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ELEMENTS OF ELECTRICAL ENGINEERING

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Concept of Phasor and Phasor Diagram ; Mathematical representation of a Phasor

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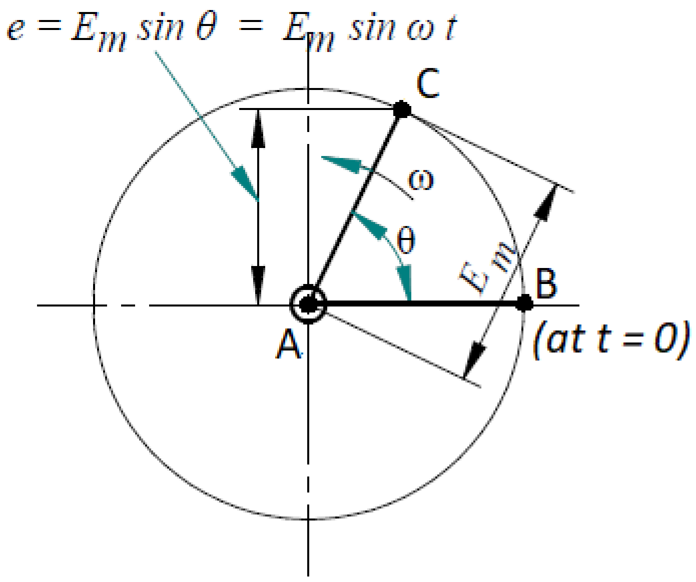
Concept of Phasor

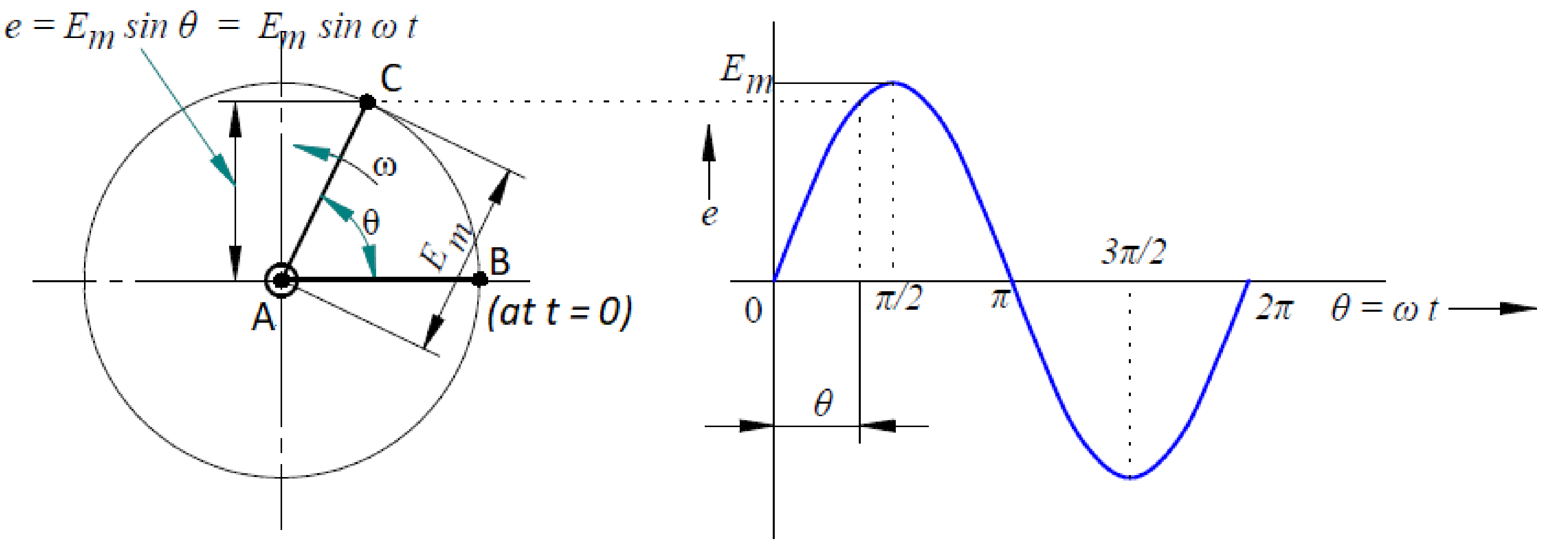
Let us consider a rotating crank of length E_m lying at 0° position at t = 0 and rotating anticlockwise at an angular speed of 'ω' rad/s.



At general time 't', it would be at an angle $\theta = \omega t$

Its vertical projection defines a sinusoidal function.

