np.exp(z_shifted)
    return exp_z / np.sum(exp_z, axis=1, keepdims=True)
```

0.1.2 Softmax Test Case:

This test case checks that each row in the resulting softmax probabilities sums to 1, which is the fundamental property of softmax.

[ ]:
```
# Example test case
z_test = np.array([[2.0, 1.0, 0.1], [1.0, 1.0, 1.0]])
softmax_output = softmax(z_test)
```

```python
# Verify if the sum of probabilities for each row is 1 using assert
row_sums = np.sum(softmax_output, axis=1)

# Assert that the sum of each row is 1
assert np.allclose(row_sums, 1), f"Test failed: Row sums are {row_sums}"

print("Softmax function passed the test case!")
```

```
Softmax function passed the test case!
```

[ ]:
```python
from google.colab import drive
drive.mount('/content/drive')
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call
drive.mount("/content/drive", force_remount=True).
```

0.1.3 Prediction Function:

[ ]:
```python
def predict_softmax(X, W, b):
    """
    Predict the class labels for a set of samples using the trained softmax␣
 ↪model.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d), where n is the number of␣
 ↪samples and d is the number of features.
    W (numpy.ndarray): Weight matrix of shape (d, c), where c is the number of␣
 ↪classes.
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    numpy.ndarray: Predicted class labels of shape (n,), where each value is␣
 ↪the index of the predicted class.
    """

    z = np.dot(X, W) + b  # Compute the scores (logits)
    y_pred = softmax(z)  # Get the probabilities using the softmax function

    predicted_classes = np.argmax(y_pred, axis=1)

    return predicted_classes
```

0.1.4 Test Function for Prediction Function:

The test function ensures that the predicted class labels have the same number of elements as the input samples, verifying that the model produces a valid output shape.

```
[ ]: # Define test case
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]])  # Feature matrix (3␣
 ↪samples, 2 features)
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]])  # Weights (2 features, 3␣
 ↪classes)
b_test = np.array([0.1, 0.2, 0.3])  # Bias (3 classes)

# Expected Output:
# The function should return an array with class labels (0, 1, or 2)

y_pred_test = predict_softmax(X_test, W_test, b_test)

# Validate output shape
assert y_pred_test.shape == (3,), f"Test failed: Expected shape (3,), got␣
 ↪{y_pred_test.shape}"

# Print the predicted labels
print("Predicted class labels:", y_pred_test)
```

```
Predicted class labels: [1 1 0]
```

0.1.5 Loss Function:

```
[ ]: def loss_softmax(y_pred, y):
    """
    Compute the cross-entropy loss for a single sample.

    Parameters:
    y_pred (numpy.ndarray): Predicted probabilities of shape (c,) for a single␣
 ↪sample,
                            where c is the number of classes.
    y (numpy.ndarray): True labels (one-hot encoded) of shape (c,), where c is␣
 ↪the number of classes.

    Returns:
    float: Cross-entropy loss for the given sample.
    """

    epsilon = 1e-12  # To avoid log(0)
    y_pred = np.clip(y_pred, epsilon, 1.0 - epsilon)  # Prevent log(0) by␣
 ↪clipping values
    n = y.shape[0]  # Number of samples
    loss = -np.sum(y * np.log(y_pred)) / n
    return loss
```

0.2 Test case for Loss Function:

This test case Compares loss for correct vs. incorrect predictions. * Expects low loss for correct predictions. * Expects high loss for incorrect predictions.

[ ]:
```
import numpy as np

# Define correct predictions (low loss scenario)
y_true_correct = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])  # True one-hot␣
↪labels
y_pred_correct = np.array([[0.9, 0.05, 0.05],
                           [0.1, 0.85, 0.05],
                           [0.05, 0.1, 0.85]])  # High confidence in the␣
↪correct class

# Define incorrect predictions (high loss scenario)
y_pred_incorrect = np.array([[0.05, 0.05, 0.9],  # Highly confident in the␣
↪wrong class
                             [0.1, 0.05, 0.85],
                             [0.85, 0.1, 0.05]])

# Compute loss for both cases
loss_correct = loss_softmax(y_pred_correct, y_true_correct)
loss_incorrect = loss_softmax(y_pred_incorrect, y_true_correct)

# Validate that incorrect predictions lead to a higher loss
assert loss_correct < loss_incorrect, f"Test failed: Expected loss_correct <␣
↪loss_incorrect, but got {loss_correct:.4f} >= {loss_incorrect:.4f}"

