ELEMENTS OF ELECTRICAL ENGINEERING (UE24EE141B)

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Unit 2 – Single Phase AC Circuits – Lecture 20 - Average & RMS Values of a Sine Wave; Concept of Phase Lag and Phase Lead

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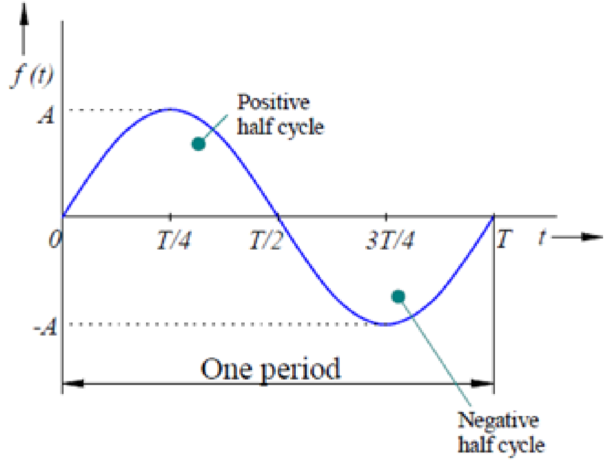
Average value of a Sinusoidal Function

The average value of an AC waveform f(t) is given by

$$F_{avg} = \frac{1}{T}\int_0^T f(t)dt$$

The average value of a sinusoidal function f(t) = Asin(ωt) is

$$F_{avg} = \frac{1}{T}\int_0^T A\sin(\omega t)dt = \frac{A}{T}\left(\frac{-\cos(\omega t)}{\omega}\right)_0^T = 0$$



Net Area over one period = 0

Hence, F_{avg} = 0

Effective (or) Root Mean Square (RMS) Value of an AC function

How can we represent an AC waveform effectively by one value?

Consider an AC Voltage v(t) connected across a resistor R for ‘T’ seconds.

Energy consumed by the resistor during this period is

$$E_{AC} = \int_0^T p(t)dt \quad = \int_0^T \frac{[v(t)]^2}{R} dt \qquad \text{-------} \ (1)$$

Now, excite this resistor using a DC Voltage source of voltage ‘V’ for same time ‘T’ seconds.

Energy consumed by the resistor in this case is

$$E_{DC} = \frac{V^2}{R}.T \qquad \text{-------} \ (2)$$

Effective (or) Root Mean Square (RMS) Value of an AC function

That value of DC voltage 'V' for which $E_{AC} = E_{DC}$ is said to be the Effective value of the AC voltage v(t).

Hence, $\int_0^T \frac{[v(t)]^2}{R} dt = \frac{V^2}{R} . T$

Therefore, Effective value $V = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2 dt}$

Mathematically the operations involved are

i) Square of the function
ii) Mean (Average) of the function
iii) Square root of the function

Hence, it is also called Root Mean Square (RMS) value.

Effective (or) Root Mean Square (RMS) Value of Sine Wave

Consider a sinusoidal voltage $v(t)=V_m\sin(\omega t)$

Its RMS value, $V = \sqrt{\frac{1}{T}\int_0^T [V_m \sin\omega t]^2 dt}$

$$= \sqrt{\frac{V_m^{\ 2}}{T}\int_0^T [\sin^2\omega t] dt}$$

$$= \sqrt{\frac{V_m^{\ 2}}{T} * \frac{T}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

Finding Power and Energy consumed using RMS value

Major advantage of finding effective (or) RMS value of an AC function is that it makes power calculations easy.