Concept of Phasor



(a) Crank

(b) Sinusoidal waveform

Thus the above rotating crank represents a sinusoidal function of the form $E_m \sin(\omega t)$

Concept of Phasor

Similarly, a sinusoidal function of the form $E_m\sin(\omega t + \phi)$ can be represented by another rotating crank of same length ‘E_m’ and rotating with same angular speed ‘ω’ rad/s anticlockwise but lying at an angle ‘ϕ’ at $t = 0$.

Thus, any sinusoidal function can be represented by a rotating crank and it is called ‘**Phasor representation**’ of a sinusoidal function.

A **Phasor** is a rotating vector which effectively represents a sinusoidal function.

Phasor Diagram

When a number of sinusoidal functions are to be represented as phasors, it is represented using a diagram called **phasor diagram**.

While drawing a phasor diagram, all phasors must be represented corresponding to same point in time. It is usually preferred to represent them at a time t = 0. Then, angular position of each sinusoidal function corresponds to its phase angle.

Note: Only sinusoidal functions of same frequency can be represented together as a phasor diagram. Also, the length of the phasor is its RMS value.

Phasor Diagram – Example

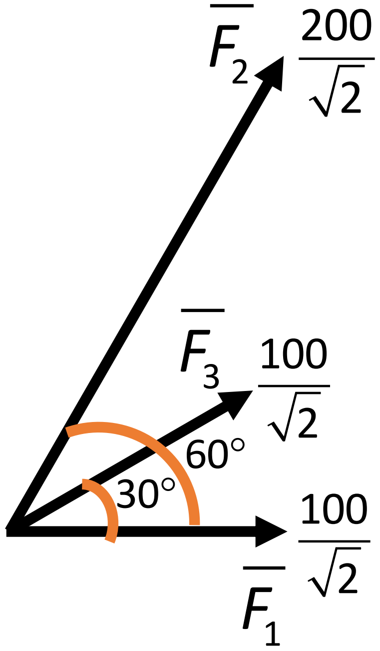
Consider the following sinusoidal functions

i) $f_1(t) = 100\sin(100\pi t)$

ii) $f_2(t) = 200\sin(100\pi t + 60°)$

iii) $f_3(t) = 100\cos(100\pi t - 60°)$

Let us represent them using a phasor diagram.



Note: Convert a cosine function to sine form before representing as a phasor.

For instance,

$$f_3(t) = 100\cos(100\pi t - 60°)$$
$$= 100\sin(100\pi t - 60° + 90°)$$
$$= 100\sin(100\pi t + 30°)$$

Mathematical Representation of a Phasor

A phasor is mathematically represented as

$$\text{Phasor} = \text{Magnitude} \angle \text{Phase Angle}$$

Where, magnitude is the RMS value.

For instance, Consider these sinusoidal functions

i) $f_1(t) = 100\sin(100\pi t)$ ii) $f_2(t) = 200\sin(100\pi t + 60°)$

iii) $f_3(t) = 100\cos(100\pi t - 60°)$

Let us represent them using phasor representation.

$$f_1(t) = 100\sin(100\pi t) \Rightarrow \overline{F_1} = \frac{100}{\sqrt{2}}\angle 0°$$

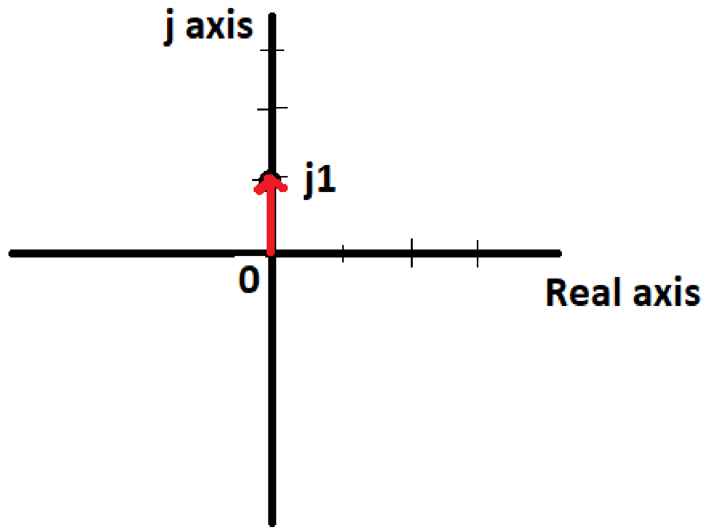
$$f_2(t) = 200\sin(100\pi t + 60°) \Rightarrow \overline{F_2} = \frac{200}{\sqrt{2}}\angle 60°$$

$$f_3(t) = 100\cos(100\pi t - 60°) = 100\sin(100\pi t + 30°)$$

$$\Rightarrow \overline{F_3} = \frac{100}{\sqrt{2}}\angle 30°$$

j operator

‘j’ operator in phasor representation is analogous to ‘i’ operator in complex mathematics.



