Classification of Gyrogroups of Small Orders

Kurosh Mavaddat Nezhaad*, **Ali Reza Ashrafi, Mohammad Ali Salahshour**

Department of Pure Mathematics, Faculty of Mathematical Sciences, University of Kashan, Kashan 87317-53153, I. R. Iran

kuroshmavaddat@gmail.com

Suppose $(G, \oplus)$ is a groupoid and $Aut(G)$ denotes the set of all automorphisms of G. The pair $(G, \oplus)$ is called a *gyrogroup* if and only if $i)$ there exists an element $0 \in G$ such that for all $x \in G$, $0 \oplus x = x$; $ii)$ for each $a \in G$, there exists $b \in G$ such that $b \oplus a = 0$; $iii)$ there exists a function $gyr : G \times G \longrightarrow Aut(G)$ such that for every $a, b, c \in G$, $a \oplus (b \oplus c) = (a \oplus b) \oplus gyr[a, b]c$, where $gyr[a, b]c = gyr(a, b)(c)$; $iv)$ for each $a, b \in G$, $gyr[a, b] = gyr[a \oplus b, b]$. The notion of a gyrogroup was introduced by Abraham Ungar in $[1, 2]$. The gyrogroup is the closest algebraic structure to the group ever discovered. Since each gyrogroup is a Bol loop, we can apply Burn's results in Bol loops to deduce that all gyrogroups of order p, $2p$, and p^2 are groups. In this talk, we report the classifications of gyrogroups of orders 8, 12, 15, 18, 20, 21, and 28. This talk will also give functions in the GAP Program that help us to do calculations with gyrogroups.

References

[1] A. A. Ungar, Thomas rotation and the parametrization of the Lorentz transformation group, *Found. Phys. Lett.* **1**(1) (1988) 57–89.
[2] A. A. Ungar, The Thomas rotation formalism underlying a nonassociative group structure for relativistic velocities, *Appl. Math. Lett.* **1** (4) (1988), 403–405.
[3] R. P. Burn, Finite Bol loops, *Mathematical Proceedings of the Cambridge Philosophical Society* **84** (3) (1978) 377 – 385.
[4] R. P. Burn, Finite Bol loops II, *Mathematical Proceedings of the Cambridge Philosophical Society* **89** (3) (1981) 445 – 455.
[5] R. P. Burn, Finite Bol loops III, *Mathematical Proceedings of the Cambridge Philosophical Society* **97** (2) (1985) 219 – 223.
[6] R. P. Burn, Corrigenda to: Finite Bol loops III, *Mathematical Proceedings of the Cambridge Philosophical Society* **98** (3) (1985) 219 – 223.
[7] The GAP Group, GAP – Groups, Algorithms, and Programming, Version 4.11.1; 2021. (https://www.gap-system.org).