Power consumed in AC circuits, $p(t) = v(t)*i(t)$

Average power consumed, $P = \dfrac{\int_0^T p(t)dt}{T} = \dfrac{\int_0^T \frac{[v(t)]^2}{R}}{T} = \dfrac{\int_0^T \frac{[v(t)]^2}{T}}{R} = \dfrac{V^2}{R}$

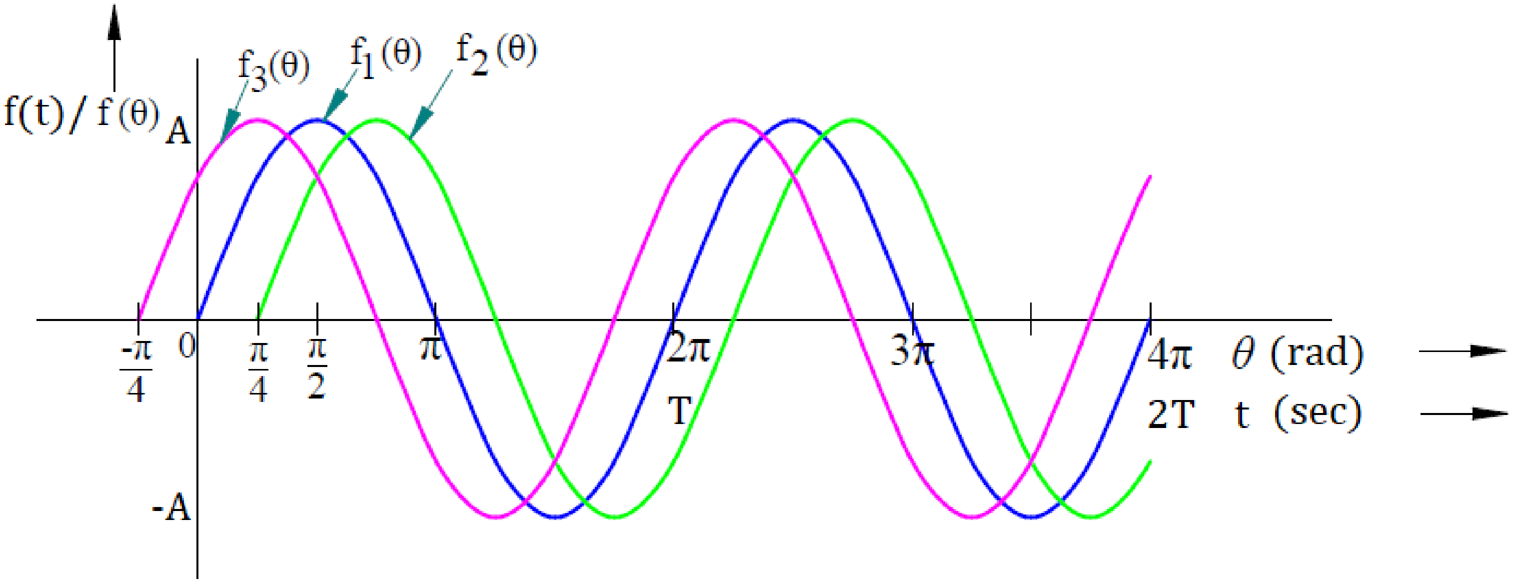
Where V = RMS value of voltage.

Similarly, average power consumed is also equal to (I^2*R) where I = RMS current.

