Chapter 6

Force and Motion–II

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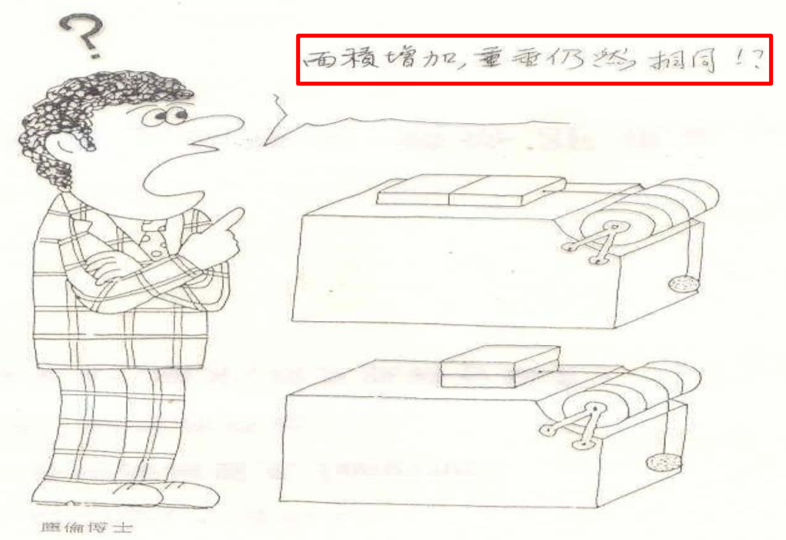
6-1 Friction

Learning Objectives

6.01 Distinguish between friction in a static situation and a kinetic situation.

6.02 Determine direction and magnitude of a frictional force.

6.03 For objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law.



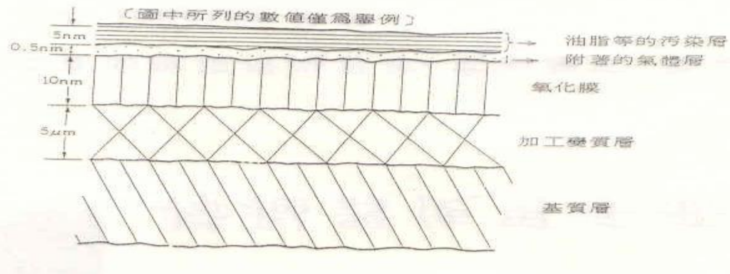
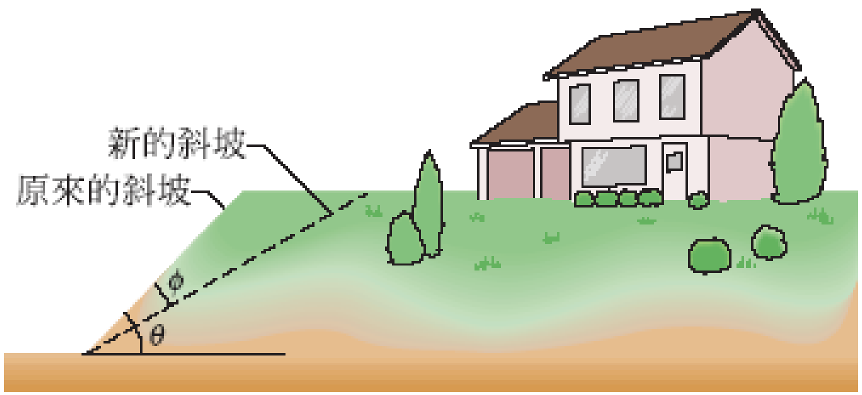


圖 1-3　金屬表層的構造

6-1 Friction

- Friction forces are essential:
  - Picking things up
  - Walking, biking, driving anywhere
  - Writing with a pencil
  - Building with nails, weaving cloth



- But overcoming friction forces is also important:
  - Efficiency in engines
  - (20% of the gasoline used in an automobile goes to counteract friction in the drive train)
  - Roller skates, fans
  - Anything that we want

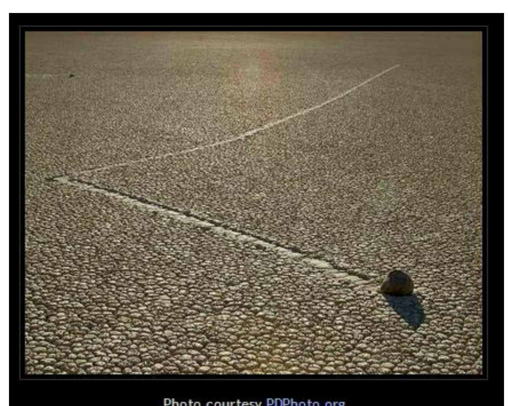
  to remain in motion

6-1 Friction

- The mysterious sliding stones. :
  - Along the remote Racetrack Playa(乾鹽湖)in Death Valley, California (加利福尼亞州死亡谷), stones sometimes gouge out prominent trails in the desert floor, as if the stones had been migrating (see Figures). For years curiosity mounted about why the stones moved. One explanation was that strong winds during occasional rainstorms would drag the rough stones over ground softened by rain. When the desert dried out, the trails behind the stones were hard-baked in place. According to measurements, the coefficient of kinetic friction between the stones and the wet playa ground is about 0.80. What horizontal force must act on a 20 kg stone (a typical mass) to maintain the stone's motion once a gust has started it moving?

What moved the stone?

~假設地面動摩擦係數:0.8，石頭為密度:2.5 g/cm^3 之立方體，估計風速? m/s，相當於幾級風?



6-1 Friction

- Two types of friction
- The **static frictional force**:
  - The opposing force that prevents an object from moving
  - Can have any magnitude from 0 N up to a maximum
  - Once the maximum is reached, forces are no longer in equilibrium and the object slides
- The **kinetic frictional force**:
  - The opposing force that acts on an object in motion
  - Has only one value
  - Generally smaller than the maximum static frictional force

