ELEMENTS OF ELECTRICAL ENGINEERING (UE24EE141B)

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Unit 2 – Lectures 27 to 29 - Analysis of Series RLC circuit ; Impedance and Power Triangles

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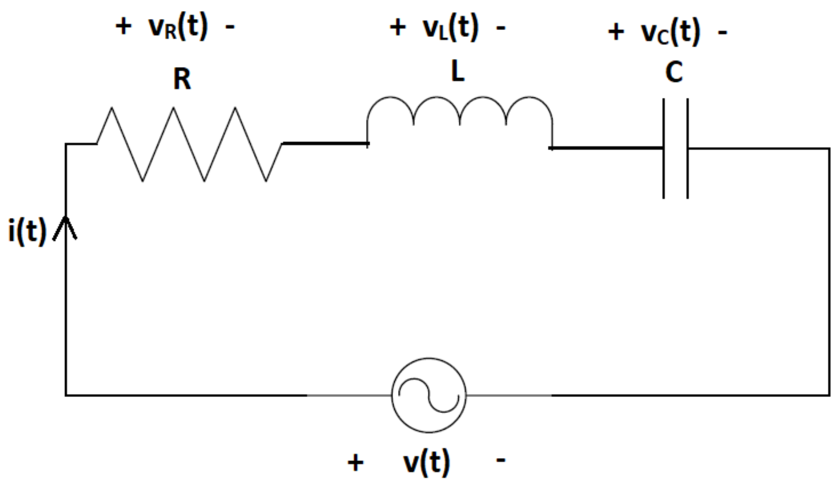
Series RLC Circuit



By KVL, $v(t) = v_R(t) + v_L(t) + v_c(t)$

In Phasor form, $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$

$$\bar{V}_R = \bar{I} * R \qquad \bar{V}_L = \bar{I} * (jX_L) \qquad \bar{V}_C = \bar{I} * (-jX_C)$$

$$\bar{V} = \bar{I} * (R + jX_L - jX_C)$$

$$Z_T = \frac{\bar{V}}{\bar{I}} = (R+jX_L-jX_C) = \sqrt{R^2+(X_L-X_C)^2}\angle \mathrm{Tan}^{-1}\left(\frac{X_L-X_C}{R}\right)$$

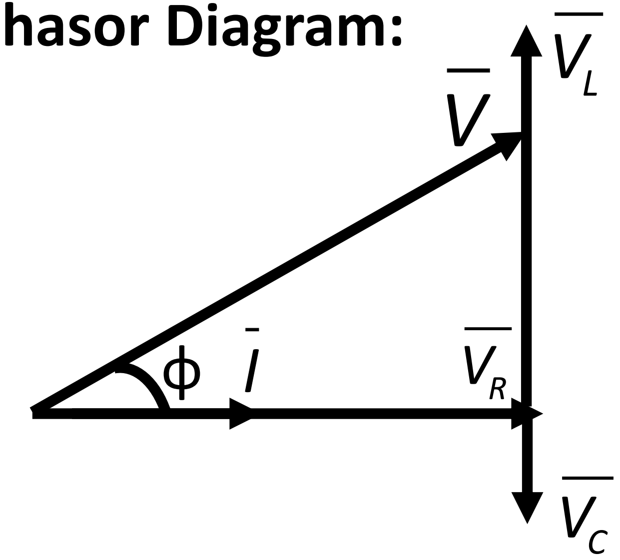
Case 1: $X_L > X_C$

If $X_L > X_C$ then $IX_L > IX_C$

i.e., $|\overline{V_L}| > |\overline{V_C}|$

The circuit behaves effectively as inductive circuit i.e., series RL type.

Phasor Diagram:



$$\phi = Tan^{-1}\left(\frac{|\overline{V_L}|-|\overline{V_C}|}{|\overline{V_R}|}\right) = Tan^{-1}\left(\frac{V_L-V_C}{V_R}\right)$$

$$= Tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

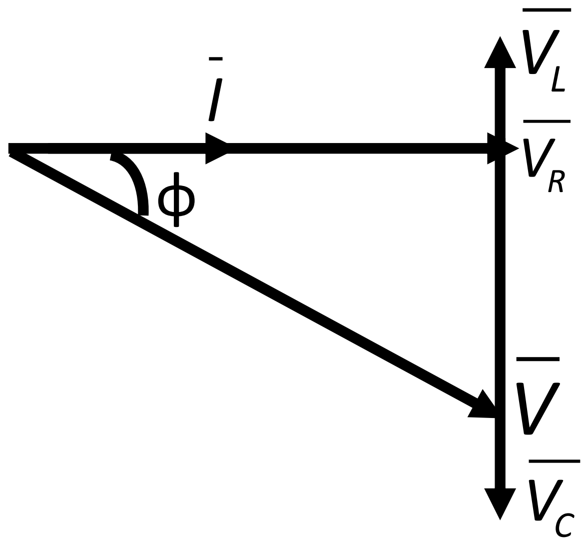
Case 2: $X_C > X_L$

If $X_C > X_L$ then $IX_C > IX_L$

i.e., $\left|\overline{V}_C\right| > \left|\overline{V}_L\right|$

The circuit behaves effectively as a capacitive circuit i.e., series RC type.

