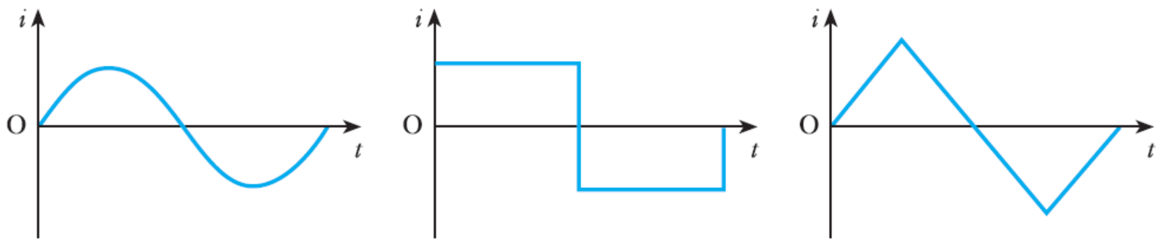
Due to the invention of transformer, AC systems have gained popularity over DC Systems for Power Generation, Transmission and Distribution.

AC Stands for ‘Alternating Current’. An AC waveform is a periodic waveform which alternates i.e., which has alternately positive and negative portions in the waveform.

For instance, the following waveforms are examples of AC waveforms.



Sinusoidal wave Square wave Triangular wave

Some Basic Definitions:

1) Periodic waveform:

A periodic waveform is one which repeats itself after certain time interval.

2) Time Period(T):

The time taken to complete one cycle of a periodic waveform is termed as its time period. It is measured in Seconds.

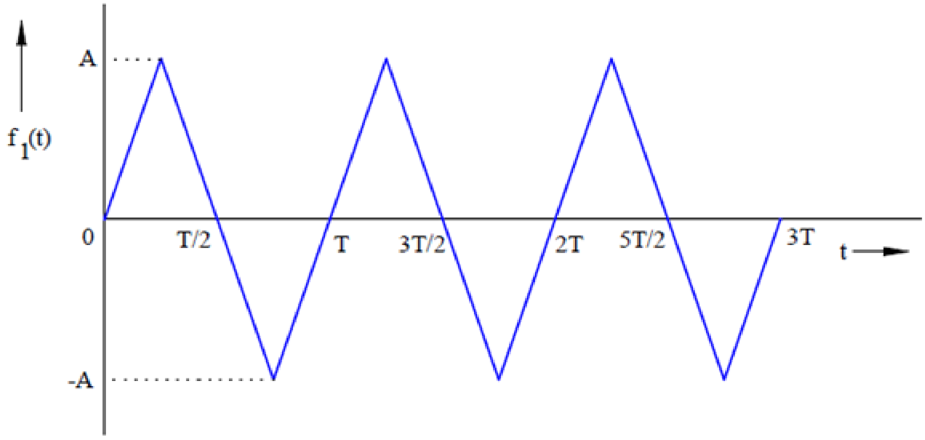
3) **Frequency(f):**

The number of cycles completed in one second of a periodic waveform is termed as its frequency. It is measured in Hz (or) cycles/sec.

Concept of Pure AC waveform:

A pure AC waveform is one whose average value is zero i.e., each positive area is equally matched by a corresponding negative area over one Time period.

For instance, consider the following waveform $f_1(t)$:



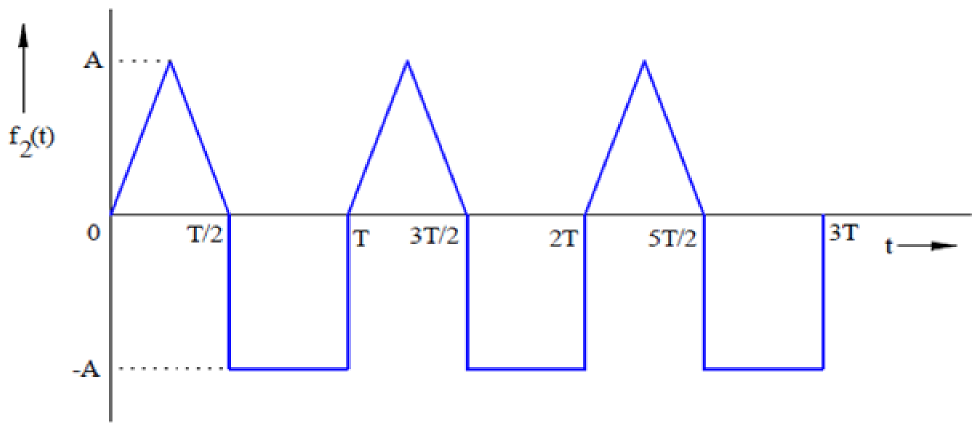
Positive Area = (1/2)*(T/2)*A = AT/4

Negative Area = (1/2)*(T/2)*(-A) = -AT/4

Net Area over one Time Period = 0

Hence, Average value is zero. Therefore, it is a Pure AC waveform.

Consider another waveform $f_2(t)$ as shown below:



Positive Area = (1/2)*(T/2)*A = AT/4

Negative Area = (T/2)*(-A) = -AT/2

Net Area over one Time Period = -AT/4

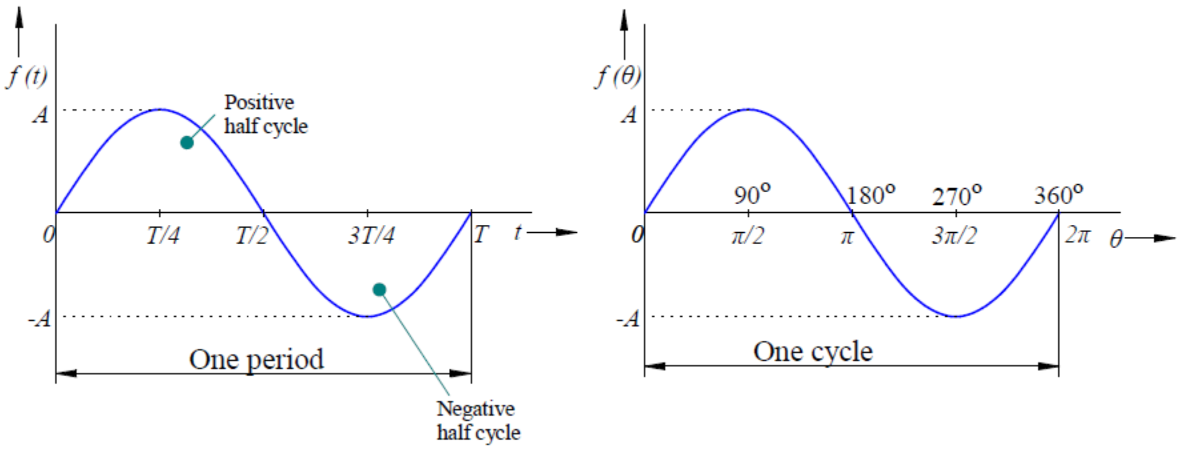
Hence, Average value is non-zero. Therefore, it is not a Pure AC waveform.

Sinusoidal waveform:

Since power generation, transmission and distribution happens as sinusoidal AC power, our discussion in this chapter confines to Sinusoidal waveform.

A sinusoidal waveform for one complete cycle is shown below:

It can be expressed as a function of time in seconds (or) angle in radians.



Accordingly, one cycle completes in T seconds (or) 2π radians.

The following table gives relation between time in seconds and angle in radians.

Time (sec)	Angle θ (Rad)
T	2π
T/2	Π
1	(2π/T)
t	(2π/T)*t

From the above table it can be concluded that at a general angle 't' seconds, the corresponding angle θ is (2π/T)*t radians.

A sinusoidal function is usually expressed as a function of angle as

$e(\theta) = E_m\sin(\theta)$

$= E_m\sin((2\pi/T)*t) = E_m\sin(\omega t) = e(t)$

where, $\omega = 2\pi/T = 2\pi f$ is called the angular frequency of the sine wave in rad/s.

In general, the standard representation of a sinusoidal function is $E_m\sin(\omega t+\phi)$ where ϕ is called the phase angle which can be either positive or negative.

Numerical Example 1

Question:

For a Sinusoidal function of frequency 50 Hz, find

i) Half time period

ii) Angular frequency

Solution:

Time period, T = 1/f = 1/50 = 0.02s = 20 ms

i) Half time period T/2 = 20/2=10 ms

ii) Angular frequency (ω)

$\omega = 2\pi f = 2\pi(50) = 100\,\pi = 314.159$ rad/sec

Numerical Example 2

Question:

The maximum value of a sinusoidal alternating current of frequency 50Hz is 25 A. Write the equation for the instantaneous expression of current,. Determine its value at 3ms and 14 ms.

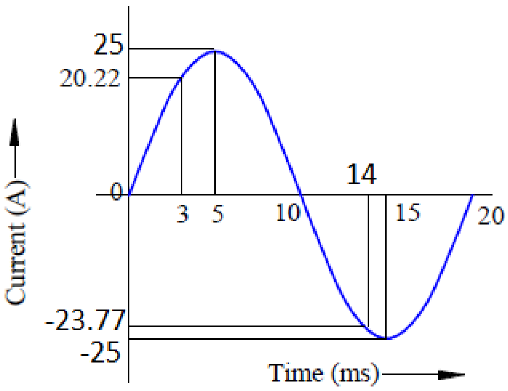
Solution:

Angular frequency, $\omega = 2\pi f = 100\pi$rad/s

$i(t) = 25\sin(100\pi t)$ A

$i(3ms) = 25\sin(100*\pi*0.003) = 20.22A$

Similarly, $i(14ms) = -23.77A$



Note: If radian scale is selected then substitute 'π' symbol in above equation. If degree scale is selected then don't use 'π' symbol, but substitute 180 in place of 'π'.

Average value of a Sinusoidal Function:

In general, the average value of an AC waveform f(t) is given by

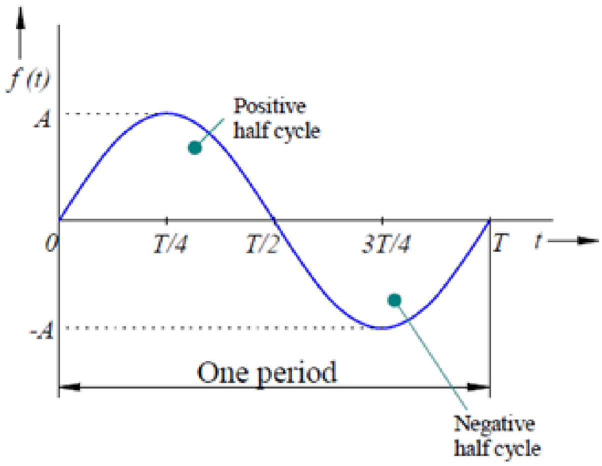
$$F_{avg} = \frac{1}{T}\int_0^T f(t)dt$$

The average value of a sinusoidal function f(t) = Asin(ωt) is

$$F_{avg} = \frac{1}{T}\int_0^T A\sin(\omega t)dt$$

$$= \frac{A}{T}\left(\frac{-\cos(\omega t)}{\omega}\right)_0^T = 0$$

Also, we can conclude the same result from the waveform as well.



Over one Time period, Net Area = 0. Hence, $F_{avg} = 0$

Effective (or) Root Mean Square (RMS) Value of an AC function:

Consider an AC Voltage v(t) connected across a resistor R for ‘T’ seconds.

Energy consumed by the resistor during this period is

$$E_{AC} = \int_0^T p(t)dt$$

$$= \int_0^T \frac{[v(t)]^2}{R}dt$$

Now, excite this resistor using a DC Voltage source of voltage ‘V’ for same time ‘T’ seconds.

Energy consumed by the resistor in this case is

$$E_{DC} = \frac{V^2}{R}.T$$

That value of DC voltage 'V' for which $E_{AC} = E_{DC}$ is said to be the Effective value of the AC voltage v(t).

Hence,

$$\int_0^T \frac{[v(t)]^2}{R} dt = \frac{V^2}{R}.T$$

Therefore, Effective value

$$V = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2 dt}$$

Mathematically the operations involved are

i) Square of the function

ii) Mean (Average) of the function

iii) Square root of the function

Hence, it is also called Root Mean Square (RMS) value.

Effective (or) Root Mean Square (RMS) Value of Sine Wave:

Consider a sinusoidal voltage

$$v(t) = V_m \sin(\omega t)$$

Its RMS value, $V = \sqrt{\frac{1}{T}\int_0^T [V_m \sin\omega t]^2 dt}$

$$=\sqrt{\frac{V_m^{\ 2}}{T}\int_0^T[\sin^2\omega t]dt}$$

$$=\sqrt{\frac{V_m^{\ 2}}{T}*\frac{T}{2}}$$

$$=\frac{V_m}{\sqrt{2}}$$

Major advantage of finding effective (or) RMS value of an AC function is that it makes power calculations easy.

Power consumed in AC circuits, p(t) = v(t)*i(t)

Average power consumed, P = $\dfrac{\int_0^T p(t)dt}{T}=\dfrac{\int_0^T \frac{[v(t)]^2}{R}}{T}=\dfrac{\int_0^T \frac{[v(t)]^2}{T}}{R}=\dfrac{V^2}{R}$

Where V = RMS value of voltage.

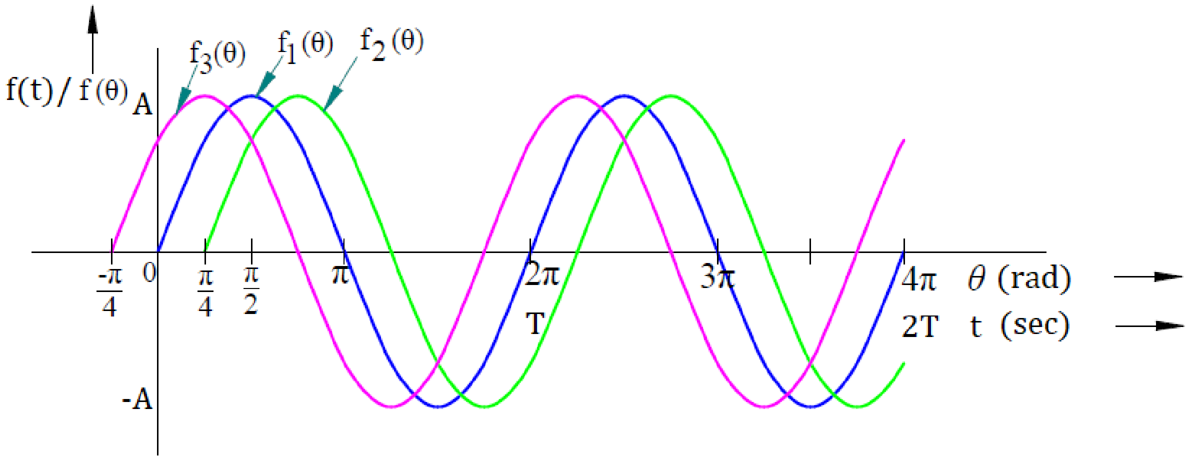
Similarly, average power consumed is also equal to I^2*R where, I = RMS current.