6-1 Friction

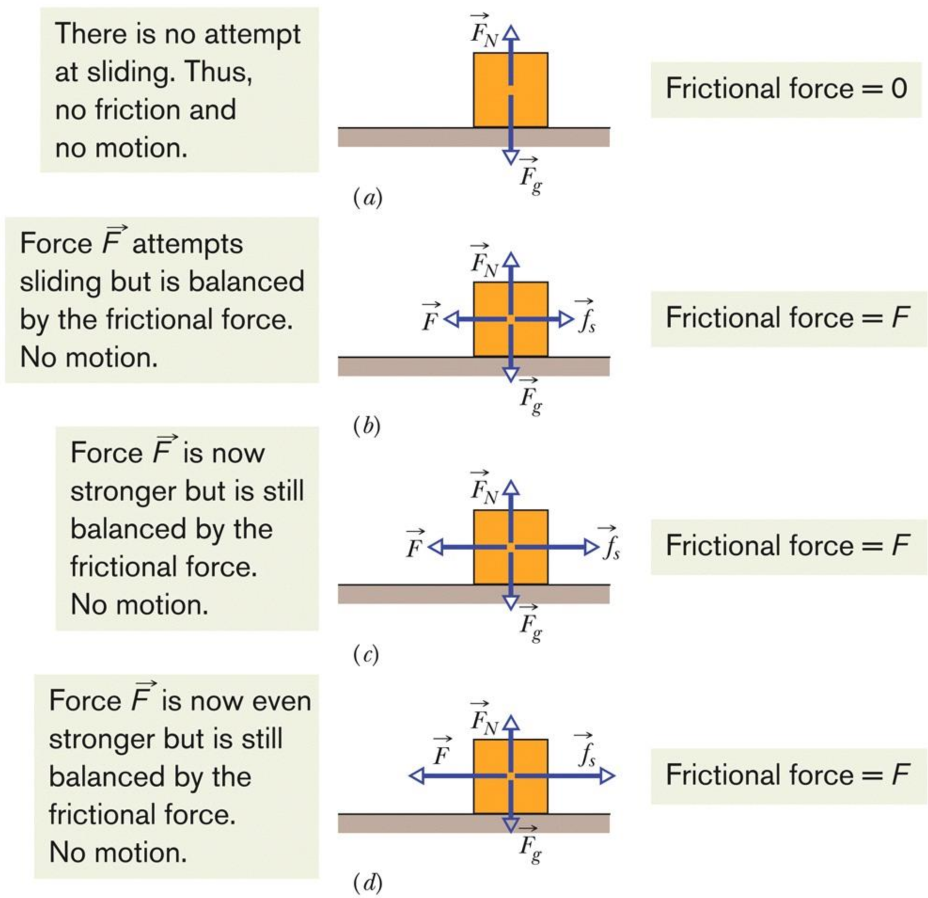
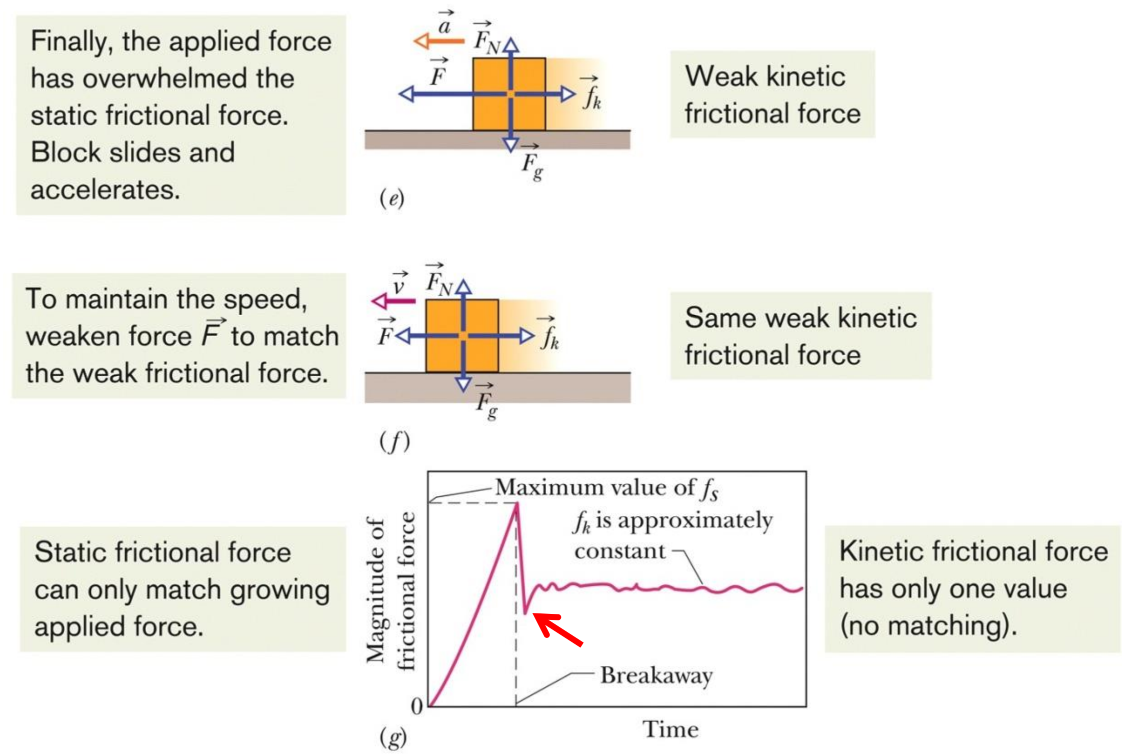


Figure 6-1



6-1 Friction

- Microscopic picture: surfaces are bumpy
- Friction occurs as contact points slide over each other
- Two specially prepared metal surfaces can *cold-weld* together and become impossible to slide, because there is so much contact between the surfaces
- Greater force normal to the contact plane increases the friction because the surfaces are pressed together and make more contact
- Sliding that is jerky, due to the ridges on the surface, produces squeaking/squealing/sound

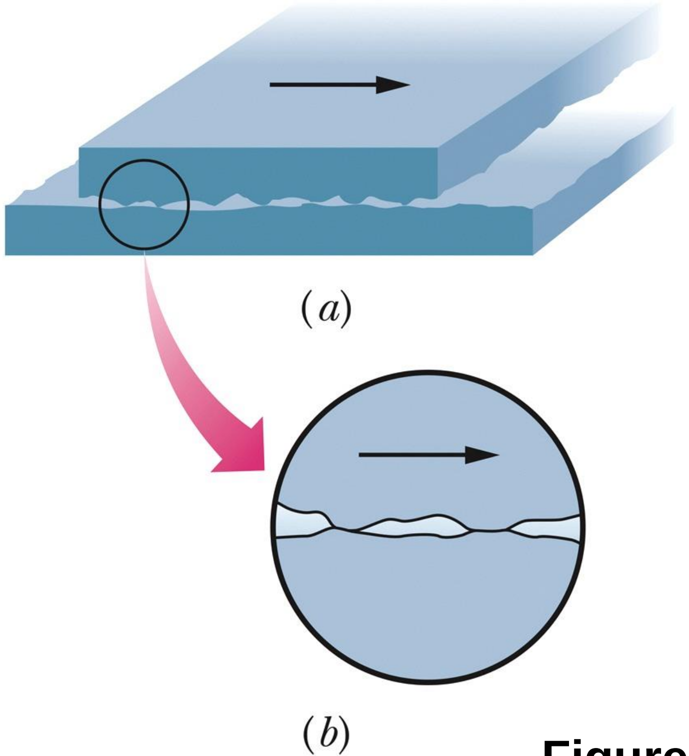


Figure 6-2

6-1 Friction

- The properties of friction
  1. If the body does not move, then the applied force and frictional force balance along the direction parallel to the surface: equal in magnitude, opposite in direction
  2. The magnitude of f_s has a maximum $f_{s,max}$ given by:

$$f_{s,\max} = \mu_s F_N, \qquad \text{Eq. (6-1)}$$

where μ_s is the **coefficient of static friction**. If the applied force increases past $f_{s,max}$, sliding begins.

6-1 Friction

- The properties of friction

  3. Once sliding begins, the frictional force decreases to f_k given by:

$$f_k = \mu_k F_N, \quad \text{Eq. (6-2)}$$

  where μ_k is the **coefficient of kinetic friction**.

- Magnitude F_N of the normal force measures how strongly the surfaces are pushed together
- The values of the friction coefficients are unitless and must be determined experimentally

Summary of Forces of Friction摩擦力

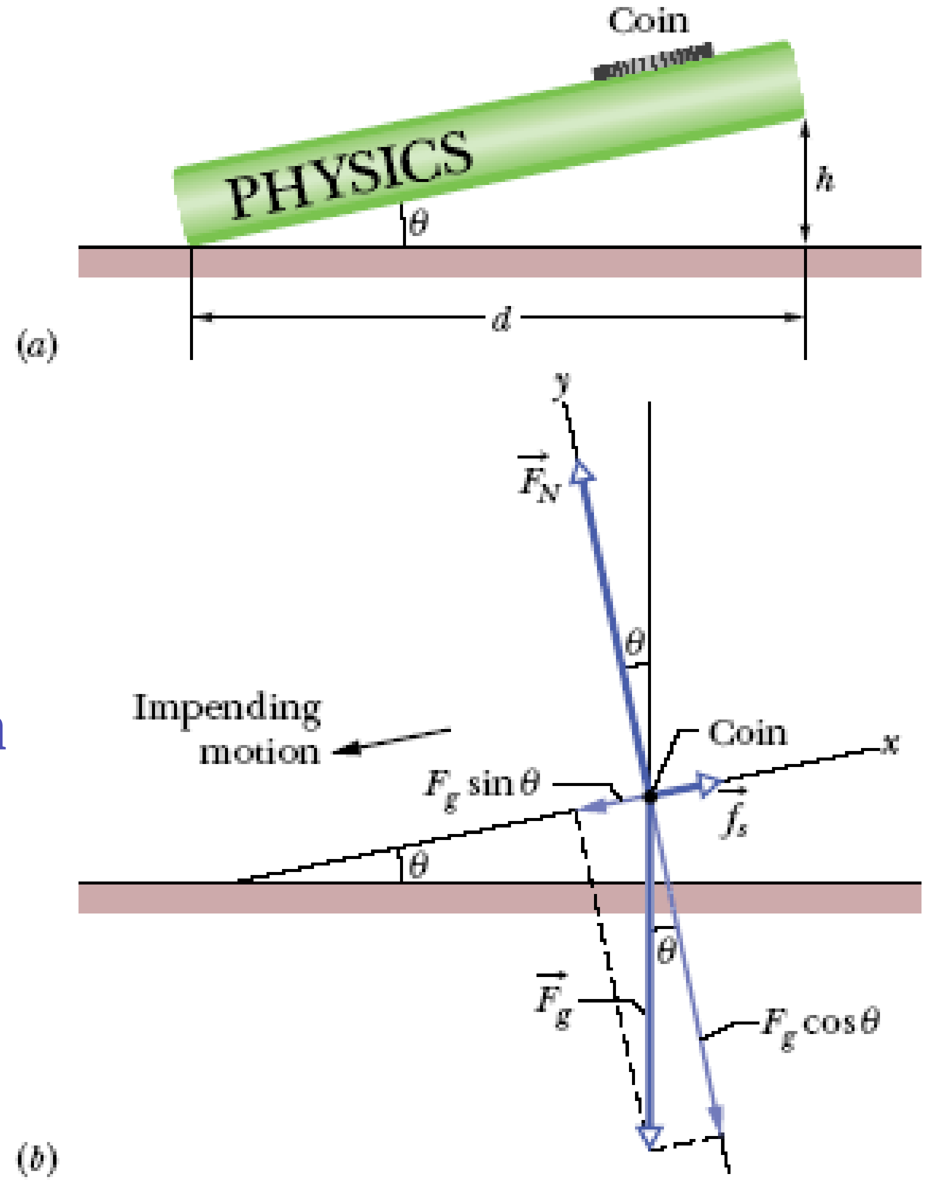
- The force of static friction, f_s, is generally greater than the force of kinetic friction, f_k (靜摩擦力 f_s >動摩擦力 f_k)
- The coefficient of friction (μ) depends on the surfaces in contact , and is nearly independent of the area of contact. (摩擦係數 μ 與接觸面之粗糙度相關，與接觸面之面積大小無關!)
- Friction is proportional to the normal force
  - $f_s \le \mu_s F_N$ and $f_k = \mu_k F_N$, F_N: *normal force, 正向力。*