Phasor Diagram:



$$\phi = Tan^{-1}\left(\frac{\left|\overline{V}_L\right| - \left|\overline{V}_C\right|}{\left|\overline{V}_R\right|}\right) = Tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$= Tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

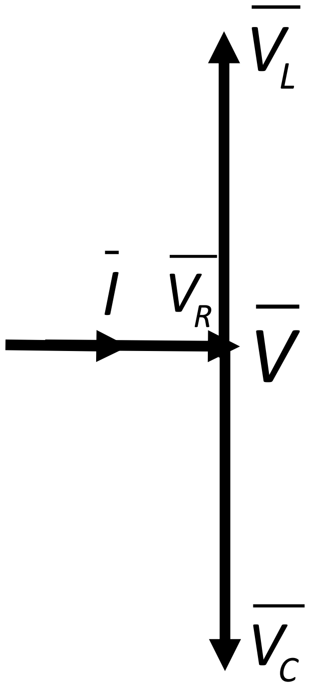
Note: ϕ will be negative in this case since $X_L < X_C$

Case 3: $X_L = X_C$

If $X_L = X_C$ then $IX_L = IX_C$ i.e., $|\overline{V_L}| = |\overline{V_C}|$

The circuit behaves effectively as a purely resistive circuit. This case is called **'Series Resonance'** case.

Phasor Diagram:



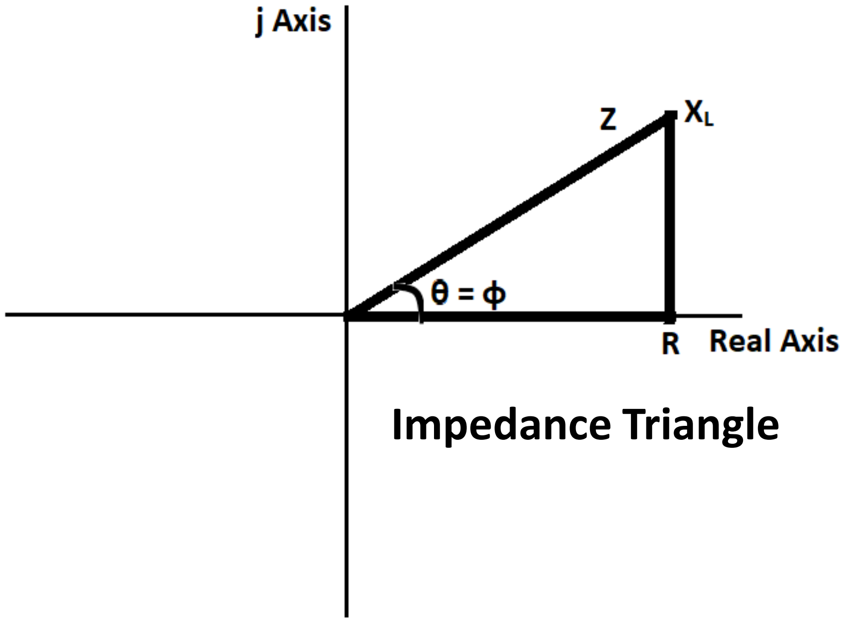
$$\overline{V} = \overline{V_R}$$

$$Z = R$$

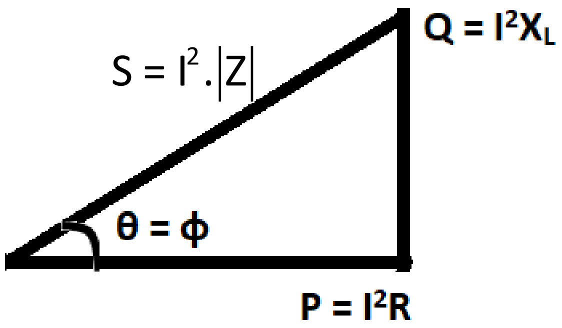
$$\phi = 0^\circ$$

Impedance & Power Triangles – Series RL Circuit

For a series RL circuit, $Z = R + jX_L = \sqrt{R^2 + X_L^2} \angle Tan^{-1}(\frac{X_L}{R})$



Impedance Triangle



Power Triangle

Impedance Triangle of a series RL circuit lies Quadrant I of complex plane.