# Print results
print(f"Cross-Entropy Loss (Correct Predictions): {loss_correct:.4f}")
print(f"Cross-Entropy Loss (Incorrect Predictions): {loss_incorrect:.4f}")
```

```
Cross-Entropy Loss (Correct Predictions): 0.1435
Cross-Entropy Loss (Incorrect Predictions): 2.9957
```

0.2.1 Cost Function:

[ ]:
```
def cost_softmax(X, y, W, b):
    """
    Compute the average softmax regression cost (cross-entropy loss) over all␣
↪samples.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d), where n is the number of␣
↪samples and d is the number of features.
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c), where n␣
↪is the number of samples and c is the number of classes.
```

```
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    float: Average softmax cost (cross-entropy loss) over all samples.
    """

    n = X.shape[0]  # Number of samples
    z = np.dot(X, W) + b
    y_pred = softmax(z)

    total_loss = loss_softmax(y_pred, y)

    # Return average loss
    return total_loss / n
```

0.2.2 Test Case for Cost Function:

The test case assures that the cost for the incorrect prediction should be higher than for the correct prediction, confirming that the cost function behaves as expected.

[ ]:
```
import numpy as np

# Example 1: Correct Prediction (Closer predictions)
X_correct = np.array([[1.0, 0.0], [0.0, 1.0]])  # Feature matrix for correct␣
 ↪predictions
y_correct = np.array([[1, 0], [0, 1]])  # True labels (one-hot encoded,␣
 ↪matching predictions)
W_correct = np.array([[5.0, -2.0], [-3.0, 5.0]])  # Weights for correct␣
 ↪prediction
b_correct = np.array([0.1, 0.1])  # Bias for correct prediction

# Example 2: Incorrect Prediction (Far off predictions)
X_incorrect = np.array([[0.1, 0.9], [0.8, 0.2]])  # Feature matrix for␣
 ↪incorrect predictions
y_incorrect = np.array([[1, 0], [0, 1]])  # True labels (one-hot encoded,␣
 ↪incorrect predictions)
W_incorrect = np.array([[0.1, 2.0], [1.5, 0.3]])  # Weights for incorrect␣
 ↪prediction
b_incorrect = np.array([0.5, 0.6])  # Bias for incorrect prediction

# Compute cost for correct predictions
cost_correct = cost_softmax(X_correct, y_correct, W_correct, b_correct)

# Compute cost for incorrect predictions
cost_incorrect = cost_softmax(X_incorrect, y_incorrect, W_incorrect,␣
 ↪b_incorrect)
```

```python
# Check if the cost for incorrect predictions is greater than for correct␣
↪predictions
assert cost_incorrect > cost_correct, f"Test failed: Incorrect cost␣
↪{cost_incorrect} is not greater than correct cost {cost_correct}"

# Print the costs for verification
print("Cost for correct prediction:", cost_correct)
print("Cost for incorrect prediction:", cost_incorrect)

print("Test passed!")
```

```
Cost for correct prediction: 0.00031171820666746‍62
Cost for incorrect prediction: 0.14965430679723057
Test passed!
```

0.2.3 Computing Gradients:

[ ]:
```python
def compute_gradient_softmax(X, y, W, b):
    """
    Compute the gradients of the cost function with respect to weights and␣
↪biases.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    tuple: Gradients with respect to weights (d, c) and biases (c,).
    """

    n, d = X.shape
    z = np.dot(X, W) + b
    y_pred = softmax(z)

    grad_W = np.dot(X.T, (y_pred - y)) / n  # Gradient with respect to weights
    grad_b = np.sum(y_pred - y, axis=0) / n  # Gradient with respect to biases

    return grad_W, grad_b
```

0.2.4 Test case for compute_gradient function:

The test checks if the gradients from the function are close enough to the manually computed gradients using np.allclose, which accounts for potential floating-point discrepancies.

[ ]:

```python
import numpy as np

# Define a simple feature matrix and true labels
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]])  # Feature matrix (3␣
↪samples, 2 features)
y_test = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])  # True labels (one-hot␣
↪encoded, 3 classes)

# Define weight matrix and bias vector
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]])  # Weights (2 features, 3␣
↪classes)
b_test = np.array([0.1, 0.2, 0.3])  # Bias (3 classes)

# Compute the gradients using the function
grad_W, grad_b = compute_gradient_softmax(X_test, y_test, W_test, b_test)

# Manually compute the predicted probabilities (using softmax function)
z_test = np.dot(X_test, W_test) + b_test
y_pred_test = softmax(z_test)