Some Coefficients of Friction

TABLE 5.1 Coefficients of Friction

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

Experimental Determination of μ_S and μ_k?



μ_S = tan θ_C

***μ_k = tan θ'*, with constant speed**

6-1 Friction

Example For a force applied at an angle:

- Decompose the force into *x* and *y* components
- Balance the vertical components (F_N, F_g, F_y)
- Balance the horizontal components (f, F_x)
- Solve for your unknown, noting that F_N and f are related

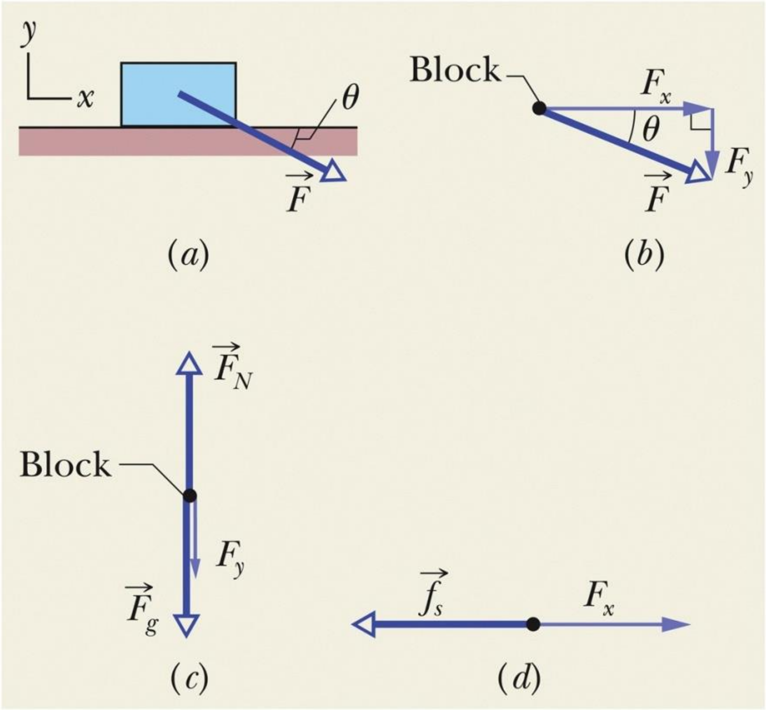
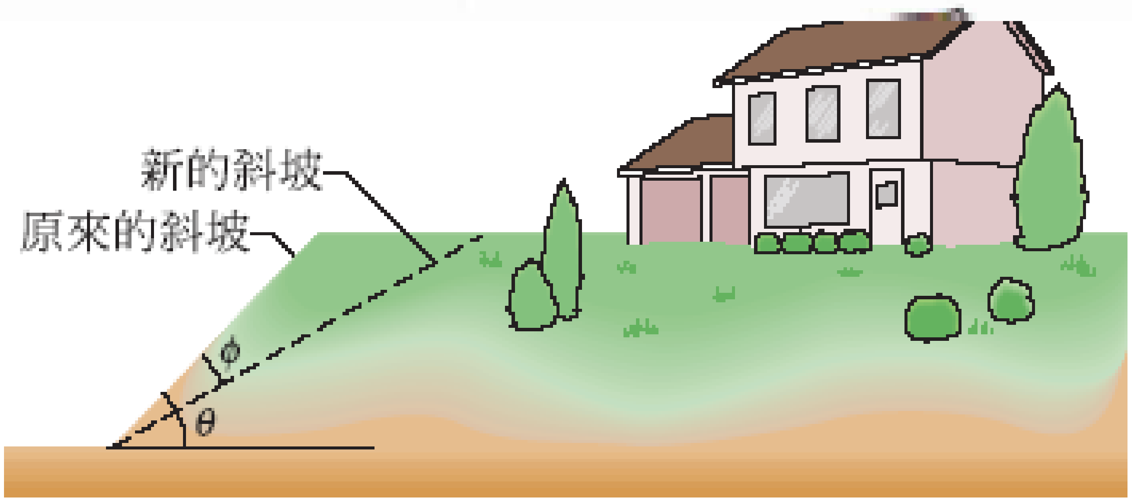


Figure 6-3

72 A house is built on the top of a hill with a nearby slope at angle $\theta = 45°$ (Fig. 6-54). An engineering study indicates that the slope angle should be reduced because the top layers of soil along the slope might slip past the lower layers. If the coefficient of static friction between two such layers is 0.5, what is the least angle ϕ through which the present slope should be reduced to prevent slippage?



6-2 The Drag Force and Terminal Speed

Learning Objectives

6.04 Apply the relationship between the drag force on an object moving through the air and the speed of the object.

6.05 Determine the terminal speed of an object falling through the air.

6-2 The Drag Force and Terminal Speed

- A **fluid** is anything that can flow (gas or liquid)
- When there is relative velocity between fluid and an object there is a **drag force**:
  - That opposes the relative motion
  - And points along the direction of the flow, relative to the body
- Here we examine the drag force for
  - Air
  - With a body that is not streamlined (流線型)
  - For motion fast enough that the air becomes turbulent (breaks into swirls~產生漩渦)

6-2 The Drag Force and Terminal Speed

- For this case, the drag force is:

$$\boxed{D = \tfrac{1}{2}C\rho A v^2,}$$

Eq. (6-14)

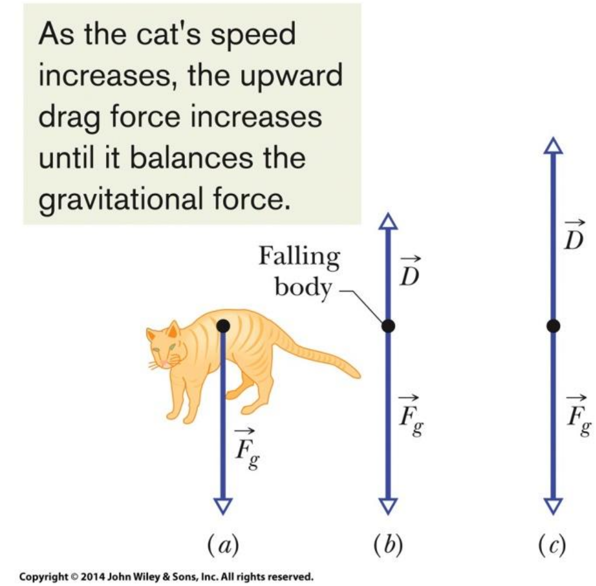


Figure 6-6

- Where:
  - *v* is the relative velocity
  - *ρ* is the air density (mass/volume)
  - *C* is the experimentally determined drag coefficient
  - *A* is the effective cross-sectional area of the body (the area taken perpendicular to the relative velocity)
- In reality, *C* is not constant for all values of *v*