Also, Energy consumed in ‘t’ seconds = P*t

i.e.,

$$(I^2R)t \ \text{(or)} \ \frac{V^2}{R}t$$

Concept of Phase Lag and Phase Lead:



$f_1(t)$ = Asin(ωt) represents a reference sine wave.

$f_2(t)$ = Asin(ωt - $\frac{\pi}{4}$) lags reference sine wave by $\frac{\pi}{4}$ rad.

$f_3(t)$ = Asin(ωt + $\frac{\pi}{4}$) leads reference sine wave by $\frac{\pi}{4}$ rad.

Also, $f_2(t)$ lags $f_3(t)$ by $\frac{\pi}{2}$ rad.

In general, sinusoidal function is represented as A sin(ωt + ϕ) where, ϕ represents the phase angle. If ϕ is positive, it leads the reference sine wave and lags if ϕ is negative.

Question 3:

Write an equation to represent the following sine waves of 50Hz frequency.

i) A sinusoidal current with RMS value 10A & starting at 5ms

ii) A sinusoidal current with peak value 20A & starting at -2.5ms

Also, comment on the phase relation between them.

Solution:

ω= 2πf = 100πrad/s

Case (i) : Angle = ω*t = (100π*0.005) = $\frac{\pi}{2}$ rad

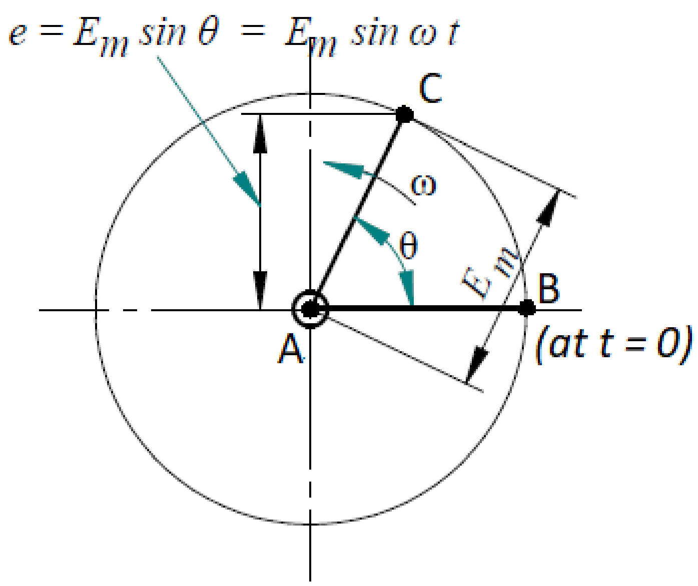
$i_1(t)=10\sqrt{2}\sin(100\pi t - \frac{\pi}{2})A$

Case (ii) : Angle = ω*t = (100π*0.0025) = $\frac{\pi}{4}$ rad

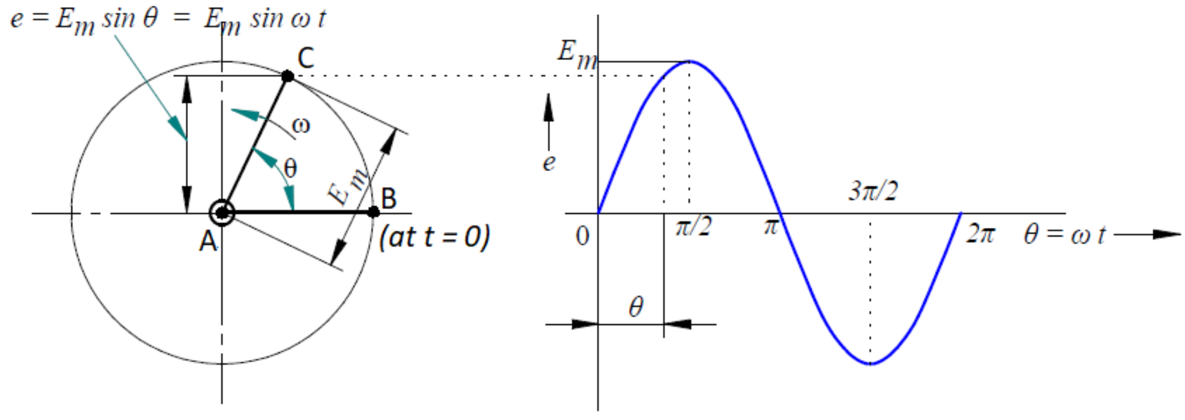
$i_2(t) = 20\sin(100\pi t + \frac{\pi}{4})A$

Concept of Phasor:

Let us consider a rotating crank of length E_m lying at 0° position at t = 0 and rotating anticlockwise at an angular speed of 'ω' rad/s.



At general time 't', it would be at an angle θ = ωt. Its vertical projection defines a sinusoidal function.



(a) Crank (b) Sinusoidal waveform

Thus the above rotating crank represents a sinusoidal function of the form $E_m\sin(\omega t)$.

Similarly, a sinusoidal function of the form $E_m\sin(\omega t + \phi)$ can be represented by another rotating crank of same length 'E_m' and rotating with same angular speed 'ω' rad/s anticlockwise but lying at an angle 'ϕ' at t = 0.

Thus, any sinusoidal function can be represented by a rotating crank and it is called '**Phasor representation**' of a sinusoidal function.

A **Phasor** is a rotating vector which effectively represents a sinusoidal function.

When a number of sinusoidal functions are to be represented as phasors, it is represented using a diagram called **phasor diagram**. While drawing a phasor diagram, all phasors must be represented corresponding to same point in time. It is usually preferred to represent them at a time t = 0. Then, angular position of each sinusoidal function corresponds to its phase angle.

Note: Only sinusoidal functions of same frequency can be represented together as a phasor diagram. Also, the length of the phasor is its RMS value.

Question 4:

Consider the following sinusoidal functions

i) $f_1(t) = 100\sin(100\pi t)$

ii) $f_2(t) = 200\sin(100\pi t + 60°)$

iii) $f_3(t) = 100\cos(100\pi t - 60°)$

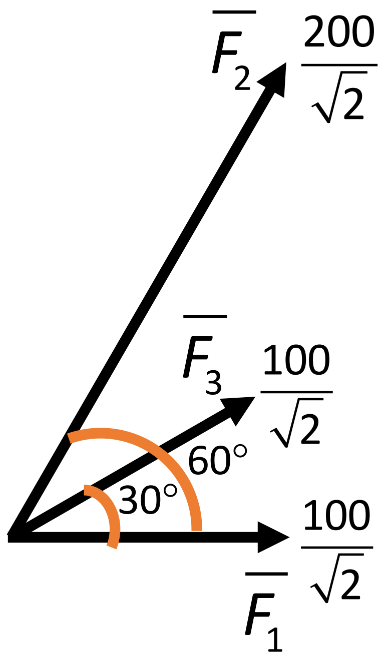
Let us represent them using a phasor diagram.

Solution:

Note: Convert a cosine function to sine form before representing as a phasor.

For instance,

$$f_3(t) = 100\cos(100\pi t - 60°)$$
$$= 100\sin(100\pi t - 60° + 90°)$$
$$= 100\sin(100\pi t + 30°)$$



Mathematical Representation of a Phasor:

A phasor is mathematically represented as

Phasor = Magnitude $\angle$Phase Angle

where, magnitude is the RMS value.

For instance, Consider these sinusoidal functions

i) $f_1(t) = 100\sin(100\pi t)$
ii) $f_2(t) = 200\sin(100\pi t + 60°)$
iii) $f_3(t) = 100\cos(100\pi t - 60°)$

Let us represent them using phasor representation.

$$f_1(t) = 100\sin(100\pi t) \quad \Rightarrow \overline{F_1} = \frac{100}{\sqrt{2}}\angle 0°$$

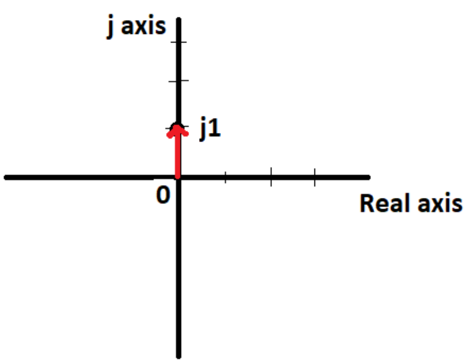
$$f_2(t) = 200\sin(100\pi t + 60°) \quad \Rightarrow \overline{F_2} = \frac{200}{\sqrt{2}}\angle 60°$$

$f_3(t) = 100\cos(100\pi t - 60°) = 100\sin(100\pi t + 30°)$

$$\Rightarrow \overline{F_3} = \frac{100}{\sqrt{2}}\angle 30°$$

j operator

'j' operator in phasor representation is analogous to 'i' operator in complex mathematics.



In rectangular form, j = (0 + j1)

In polar form, j = $1\angle 90^\circ$

Conversion between the forms

Polar to Rectangular conversion :

Let us consider a polar number $r\angle\theta$

It can be converted to rectangular form (A + jB) using

A = rcosθ ; B = rsinθ

Rectangular to Polar conversion :

Let us consider a rectangular number (A + jB)

It can be converted to polar form $r\angle\theta$ using

$$r = \sqrt{A^2+B^2}\ ;\ \theta = Tan^{-1}\left(\frac{B}{A}\right)$$

θ will be positive if 'B' is positive and it is negative if 'B' is negative.

Addition, Subtraction, Multiplication & Division of Phasors:

Addition & Subtraction of Phasors:

Addition & subtraction of phasors would be easier in rectangular form.

For instance, let $\overline{F_1}=(A_1+jB_1)$ & $\overline{F_2}=(A_2+jB_2)$

$$\overline{F_1}+\overline{F_2} = (A_1+A_2)+j(B_1+B_2)$$

$$\overline{F_1}-\overline{F_2} = (A_1-A_2)+j(B_1-B_2)$$

Multiplication & Division of Phasors:

Multiplication & Division of phasors would be easier in Polar form.

For instance, let $\overline{F_1} = r_1\angle\theta_1$ & $\overline{F_2} = r_2\angle\theta_2$

$$\overline{F_1}*\overline{F_2} = r_1*r_2\angle(\theta_1+\theta_2)$$

$$\frac{\overline{F_1}}{\overline{F_2}} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

j operator properties:

'j' operator when multiplied to a phasor, does not change the magnitude of the phasor but rotates the phasor anticlockwise by 90°

For instance, if $\overline{F} = 3\angle 60°$, then $j\overline{F} = 1\angle 90° * 3\angle 60° = 3\angle 150°$

$$j^2 = 1\angle 90° * 1\angle 90° = 1\angle 180° = \cos(180°) + j\sin(180°) = -1$$

Similarly, $j^3 = j^2 * j = -j$

And $j^4 = j^2 * j^2 = 1$

Question 5:

There are 3 conducting wires connected to form a junction. The currents flowing into the junction in two wires are $i_1 = 10\sin 314t$ A and $i_2 = 15\cos(314t - 45°)$A. What is the current leaving the junction in the third wire? What is its value at t=0?

Solution: 1) Using Time-Domain Method

By KCL at the junction, $i_3(t) = i_1(t) + i_2(t)$

$$i_3(t) = 10\sin(314t) + 15\cos(314t - 45°)$$

$$i_3(t) = 10\sin(314t) + 15*(\cos 314t * \cos 45° + \sin 314t * \sin 45°)$$

$$i_3(t) = 20.61\sin(314t) + 10.61\cos(314t)$$

$$i_3(t) = 23.18*\left(\frac{20.61}{23.18}\sin(314t) + \frac{10.61}{23.18}\cos(314t)\right)$$

$$i_3(t) = 23.18*(\cos(27.24°)*\sin(314t) + \sin(27.24°)\cos(314t))$$

$$i_3(t) = 23.18*\sin(314t + 27.24°) \text{ A}$$

Its value at t = 0 is $i_3(0) = 23.18\sin(27.24°) = 10.61A$

Solution: 2) Using Phasor Domain Method

By KCL at the junction, $i_3(t) = i_1(t) + i_2(t)$

In Phasor form, $\bar{I}_3 = \bar{I}_1 + \bar{I}_2$

$i_1(t) = 10\sin(314t) \Rightarrow \bar{I}_1 = \frac{10}{\sqrt{2}}\angle 0°A$

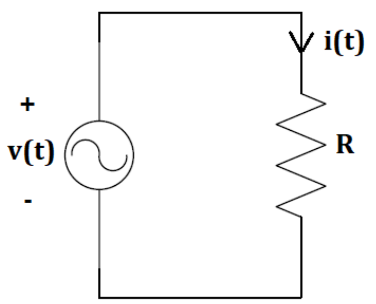
$i_2(t) = 15\cos(314t - 45°) = 15\sin(314t + 45°) \Rightarrow \bar{I}_2 = \frac{15}{\sqrt{2}}\angle 45°A$

$$\bar{I}_3 = \frac{10}{\sqrt{2}}\angle 0° + \frac{15}{\sqrt{2}}\angle 45° = 16.39\angle 27.24°A$$

$i_3(t) = 23.18*\sin(314t+27.24°)$ A

Its value at t = 0 is $i_3(0) = 23.18\sin(27.24°) = 10.61A$

Response of Resistive Load to Sinusoidal Supply:



Let the supply voltage be $v(t) = V_m\sin(\omega t)$

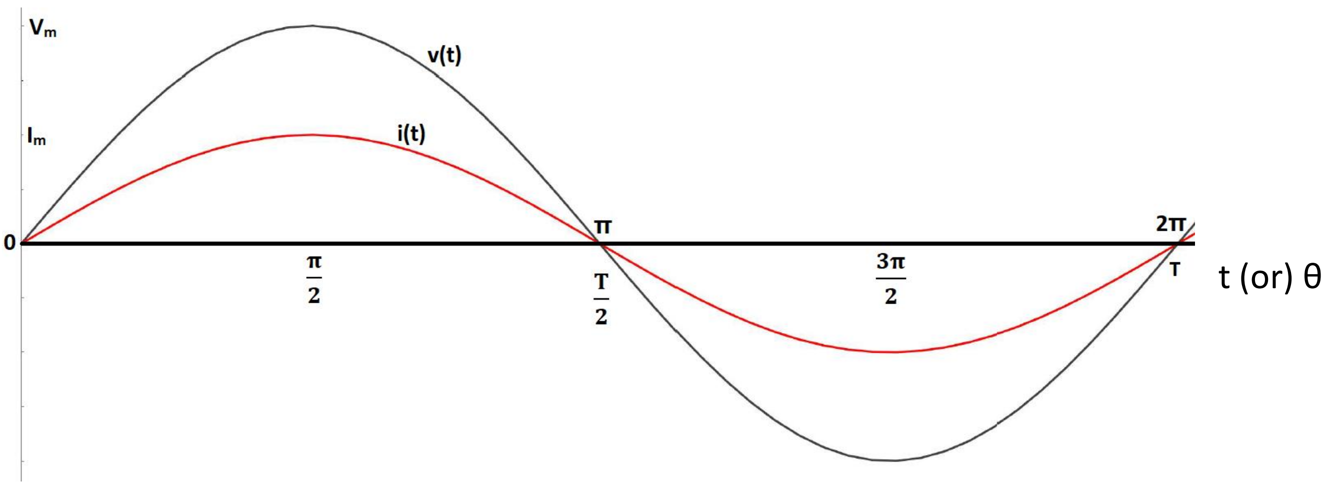
where, V_m is the peak value of voltage

By Ohm's Law, $i(t) = \frac{v(t)}{R}$

Hence current will be of the form, $i(t) = I_m\sin(\omega t)$

where, $I_m = \frac{V_m}{R}$ is the peak value of current

Time domain representation:



