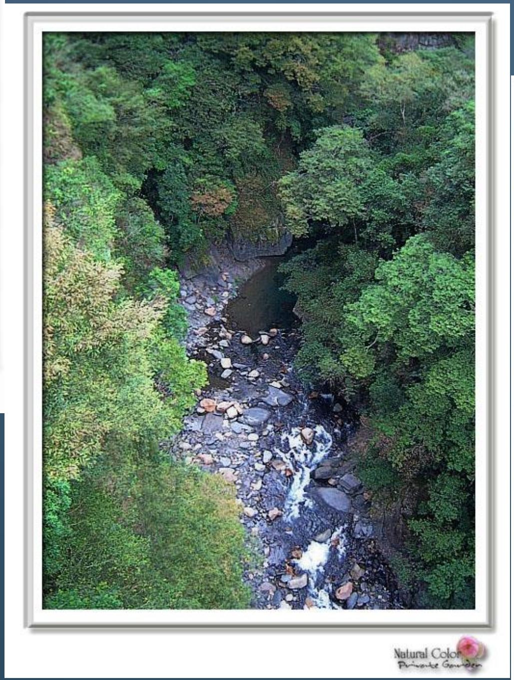
Chapter 8

Potential Energy and Conservation of Energy



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8-1 Potential Energy

Learning Objectives

8.01 Distinguish a conservative force from a nonconservative force.

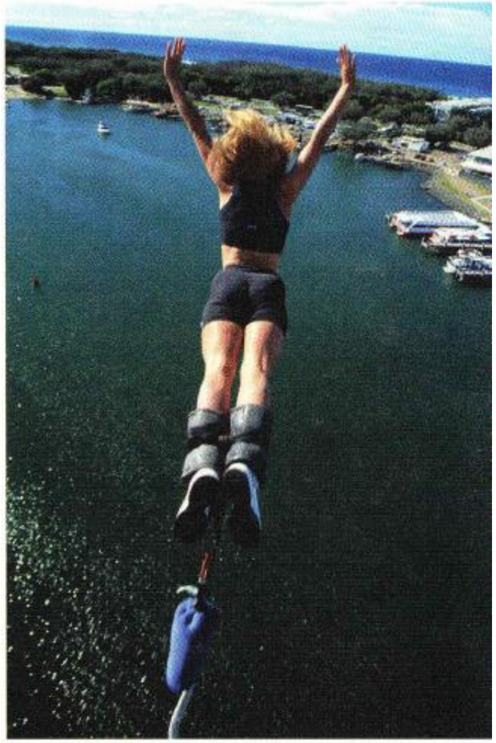
8.02 For a particle moving between two points, identify that the work done by a conservative force does not depend on which path the particle takes.

8.03 Calculate the gravitational potential energy of a particle (or, more properly, a particle-Earth system).

8.04 Calculate the elastic potential energy of a block-spring system.

8-1 Potential Energy

- **Potential energy** *U* is energy that can be associated with the configuration of a system of objects that exert forces on one another
- A system of objects may be:
  - Earth and a bungee jumper**(高空彈跳)**
  - **Gravitational potential energy** accounts for kinetic energy increase during the fall
  - **Elastic potential energy** accounts for deceleration by the bungee cord
- Physics determines how potential energy is calculated, to account for stored energy



8-1 Potential Energy

- For an object being raised or lowered:

$$\boxed{\Delta U = -W.} \qquad \text{Eq. (8-1)}$$

- The change in gravitational potential energy is the negative of the work done
- This also applies to an elastic block-spring system

