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15MAT31

# Third Semester B.E. Degree Examination, Dec.2017/Jan.2018 Engineering Mathematics - III

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

# Module-1

- a. Express  $f(x) = (\pi x)^2$  as a Fourier series of period  $2\pi$  in the interval  $0 < x < 2\pi$ . Hence deduce the sum of the series  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  (08 Marks)
  - b. The turning moment T units of the Crank shaft of a steam engine is a series of values of the crank angle  $\theta$  in degrees. Find the first four terms in a series of sines to represent T. Also calculate T when  $\theta = 75^{\circ}$ .

 θ:
 0°
 30°
 60°
 90°
 120°
 150°
 180°

 T:
 0
 5224
 8097
 7850
 5499
 2626
 0

#### OR

2 a. Find the Fourier Series expansion of the periodic function,

 $f(x) = \begin{cases} l+x, & -l \le x \le 0 \\ l-x, & 0 \le x \le l \end{cases}$ 

(06 Marks)

b. Obtain a half-range cosine series for  $f(x) = x^2$  in  $(0, \pi)$ .

(05 Marks)

c. The following table gives the variations of periodic current over a period:

t sec:	.0	T T		Т	2T	5T	
		6	3	2	3	6	
A amp:	1.98	1.30	1.05	1.30	-0.88	-0.25	

Show that there is a direct current part 0.75 amp in the variable current and obtain the amplitude of the first harmonic. (05 Marks)

#### Module-2

- 3 a. Find the Fourier transform of  $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$  and evaluate  $\int_{0}^{\infty} \left(\frac{\sin x}{x}\right) dx$  (06 Marks)
  - b. Find the Fourier cosine transform of,  $f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$  (05 Marks)
  - c. Obtain the inverse Z-transform of the following function,  $\frac{z}{(z-2)(z-3)}$ . (05 Marks)

## OR

4 a. Find the Z-transform of  $\cos\left(\frac{n\pi}{2} + \alpha\right)$ .

(66 Marks)

b. Solve  $u_{n+2} - 5u_{n+1} + 6u_n = 36$  with  $u_0 = u_1 = 0$ , using Z-transforms.

(05 Marks)

c. If Fourier sine transform of f(x) is  $\frac{e^{-a\alpha}}{\alpha}$ ,  $\alpha \neq 0$ . Find f(x) and hence obtain the inverse

Fourier sine transform of  $\frac{1}{\alpha}$ .

(05 Marks)

portant Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.

2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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#### Module-3

5 a. Calculate the Karl Pearson's co-efficient for the following ages of husbands and wives:
(06 Marks)

Husband's age x:	23	27	28	28	29	30	31	33	35	36
Wife's age v:	18	20	22	27	21	29	27	29	28	29

b. By the method of least square, find the parabola y = ax² + bx + c that best fits the following data:
 (05 Marks)

x: 10 12 15 23 20 y: 14 17 23 25 21

C. Using Newton-Raphson method, find the real root that lies near x = 4.5 of the equation tan x = x correct to four decimal places. (Here x is in radians). (05 Marks)

#### OF

- 6 a. In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as 4x 5y + 33 = 0 and 20x 9y = 107 respectively. Calculate  $\overline{x}$ ,  $\overline{y}$  and the coefficient of correlation between x and y.

  (06 Marks)
  - b. Find the curve of best fit of the type  $y = ae^{bx}$  to the following data by the method of least squares: (05 Marks)

 x:
 1
 5
 7
 9
 12

 y:
 10
 15
 12
 15
 21

c. Find the real root of the equation  $xe^x - 3 = 0$  by Regula Falsi method, correct to three decimal places. (05 Marks)

#### Module-4

7 a. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46: (06 Marks)

 Age:
 45
 50
 55
 60
 65

 Premium (in Rupees):
 114.84
 96.16
 83.32
 74.48
 68.48

b. Using Newton's divided difference interpolation, find the polynomial of the given data:

(05 Marks)

 x
 3
 7
 9
 10

 f(x)
 168
 120
 72
 63

c. Using Simpson's  $\left(\frac{1}{3}\right)^{nd}$  rule to find  $\int_{0}^{0.6} e^{-x^2} dx$  by taking seven ordinates. (05 Marks)

#### OR

8 a. Find the number of men getting wages below ₹ 35 from the following data: (06 Marks)

Wages in ₹: 0 10 10 -20 20 -30 30 -40 Frequency: 9 30 35 42

b. Find the polynomial f(x) by using Lagrange's formula from the following data: (05 Marks)

x: 0 1 2 5 f(x): 2 3 12 147

c. Compute the value of  $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x) dx$  using Simpson's  $\left(\frac{3}{8}\right)^{th}$  rule. (05 Marks)

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# Module-5

- 9 a. A vector field is given by  $\vec{F} = \sin y \hat{i} + x(1 + \cos y)\hat{j}$ . Evaluate the line integral over a circular path given by  $x^2 + y^2 = a^2$ , z = 0.
  - b. If C is a simple closed curve in the xy-plane not enclosing the origin. Show that  $\int F dR = 0$ ,

where  $\vec{F} = \frac{y\hat{i} - x\hat{j}}{x^2 + y^2}$ .

(05 Marks)

c. Derive Euler's equation in the standard form viz.,  $\frac{\partial f}{\partial y} - \frac{d}{dx} \left[ \frac{\partial f}{\partial y} \right] = 0$ . (05 Marks)

OR

- 10 a. Use Stoke's theorem to evaluate  $\int_{C} \vec{F} \cdot d\vec{R}$  where  $\vec{F} = (2x y)\hat{i} yz^2\hat{j} y^2z\hat{k}$  over the upper half surface of  $x^2 + y^2 + z^2 = 1$  bounded by its projection on the xy-plane. (06 Marks)
  - b. Show that the geodesics on a plane are straight lines. (05 Marks)
  - c. Find the curves on which the functional  $\int_{0}^{1} ((y')^{2} + 12xy) dx$  with y(0) = 0 and y(1) = 1 can be extremized. (05 Marks)