Phasor Representation and Phasor Diagram:

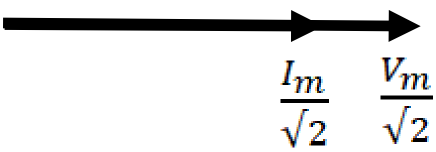
$$v(t)=V_m\sin(\omega t) \Rightarrow \bar{V}=\frac{V_m}{\sqrt{2}}\angle 0^\circ$$

$$i(t)=I_m\sin(\omega t) \Rightarrow \bar{I}=\frac{I_m}{\sqrt{2}}\angle 0^\circ$$

$$\text{Impedance,}\quad Z=\bar{V}/\bar{I}=R\angle 0^\circ=R\ \Omega$$

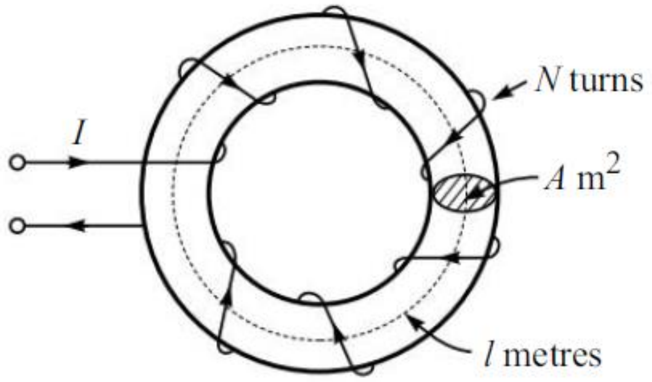
Impedance is the ratio of voltage phasor to current phasor. It is analogous to resistance in DC networks. It is the opposition offered by an AC element or network to the flow of AC currents.

Phasor Diagram:



Inductor & the concept of inductance:

An inductor is obtained by winding the conductor into a coil.



A current carrying coil sets up a magnetic field around it.

Magnetic field is expressed as magnetic flux ϕ around the coil.

Magnetic flux $\phi = \dfrac{\text{Magnetomotive Force}}{\text{Reluctance}} = \dfrac{N*I}{S}$

Where, $S = \text{Reluctance} = \dfrac{\text{length}}{(\text{Permeability*Area})}$

Magnetic flux, ϕ is directly proportional to the current in the inductor coil.

i.e., $\phi \propto i$

i.e., $N\phi \propto i$

Where, Nϕ is called **flux linkages** denoted by ψ

Therefore, ψ is proportional to i

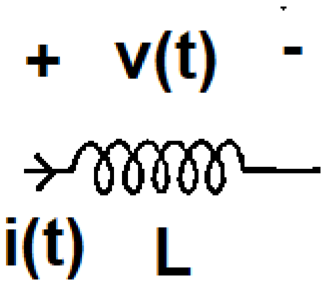
$\Rightarrow \psi = Li$

Where, L is the proportionality constant called 'Inductance' of the inductor.

$$L = \frac{\psi}{i} = \frac{N\phi}{i}$$

Inductance is measured in Henrys (H).

Voltage – Current relationship in an inductor:



The voltage across the terminals of an inductor is directly proportional to rate of change of flux linkages.

i.e., $v(t) \propto \frac{d}{dt}(\psi)$

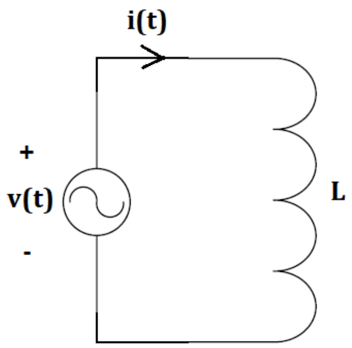
$$v(t) = \frac{d}{dt}(\psi) = \frac{d}{dt}(N\phi) = N\frac{d\phi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$$

i.e., voltage v(t) is related to current i(t) as $v(t) = L\frac{di(t)}{dt}$

Therefore, i(t) can be expressed as

$$i(t) = \frac{1}{L}\int v(t)dt$$

Response of Pure Inductive Load to Sinusoidal Supply:



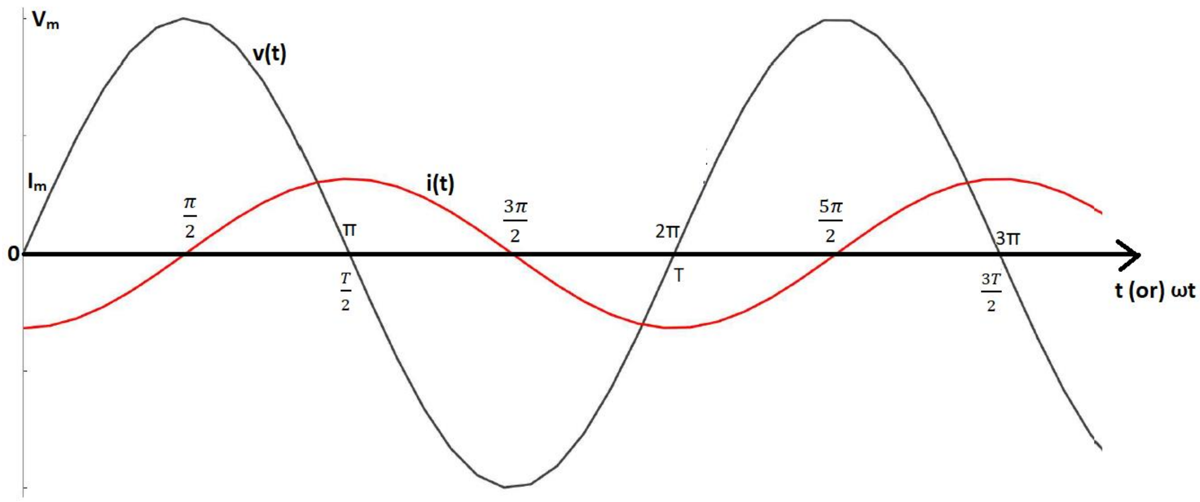
Let the supply voltage be $v(t) = V_m \sin(\omega t)$

In a pure inductor, $i(t) = \frac{1}{L}\int v(t)dt$

$$= \frac{-V_m}{\omega L}\cos(\omega t)$$

$$= I_m \sin(\omega t - 90°)$$

where, $I_m = \frac{V_m}{\omega L}$ is the peak value of current

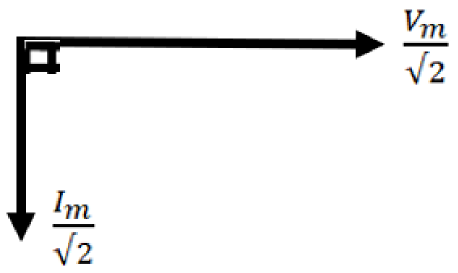


In a pure inductor, current **lags** voltage by 90°

Phasor Diagram:

$$v(t)=V_m\sin(\omega t) \quad \Rightarrow \quad \bar{V}=\frac{V_m}{\sqrt{2}}\angle 0^\circ$$

$$i(t)=I_m\sin(\omega t-90^\circ) \quad \Rightarrow \quad \bar{I}=\frac{I_m}{\sqrt{2}}\angle -90^\circ$$

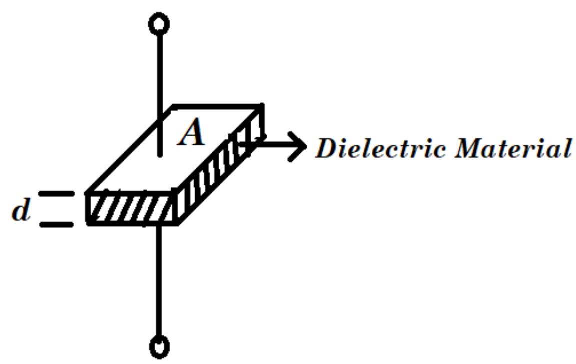


$$Z=\frac{\bar{V}}{\bar{I}}=\frac{\frac{V_m}{\sqrt{2}}\angle 0^\circ}{\frac{I_m}{\sqrt{2}}\angle -90^\circ}=\omega L\angle 90^\circ=jX_L \quad \Omega$$

Where, X_L is called '**Inductive Reactance**'.

Capacitor & the concept of Capacitance:

A Capacitor is obtained by placing a dielectric medium between the conducting plates.



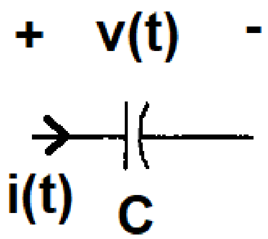
Capacitance, C = $\frac{\in A}{d}$ Farad

Where, A is the area of each of the plates in m^2

d is the distance between the plates in m

ε is the permittivity of the dielectric medium in F/m

Voltage – Current relationship in a Capacitor:



The charge on the plates of a capacitor is directly proportional to the voltage across its terminals.

i.e., $q(t) \propto v(t) \Rightarrow q(t) = Cv(t)$

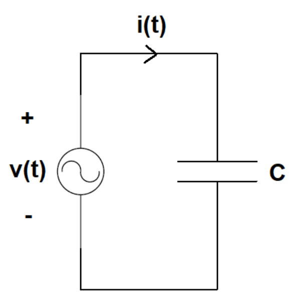
The constant of proportionality 'C' is called Capacitance of the Capacitor.

Hence, current, $i(t) = \frac{dq(t)}{dt} = C\frac{dv(t)}{dt}$

Therefore, v(t) can be expressed as

$$v(t) = \frac{1}{C}\int i(t)dt$$

Response of Pure Capacitor to Sinusoidal Supply:



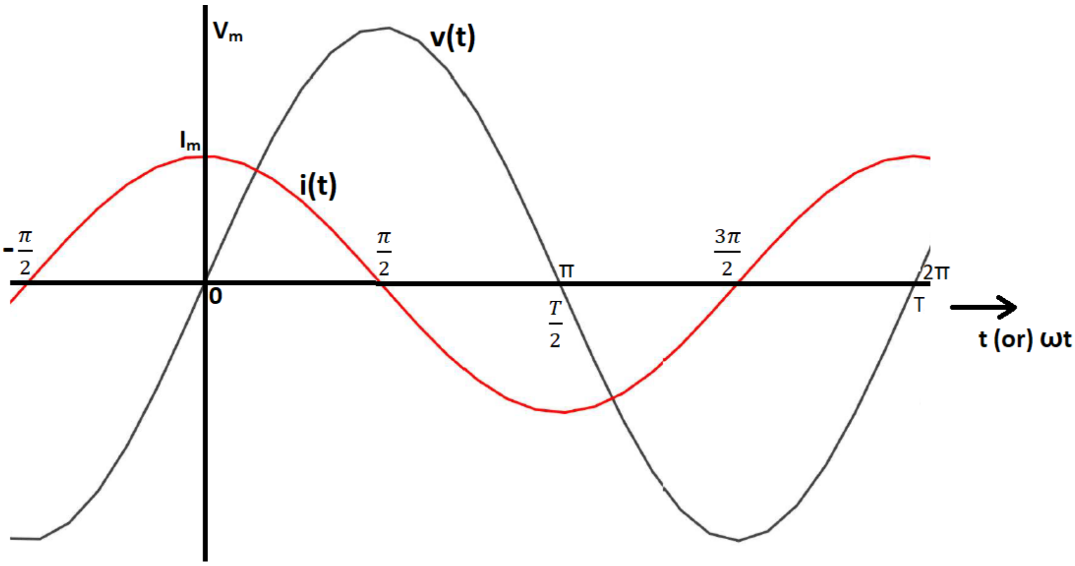
Let the supply voltage be $v(t) = V_m \sin(\omega t)$

In a pure capacitor, $i(t) = C\dfrac{dv(t)}{dt}$

$$= CV_m \omega \cos(\omega t)$$

$$= I_m \sin(\omega t + 90°)$$

Where, $I_m = V_m \omega C$ is the peak value of current

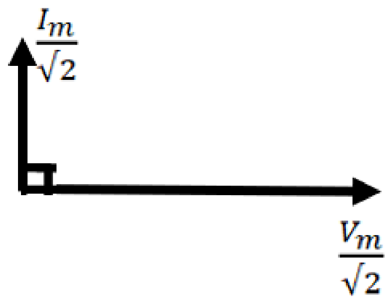


In a pure capacitor, current **leads** voltage by 90°

Phasor Diagram:

$$v(t) = V_m \sin(\omega t) \implies \bar{V} = \frac{V_m}{\sqrt{2}} \angle 0°$$

$$i(t) = I_m \sin(\omega t + 90°) \implies \bar{I} = \frac{I_m}{\sqrt{2}} \angle 90°$$



$$Z = \frac{\bar{V}}{\bar{I}} = \frac{\frac{V_m}{\sqrt{2}}\angle 0^\circ}{\frac{I_m}{\sqrt{2}}\angle 90^\circ} = \frac{1}{\omega C}\angle -90^\circ = -jX_C \ \ \Omega$$

Where, $X_C = \frac{1}{\omega C}$ is called '**Capacitive Reactance**'.

Question 6:

A Capacitor of Capacitance 100µF is connected across an AC voltage source 100sin(100πt) V. Determine

i) Capacitive Reactance

ii) Impedance

iii) Instantaneous expression for the current

Also, draw the phasor diagram.