# Compute the manually computed gradients
grad_W_manual = np.dot(X_test.T, (y_pred_test - y_test)) / X_test.shape[0]
grad_b_manual = np.sum(y_pred_test - y_test, axis=0) / X_test.shape[0]

# Assert that the gradients computed by the function match the manually␣
↪computed gradients
assert np.allclose(grad_W, grad_W_manual), f"Test failed: Gradients w.r.t. W␣
↪are not equal.\nExpected: {grad_W_manual}\nGot: {grad_W}"
assert np.allclose(grad_b, grad_b_manual), f"Test failed: Gradients w.r.t. b␣
↪are not equal.\nExpected: {grad_b_manual}\nGot: {grad_b}"

# Print the gradients for verification
print("Gradient w.r.t. W:", grad_W)
print("Gradient w.r.t. b:", grad_b)

print("Test passed!")
```

```
Gradient w.r.t. W: [[ 0.1031051   0.01805685 -0.12116196]
 [-0.13600547  0.00679023  0.12921524]]
Gradient w.r.t. b: [-0.03290036  0.02484708  0.00805328]
Test passed!
```

0.2.5 Implementing Gradient Descent:

```
[ ]: def gradient_descent_softmax(X, y, W, b, alpha, n_iter, show_cost=False):
         """
         Perform gradient descent to optimize the weights and biases.

         Parameters:
         X (numpy.ndarray): Feature matrix of shape (n, d).
         y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
         W (numpy.ndarray): Weight matrix of shape (d, c).
         b (numpy.ndarray): Bias vector of shape (c,).
         alpha (float): Learning rate.
         n_iter (int): Number of iterations.
         show_cost (bool): Whether to display the cost at intervals.

         Returns:
         tuple: Optimized weights, biases, and cost history.
         """
         cost_history = []

         for i in range(n_iter):
             # Compute gradients
             grad_W, grad_b = compute_gradient_softmax(X, y, W, b)
             # Update weights and biases using the gradients
             W -= alpha * grad_W
             b -= alpha * grad_b

             # Compute and store cost
             cost = cost_softmax(X, y, W, b)
             cost_history.append(cost)

             # Print cost at regular intervals
             if show_cost and (i % 100 == 0 or i == n_iter - 1):
                 print(f"Iteration {i}: Cost = {cost:.6f}")
         return W, b, cost_history
```

0.3 Preparing Dataset:

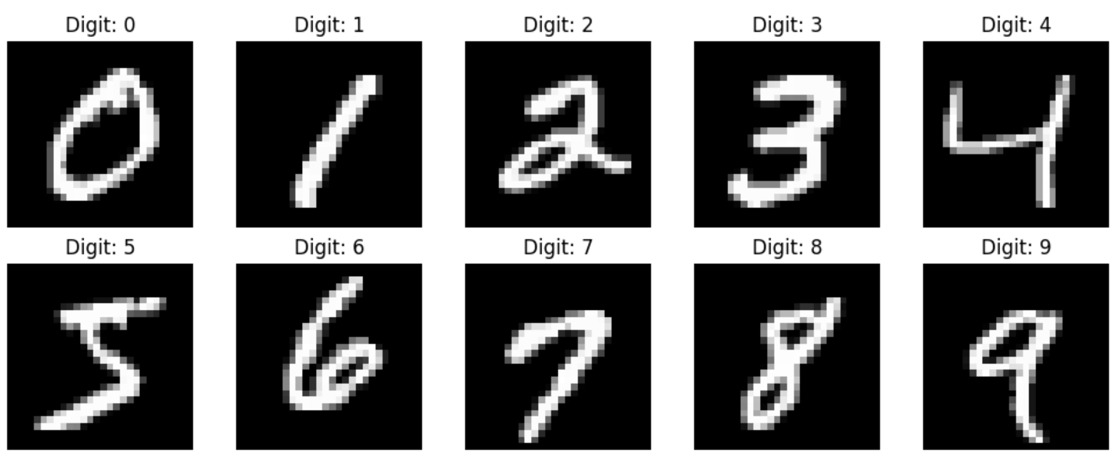
```
[ ]: import pandas as pd
     import numpy as np
     import matplotlib.pyplot as plt
     from sklearn.model_selection import train_test_split

     def load_and_prepare_mnist(csv_file, test_size=0.2, random_state=42):
         """
         Reads the MNIST CSV file, splits data into train/test sets, and plots one␣
      ↪image per class.
```

```
    Arguments:
    csv_file (str)        : Path to the CSV file containing MNIST data.
    test_size (float)     : Proportion of the data to use as the test set␣
↪(default: 0.2).
    random_state (int)    : Random seed for reproducibility (default: 42).