6-2 The Drag Force and Terminal Speed

- The drag force from the air opposes a falling object

$$F_g - D = ma$$ Eq. (6-15)

- Once the drag force equals the gravitational force, the object falls at a constant **terminal speed**:

$$\boxed{v_t = \sqrt{\frac{2F_g}{C\rho A}}.}$$ Eq. (6-16)

- Terminal speed can be increased by reducing A
- Terminal speed can be decreased by increasing A
- Skydivers use this to control descent

6-2 The Drag Force and Terminal Speed

Example Speed of a rain drop:

- Spherical drop feels gravitational force *F* = *mg*:
  - Express in terms of density of water

$$F_g = V\rho_w g = \tfrac{4}{3}\pi R^3 \rho_w g.$$

- So plug in to the terminal velocity equation using the values provided in the text:
  - Use $A = \pi R^2$ for the cross-sectional area

$$v_t = \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}}$$

$$= \sqrt{\frac{(8)(1.5 \times 10^{-3}\ \text{m})(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)}{(3)(0.60)(1.2\ \text{kg/m}^3)}}$$

$$= 7.4\ \text{m/s} \approx 27\ \text{km/h}. \qquad \text{(Answer)}$$

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Supplement the drag force in air:

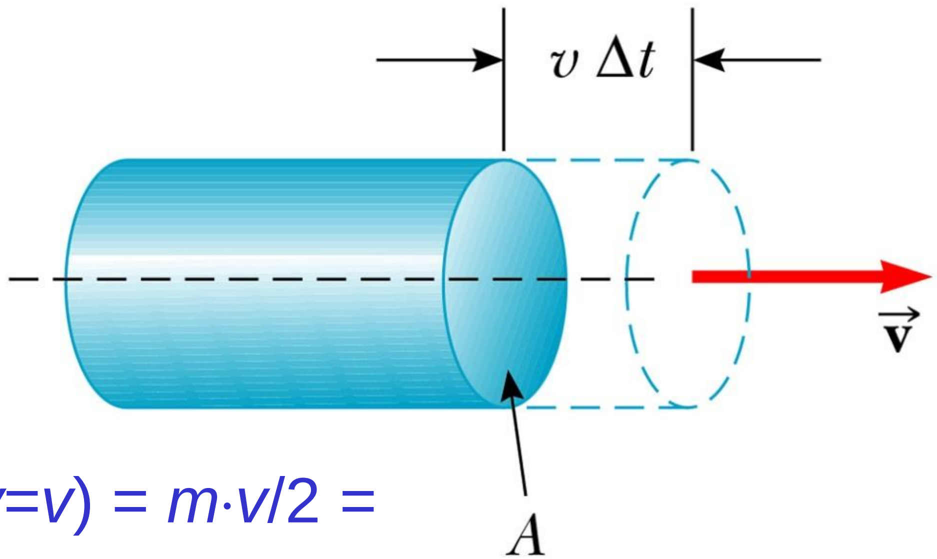
Proof: F = ½ $C\rho A v^2$, and Solve the time-dependent v, $v(t)$:

$t \rightarrow 0$, $v = 0$;

$t \rightarrow \infty$,

$$\boxed{v_t = \sqrt{\frac{2mg}{C\rho A}}}$$

With C = 1



$\Delta P = m\,\Delta v_{average}\,(v{=}0 \rightarrow v{=}v) = m \cdot v/2 =$

½ $\rho A v \Delta t \cdot v$

$F = \Delta P/\Delta t =$ ½ $\rho A v^2$

一般而言，考慮其等效面積: $A_{eff} = CA \rightarrow F =$ ½ $C\rho A v^2$，通常C <1!

TABLE 6-1 Some Terminal Speeds in Air

Object	Terminal Speed (m/s)	95% Distance[a] (m)
Shot (from shot put)	145	2500
Sky diver (typical)	60	430
Baseball	42	210
Tennis ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop (radius = 1.5 mm)	7	6
Parachutist (typical)	5	3

[a]This is the distance through which the body must fall from rest to reach 95% of its terminal speed.

Source: Adapted from Peter J. Brancazio, *Sport Science*, 1984, Simon & Schuster, New York.

TABLE 6.1 Terminal Speed for Various Objects Falling Through Air

Object	Mass (kg)	Cross-Sectional Area (m^2)	v_t (m/s)
Sky diver	75	0.70	60
Baseball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hailstone (radius 0.50 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop (radius 0.20 cm)	3.4×10^{-5}	1.3×10^{-5}	9.0

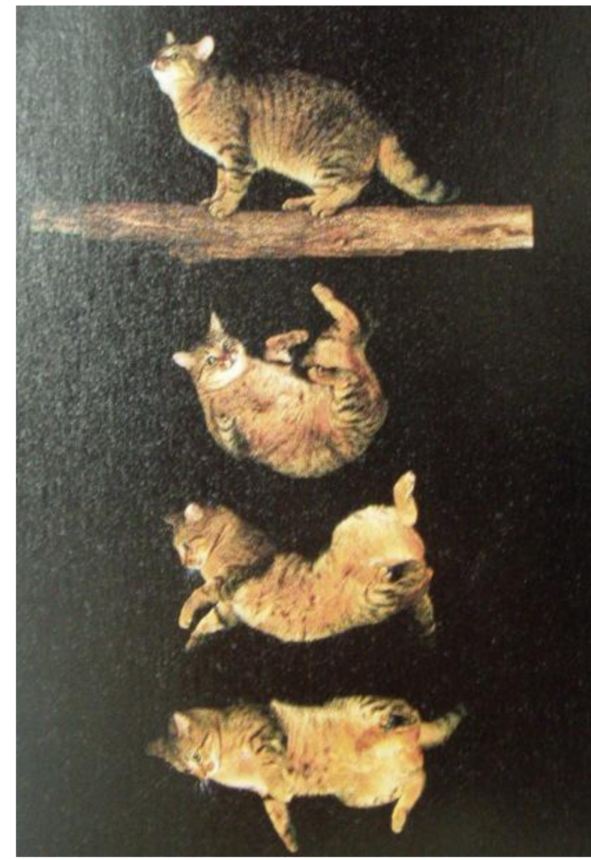
$$\boxed{v_t = \sqrt{\frac{2mg}{C\rho A}}}$$

V_t 與“**有效截面積*A***”相關!

Aerodynamic car. A streamlined body reduces air drag and increases fuel efficiency.

應用~ 天空跳傘與“九命怪貓”:

Fig. 6-8 A sky diver in a horizontal “spread eagle” maximizes air drag.



驚險的跳傘表演

https://www.bilibili.com/video/BV1dx411M7zt/

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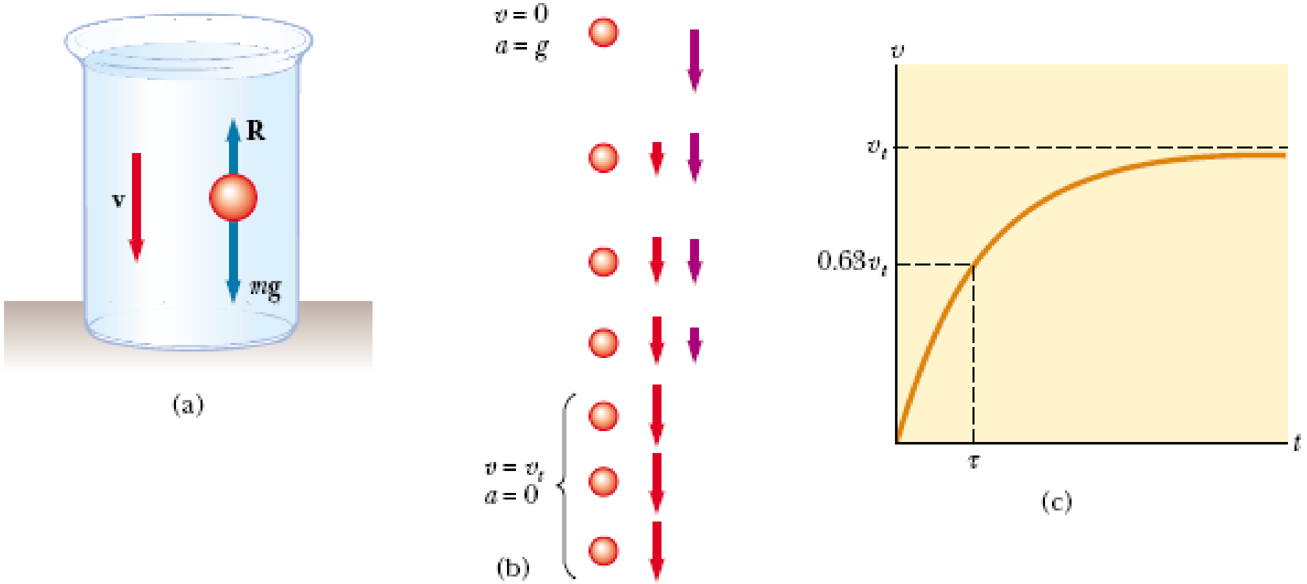
[是跳還是死？白頰黑雁雛雁在120m的終極選擇]

https://www.fullyu.com/article/2364/%E7%99%BD%E9%A0%B0%E9%BB%91%E9%9B%81%E8%B7%B3%E5%B4%96/