Solution:

Given, V(t) = 100sin(100πt) V

Hence, $\omega = 100\pi$ rad/s

i) Capacitive Reactance, $X_C = \frac{1}{\omega C} = 31.83\Omega$

ii) Impedance, $Z = -jX_C = -j31.83\Omega$

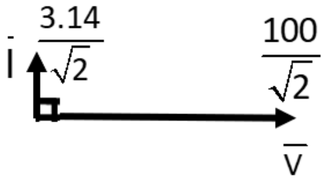
iii) Instantaneous current, $i(t) = V_m\omega C\sin(\omega t + 90^\circ)$ A

$= 3.14\sin(\omega t + 90^\circ)$ A

Phasor Diagram:

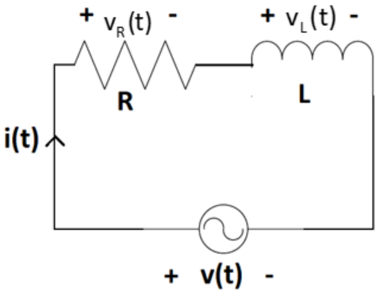
$$\overline{V} = \frac{100}{\sqrt{2}}\angle 0^{\circ}\ \text{V}$$

$$\dot{I} = \frac{3.14}{\sqrt{2}}\angle 90^{\circ}\ \text{A}$$



Analysis of Series RL and Series RC Circuits:

Series RL Circuit:



By KVL, $v(t) = v_R(t) + v_L(t)$

In Phasor form, $\overline{V} = \overline{V_R} + \overline{V_L}$

In general for any element,

(Voltage Phasor) = (Current Phasor)*(Impedance)

$$\overline{V_R} = \bar{I} * R$$

$$\overline{V_L} = \bar{I} * (jX_L)$$

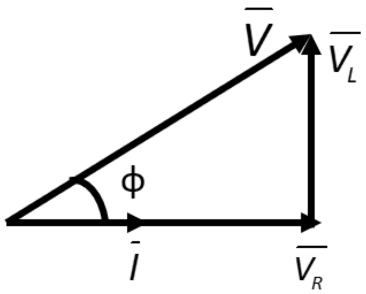
$$\overline{V} = \bar{I} * (R + jX_L)$$

$$Z_T = \frac{\overline{V}}{\dot{I}} = (R + jX_L) = \sqrt{R^2 + X_L^2}\angle \text{Tan}^{-1}\left(\frac{X_L}{R}\right)$$

It can be observed that the total impedance of a series AC network is equal to the sum of individual element impedances.

Phasor Diagram:

Note: While drawing phasor diagram for a series AC network, considering current phasor as reference is preferable.

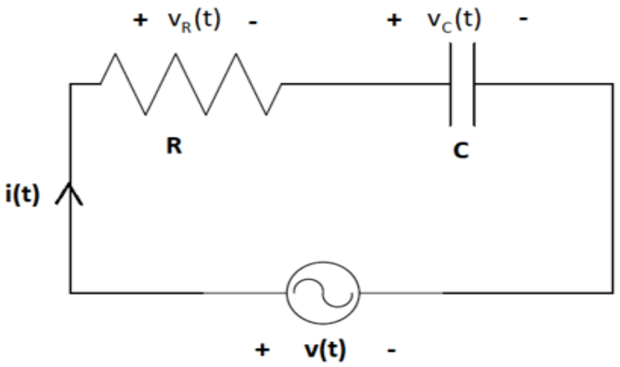


$$\phi = Tan^{-1}\left(\frac{|\bar{V}_L|}{|\bar{V}_R|}\right) = Tan^{-1}\left(\frac{V_L}{V_R}\right)$$

$$= Tan^{-1}\left(\frac{X_L}{R}\right)$$

+ve phase angle means voltage leads current.

Series RC Circuit:



By KVL, $v(t) = v_R(t) + v_C(t)$

In Phasor form, $\bar{V} = \bar{V}_R + \bar{V}_C$

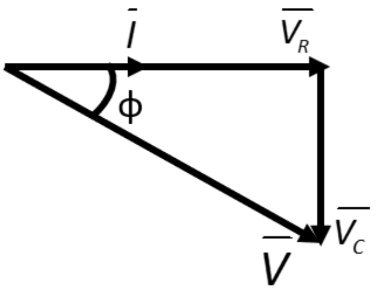
$$\bar{V}_R = \bar{I} * R$$

$$\bar{V}_C = \bar{I} * (-jX_C)$$

$$\bar{V} = \bar{I} * (R - jX_C)$$

$$Z_T = \frac{\bar{V}}{\bar{I}} = (R - jX_C) = \sqrt{R^2 + X_C^2}\angle -Tan^{-1}\left(\frac{X_C}{R}\right)$$

Phasor Diagram:



Phase angle of a network is found as

$$\phi = \angle\bar{V} - \angle\bar{I}$$

$$\phi = -Tan^{-1}\left(\frac{|\overline{V_C}|}{|\overline{V_R}|}\right) = -Tan^{-1}\left(\frac{V_C}{V_R}\right) = -Tan^{-1}\left(\frac{X_C}{R}\right)$$

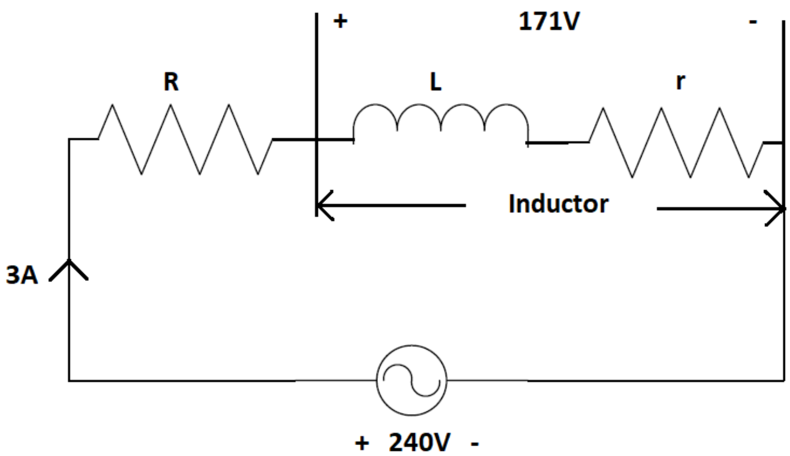
Negative phase angle means voltage lags current.

In series AC networks, phase angle = Impedance angle.

Question 7:

When a resistor and an inductor in series are connected to a 240V supply, a current of 3A flows lagging 37^0 behind the supply voltage, while the voltage across the inductor is 171V. Find the resistance of the resistor, and the resistance and reactance of the inductor. Find the power factor of the circuit.

Solution:



Note: In AC systems, if voltage and current are given as numerical values, they represent RMS values**.**

Let us consider current as reference.

i.e., $\bar{I} = 3\angle 0° A$

Therefore, supply voltage phasor, $\bar{V} = 240\angle 37°$ V

$$Z_T = \frac{\bar{V}}{\bar{I}} = \frac{240\angle 37°}{3\angle 0°}$$

$= 80\angle 37°\ \Omega$

$= (63.89 + j48.14)\ \Omega$ ---- (1)

$= R + (r + jX_L)$ ---(2)

Comparing Real and Imaginary parts in (1) & (2), $X_L = 48.14\Omega$

Also, $(R+r) = 63.89\Omega$ --- (3)

Across Inductor, $\left|\bar{V}_{inductor}\right| / \left|\bar{I}\right| = \frac{171}{3} = \sqrt{r^2 + X_L^2}$ ---(4)

Solving (3) & (4), $r = 30.52\Omega$; $R = 33.37\Omega$

Power factor = $(R+r) / |Z_T|$ = 0.798 Lag

Concept of Active, Reactive and Apparent Powers

Average Power in AC Circuits:

In AC systems, since both voltage v(t) and current i(t) are time varying, power is also time varying in nature.

Instantaneous power, $p(t) = v(t)*i(t)$

For the sake of energy calculations, it is useful to find average power.

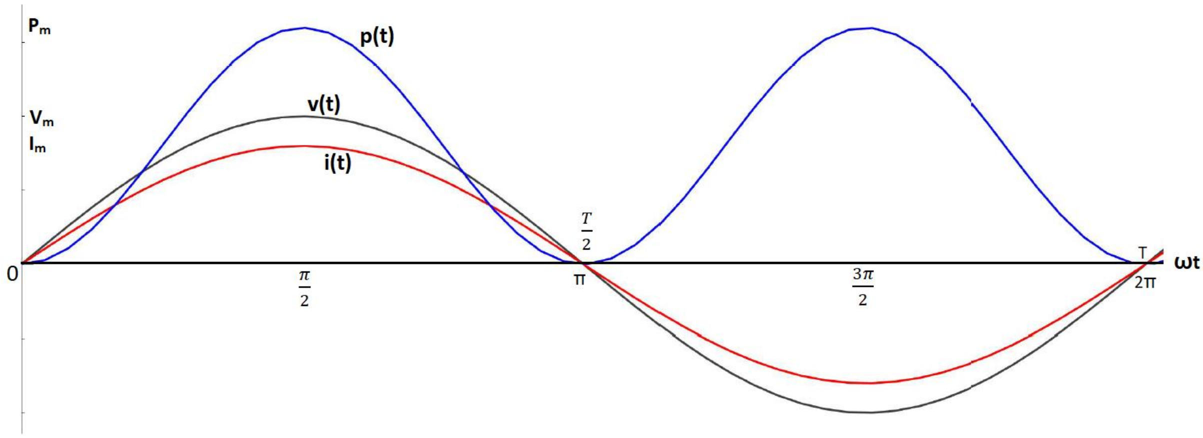
Average power, denoted by P is found out using the following equation:

$$P = \frac{1}{T}\int_0^T p(t)dt$$

It is measured in Watts (W).

Case 1: Resistive Load

For $v(t) = V_m \sin(\omega t)$, the current $i(t) = I_m \sin(\omega t)$; $p(t) = v(t)*i(t)$



$$P = \frac{1}{T}\int_0^T v(t)*i(t)dt = \frac{1}{T}\int_0^T V_m I_m \sin^2(\omega t)dt$$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} = V*I \quad \text{Watts}$$

where, V = RMS voltage and I = RMS current

In a resistive load, instantaneous power p(t) is always positive because a resistor consumes the power given to it by the source. A resistor dissipates the power absorbed as heat.

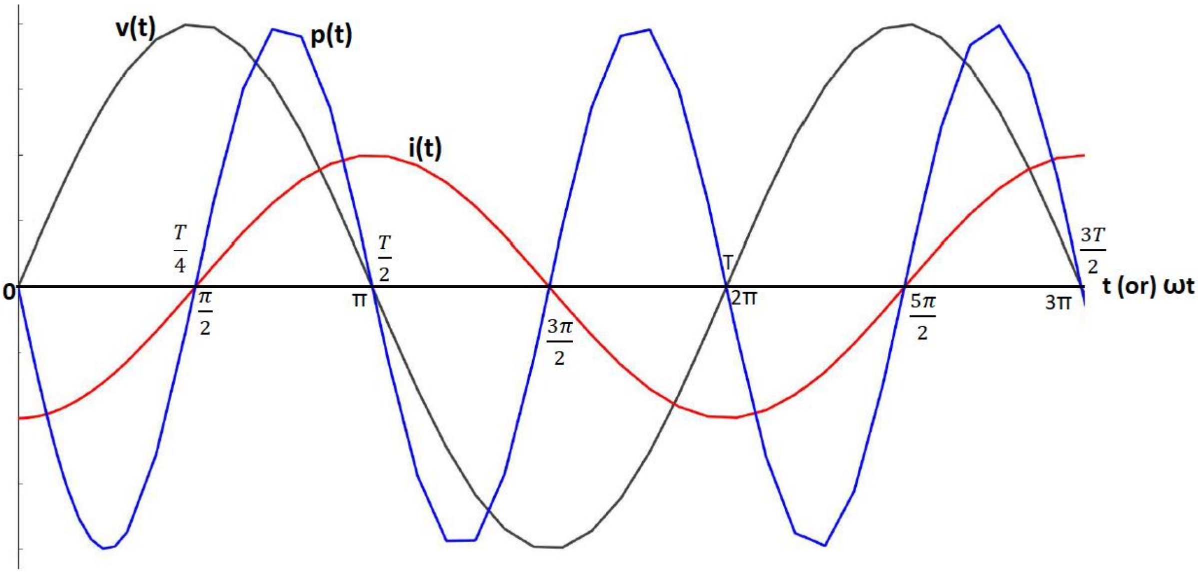
Average power is also called **active power** (or) **real power** and it is measured in Watts(W).

Case 2: Purely Inductive Load

For $v(t) = V_m \sin(\omega t)$, the current $i(t) = I_m \sin(\omega t - 90^\circ)$; $p(t) = v(t)*i(t)$

$$P = \frac{1}{T}\int_0^T v(t)*i(t)dt = \frac{1}{T}\int_0^T V_m I_m \sin(\omega t)\sin(\omega t - 90^\circ)dt$$

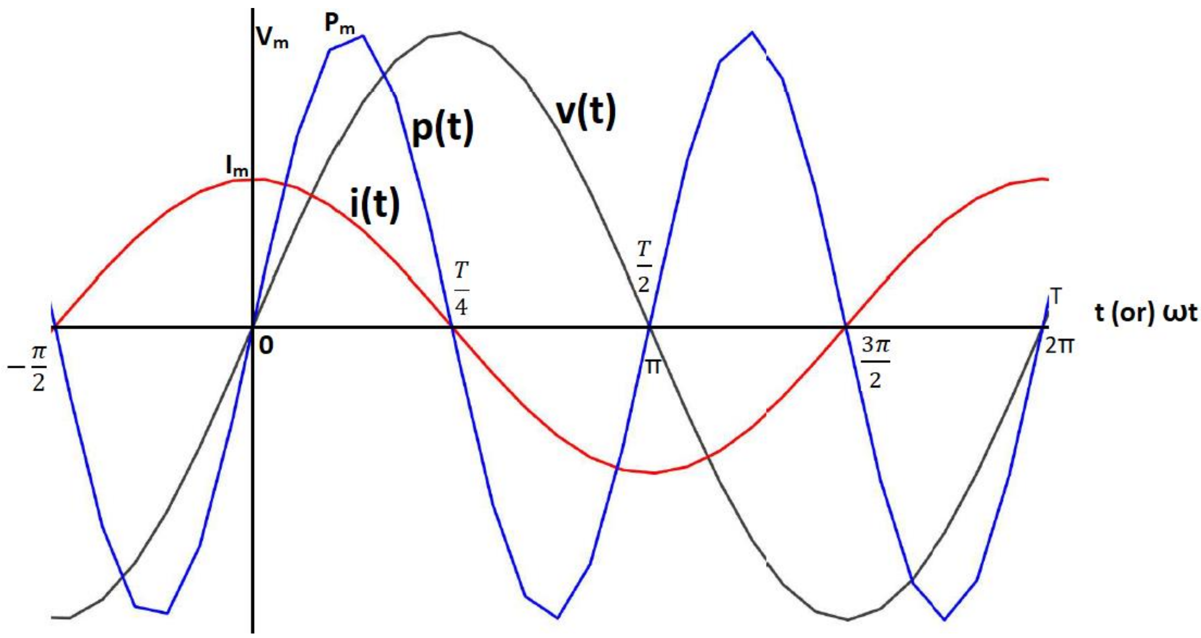
$$= 0$$



In a pure inductor, during one quarter cycle of voltage, power is positive i.e., it flows from source to inductor and gets stored in the magnetic field of the inductor & during the next quarter cycle of voltage, power is negative i.e. stored energy in the inductor flows back to the source.