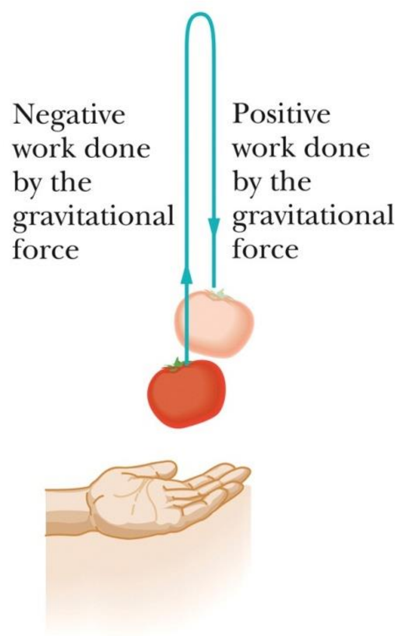


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Figure 8-2

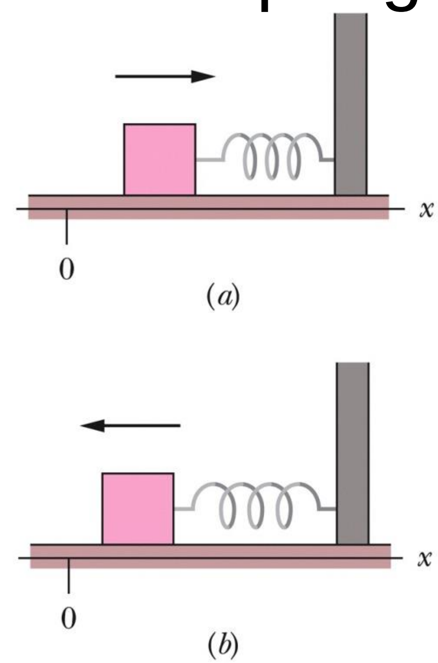

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Figure 8-3

8-1 Potential Energy

- Key points:
  1. The *system* consists of two or more objects
  2. A *force* acts between a particle (tomato/block) and the rest of the system
  3. When the configuration changes, the force does *work* $W_1 = -mgh$ (重力做負功) changing kinetic energy to another form
  4. When the configuration change is reversed, the force reverses the energy transfer , doing work $W_2 = mgh$ (重力儲能→做功)
- Thus the kinetic energy of the tomato/block becomes potential energy, and then kinetic energy again

$$\boxed{\Delta U = -W}$$

8-1 Potential Energy

Conservative forces are forces for which $W_1 = -W_2$ is always true **($W_1 = -W_2$恆成立: 守恆力)**

- Examples: gravitational force, spring force
- Otherwise we could ***not*** speak of their potential energies

- **Nonconservative forces** are those for which it is false
  - Examples: kinetic friction force, drag force
  - Kinetic energy of a moving particle is transferred to heat by friction
  - Thermal energy cannot be recovered back into kinetic energy of the object via the friction force
  - Therefore the force is not conservative, thermal energy is not a potential energy ~摩擦力、拖曳力屬之。例如: 動能→摩擦力→熱→(x) 動能。

8-1 Potential Energy

- When only conservative forces act on a particle, we find many problems can be simplified:

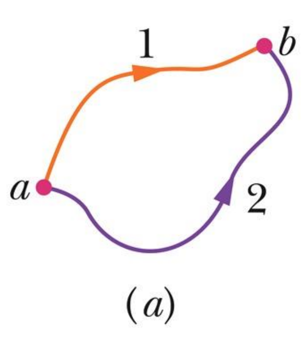
> The net work done by a conservative force on a particle moving around any closed path is zero.

守恆力作功於一封閉路徑---作功 = 0!

- A result of this is that:

> The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

守恆力作功---與路徑無關!



The force is conservative. Any choice of path between the points gives the same amount of work.

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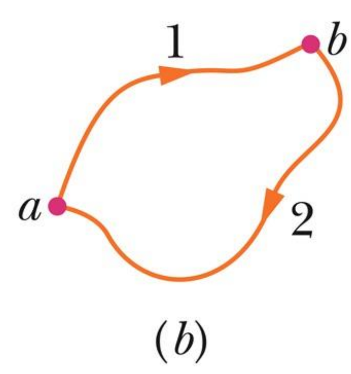

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And a round trip gives a total work of zero.

Figure 8-4

8-1 Potential Energy

- For the general case, we calculate work as:

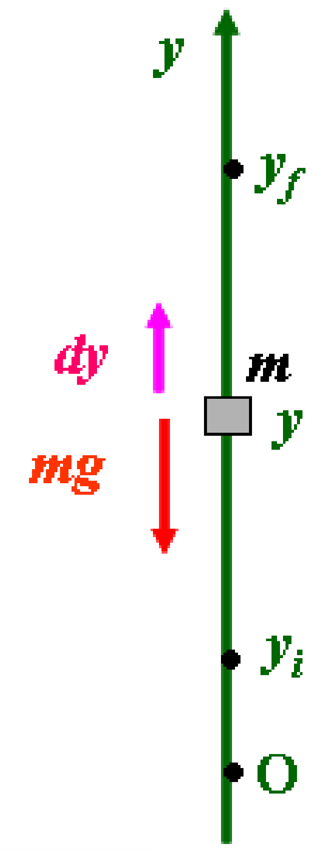
$$W = \int_{x_i}^{x_f} F(x)\, dx. \qquad \textbf{Eq. (8-5)}$$

- So we calculate potential energy as:
- $$\boxed{\Delta U = -\int_{x_i}^{x_f} F(x)dx} \qquad \textbf{Eq. (8-6)}$$

- Using this to calculate gravitational PE, relative to a **reference configuration** with **reference point** $y_i = 0$
- :

$$\boxed{U(y) = mgy} \qquad \textbf{Eq. (8-9)}$$

- : 1樓 →10 樓: ΔU >0; 而:$\boldsymbol{F_g}(y)$向下，$\boldsymbol{dy}$向上➔$\boldsymbol{F_g}(y)\boldsymbol{dy}$ <0 ➔ – $\boldsymbol{F_g}(y)\boldsymbol{dy}$ > **0!**