Also, Energy consumed in 't' seconds = P*t

i.e., $(I^2R)t$ (or) $\dfrac{V^2}{R}t$

Concept of Phase Lag and Phase Lead



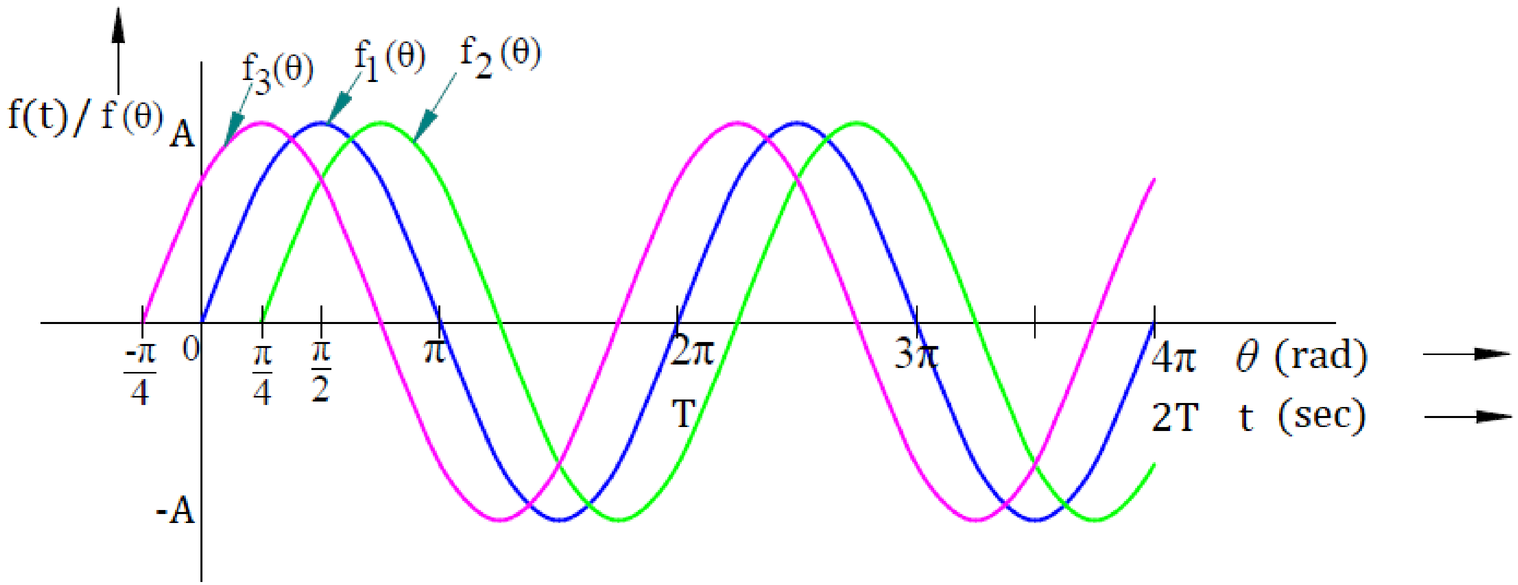
$f_1(t) = A\sin(\omega t)$ represents a reference sine wave.

$f_2(t) = A\sin(\omega t - \frac{\pi}{4})$ lags reference sine wave by $\frac{\pi}{4}$ rad.

$f_3(t) = A\sin(\omega t + \frac{\pi}{4})$ leads reference sine wave by $\frac{\pi}{4}$ rad.

Also, $f_2(t)$ lags $f_3(t)$ by $\frac{\pi}{2}$ rad.

Concept of Phase Lag and Phase Lead



In general, sinusoidal function is represented as

A sin(ωt + ϕ) where ϕ represents the phase angle.

If ϕ is positive, it leads the reference sine wave and lags if ϕ is negative.

Numerical Example

Question:

Write an equation to represent the following sine waves of 50Hz frequency.

i) A sinusoidal current with RMS value 10A & starting at 5ms

ii) A sinusoidal current with peak value 20A & starting at -2.5ms

Also, comment on the phase relation between them.

Solution:

$\omega = 2\pi f = 100\pi \text{rad/s}$

Case (i) : Angle = $\omega * t = (100\pi * 0.005) = \frac{\pi}{2} \text{rad}$

$i_1(t) = 10\sqrt{2}\sin(100\pi t - \frac{\pi}{2})\text{A}$

Case (ii) : Angle = $\omega * t = (100\pi * 0.0025) = \frac{\pi}{4} \text{rad}$

$i_2(t) = 20\sin(100\pi t + \frac{\pi}{4})\text{A}$

Text Book & References

Text Book:

"Electrical and Electronic Technology" E. Hughes (Revised by J. Hiley, K. Brown & I.M Smith), 11th Edition, Pearson Education, 2012.

Reference Books:

1. "Basic Electrical Engineering - Revised Edition", D. C. Kulshreshta, Tata- McGraw-Hill, 2012.
2. "Basic Electrical Engineering", K Uma Rao, Pearson Education, 2011.
3. "Engineering Circuit Analysis", William Hayt Jr., Jack E. Kemmerly & Steven M. Durbin, 8th Edition, McGraw-Hill, 2012.

THANK YOU

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