Case 3: Purely Capacitive Load

For $v(t) = V_m\sin(\omega t)$, the current $i(t) = I_m\sin(\omega t+90^\circ)$; $p(t) = v(t)*i(t)$



$$P = \frac{1}{T}\int_0^T v(t)*i(t)dt = \frac{1}{T}\int_0^T V_m I_m \sin(\omega t)\sin(\omega t+90°)dt$$

$$= 0$$

In a pure capacitor, during one quarter cycle of voltage, power is positive i.e., it flows from source to capacitor and gets stored in the electric field of the capacitor & during the next quarter cycle of voltage, power is negative i.e. stored energy in the capacitor flows back to the source.

Concept of Reactive Power:

In case of pure inductor and pure capacitor, average power is zero. In these elements, power circulates between the source and element but is not consumed. This type of AC power which is not consumed but circulates between the source and the element is called **Reactive Power.**

Accordingly, inductor and capacitor are called reactive elements.

Reactive Power is denoted by **Q** and is measured in **Volt-Amperes Reactive (VAR).**

Concept of Apparent Power:

The product of RMS value of voltage and RMS value of current of an element is called the **Apparent Power.**

Apparent Power is denoted by **S** and measured in **Volt-Amperes (VA).**

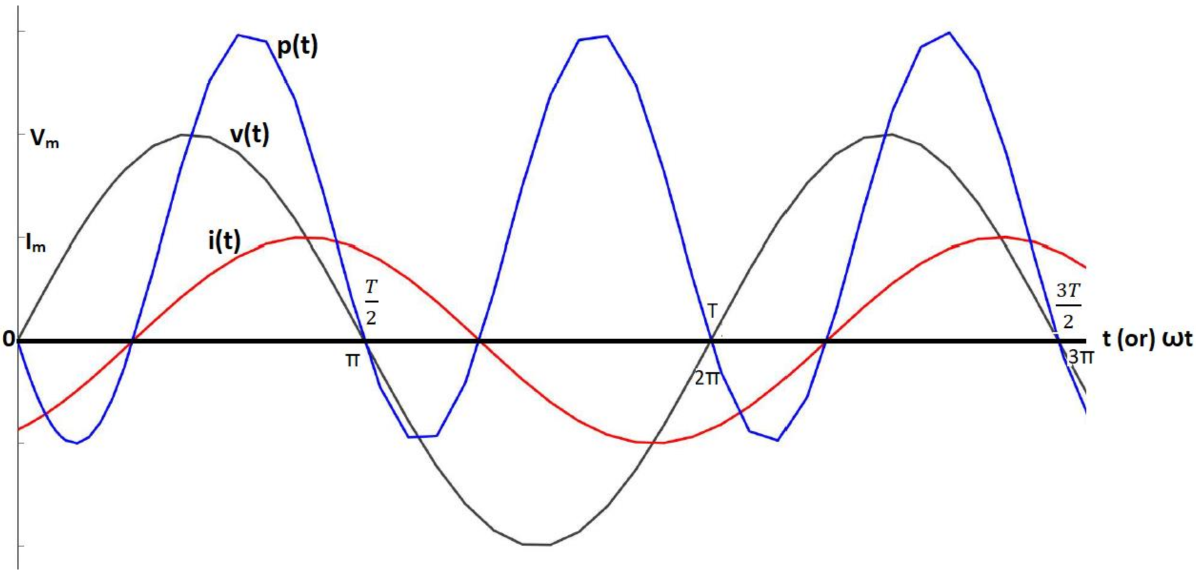
In case of a resistor, power absorbed is consumed. Hence, its reactive power is zero. Its power is completely in active form.

In case of a pure inductor or pure capacitor, power consumed is zero. Hence, its active power is zero. Its power is completely in reactive form i.e., it circulates between the source and the element.

Case iv) : General AC circuit:

Let us consider a general AC circuit, for instance series RL circuit.

For $v(t) = V_m \sin(\omega t)$, the current $i(t) = I_m \sin(\omega t+\phi)$ where ϕ is -ve.



In this case, average power is non-zero. This is because Resistor consumes power. Also, there is negative portion of power which is due to inductor. Thus, in this case there will be both active and reactive power.

$$P = \frac{1}{T}\int_0^T v(t)*i(t)dt = \frac{1}{T}\int_0^T V_m I_m \sin(\omega t)\sin(\omega t+\phi)dt$$

$$= \frac{V_m I_m}{2}\cos(\phi) = \frac{V_m}{\sqrt{2}}*\frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi \quad W$$

Thus, in general in an AC circuit, active power is given by

$$P = VI\cos\phi \quad W$$

Where, V is the RMS voltage, I is the RMS current and ϕ is the angle between the voltage and current.

Similarly, reactive power in an AC circuit is

$$Q = VI\sin\phi \quad VAR$$

Thus, Apparent power is

$$S = \sqrt{P^2+Q^2} = VI \quad \text{Volt-Amperes}$$

Power Expressions for various cases:

Element/ Network	**Phase Angle (ϕ)**	**Active Power ($P = VI\cos\phi$)**	**Reactive Power ($Q = VI\sin\phi$)**	**Apparent Power ($S = VI$)**
R	0°	VI	0	VI
L	90°	0	VI	VI
C	-90°	0	-VI	VI
Series RL Circuit	$\tan^{-1}\left(\frac{X_L}{R}\right)$	**$VI\cos\phi$**	**$VI\sin\phi$ (+ve)**	VI
Series RC Circuit	$-\tan^{-1}\left(\frac{X_C}{R}\right)$	**$VI\cos\phi$**	**$VI\sin\phi$ (-ve)**	VI

Note: Conventionally, Inductive reactive power is positive and capacitive reactive power is negative.

Power factor in AC circuits

The ratio of Active Power to Apparent Power is termed as **Power factor** in AC systems.

$$\text{Power factor} = \frac{P}{S} = \cos\phi$$

Power factor is the cosine of phase angle of the network.

In case of series AC circuits, power factor can also be found as

$$\text{Power factor} = \frac{R_T}{|Z_T|}$$

Where, R_T is the total resistance in series and $|Z_T|$ is the magnitude of total impedance in series.

The power factor of a purely resistive circuit is **unity** since S = P for resistive circuits.

The power factor of a pure inductor is **Zero Lag** since P = 0 for a pure inductor. The word ‘Lag’ in the power factor indicates that current lags voltage.

The power factor of a pure Capacitor is **Zero Lead** since P = 0 for a pure capacitor.

The power factor of inductive circuits is between Zero and One and Lagging type & for capacitive circuits it is leading type and between Zero and One.

Ideally power factor must be as close to unity as possible.

Question 8:

A series RL circuit is connected to a sinusoidal voltage source v(t) = 100sin(ωt) V. It draws a current of 10sin(ωt - 60°) A.

Determine

i) Active, Reactive and Apparent Powers.

ii) Power factor of the circuit.

Solution:

$$V = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} V$$

$$I = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} A$$

Phase Angle, $\phi = \angle\bar{V} - \angle\bar{I} = 0° - (-60°) = 60°$

i) $P = VI\cos\phi = 250W$

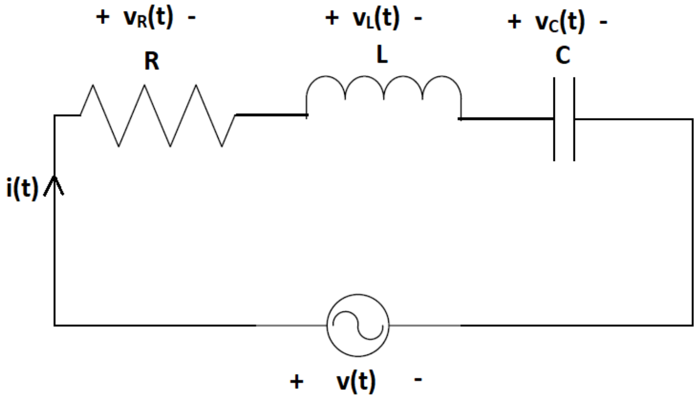
$Q = VI\sin\phi = 433\ VAR$

$S = VI = 500\ VA$

ii) Power factor = $\cos\phi = \frac{P}{S} = 0.5Lag$

Analysis of Series RLC circuit & Impedance and Power Triangles

Series RLC Circuit:



By KVL, $v(t) = v_R(t) + v_L(t) + v_c(t)$

In Phasor form, $\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$

$\overline{V_R} = \bar{I} * R \qquad \overline{V_L} = \bar{I} * (jX_L) \qquad \overline{V_C} = \bar{I} * (-jX_C)$

$\overline{V} = \bar{I} * (R + jX_L - jX_C)$

$$Z_T = \frac{\overline{V}}{\bar{I}} = (R+jX_L-jX_C) = \sqrt{R^2+(X_L-X_C)^2}\angle Tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

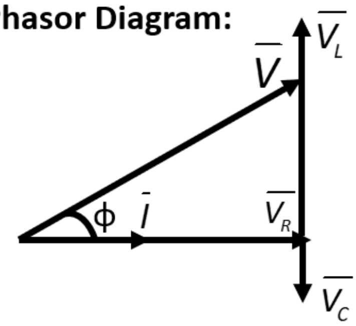
Case 1: $X_L > X_C$

If $X_L > X_C$ then $IX_L > IX_C$

i.e., $|\overline{V_L}| > |\overline{V_C}|$

The circuit behaves effectively as inductive circuit i.e., series RL type.

Phasor Diagram:



$$\phi = Tan^{-1}\left(\frac{|\overline{V_L}|-|\overline{V_C}|}{|\overline{V_R}|}\right) = Tan^{-1}\left(\frac{V_L-V_C}{V_R}\right)$$

$$= Tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

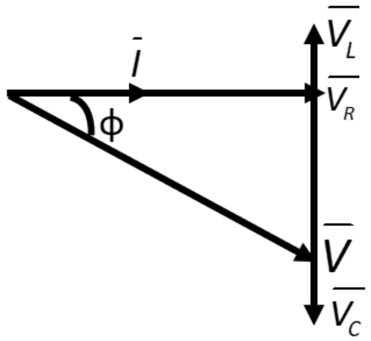
Case 2: $X_C > X_L$

If $X_C > X_L$ then $IX_C > IX_L$

i.e., $|\overline{V_C}| > |\overline{V_L}|$

The circuit behaves effectively as a capacitive circuit i.e., series RC type.

Phasor Diagram:



$$\phi = Tan^{-1}\left(\frac{|\overline{V_L}| - |\overline{V_C}|}{|\overline{V_R}|}\right) = Tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$= Tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

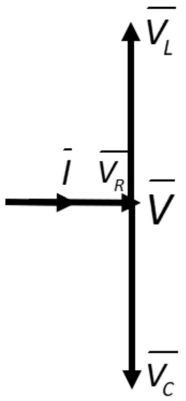
Note: ϕ will be negative in this case since $X_L < X_C$

Case 3: $X_L = X_C$

If $X_L = X_C$ then $IX_L = IX_C$ i.e., $|\overline{V_L}| = |\overline{V_C}|$

The circuit behaves effectively as a purely resistive circuit. This case is called **'Series Resonance'** case.

Phasor Diagram:



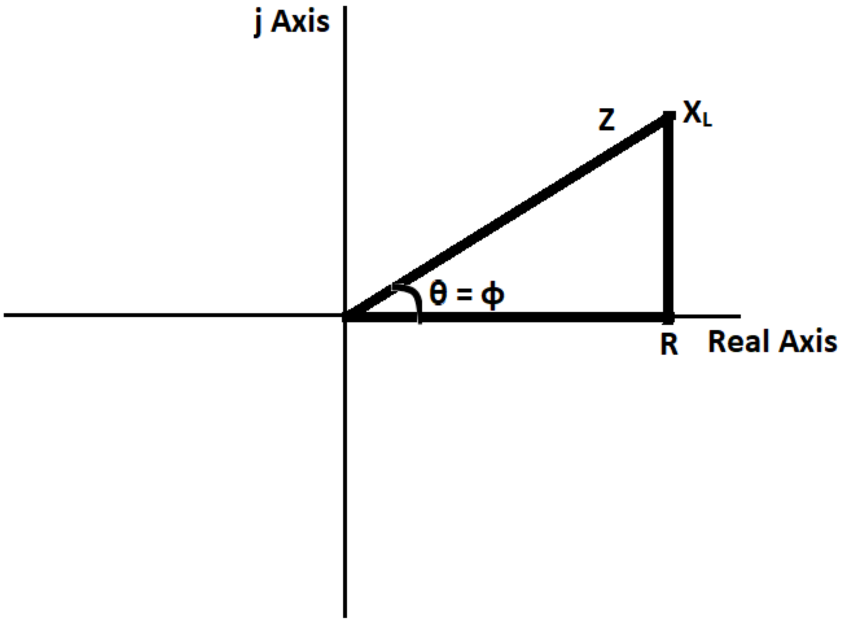
$$\overline{V} = \overline{V_R}$$

$$Z = R$$

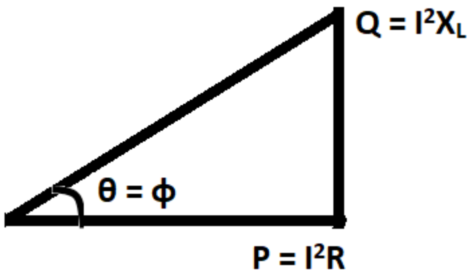
$$\phi = 0°$$

Impedance & Power Triangles – Series RL Circuit

For a series RL circuit, $Z = R + jX_L = \sqrt{R^2+X_L^2}\angle Tan^{-1}(\frac{X_L}{R})$



Impedance Triangle of a series RL circuit lies Quadrant I of complex plane.



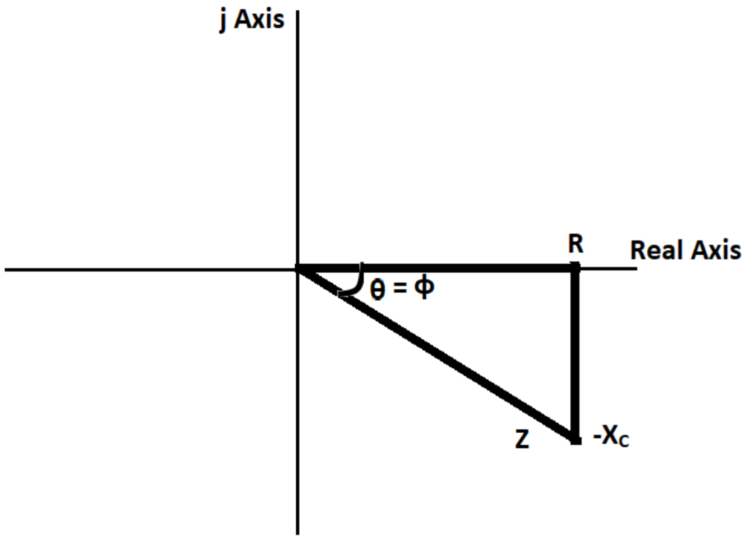
$$P = VI\cos\phi = (I.|Z|).I.\frac{R}{|Z|} = I^2R$$

$$Q = VI\sin\phi = (I.|Z|).I.\frac{X_L}{|Z|} = I^2X_L$$

$$S = VI = (I.|Z|).I = I^2|Z|$$

Impedance & Power Triangles – Series RC Circuit

For a series RC circuit, $Z = R - jX_C = \sqrt{R^2 + X_C^2}\angle -Tan^{-1}(\frac{X_C}{R})$



Impedance Triangle of a series RC circuit lies Quadrant IV of complex plane.