流體中“慢速”下落之終端速度:



Resistive Force Proportional to Object Speed

If we assume that the resistive force acting on an object moving through a liquid or gas is proportional to the object's speed, then the magnitude of the resistive force can be expressed as

$$\boxed{R = bv} \tag{6.2}$$

$$mg - bv = ma = m\frac{dv}{dt}$$

where the acceleration dv/dt is downward. Solving this expression for the acceleration gives

$$\frac{dv}{dt} = g - \frac{b}{m}v$$

it continues to move at this speed with zero acceleration, as shown in Figure 6.15b. We can obtain the terminal speed from Equation 6.3 by setting $a = dv/dt = 0$. This gives

$$mg - bv_t = 0 \qquad \text{or} \qquad v_t = mg/b$$

$v=v_t$(終端速度)；

$a= dv/dt =0$

The expression for v that satisfies Equation 6.4 with $v = 0$ at $t = 0$ is

$$v = \frac{mg}{b}(1 - e^{-bt/m}) = v_t(1 - e^{-t/\tau})$$

This function is plotted in Figure 6.15c. The **time constant** $\tau = m/b$ (Greek letter tau) is the time it takes the sphere to reach 63.2% ($= 1 - 1/e$) of its terminal speed. This can be seen by noting that when $t = \tau$, Equation 6.5 yields $v = 0.632v_t$.

We can check that Equation 6.5 is a solution to Equation 6.4 by direct differentiation:

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{mg}{b} - \frac{mg}{b}e^{-bt/m}\right) = -\frac{mg}{b}\frac{d}{dt}e^{-bt/m} = ge^{-bt/m}$$

SOLUTION We start with

$$\frac{dv}{dt} = g - \frac{b}{m} v.$$

There are two variables, v and t. We collect variables of the same type on one or the other side of the equation:

$$\frac{dv}{g - \frac{b}{m} v} = dt \qquad \text{or} \qquad \frac{dv}{v - \frac{mg}{b}} = -\frac{b}{m} dt.$$

Now we can integrate, remembering $v = 0$ at $t = 0$:

$$\int_0^v \frac{dv}{v - \frac{mg}{b}} = -\frac{b}{m} \int_0^t dt$$

$$\ln\left(v - \frac{mg}{b}\right) - \ln\left(-\frac{mg}{b}\right) = -\frac{b}{m} t$$

or

$$\ln \frac{v - mg/b}{-mg/b} = -\frac{b}{m} t.$$

We raise each side to the exponential [note that the natural log and the exponential are inverse operations of each other: $e^{\ln x} = x$, or $\ln\left(e^x\right) = x$] and obtain

$$v - \frac{mg}{b} = -\frac{mg}{b} e^{-\frac{b}{m}t} \qquad \text{or finally,} \qquad v = \frac{mg}{b}\left(1 - e^{-\frac{b}{m}t}\right).$$

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In addition: solve $v(t)$ for $F_{drag} = \frac{1}{2} C\rho A v^2$??

~Using $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} ln(\frac{a+x}{a-x})$:

Ans:

$V(t) = \sqrt{\frac{mg}{k}} \frac{1-e^{-t/K}}{1+e^{-t/K}}$, where $k = \frac{1}{2} C\rho A$, and $K = \frac{1}{2}\sqrt{\frac{m}{gk}}$

➔ Estimate the 95% distance in Table 6-1?

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6-3 Uniform Circular Motion

Learning Objectives

6.06 Sketch the path taken in uniform circular motion and explain the velocity, acceleration, and force vectors (magnitudes and directions) during the motion.

6.07 Identify that unless there is a radially inward net force (a centripetal force), an object cannot move in circular motion.

6.08 For a particle in uniform circular motion, apply the relationship between the radius of the path, the particle's speed and mass, and the net force acting on the particle.

6-3 Uniform Circular Motion

- Recall that circular motion requires a centripetal acceleration

$$a = \frac{v^2}{R}$$ Eq. (6-17)

Examples You are a passenger:

- For a car, rounding a curve, the car accelerates toward the center of the curve due to a **centripetal force** provided by the inward friction on the tires. Your inertia makes you want to go straight ahead so you may feel friction from your seat and may also be pushed against the side of the car. These inward forces keep you in uniform circular motion in the car.
- For a space shuttle, the shuttle is kept in orbit by the gravitational pull of Earth acting as a centripetal force. This force also acts on every atom in your body, and keeps you in orbit around the Earth. You float with no sensation of force, but are subject to a centripetal acceleration.

6-3 Uniform Circular Motion

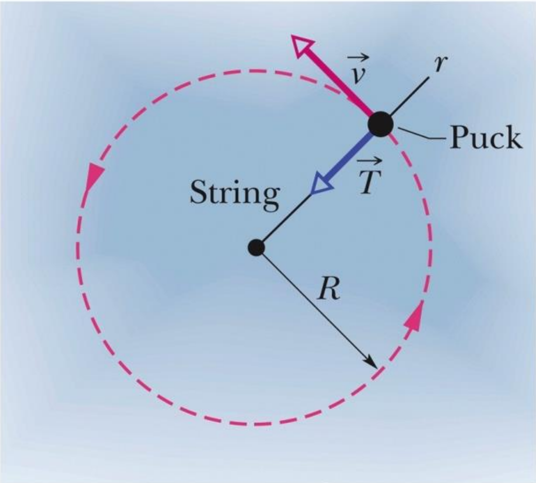
- Centripetal force is not a new *kind* of force, it is simply an application of force

$$\boxed{F = m\frac{v^2}{R}}$$

Eq. (6-18)

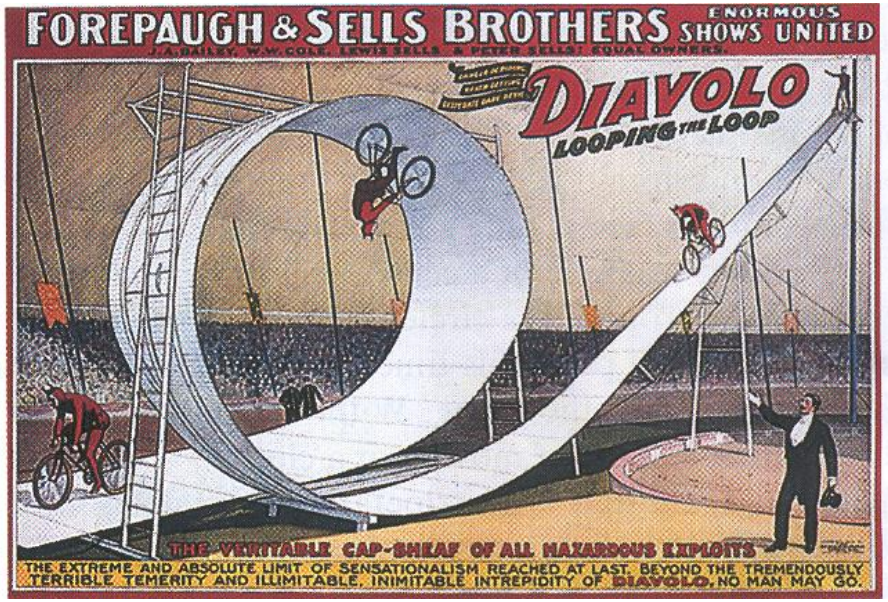
A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

- For the puck on a string, the string tension supplies the centripetal force necessary to maintain circular motion

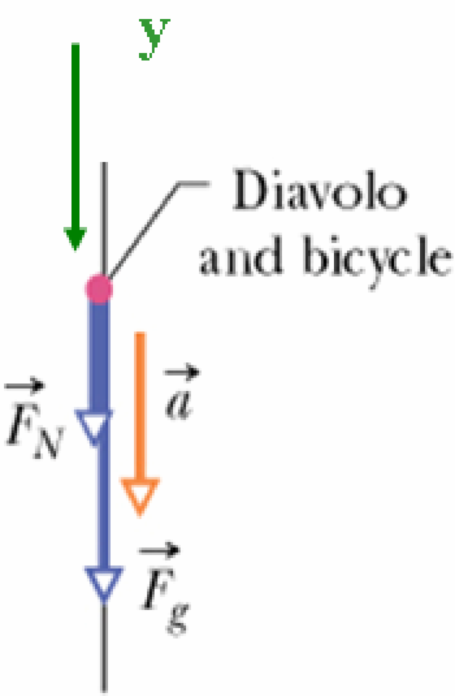


The puck moves in uniform circular motion only because of a toward-the-center force.