In rectangular form, j = (0 + j1)

In polar form, $j = 1\angle 90^\circ$

Conversion between the forms

Polar to Rectangular conversion :

Let us consider a polar number $r\angle\theta$

It can be converted to rectangular form (A + jB) using

$A = r\cos\theta \; ; \; B = r\sin\theta$

Rectangular to Polar conversion :

Let us consider a rectangular number (A + jB)

It can be converted to polar form $r\angle\theta$ using

$$r = \sqrt{A^2 + B^2} \; ; \; \theta = \mathrm{Tan}^{-1}\left(\frac{B}{A}\right)$$

θ will be positive if 'B' is positive and it is negative if 'B' is negative.

Addition, Subtraction, Multiplication & Division of Phasors

Addition & Subtraction of Phasors:

Addition & subtraction of phasors would be easier in rectangular form.

For instance, let $\overline{F_1}=(A_1+jB_1)$ & $\overline{F_2}=(A_2+jB_2)$

$$\overline{F_1}+\overline{F_2} = (A_1+A_2) + j(B_1+B_2)$$

$$\overline{F_1}-\overline{F_2} = (A_1-A_2) + j(B_1-B_2)$$

Multiplication & Division of Phasors:

Multiplication & Division of phasors would be easier in Polar form.

For instance, let $\overline{F_1} = r_1\angle\theta_1$ & $\overline{F_2} = r_2\angle\theta_2$

$$\overline{F_1} * \overline{F_2} = r_1 * r_2\angle(\theta_1+\theta_2)$$

$$\frac{\overline{F_1}}{\overline{F_2}} = \frac{r_1}{r_2}\angle(\theta_1-\theta_2)$$

j operator properties

'j' operator when multiplied to a phasor, does not change the magnitude of the phasor but rotates the phasor anticlockwise by 90°

For instance, if $\bar{F} = 3\angle 60°$, then $j\bar{F} = 1\angle 90° * 3\angle 60° = 3\angle 150°$

$$j^2 = 1\angle 90° * 1\angle 90° = 1\angle 180° = \cos(180°) + j\sin(180°) = -1$$

Similarly, $j^3 = j^2 * j = -j$

And $j^4 = j^2 * j^2 = 1$

Numerical Example

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Question:

There are 3 conducting wires connected to form a junction. The currents flowing into the junction in two wires are i_1 =10sin314t A and i_2 = 15cos(314t - 45^o)A. What is the current leaving the junction in the third wire? What is its value at t=0?

Solution: 1) Using Time-Domain Method

By KCL at the junction, $i_3(t) = i_1(t) + i_2(t)$

$i_3(t)=10\sin(314t)+15\cos(314t-45°)$

$i_3(t)=10\sin(314t)+15*(\cos314t*\cos45°+\sin314t*\sin45°)$

$i_3(t)=20.61\sin(314t)+10.61\cos(314t)$

$$i_3(t)=23.18*(\frac{20.61}{23.18}\sin(314t)+\frac{10.61}{23.18}\cos(314t))$$

$i_3(t) = 23.18*(\cos(27.24°)*\sin(314t)+\sin(27.24°)\cos(314t))$

$i_3(t) = 23.18*\sin(314t+27.24°)$ A

Its value at $t = 0$ is $i_3(0) = 23.18\sin(27.24°) = 10.61$A

Numerical Example

Solution: 2) Using Phasor Domain Method

By KCL at the junction, $i_3(t) = i_1(t) + i_2(t)$

In Phasor form, $\bar{I}_3 = \bar{I}_1 + \bar{I}_2$

$i_1(t) = 10\sin(314t) \Rightarrow \bar{I}_1 = \frac{10}{\sqrt{2}}\angle 0°A$

$i_2(t) = 15\cos(314t - 45°) = 15\sin(314t + 45°) \Rightarrow \bar{I}_2 = \frac{15}{\sqrt{2}}\angle 45°A$

$$\bar{I}_3 = \frac{10}{\sqrt{2}}\angle 0° + \frac{15}{\sqrt{2}}\angle 45° = 16.39\angle 27.24°A$$

$i_3(t) = 23.18*\sin(314t+27.24°)$ A

Its value at t = 0 is $i_3(0) = 23.18\sin(27.24°) = 10.61A$

Text Book & References

Text Book:

"Electrical and Electronic Technology" E. Hughes (Revised by J. Hiley, K. Brown & I.M Smith), 11th Edition, Pearson Education, 2012.

Reference Books:

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2. "Basic Electrical Engineering", K Uma Rao, Pearson Education, 2011.
3. "Engineering Circuit Analysis", William Hayt Jr., Jack E. Kemmerly & Steven M. Durbin, 8th Edition, McGraw-Hill, 2012.

THANK YOU

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