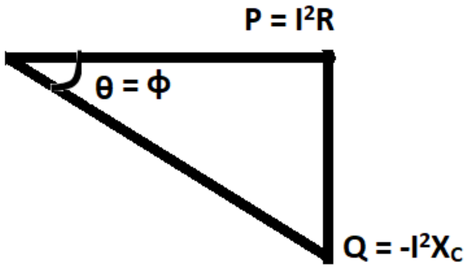


Fig: Power Triangle

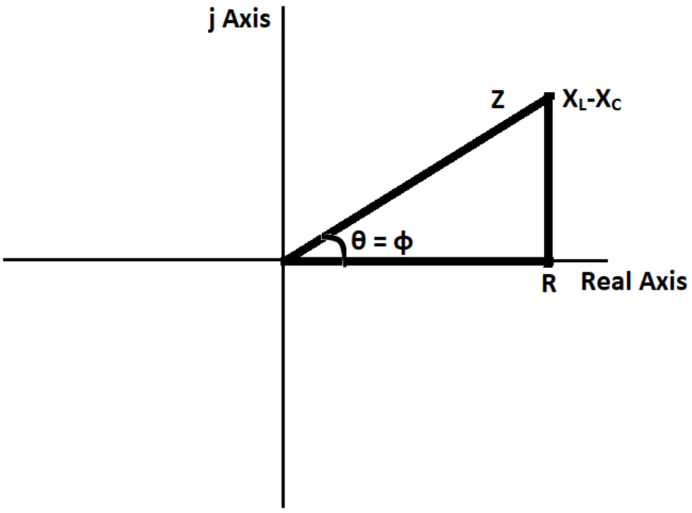
$P = VIcos\phi = (I.|Z|).I.\frac{R}{|Z|} = I^2R$

$Q = VIsin\phi = (I.|Z|).I.\frac{-X_C}{|Z|} = -I^2X_C$

$S = VI = (I.|Z|).I = I^2\,|Z|$

Impedance & Power Triangles – Series RLC Circuit

For a series RLC circuit, $Z = R + j(X_L - X_C) = \sqrt{R^2 + (X_L - X_C)^2} \angle Tan^{-1}(\frac{X_L - X_C}{R})$



Impedance Triangle of a series RLC circuit for $X_L > X_C$ lies in Quadrant I of complex plane.

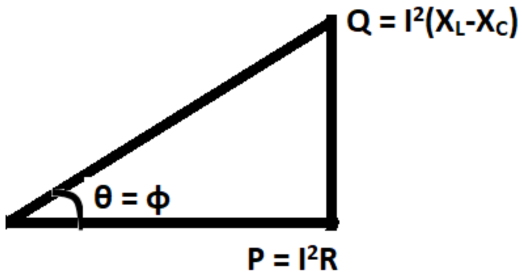


Fig: Power Triangle

$$P = VIcos\phi = (I.|Z|).I.\frac{R}{|Z|} = I^2R$$

$$Q = VIsin\phi = I^2(X_L - X_C)$$

$$S = VI = (I.|Z|).I = I^2|Z|$$

Question 9:

A series RLC circuit draws a current of 20A when connected to 200V, 50Hz supply. If the total active power drawn from the source is 500W and the circuit behaves effectively like an inductive circuit (series RL type), determine

i) Power factor of the circuit

ii) Inductance in the circuit if Capacitance is 100µF

Solution:

Given, V = 200V , I = 20A & P = 500W

i) Since $P = I^2R$,

$R = 1.25\Omega$

$$|Z| = \frac{V}{I} = 10\Omega$$

Therefore, Power factor $= \frac{R}{|Z|} = 0.125$ Lag

ii) Net Reactance, $X = (X_L - X_C) = \sqrt{Z^2 - R^2} = 9.92\Omega$

$X_C = 31.83\Omega$

Hence, $X_L = 41.75\Omega$

Therefore, L = 132.89mH

Concept of Admittance

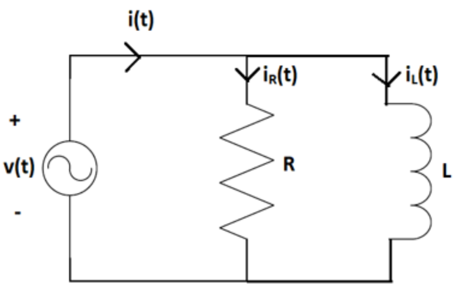
Admittance of an element is equal to the reciprocal of its impedance.

$$\text{Admittance, } Y = \frac{1}{Z}$$

It is measured in Siemens (S) or Mho ( ).

Element	Impedance (Z)	Admittance (Y)	Remarks
Resistor (R)	R	$\frac{1}{R} = G$	G is the conductance
Inductor (L)	jX_L	$\frac{1}{jX_L} = -jB_L$	B_L is the Inductive Susceptance
Capacitor (C)	$-jX_C$	$\frac{1}{-jX_c} = jB_c$	B_C is the Capacitive Susceptance

Parallel RL Circuit



By KCL, $i(t) = i_R(t) + i_L(t)$

In Phasor form, $\dot{I} = \overline{I_R} + \overline{I_L}$

$$\overline{I_R} = \overline{V} * G \qquad \overline{I_L} = \overline{V} * (-jB_L)$$

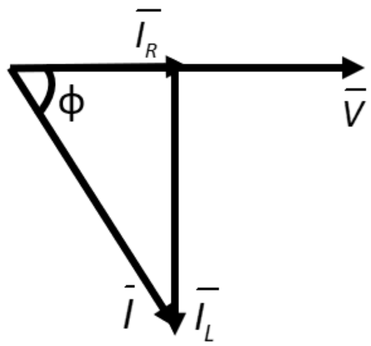
$$\bar{I} = \overline{V} * (G - jB_L)$$

$$Y_T = \frac{\bar{I}}{\overline{V}} = (G\text{-}jB_L) = \sqrt{G^2 + B_L^2} \angle -\text{Tan}^{-1}\left(\frac{B_L}{G}\right)$$

In a parallel circuit, the total admittance is equal to the sum of individual branch admittances.

Phasor Diagram:

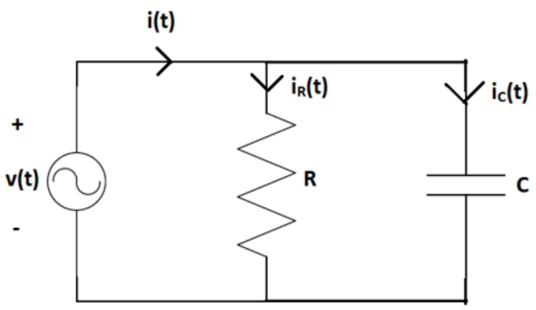
Note: In parallel AC circuits, it is preferable to consider the supply voltage as reference phasor while drawing phasor diagram.



$$\phi = Tan^{-1}\left(\frac{|\bar{I}_L|}{|\bar{I}_R|}\right) = Tan^{-1}\left(\frac{I_L}{I_R}\right)$$

$$= Tan^{-1}\left(\frac{B_L}{G}\right)$$

Parallel RC Circuit



By KCL, $i(t) = i_R(t) + i_C(t)$

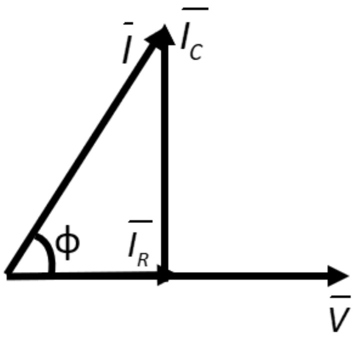
In Phasor form, $\bar{I} = \bar{I}_R + \bar{I}_C$

$$\bar{I}_R = \bar{V} * G \qquad \bar{I}_C = \bar{V} * (jB_C)$$

$$\bar{I} = \bar{V} * (G + jB_C)$$

$$Y_T = \frac{\bar{I}}{\bar{V}} = (G+jB_C) = \sqrt{G^2 + B_C{}^2} \angle Tan^{-1}\left(\frac{B_C}{G}\right)$$

Phasor Diagram:



$$\phi = -\tan^{-1}\left(\frac{|\bar{I_C}|}{|\bar{I_R}|}\right) = -\tan^{-1}\left(\frac{I_C}{I_R}\right)$$

$$= -\tan^{-1}\left(\frac{B_C}{G}\right)$$

Note: Phase Angle of a network is equal to impedance angle (or) negative of admittance angle.

Question 10:

The terminal voltage and current for a parallel circuit are 141.4sin 2000t V and 7.07sin $(2000t+36^0)$A.

Obtain the simplest two element parallel circuit, which would have the above relationship.

Solution:

To find the elements in a network, use the impedance form if it is a series network and use the admittance form if it is a parallel network.

$$v(t) = 141.4\sin(2000t)\text{ V} \quad \Rightarrow \bar{V} = \frac{141.4}{\sqrt{2}}\angle 0^\circ\text{ V}$$

$$i(t) = 7.07\sin(2000t+36^\circ)\text{ A} \quad \Rightarrow \bar{I} = \frac{7.07}{\sqrt{2}}\angle 36^\circ\text{ A}$$

$$\text{Admittance}, Y = \frac{\bar{I}}{\bar{V}} = 0.05\angle 36^\circ\text{S} = (0.04 + j0.029)\text{ S}$$

Comparing with the standard form $(G+jB_C)$,
$G = 0.04S$; $B_C = 0.029S$

Hence, it is a parallel RC network

$$R = \frac{1}{G} = 25\Omega \text{ and } C = \frac{B_C}{\omega} = \frac{0.029}{2000} = 14.5\mu F$$

Question 11:

A resistor of 30Ω and a capacitor of unknown value are connected in parallel across a 110V, 50Hz Supply. The combination draws a current of 5A from the supply. Find the value of unknown Capacitance.

Solution:

$$|Y_T| = \frac{|I|}{|V|} = \frac{5}{110} = 0.045\ \text{S} \quad \text{-----}\ (1)$$

For a parallel RC network, $|Y_T| = \sqrt{G^2 + B_C^{\ 2}}$ ----- (2)

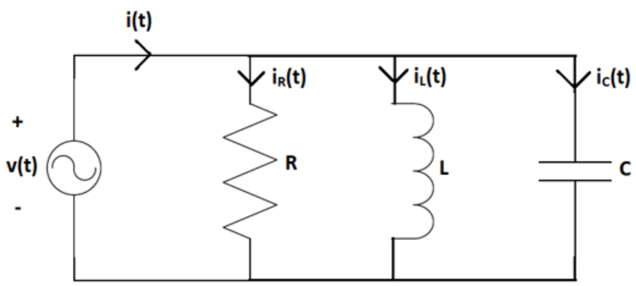
$$G = \frac{1}{R} = 0.033\text{S}$$

Substituting G in (2) and equating (1) & (2),

$B_C = 0.0306\text{S}$

$$C = \frac{B_C}{\omega} = \frac{0.0306}{100\pi} = 97.38\mu\text{F}$$

Parallel RLC Circuit:



By KCL, $i(t) = i_R(t) + i_L(t) + i_c(t)$

In Phasor form, $\dot{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$

$$\bar{I}_R = \bar{V} * G \qquad \bar{I}_L = \bar{V} * (-jB_L) \qquad \bar{I}_C = \bar{V} * (jB_C)$$

$$\bar{I} = \bar{V} * (G - jB_L + jB_C)$$

$$Y_T = \frac{\bar{I}}{\bar{V}} = (G\text{-}jB_L + jB_C) = \sqrt{G^2 + (B_L - B_C)^2} \angle \text{Tan}^{-1}\left(\frac{B_C - B_L}{G}\right)$$

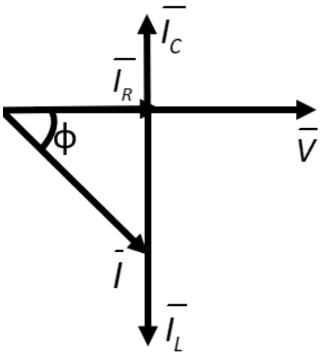
Case 1: $B_L > B_C$

If $B_L > B_C$ then $VB_L > VB_C$

i.e., $|\bar{I}_L| > |\bar{I}_C|$

The circuit behaves effectively as inductive circuit i.e., parallel RL type.

Phasor Diagram:



$$\phi = Tan^{-1}\left(\frac{|\bar{I}_L| - |\bar{I}_C|}{|\bar{I}_R|}\right) = Tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

$$= Tan^{-1}\left(\frac{B_L - B_C}{G}\right)$$

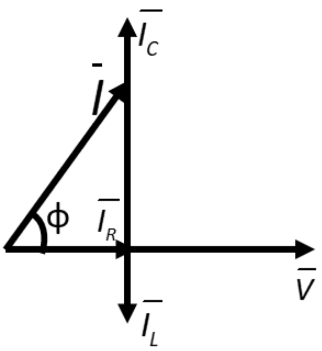
Case 2: $B_C > B_L$

If $B_C > B_L$ then $VB_C > VB_L$

i.e., $|\bar{I}_C| > |\bar{I}_L|$

The circuit behaves effectively as a capacitive circuit i.e., parallel RC type.

Phasor Diagram:



$$\phi = Tan^{-1}\left(\frac{|\bar{I}_L| - |\bar{I}_C|}{|\bar{I}_R|}\right) = Tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

$$= Tan^{-1}\left(\frac{B_L - B_C}{G}\right)$$

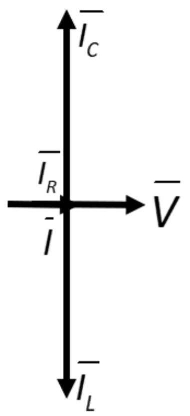
Note: ϕ will be negative in this case since $B_L < B_C$

Case 3: $B_L = B_C$

If $B_L = B_C$ then $VB_L = VB_C$ i.e., $|\overline{I_L}| = |\overline{I_C}|$

The circuit behaves effectively as a purely resistive circuit. This case is called **'Parallel Resonance'** case.