Figure 6-8



Sample problem 6.04: In a 1901 circus performance Allo Diavolo introduced the stunt of riding a bicycle in a looping-the-loop. The loop sis a circle of radius *R*. We are asked to calculate the minimum speed *v* that Diavolo should have at the top of the loop and not fall We draw a free body diagram for Diavolo when he is at the top of the loop. Two forces are acting along the y-axis: Gravitational force F_g and the normal reaction F_N from the loop. When Diavolo has the minimum speed *v* he has just lost contact with the loop and thus $F_N = 0$. The only force acting on Diavolo is F_g The gravitational force F_g is the centripetal force.



C

Figure 6-9

Thus: $F_{ynet} = mg = \dfrac{mv_{\min}^2}{R} \rightarrow v_{\min} = \sqrt{Rg}$

$$v = \sqrt{gR} = \sqrt{(9.8\ \text{m/s}^2)(2.7\ \text{m})}$$
$$= 5.1\ \text{m/s}.$$

Sample Problem 6.06:

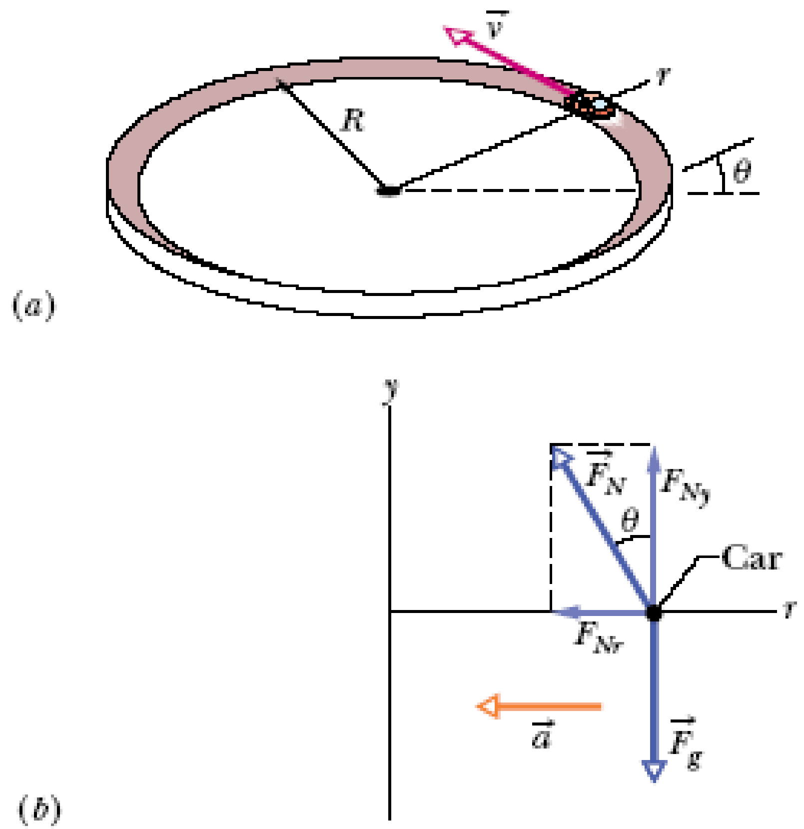


Fig. 6-13 (*a*) A car moves around a curved banked road at constant speed v. The bank angle is exaggerated for clarity. (*b*) A free-body diagram for the car, assuming that friction between tires and road is zero and that the car lacks negative lift. The radially inward component F_{Nr} of the normal force (along radial axis r) provides the necessary centripetal force and radial acceleration.

6-3 Uniform Circular Motion

Example Car in a banked circular turn:

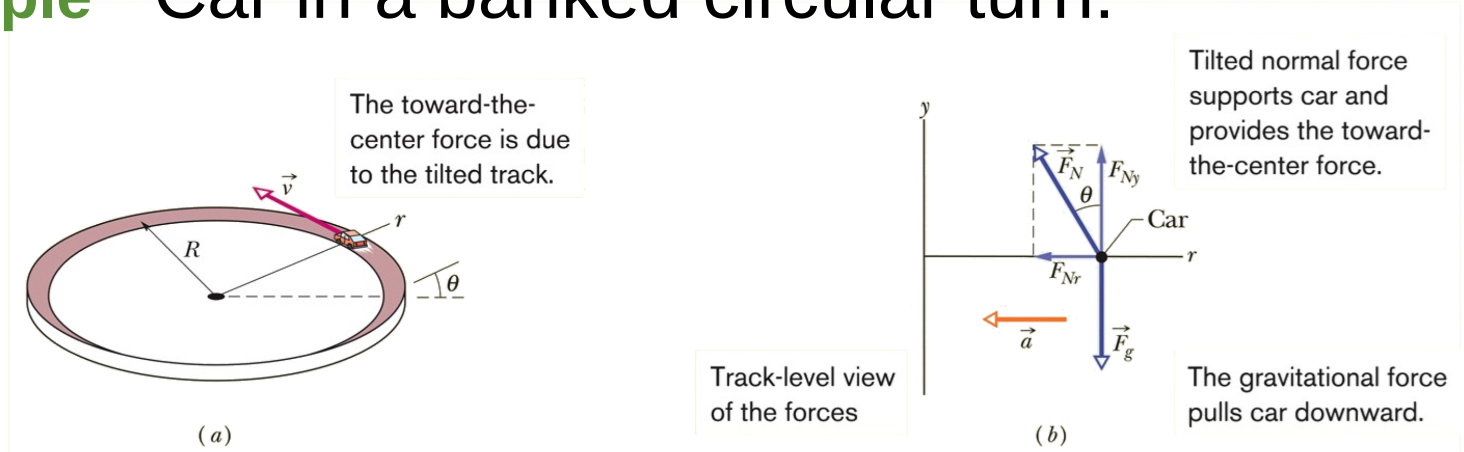


Figure 6-11

- Sum components along the radial direction:

$$-F_N \sin\theta = m\left(-\frac{v^2}{R}\right). \quad \text{Eq. (6-23)}$$

- Sum components along the vertical direction:

$$F_N \cos\theta = mg. \quad \text{Eq. (6-24)}$$

- Divide and replace (sin *θ*)/(cos *θ*) with tangent

$$\theta = \tan^{-1}\frac{v^2}{gR}$$

- ~ The angle prevents sliding!.