    Returns:
    X_train, X_test, y_train, y_test : Split dataset.
    """

    # Load dataset
    df = pd.read_csv("/content/drive/MyDrive/AI-Shivkumar/Week2/Workshop2/
↪mnist_dataset.csv")

    # Separate labels and features
    y = df.iloc[:, 0].values  # First column is the label
    X = df.iloc[:, 1:].values  # Remaining columns are pixel values

    # Normalize pixel values (optional but recommended)
    X = X / 255.0  # Scale values between 0 and 1

    # Split data into train and test sets
    X_train, X_test, y_train, y_test = train_test_split(X, y,␣
↪test_size=test_size, random_state=random_state)

    # Plot one sample image per class
    plot_sample_images(X, y)

    return X_train, X_test, y_train, y_test

def plot_sample_images(X, y):
    """
    Plots one sample image for each digit class (0-9).

    Arguments:
    X (np.ndarray): Feature matrix containing pixel values.
    y (np.ndarray): Labels corresponding to images.
    """

    plt.figure(figsize=(10, 4))
    unique_classes = np.unique(y)  # Get unique class labels

    for i, digit in enumerate(unique_classes):
        index = np.where(y == digit)[0][0]  # Find first occurrence of the class
        image = X[index].reshape(28, 28)  # Reshape 1D array to 28x28
```

```python
        plt.subplot(2, 5, i + 1)
        plt.imshow(image, cmap='gray')
        plt.title(f"Digit: {digit}")
        plt.axis('off')

    plt.tight_layout()
    plt.show()
```

[ ]:

```python
csv_file_path = "/content/drive/MyDrive/AI-Shivkumar/Week2/Workshop2/mnist_dataset.csv"  # Path to saved dataset
X_train, X_test, y_train, y_test = load_and_prepare_mnist(csv_file_path)
```



0.3.1 A Quick debugging Step:

[ ]:

```python
# Assert that X and y have matching lengths
assert len(X_train) == len(y_train), f"Error: X and y have different lengths! X={len(X_train)}, y={len(y_train)}"
print("Move forward: Dimension of Feture Matrix X and label vector y matched.")
```

```
Move forward: Dimension of Feture Matrix X and label vector y matched.
```

0.4 Train the Model:

[ ]:

```python
print(f"Training data shape: {X_train.shape}")
print(f"Test data shape: {X_test.shape}")
```

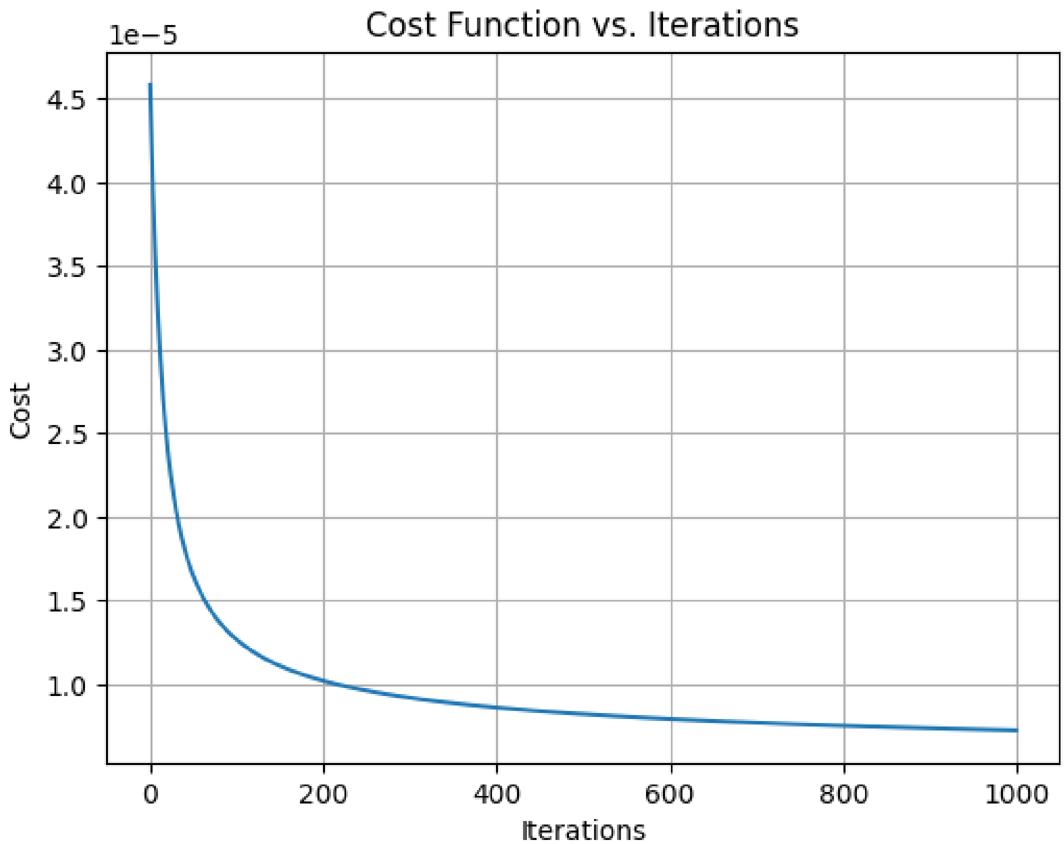
```
Training data shape: (48000, 784)
Test data shape: (12000, 784)
```

[ ]:

```python
from sklearn.preprocessing import OneHotEncoder
```

```python
# Check if y_train is one-hot encoded
if len(y_train.shape) == 1:
    encoder = OneHotEncoder(sparse_output=False)  # Use sparse_output=False for␣
 ↪newer versions of sklearn
    y_train = encoder.fit_transform(y_train.reshape(-1, 1))  # One-hot encode␣
 ↪labels
    y_test = encoder.transform(y_test.reshape(-1, 1))  # One-hot encode test␣
 ↪labels

# Now y_train is one-hot encoded, and we can proceed to use it
d = X_train.shape[1]  # Number of features (columns in X_train)
c = y_train.shape[1]  # Number of classes (columns in y_train after one-hot␣
 ↪encoding)

# Initialize weights with small random values and biases with zeros
W = np.random.randn(d, c) * 0.01  # Small random weights initialized
b = np.zeros(c)  # Bias initialized to 0

# Set hyperparameters for gradient descent
alpha = 0.1  # Learning rate
n_iter = 1000  # Number of iterations to run gradient descent

# Train the model using gradient descent
W_opt, b_opt, cost_history = gradient_descent_softmax(X_train, y_train, W, b,␣
 ↪alpha, n_iter, show_cost=True)

# Plot the cost history to visualize the convergence
plt.plot(cost_history)
plt.title('Cost Function vs. Iterations')
plt.xlabel('Iterations')
plt.ylabel('Cost')
plt.grid(True)
plt.show()
```