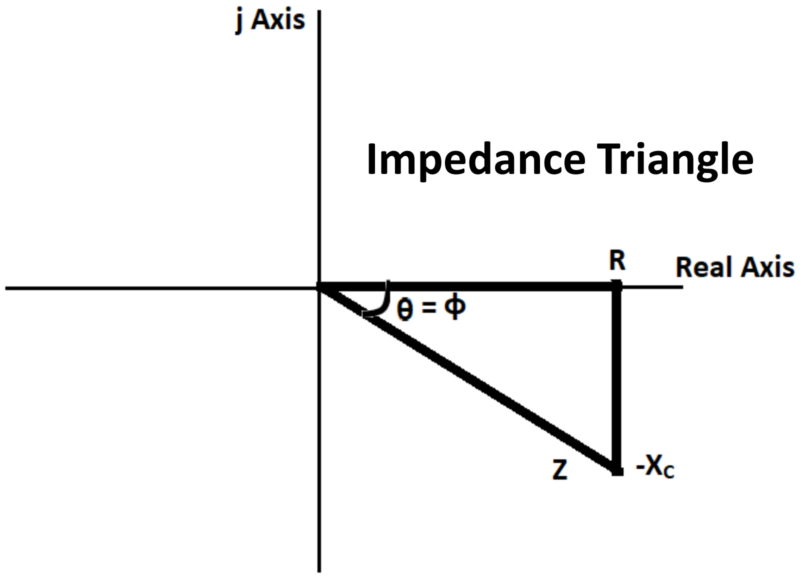
$$P = VI\cos\phi = (I.|Z|).I.\frac{R}{|Z|} = I^2R$$

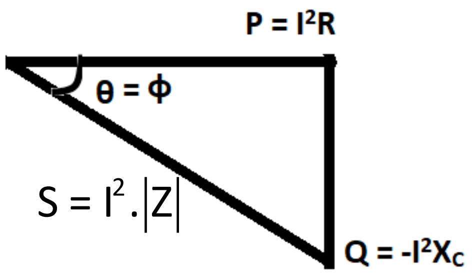
$$Q = VI\sin\phi = (I.|Z|).I.\frac{X_L}{|Z|} = I^2X_L$$

$$S = VI = (I.|Z|).I = I^2|Z|$$

Impedance & Power Triangles – Series RC Circuit

For a series RC circuit, $Z = R - jX_C = \sqrt{R^2 + X_C^{\,2}} \angle -\tan^{-1}\left(\frac{X_C}{R}\right)$





Power Triangle

$$P = VI\cos\phi = (I.|Z|).I.\frac{R}{|Z|} = I^2R$$

$$Q = VI\sin\phi = (I.|Z|).I.\frac{-X_C}{|Z|} = -I^2X_C$$

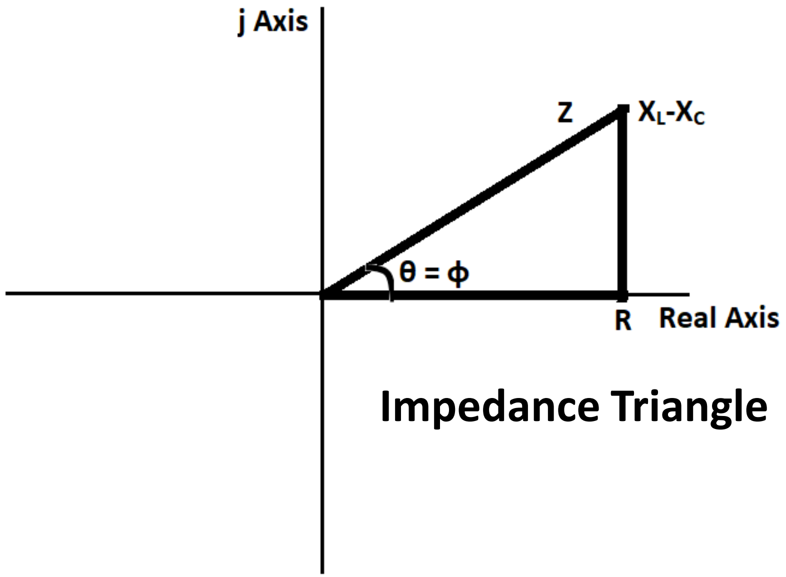
$$S = VI = (I.|Z|).I = I^2|Z|$$

Impedance Triangle of a series RC circuit lies Quadrant IV of complex plane.