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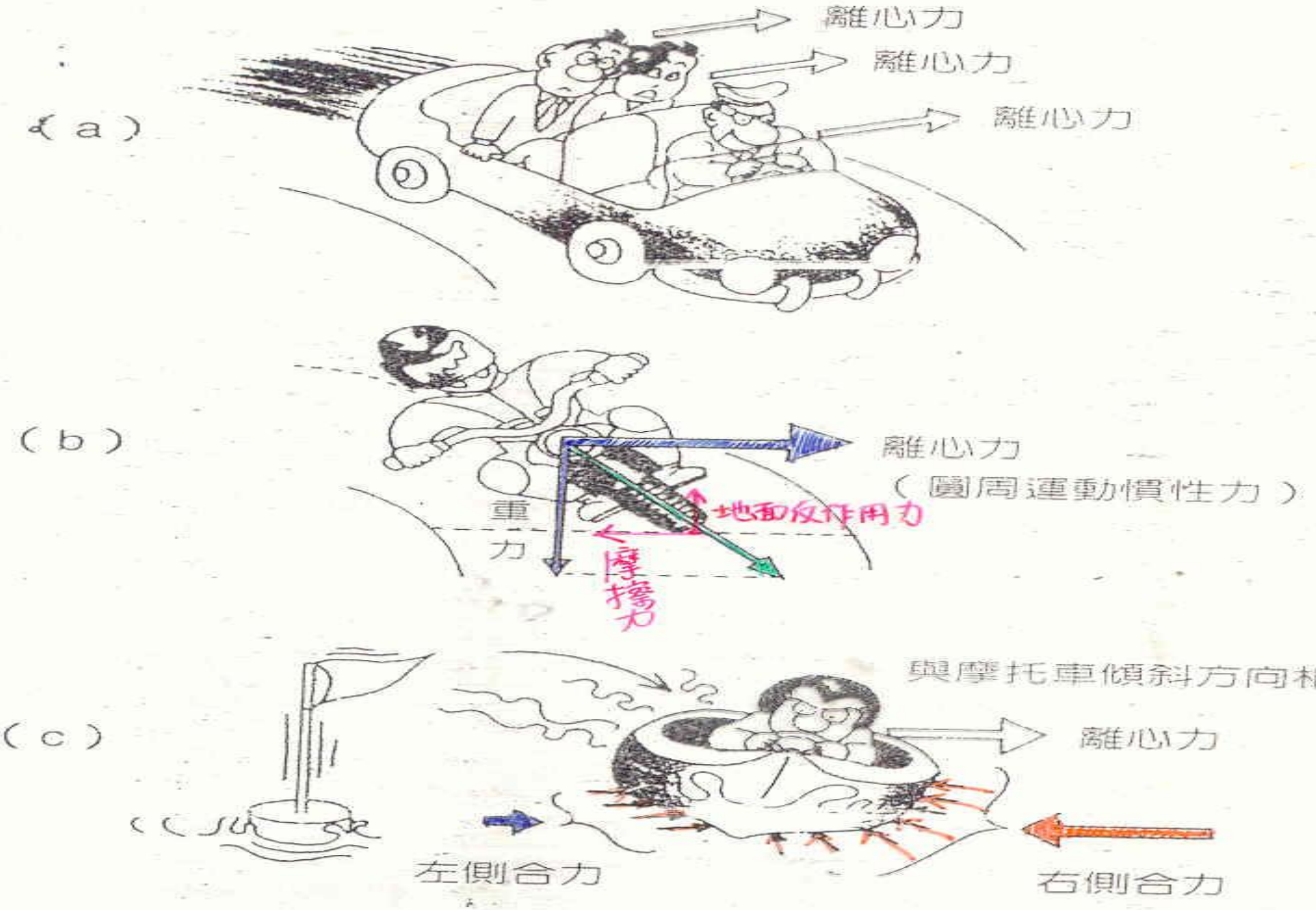
拐彎時受指向外側的力