```
Iteration 0: Cost = 0.000046
Iteration 100: Cost = 0.000013
Iteration 200: Cost = 0.000010
Iteration 300: Cost = 0.000009
Iteration 400: Cost = 0.000009
Iteration 500: Cost = 0.000008
Iteration 600: Cost = 0.000008
Iteration 700: Cost = 0.000008
Iteration 800: Cost = 0.000008
Iteration 900: Cost = 0.000007
Iteration 999: Cost = 0.000007
```



0.5 Evaluating the Model:

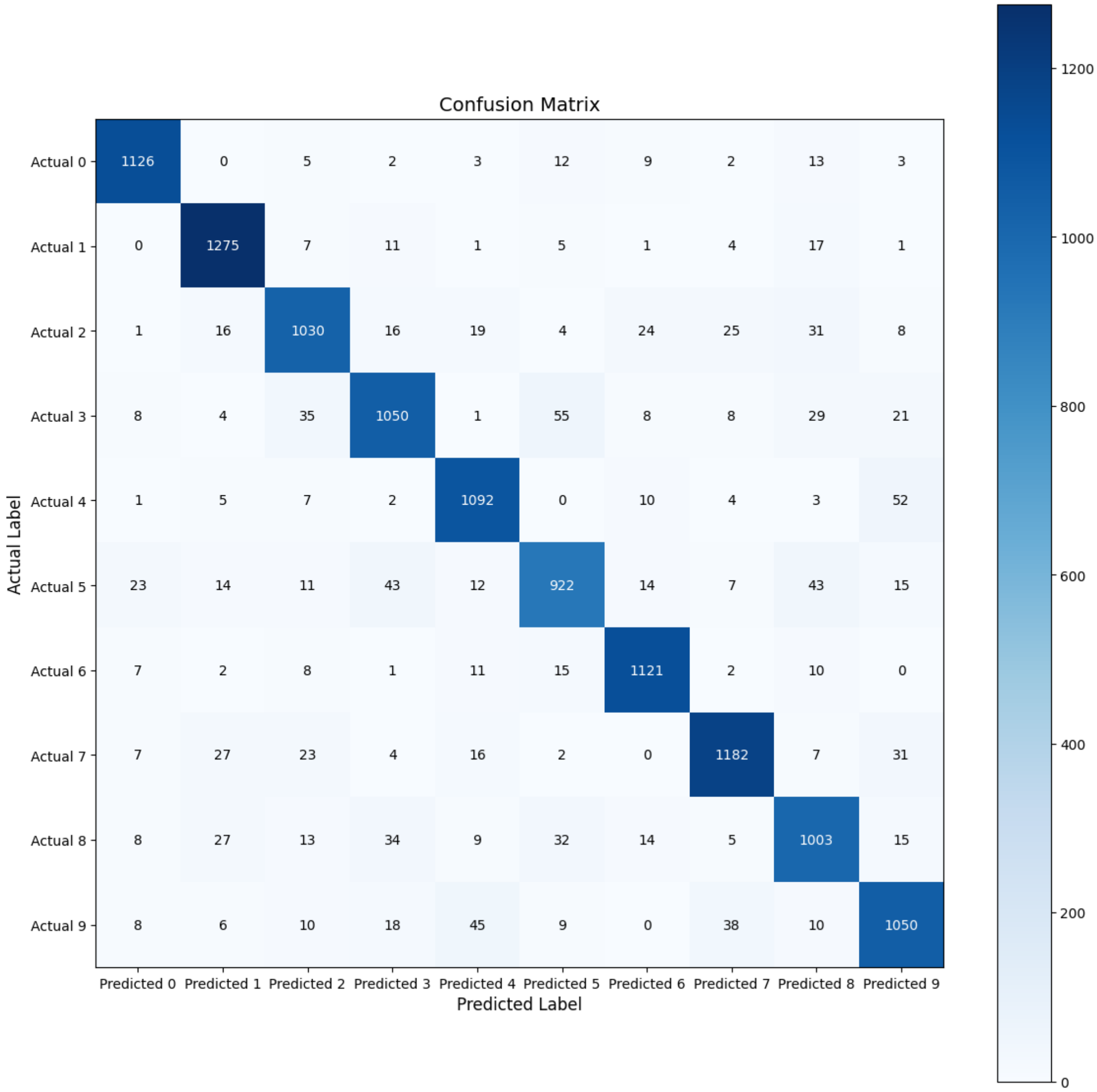
[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.metrics import confusion_matrix, precision_score, recall_score,␣
↪f1_score

# Evaluation Function
def evaluate_classification(y_true, y_pred):
    """
    Evaluate classification performance using confusion matrix, precision,␣
↪recall, and F1-score.

    Parameters:
    y_true (numpy.ndarray): True labels
    y_pred (numpy.ndarray): Predicted labels

    Returns:
    tuple: Confusion matrix, precision, recall, F1 score
```

```python
    """
    # Compute confusion matrix
    cm = confusion_matrix(y_true, y_pred)

    # Compute precision, recall, and F1-score
    precision = precision_score(y_true, y_pred, average='weighted')
    recall = recall_score(y_true, y_pred, average='weighted')
    f1 = f1_score(y_true, y_pred, average='weighted')

    return cm, precision, recall, f1
```