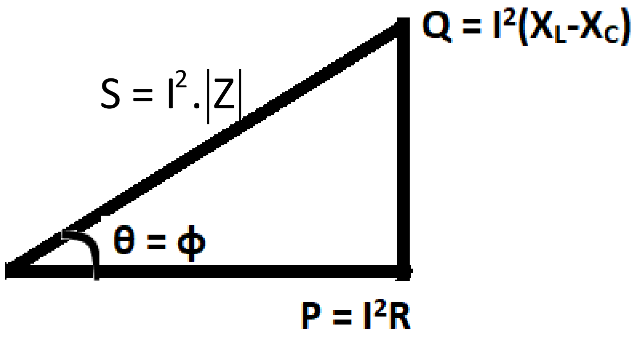
Impedance & Power Triangles – Series RLC Circuit

For a series RLC circuit, $Z = R + j(X_L - X_C) = \sqrt{R^2 + (X_L - X_C)^2}\angle \mathrm{Tan}^{-1}\left(\frac{X_L - X_C}{R}\right)$

Case i) $X_L > X_C$



Impedance Triangle



Power Triangle

Impedance Triangle of a series RLC circuit for $X_L > X_C$ lies in Quadrant I of complex plane.

$P = VI\cos\phi = I^2R$

$Q = VI\sin\phi = I^2(X_L - X_C)$

$S = VI = I^2|Z|$

Numerical Example

Question:

A series RLC circuit draws a current of 20A when connected to 200V, 50Hz supply. If the total active power drawn from the source is 500W and the circuit behaves effectively like an inductive circuit (series RL type), determine

i) Power factor of the circuit

ii) Inductance in the circuit if Capacitance is 100µF

Numerical Example

Solution:

Given, V = 200V , I = 20A & P = 500W

i) Since $P = I^2R$,

$R = 1.25\Omega$

$$|Z| = \frac{V}{I} = 10\Omega$$

Therefore, Power factor = $\frac{R}{|Z|}$ = 0.125 Lag

ii) Net Reactance, $X = (X_L - X_C) = \sqrt{Z^2 - R^2} = 9.92\Omega$

$X_C = 31.83\Omega$

Hence, $X_L = 41.75\Omega$

Therefore, L = 132.89mH

Numerical Example

1. A non-inductive resistor is connected in series with a coil and capacitor of 25.5μF. The current in the circuit is 0.4A and the potential difference across the non-inductive resistor is 20V, across the coil is 35V, across the capacitor is 50V and across the combination of non-inductive resistor and coil is 45V. Find the resistance and inductance of the coil. Also find the applied voltage, frequency and the power dissipated in the coil and the whole circuit.

Numerical Example

Q7. A coil of power factor 0.6 is in series with a 100μF capacitor. When connected to a 50Hz supply, the potential difference across the coil is equal to the potential difference across the capacitor. Find the resistance and inductance of the coil.

Numerical Example

Q8. An emf whose instantaneous value at time t is given by 283 sin($100\pi t + \pi/4$) Volts is applied to an inductive circuit and the current in the circuit is 5.66 sin($100\pi t - \pi/6$) Amperes. Determine i) the frequency of the emf, ii) the resistance and inductance of the circuit, iii) the active power absorbed. If series capacitance is added so as to bring the circuit into resonance at this frequency and the above emf is applied to the resonant circuit, find the corresponding expression for the instantaneous value of the current and also find the value of the series capacitance. Draw the phasor diagram representing the circuit before and at resonance.

Numerical Example

A series RLC circuit draws 400 W from a 200 V, 50 Hz supply. If the overall resistance is 4 Ω & the overall circuit behaves as inductive type, determine

i) Power factor of the network.

ii) Inductance in the network if capacitance is 1mF

iii) What must be the value of capacitance to bring the circuit into resonance?

Numerical Example

A series RLC network draws a net reactive power of 3KVAR from a 500V, 50Hz AC supply and has an overall powerfactor of 0.8 Lag. Determine

i) Total resistance in the network

ii) Inductance if the Capacitance is 159.15μF

iii) What is the new powerfactor if an extra resistance of 10Ω is added in series in the existing network?

b)	A series RLC circuit consumes 2KW of power when connected across 200V, 50Hz Single phase AC supply. If the overall resistance of the circuit is 5Ω and the circuit behaves effectively as Capacitive type (series RC type), determine i) Power factor of the network ii) Total Reactive Power iii) Capacitance, if the inductance is 10mH What is the value of extra inductance to be connected in series so that circuit will be in resonance?	8M

Text Book & References

Text Book:

"Electrical and Electronic Technology" E. Hughes (Revised by J. Hiley, K. Brown & I.M Smith), 11th Edition, Pearson Education, 2012.h

Reference Books:

1. "Basic Electrical Engineering - Revised Edition", D. C. Kulshreshta, Tata- McGraw-Hill, 2012.
2. "Basic Electrical Engineering", K Uma Rao, Pearson Education, 2011.
3. "Engineering Circuit Analysis", William Hayt Jr., Jack E. Kemmerly & Steven M. Durbin, 8th Edition, McGraw-Hill, 2012.

THANK YOU

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