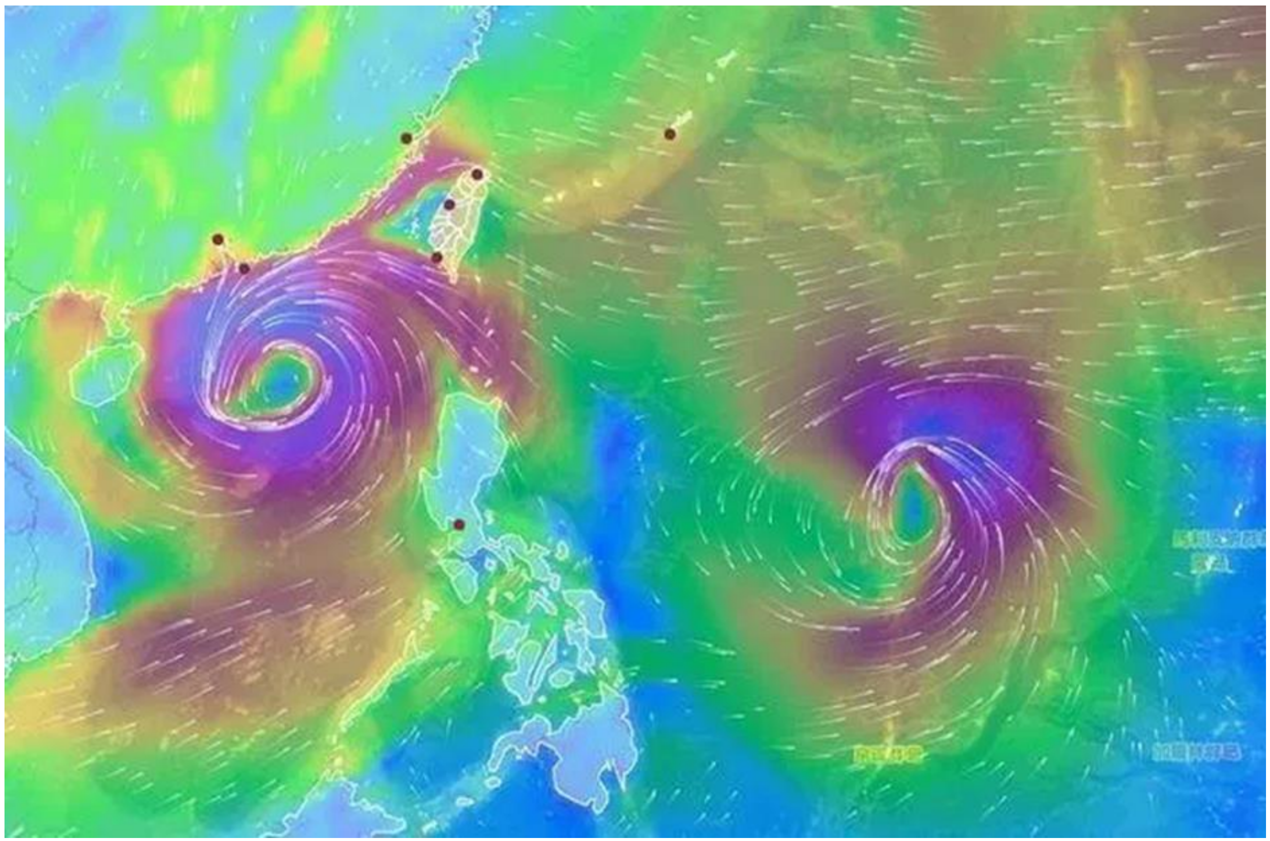


★ 作用於物體接觸面間，阻礙

$$向心力 = \frac{mv^2}{r}$$ ，

m: 質量
v: 速度
r: 迴轉半徑

氣象局風場預報顯示圖最新模擬顯示，2021國慶日當晚熱帶系統動向。

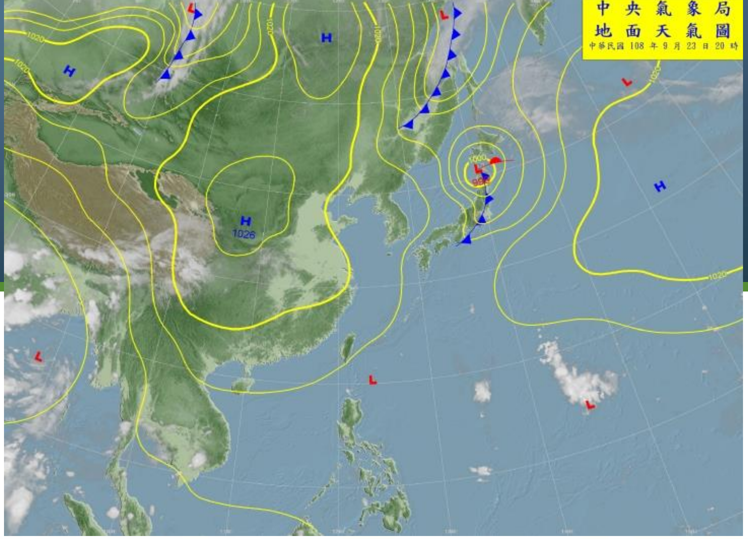


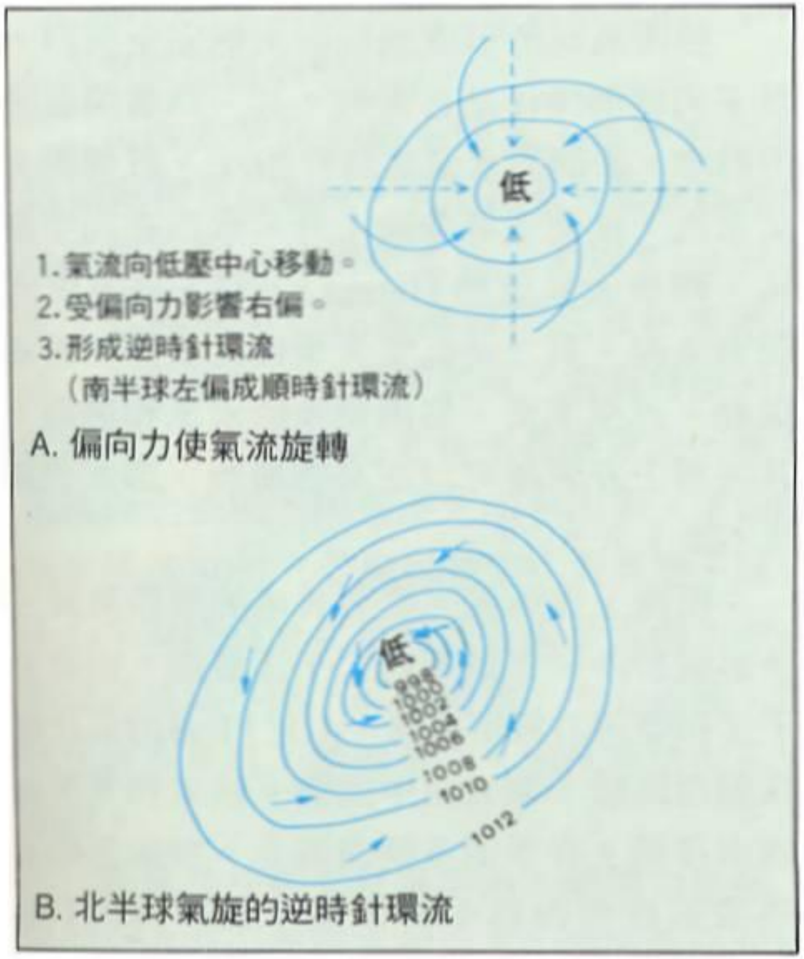
More applications:

氣旋

氣旋（cyclone），是指圍繞著一個強烈低壓中心旋轉的大氣系統。由於內外氣壓差異大，四周的空氣急促流向低壓中心；因地球自轉偏向力，在北半球形成逆時針方向的環流，南半球則為順時針方向。氣旋通常會帶來不穩定、甚至惡劣的天氣。較大規模和風速較高的有熱帶氣旋和溫帶氣旋。

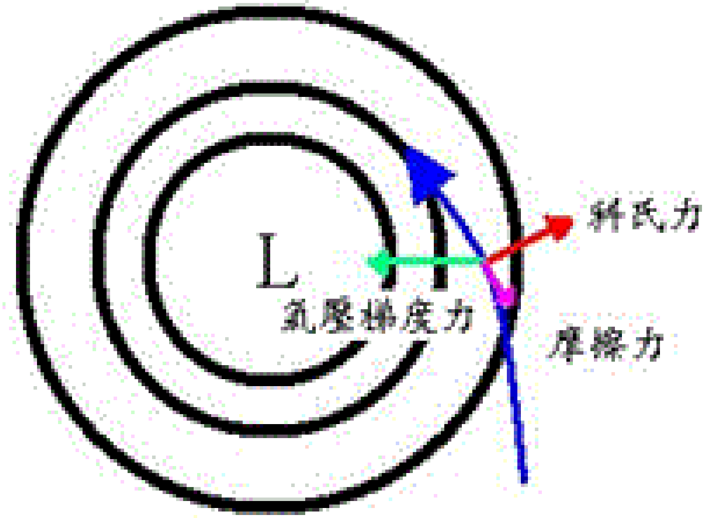
http://ihouse.hkedcity.net/~hm1203/atmosphere/wind-others.htm





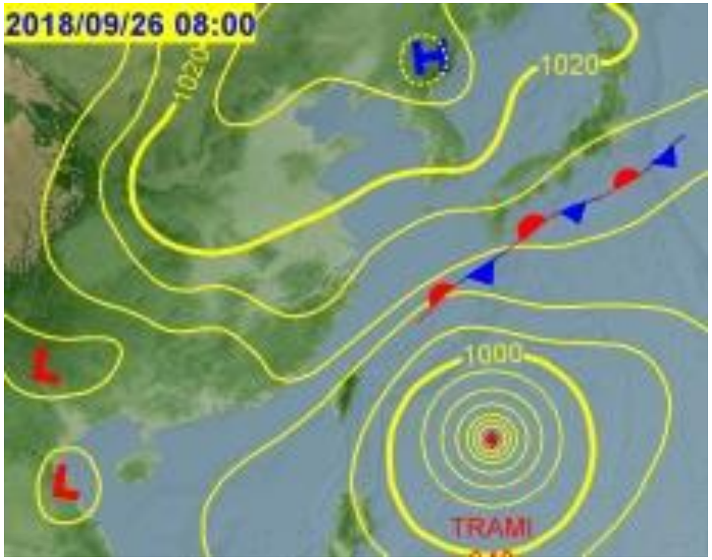
More applications:

颱風中心附近氣流，可以下圖示意，假設北半球有一等壓線為正圓的低壓系統，近中心的等壓線較外圍密集。低壓系統空氣塊的運動，受到氣壓梯度力、科氏力、摩擦力的影響，最後會使空氣塊以逆時針逐漸往中心方向靠近。為什麼外圍的空氣塊徑向運動速率會比近中心快？事實上，空氣塊在做氣旋式運動的時候，有一部份的氣壓梯度力是做為圓周運動之向心力。隨著空氣逐漸向內部運動的時候，其旋轉半徑越小，維持氣旋式旋轉所需要的向心力也就越大。因此，越往中心，向中心方向運動速率越小。那為什麼颱風眼氣壓最低，反而沒有雲？那是因為越往颱風中心，等壓線越密集而風速越大，為能維持氣旋式旋轉所需要的向心力也越大，這導致空氣最後反而不容易進入颱風中心，使得颱風中心區域雖然氣壓最低，其輻合作用反而最弱，甚至在颱風眼附近有微弱的下沉氣流。
http://highscope.ch.ntu.edu.tw/wordpress/?p=18097



~氣壓梯度力提供向心力?
~check this issue!?

Refer:民國107年09月26日14時， 中度颱風 潭美 (編號第 24 號，國際命名 TRAMI)，中心位置位於北緯 21.00 度、東經 129.10 度，向北北西緩慢進行。颱風中心氣壓 945 百帕，近中心最大風速每秒 43 公尺，瞬間之最大陣風每秒 53 公尺，七級風半徑 250 公里，十級風半徑 100 公里。



6 Summary

Friction

- Opposes the direction of motion or attempted motion
- Static if the object does not slide
- Static friction can increase to a maximum

$$f_{s,\max} = \mu_s F_N, \quad \textbf{Eq. (6-1)}$$

- Kinetic if it does slide

$$f_k = \mu_k F_N, \quad \textbf{Eq. (6-2)}$$

Drag Force

- Resistance between a fluid and an object
- Opposes relative motion
- Drag coefficient *C* experimentally determined

$$D = \tfrac{1}{2} C \rho A v^2, \quad \textbf{Eq. (6-14)}$$

- Use the effective cross-sectional area (area perpendicular to the velocity)

6 Summary

Terminal Speed

- The maximum velocity of a falling object due to drag

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}.$$

Eq. (6-16)

Uniform Circular Motion

- Centripetal acceleration required to maintain the motion

$$a = \frac{v^2}{R}$$

Eq. (6-17)

- Corresponds to a centripetal force

$$F = m\frac{v^2}{R}$$

Eq. (6-18)

- Force points toward the center of curvature

CH 6 習題:

35, 36, 48, 57, 59, and 62

記取教訓!

2018年10/21 發生普悠瑪翻車、釀18死的意外….

~普悠瑪翻覆事實報告出爐 出軌前一秒司機並無煞車… (2019/04)
https://udn.com/news/story/7934/4507382