[ ]:
```python
# Predict on the test set
y_pred_test = predict_softmax(X_test, W_opt, b_opt)

# Evaluate accuracy
y_test_labels = np.argmax(y_test, axis=1)  # True labels in numeric form

# Evaluate the model
cm, precision, recall, f1 = evaluate_classification(y_test_labels, y_pred_test)

# Print the evaluation metrics
print("\nConfusion Matrix:")
print(cm)
print(f"Precision: {precision:.2f}")
print(f"Recall: {recall:.2f}")
print(f"F1-Score: {f1:.2f}")

# Visualizing the Confusion Matrix
fig, ax = plt.subplots(figsize=(12, 12))
cax = ax.imshow(cm, cmap='Blues')  # Use a color map for better visualization

# Dynamic number of classes
num_classes = cm.shape[0]
ax.set_xticks(range(num_classes))
ax.set_yticks(range(num_classes))
ax.set_xticklabels([f'Predicted {i}' for i in range(num_classes)])
ax.set_yticklabels([f'Actual {i}' for i in range(num_classes)])

# Add labels to each cell in the confusion matrix
for i in range(cm.shape[0]):
    for j in range(cm.shape[1]):
        ax.text(j, i, cm[i, j], ha='center', va='center', color='white' if␣
 ↪cm[i, j] > np.max(cm) / 2 else 'black')

# Add grid lines and axis labels
ax.grid(False)
plt.title('Confusion Matrix', fontsize=14)
```

```python
plt.xlabel('Predicted Label', fontsize=12)
plt.ylabel('Actual Label', fontsize=12)

# Adjust layout
plt.tight_layout()
plt.colorbar(cax)
plt.show()
```

```
Confusion Matrix:
[[1126    0    5    2    3   12    9    2   13    3]
 [   0 1275    7   11    1    5    1    4   17    1]
 [   1   16 1030   16   19    4   24   25   31    8]
 [   8    4   35 1050    1   55    8    8   29   21]
 [   1    5    7    2 1092    0   10    4    3   52]
 [  23   14   11   43   12  922   14    7   43   15]
 [   7    2    8    1   11   15 1121    2   10    0]
 [   7   27   23    4   16    2    0 1182    7   31]
 [   8   27   13   34    9   32   14    5 1003   15]
 [   8    6   10   18   45    9    0   38   10 1050]]
Precision: 0.90
Recall: 0.90
F1-Score: 0.90
```

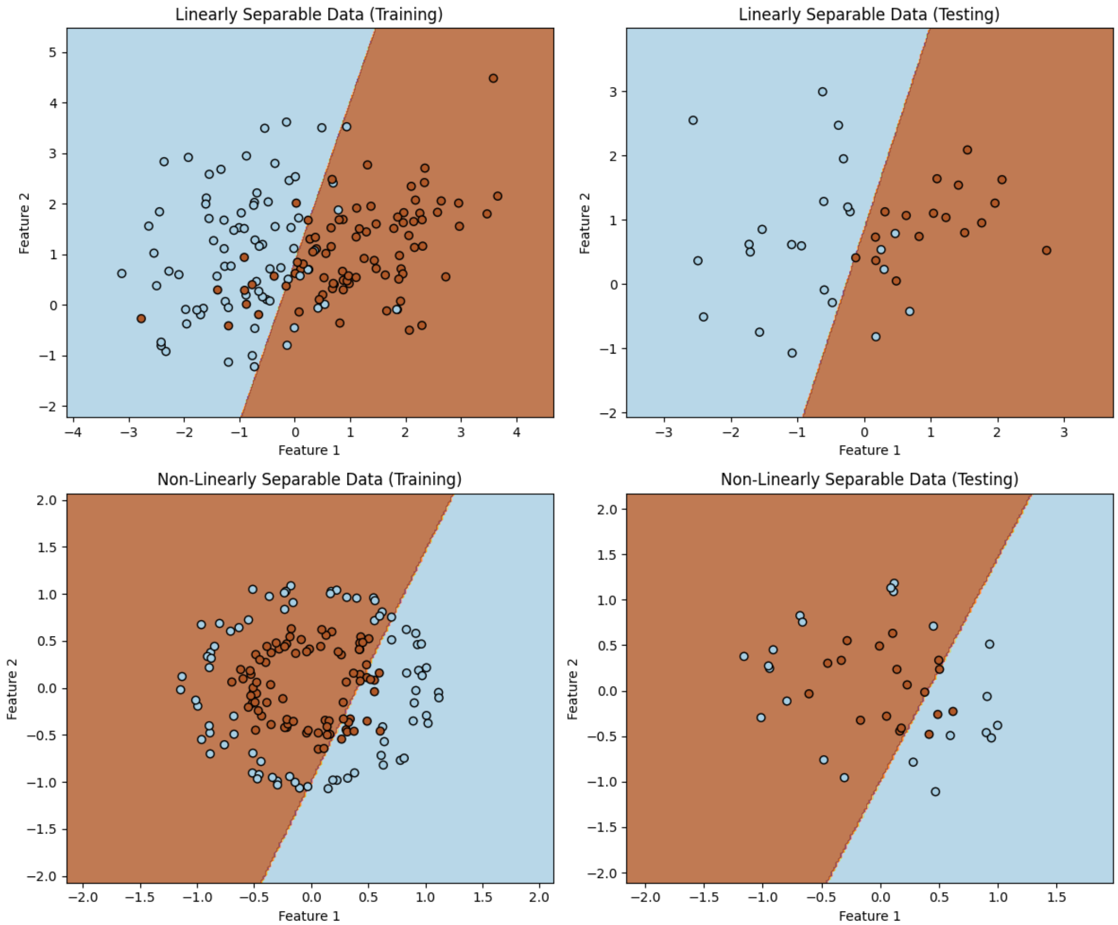


1 Linear Seperability and Logistic Regression:

[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification, make_circles
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression

# Set random seed for reproducibility
np.random.seed(42)

# Generate linearly separable dataset
```

```python
X_linear_separable, y_linear_separable = make_classification(
    n_samples=200, n_features=2, n_informative=2,
    n_redundant=0, n_clusters_per_class=1, random_state=42
)

# Split the data into training and testing sets
X_train_linear, X_test_linear, y_train_linear, y_test_linear = train_test_split(
    X_linear_separable, y_linear_separable, test_size=0.2, random_state=42
)

# Train logistic regression model on linearly separable data
logistic_model_linear_separable = LogisticRegression()
logistic_model_linear_separable.fit(X_train_linear, y_train_linear)

# Generate non-linearly separable dataset (circles)
X_non_linear_separable, y_non_linear_separable = make_circles(
    n_samples=200, noise=0.1, factor=0.5, random_state=42
)

# Split the data into training and testing sets
X_train_non_linear, X_test_non_linear, y_train_non_linear, y_test_non_linear =␣
 ↪train_test_split(
    X_non_linear_separable, y_non_linear_separable, test_size=0.2,␣
 ↪random_state=42
)

# Train logistic regression model on non-linearly separable data
logistic_model_non_linear_separable = LogisticRegression()
logistic_model_non_linear_separable.fit(X_train_non_linear, y_train_non_linear)

# Function to plot decision boundaries
def plot_decision_boundary(ax, model, X, y, title):
    h = 0.02  # Step size in the mesh
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))

    Z = model.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)

    ax.contourf(xx, yy, Z, alpha=0.8, cmap=plt.cm.Paired)
    ax.scatter(X[:, 0], X[:, 1], c=y, edgecolors='black', cmap=plt.cm.Paired)
    ax.set_title(title)
    ax.set_xlabel('Feature 1')
    ax.set_ylabel('Feature 2')

# Create subplots
```

```python
fig, axes = plt.subplots(2, 2, figsize=(12, 10))

# Plot decision boundaries
plot_decision_boundary(axes[0, 0], logistic_model_linear_separable,␣
 ↪X_train_linear, y_train_linear,
                       'Linearly Separable Data (Training)')
plot_decision_boundary(axes[0, 1], logistic_model_linear_separable,␣
 ↪X_test_linear, y_test_linear,
                       'Linearly Separable Data (Testing)')
plot_decision_boundary(axes[1, 0], logistic_model_non_linear_separable,␣
 ↪X_train_non_linear,
                       y_train_non_linear, 'Non-Linearly Separable Data␣
 ↪(Training)')
plot_decision_boundary(axes[1, 1], logistic_model_non_linear_separable,␣
 ↪X_test_non_linear,
                       y_test_non_linear, 'Non-Linearly Separable Data␣
 ↪(Testing)')

plt.tight_layout()

# Save the plots as PNG files
plt.savefig('decision_boundaries.png')
plt.show()
```



Question - 2: Provide an interpretation of the output based on your understanding.

The plots illustrate how logistic regression performs on two different types of datasets: one that is linearly separable and another that is non-linearly separable data.

Top row (Linearly Separable Data): The model works well, correctly dividing the two classes with a straight-line boundary. It performs similarly on both training and testing data, meaning it generalizes well.

Bottom row (Non-Linearly Separable Data): The model struggles because the data follows a circular pattern. A straight-line decision boundary isn't enough, leading to many misclassifications.

Question - 3: Describe any challenges you faced while implementing the code above.

One challenge I faced was making sure the model performed well on the MNIST dataset. Since logistic regression is a simple classifier, it struggled to capture complex patterns. Another issue was tuning the learning rate—if it was too high, the model wouldn't learn properly, and if too low, training was slow. Working with a large dataset also made training take longer, so I had to think about ways to make the process more efficient. Despite these challenges, it was a great learning experience in understanding how logistic regression works for digit classification.