Homework 1

Group 56

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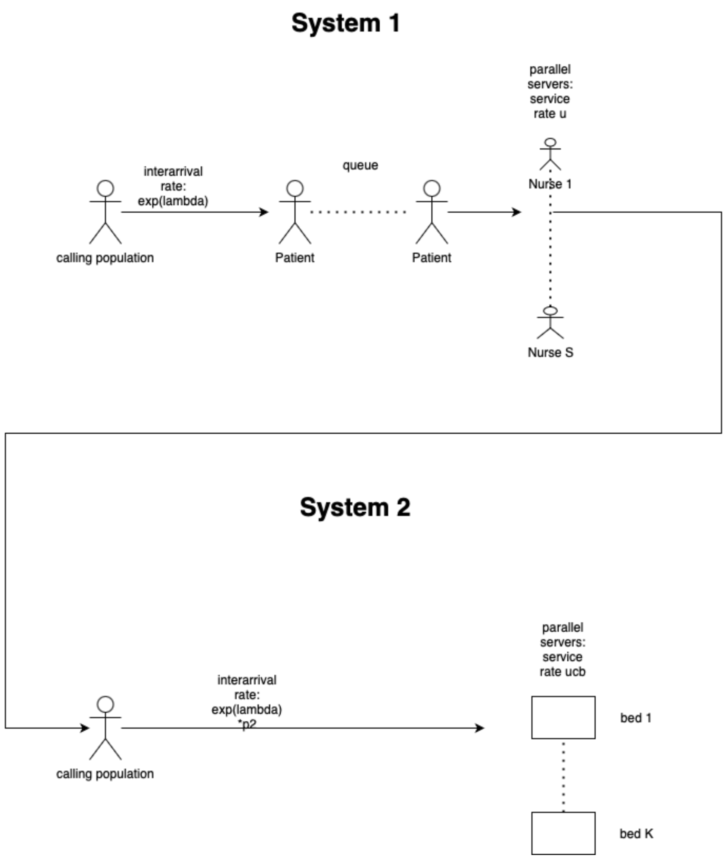
1 Introduction



In these calculations, we have:

$$\begin{aligned}
&K = 3, \\
&\mu_t = 0.416666667, \quad \text{(service rate per server in System 1)}, \\
&\lambda_A = 1, \quad \text{(arrival rate in System 1)}, \\
&p_c = 0.75, \quad \text{(fraction transitioning to System 2)}, \\
&B = 6, \quad \text{(capacity parameter in System 2)}, \\
&\mu_b = 0.15625, \quad \text{(service rate in System 2)}, \\
&\lambda_C = \lambda_A \cdot p_c = 1 \times 0.75 = 0.75.
\end{aligned}$$

2 Queueing System 1 (M/M/K)

2.1 Traffic Intensity

Traffic intensity, denoted ρ_1, is:

$$\rho_1 = \frac{\lambda_A}{K\,\mu_t}.$$

With $\lambda_A = 1$, $K = 3$, $\mu_t = 0.416666667$:

$$\rho_1 = \frac{1}{3 \times 0.416666667} \approx 0.8.$$

2.2 Expected Number of Patients in the System

$$L = \frac{\rho_1\ K\rho_1}{(1-\rho_1)\,K!}\ P_0.$$

$$L = 1.6 \times 0.04975 \approx 0.0796.$$

3 Queueing System 2 (M/M/B/B)

3.1 Traffic Intensity

For an **M/M/B/B** system, thetraffic intensity is:

$$E = \frac{\lambda_C}{\mu_b}.$$

Substituting values:

$$E = \frac{0.75}{0.15625} = 4.8.$$

3.2 Blocking Probability

The blocking probability is given by Erlang's Loss Formula:

$$P_c = \frac{E^B/B!}{\sum_{n=0}^{B} E^n/n!}.$$

Substituting values:

$$P_c = \frac{(4.8)^6/6!}{\sum_{n=0}^{6} (4.8)^n/n!}.$$

$$= \frac{110.592/720}{1 + 4.8 + \frac{4.8^2}{2!} + \frac{4.8^3}{3!} + \frac{4.8^4}{4!} + \frac{4.8^5}{5!} + \frac{4.8^6}{6!}}$$

$$= \frac{0.15333}{95.083} \approx 0.177.$$

3.3 Expected Number of Busy Servers

$$L_s = B \cdot (1 - P_c).$$

Substituting values:

$$L_s = 6 \times (1 - 0.177).$$

$$L_s = 6 \times 0.823.$$

$$L_s \approx 4.939.$$

3.4 Expected Number of Patients in the System

Since this is an M/M/B/B loss system, the expected number of patients in the system is the same as the expected number of busy servers:

$$L = L_s \approx 4.939.$$

3.5 Average Number of Patients Treated at Home

$$\text{AverageHome} = \lambda_A \times 0.25 + \lambda_C \times P_c.$$

Substituting values:

$$\text{AverageHome} = (1 \times 0.25) + (0.75 \times 0.177).$$

$$= 0.25 + 0.133.$$

$$\approx 0.383.$$

Thus, on average, 0.383 patients per hour are treated at home due to either the initial decision to stay home or the hospital reaching full capacity.