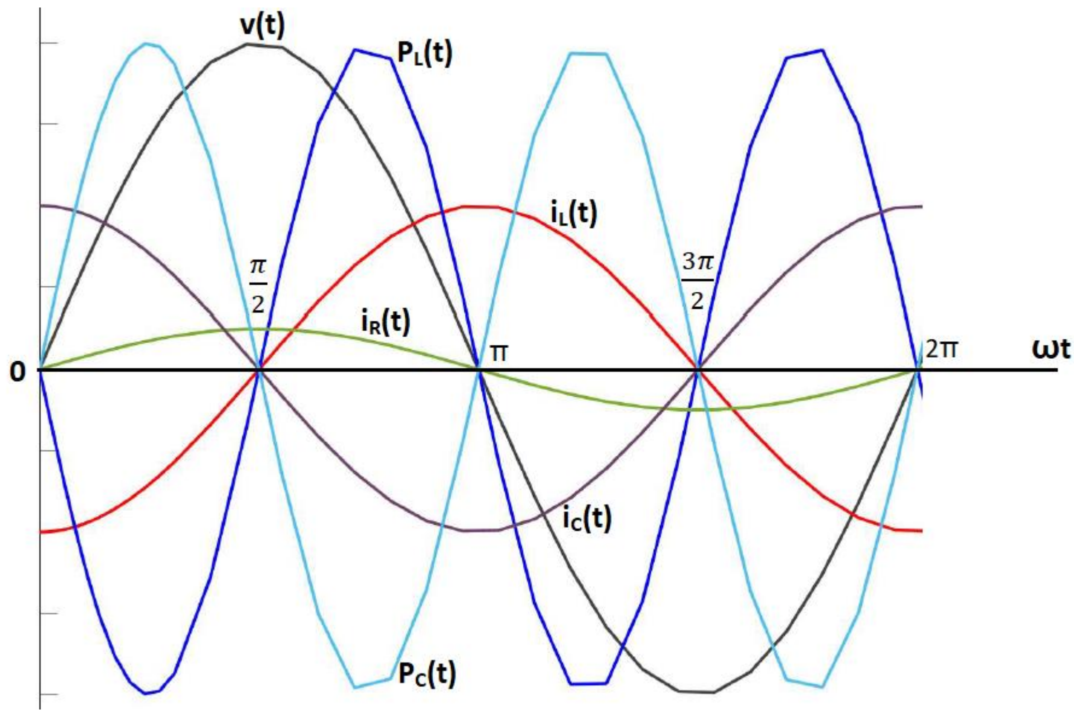
Phasor Diagram:



$\bar{I} = \overline{I_R}$

$Y = G$

$\phi = 0°$



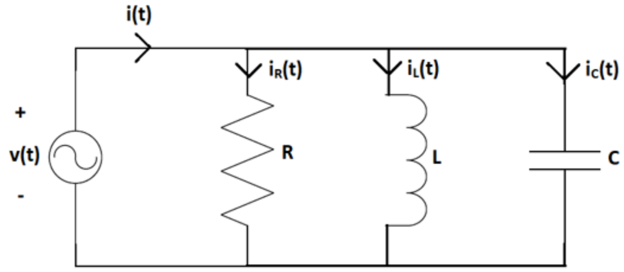
Question 12:

Three circuit elements R=2.5Ω, X_L=4Ω and X_C=10Ω are connected in parallel, the reactances being at 50Hz.

a) Determine the admittance of each element and hence obtain the input admittance.

b) If this circuit is connected across a 10V, 50Hz AC source, determine the current in each branch and the total input current.

Solution:



i) Admittance of branch1, $Y_1 = \frac{1}{Z_1} = \frac{1}{R} = G = 0.4S$

Admittance of branch2, $Y_2 = \frac{1}{Z_2} = \frac{1}{jX_L} = -jB_L = -j0.25S$

Admittance of branch3, $Y_3 = \frac{1}{Z_3} = \frac{1}{-jX_C} = jB_C = j0.1S$

Input Admittance $Y_{in} = Y_T = Y_1 + Y_2 + Y_3 = (0.4 - j0.15)S$

ii) Taking supply voltage as reference, $\bar{V} = 10\angle 0^\circ V$

current in branch 1, $\bar{I_R} = \frac{\bar{V}}{Z_1} = \bar{V}Y_1 = 10\angle 0^\circ * 0.4 = 4\angle 0^\circ A$

current in branch 2, $\bar{I_L} = \bar{V}Y_2 = 10\angle 0^\circ *(-j0.25) = 2.5\angle -90^\circ A$

current in branch 3, $\bar{I_C} = \bar{V}Y_3 = 1\angle 90^\circ A$

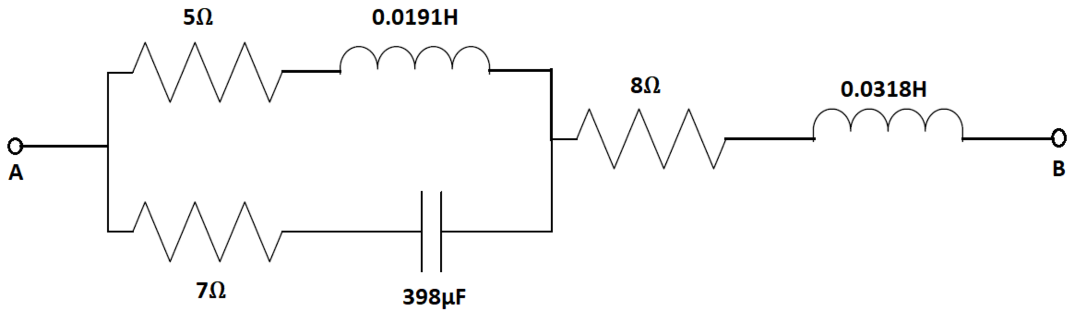
Input current, $\bar{I_S} = \bar{I_R} + \bar{I_L} + \bar{I_C} = 4.27\angle -20.55^\circ A$

Series – Parallel AC Circuits

Series - Parallel AC Circuits are those in which few elements are connected in series and few elements are connected in parallel.

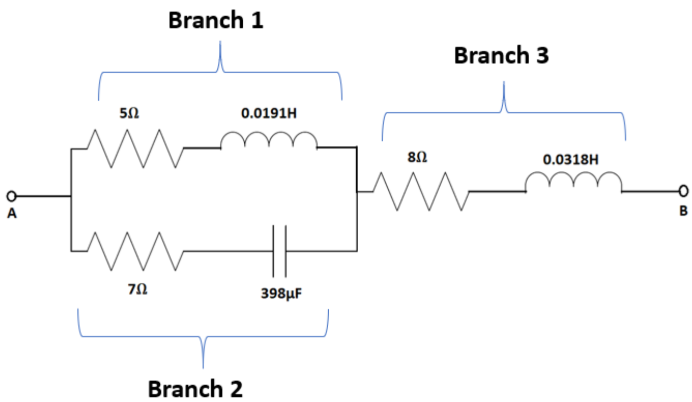
It is always advisable to solve such networks using Phasor Method. While applying Phasor method for Series – Parallel AC circuits, consider any known quantity as reference.

Question 13:



In the circuit shown, what voltage of 50Hz frequency is to be applied across A & B that will cause a current of 10A to flow in the capacitor. Also draw the phasor diagram representing the circuit.

Solution:



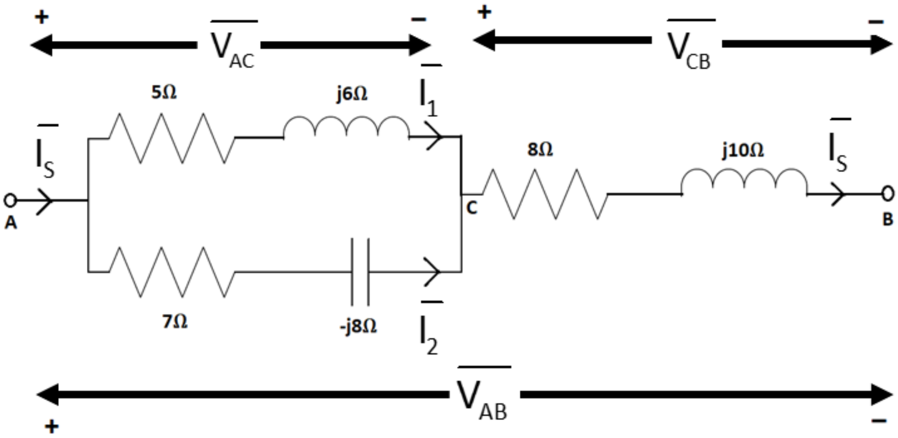
Branches 1 & 3 : Series RL branches

$\Rightarrow Z_1 = (R_1 + jX_{L1}) = 5+j(2\pi*50*0.0191) = (5+j6)\Omega$

Similarly, $Z_3 = (8+j10)\Omega$

Branch 2 : Series RC branch

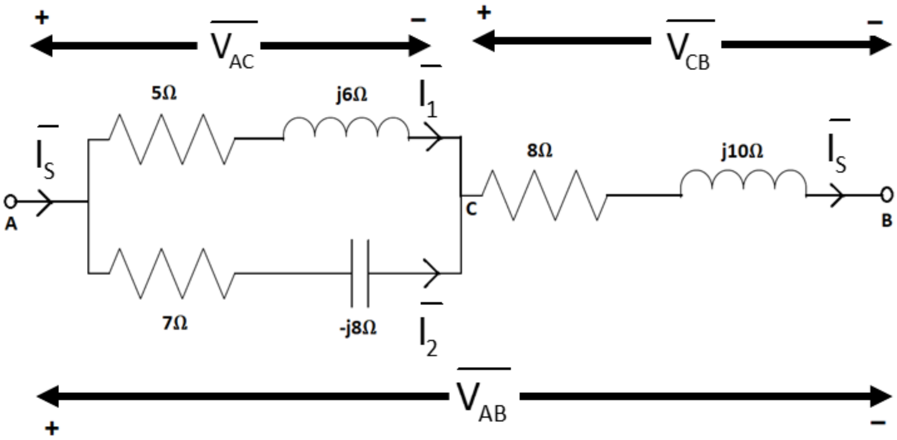
$\Rightarrow Z_2 = (R_2 - jX_{C2}) = (7-j8)\Omega$



Since current through the capacitor is known, let us take it as reference phasor.

Therefore, $\overline{I}_2 = 10\angle 0^\circ A$

Hence, $\overline{V}_{AC} = \overline{I}_2 * Z_2 = 10\angle 0^\circ * (7\text{-}j8) = 106.3\angle -48.81^\circ V$



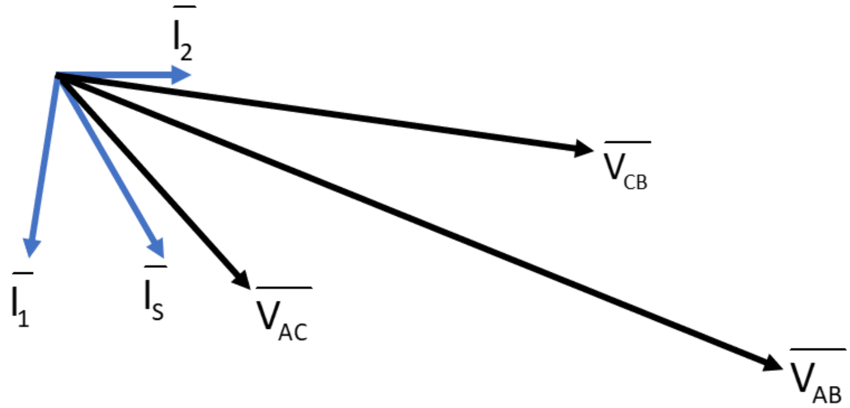
Therefore, $\overline{I}_1 = \dfrac{\overline{V}_{AC}}{Z_1} = 13.61\angle -99^\circ A$

$\Rightarrow \overline{I}_S = \overline{I}_1 + \overline{I}_2 = 15.58\angle -59.65^\circ A$

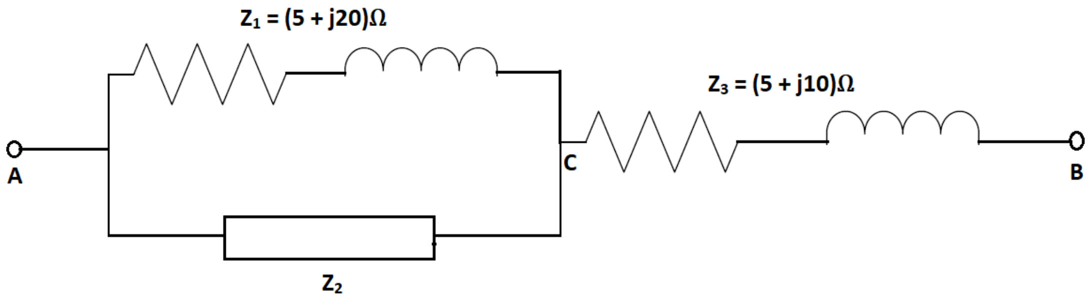
Now, $\overline{V}_{CB} = \overline{I}_S * Z_3 = 199.48\angle -8.31^\circ V$

Therefore, $\overline{V_{AB}} = \overline{V_{AC}} + \overline{V_{CB}} = 288.69\angle -22.15^{\circ}V$

Phasor Diagram :

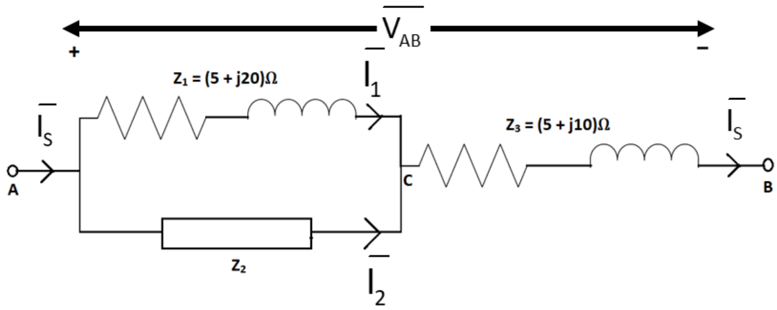


Question 14:



When a 220V AC supply is applied across terminals A & B of the circuit shown, the total power input is 3.25kW and the total current is 20A, lag. Find the complex expressions for currents through Z_1 and Z_2, taking V_{AC} as reference phasor.

Solution:



Considering supply voltage as reference, $\overline{V_{AB}} = 220\angle 0°$ V

Given, total power input = 3.25KW

i.e., $V_{AB} * I_s * \cos\phi$ = 3.25KW = 220*20*$\cos\phi$

$\Rightarrow \phi = 42.38°$

Since supply current is given as lag, $\overline{I_s} = 20\angle -42.38°$ A

$\overline{V_{CB}} = \overline{I_s} * Z_3 = 223.61\angle 21.05°$V

$\overline{V_{AC}} = \overline{V_{AB}} - \overline{V_{CB}} = 81.11\angle -81.98°$V

$\overline{I_1} = \dfrac{\overline{V_{AC}}}{Z_1} = 3.93\angle\text{-}157.95°$A

$\overline{I_2} = \overline{I_s} - \overline{I_1} = 21.98\angle -33.1°$A

We found that $\overline{V_{AC}} = 81.11\angle\text{-}81.98°$ V

To make $\overline{V_{AC}}$ as reference, add 81.98° to its phase angle.

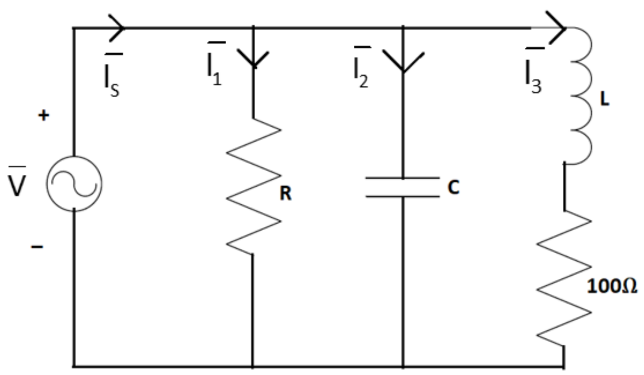
Also, Add the same angle to all other phasors.

Thus, $\overline{I_1} = 3.93\angle -75.97°$A ; $\overline{I_2} = 21.98\angle 48.88°$A

Question 15:

A voltage of 200 V is applied to a pure resistor (R), a pure capacitor, C and a lossy inductor coil with resistance of 100 Ω, all of them connected in parallel. The total current is 2.45 A, while the component currents are 1.5, 2.0 and 1.2 A respectively. Find the total power factor and also the power factor of the coil. Also find the total active and reactive power.

Solution:



Let us consider supply voltage as reference

$\Rightarrow \bar{V} = 200\angle 0°\ V$

Therefore, $\bar{I_1} = 1.5\angle 0°\ A$; $\bar{I_2} = 2\angle 90°\ A$

In branch 3, $|Z_3| = \frac{200}{1.2} = 166.66\Omega$

Therefore, $\phi_3 = \cos^{-1}(\frac{r_3}{|Z_3|}) = 53.13° \Rightarrow \bar{I_3} = 1.2\angle -53.13°\ A$

Hence, $\bar{I_S} = \bar{I_1} + \bar{I_2} + \bar{I_3} = 2.45\angle 25.1°\ A$

Phase Angle of the network = $\phi = \angle\bar{V} - \angle\bar{I_s} = -25.1°$

Overall Power factor = $\cos\phi$ = 0.905 Lead

Power factor of the coil = $\cos\phi_3$ = 0.6 Lag

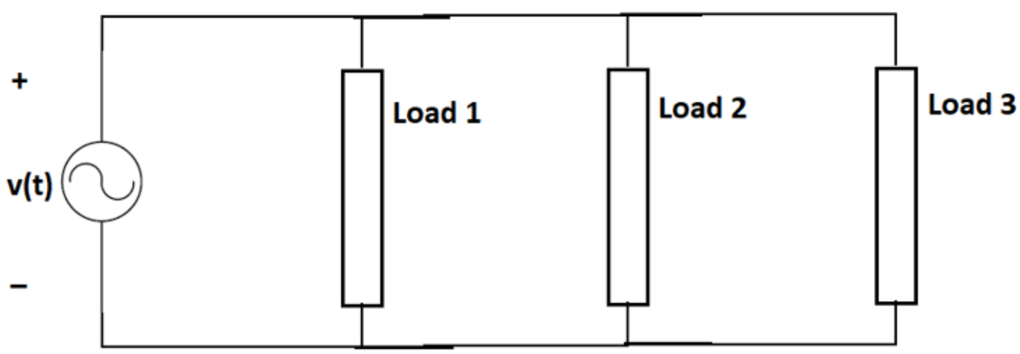
Total Active Power, $P_T = V*I_S*\cos\phi = 443.45W$

Total Reactive Power, $Q_T = V*I_S*\sin\phi = -207.85\ VAR$

Question 16:

The load connected across an AC supply consists of a heating load of 15KW, a motor load of 40KVA at 0.6 lag and a load of 20KW at 0.8 lag. Calculate the total power drawn from the supply in (KW and KVA) and its power factor. What would be the KVAR rating of a capacitor to bring the power factor to unity and how must the capacitor be connected?

Solution:



Load 1 : Heating Load $\Rightarrow$ Resistive $\Rightarrow$ $\cos\phi_1 = 1$

P_1 = 15KW (given)

$Q_1 = 0$

$S_1 = \sqrt{P_1^2 + Q_1^2}$ = 15KVA

Load 2 : Motor Load $\Rightarrow$ Inductive

S_2 = 40KVA & $\cos\phi_2$ = 0.6 Lag (given)

$P_2 = S_2\cos\phi_2$ = 24KW

$Q_2 = \sqrt{S_2^2 - P_2^2}$ = 32KVAR

Load 3 : Inductive Load

P_3 = 20KW & $\cos\phi_3$ = 0.8 Lag (given)

$S_3 = \dfrac{P_3}{\cos\phi_3}$ = 25KVA

$Q_3 = \sqrt{S_3^2 - P_3^2}$ = 15KVAR

Net Active Power, $P_T = P_1 + P_2 + P_3$ = 59KW

Net Reactive Power, $Q_T = Q_1 + Q_2 + Q_3$ = 47KVAR

Net Apparent Power, $S_T = \sqrt{P_T^2 + Q_T^2}$ = 75.43KVA

To make overall power factor unity, net reactive power must be zero. Hence, connect a capacitor of rating 47KVAR in parallel to achieve this.

Power factor Improvement

Ideally, Power factor of the system must be unity. Since most of the industrial loads are inductive in nature (Motor Loads) & they draw reactive power, power factor is usually less than unity & lagging in nature.

Electricity supply company asks industries to install capacitor banks to improve power factor by locally meeting their reactive power requirements and maintain power factor at a value above 0.9 Lag. This helps Electricity supply company in many ways

i) Overall System Size decreases, hence, saves lot of capital

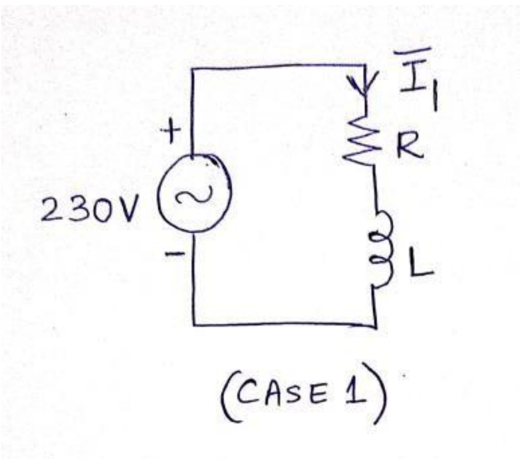
ii) Efficiency of the overall system increases

Numerical Example:

Question

The power consumed in the inductive load is 2.5 kW at 0.71 lagging power factor .The input voltage is 230 V, 50 Hz. Find the value of the capacitor C which must be placed in parallel, such that the resultant power factor of the input current is 0.866 lagging.

Solution:



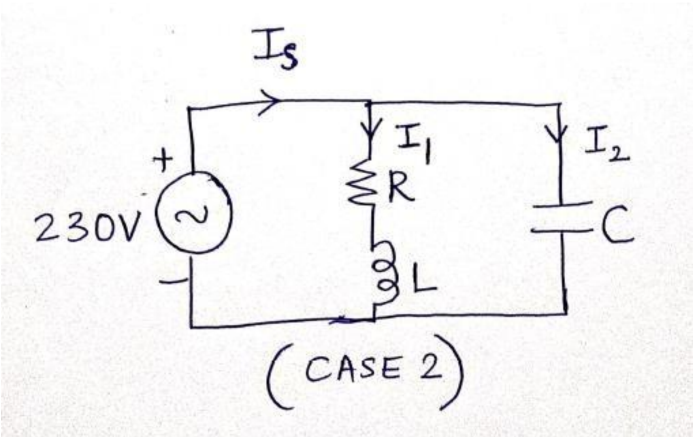
Case 1: **V** = $230\angle 0°$ V

Given, P = 2.5KW = $V*I_1*\cos(\Phi)$---------- (1)

Since, Power factor = 0.71 lag, substituting in (1),

I_1 = 15.31A & $\Phi = \cos^{-1}(0.71) = 44.76°$

Hence, $\mathbf{I_1} = 15.31\angle -44.76°$ A



Case 2: New Power factor = $\cos(\Phi')$ = 0.866 Lag

P remains same since Capacitor does not consume power

Hence, P = 2.5KW = $V*I_S*\cos(\Phi')$ --------(2)

Solving (2), I_S = 12.55A & $\Phi' = \cos^{-1}(0.866) = 30°$

Hence, $\mathbf{I_S} = 12.55\angle -30°$ A

By KCL, $\mathbf{I_S} = \mathbf{I_1} + \mathbf{I_2}$; Hence, $\mathbf{I_2} = 4.51\angle 90°$ A

Hence, X_C = (230/4.51) = 51Ω

So, C = 62.41µF

ALTERNATIVE SOLUTION:

Case 1: V = $230\angle 0°$ V

Given, P = 2.5KW = $V*I_1*\cos(\Phi)$---------- (1)

Since, Power factor = 0.71 lag, substituting in (1),

I_1 = 15.31A & $\Phi = \cos^{-1}(0.71) = 44.76°$

Hence, $\mathbf{I_1} = 15.31\angle -44.76°$ A

Hence, $\mathbf{Y_1} = \mathbf{I_1} / \mathbf{V} = 0.0665\angle -44.76°$ S

Case 2: New Power factor = $\cos(\Phi') = 0.866$ Lag

$\mathbf{Y_2} = jB_C$ & $\Phi' = \cos^{-1}(0.866) = 30°$

$\mathbf{Y_T = Y_1 + Y_2} = 0.0665\angle -44.76°\ S + jB_C$

$= 0.0472 - j0.0468 + jB_C$

Angle in the admittance is negative of phase

angle i.e., $\tan^{-1}((B_C - 0.0468)/0.0472) = -30°$

Solving, $B_C = 0.0195\ S = \omega C$; Hence, $C = 62.22\mu F$

Power factor Improvement

Ideally, Power factor of the system must be unity. Since most of the industrial loads are inductive in nature (Motor Loads) & they draw reactive power, power factor is usually less than unity & lagging in nature.

Electricity supply company asks industries to install capacitor banks to improve power factor by locally meeting their reactive power requirements and maintain power factor at a value above 0.9 Lag. This helps Electricity supply company in many ways

i) Overall System Size decreases, hence, saves lot of capital

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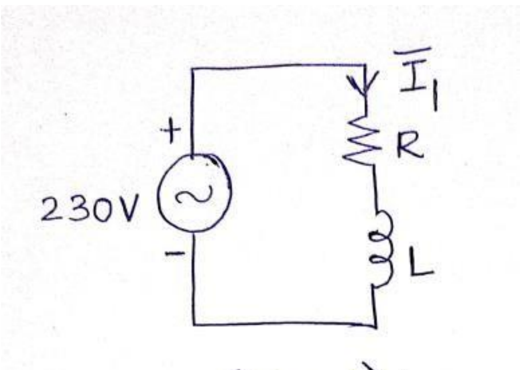
Numerical Example:

Question

The power consumed in the inductive load is 2.5 kW at 0.71 lagging power factor .The input voltage is 230 V, 50 Hz. Find the value of the capacitor C which must be placed in parallel, such that the resultant power factor of the input current is 0.866 lagging.

Solution:



(CASE 1)

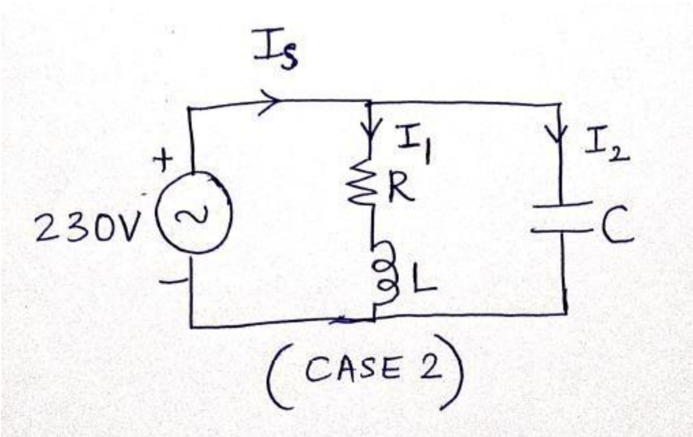
Case 1: **V** = 230∟0° V

Given, P = 2.5KW = V*I_1*cos(Φ)---------- (1)

Since, Power factor = 0.71 lag, substituting in (1),

I_1 = 15.31A & Φ = $\cos^{-1}(0.71)$ = 44.76°

Hence, $\mathbf{I_1}$ = 15.31∟-44.76° A



Case 2: New Power factor = cos(Φ') = 0.866 Lag

P remains same since Capacitor does not consume power

Hence, P = 2.5KW = V*I_S*cos(Φ') --------(2)

Solving (2), I_S = 12.55A & Φ' = $\cos^{-1}(0.866)$ = 30°

Hence, $\mathbf{I_S}$ = 12.55∟-30° A

By KCL, $\mathbf{I_S} = \mathbf{I_1} + \mathbf{I_2}$; Hence, $\mathbf{I_2}$ = 4.51∟90° A

Hence, X_C = (230/4.51) = 51Ω

So, C = 62.41µF

ALTERNATIVE SOLUTION:

Case 1: V = 230∟0° V

Given, P = 2.5KW = V*I_1*cos(Φ)---------- (1)

Since, Power factor = 0.71 lag, substituting in (1),

I_1 = 15.31A & Φ = $\cos^{-1}(0.71)$ = 44.76°

Hence, $\mathbf{I_1}$ = 15.31∟-44.76° A

Hence, $\mathbf{Y_1} = \mathbf{I_1} / \mathbf{V}$ = 0.0665∟-44.76° S

Case 2: New Power factor = $\cos(\Phi') = 0.866$ Lag

$\mathbf{Y_2} = jB_C$ & $\Phi' = \cos^{-1}(0.866) = 30°$

$\mathbf{Y_T = Y_1 + Y_2} = 0.0665\angle{-44.76°}\ S + jB_C$

$= 0.0472 - j0.0468 + jB_C$

Angle in the admittance is negative of phase

angle i.e., $\tan^{-1}((B_C - 0.0468)/0.0472) = -30°$

Solving, $B_C = 0.0195\ S = \omega C$; Hence, $C = 62.22\mu F$