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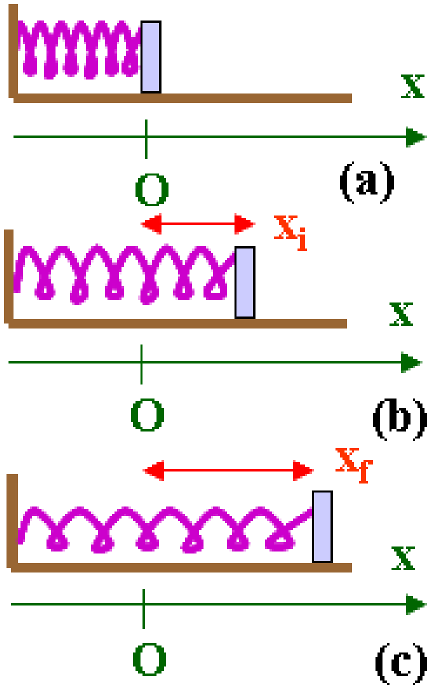
8-1 Potential Energy

- Use the same process to calculate spring PE:

- $$W = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} -kx dx = -k \int_{x_i}^{x_f} x dx \qquad \Delta U = -W \rightarrow$$



- $$\boxed{\Delta U = k\left[\frac{x^2}{2}\right]_{x_i}^{x_f} = \frac{kx_f^2}{2} - \frac{kx_i^2}{2}}$$ **Eq. (8-10)**

- With reference point $x_i = 0$ for a relaxed spring:

$$\boxed{U = \frac{kx^2}{2}}$$ **Eq. (8-11)**

8-2 Conservation of Mechanical Energy(力學能守恆!)

Learning Objectives

8.05 After first clearly defining which objects form a system, identify that the mechanical energy of the system is the sum of the kinetic energies and potential energies of those objects.

8.06 For an isolated system in which only conservative forces act, apply the conservation of mechanical energy to relate the initial potential and kinetic energies to the potential and kinetic energies at a later instant.

$$\boxed{\Delta E_{mech} = \Delta K + \Delta U = 0}$$

8-2 Conservation of Mechanical Energy

- The mechanical energy of a system is the sum of its potential energy *U* and kinetic energy *K*:力學能 =動能+位能

$$E_{mec} = K + U \quad \text{Eq. (8-12)}$$

- Work done by conservative forces increases *K* and decreases *U* by that amount, so:

$$\Delta K = -\Delta U. \quad \text{Eq. (8-15)}$$

- Using subscripts to refer to different instants of time:

$$K_2 + U_2 = K_1 + U_1 \quad \text{Eq. (8-17)}$$

- In other words:系統之力學能維持一定值!

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

8-2 Conservation of Mechanical Energy

- This is the principle of the **conservation of mechanical energy**:

$$\boxed{\Delta E_{\text{mec}} = \Delta K + \Delta U = 0.}$$ Eq. (8-18)

- This is very powerful tool:

When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion* and *without finding the work done by the forces involved.*

- One application:
  - Choose the lowest point in the system as $U = 0$
  - Then at the highest point U = max, and K = min

For an isolated system in which the forces are a mixture of conservative and non conservative forces the principle takes the following form

$$\boxed{\Delta E_{mech} = W_{nc}}$$

Here, W_{nc} is defined as the work of all the non-consrvative forces of the system

對一孤立系統:

力學能之變化量=非守恆力對系統所作之功(一般為負功):

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8-3 Reading a Potential Energy Curve

Learning Objectives

8.07 Given a particle's potential energy as a function of position *x*, determine the force on the particle.

8.08 Given a graph of potential energy versus *x*, determine the force on a particle.

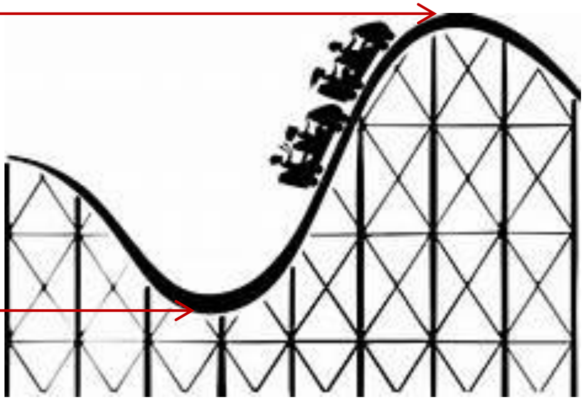
8.09 On a graph of potential energy versus *x*, superimpose a line for a particle's mechanical energy and determine kinetic energy for any given value of *x*.

8.10 If a particle moves along an x axis, use a potential-energy graph for that axis and the conservation of mechanical energy to relate the energy values at one position to those at another position.

8.11 On a potential-energy graph, identify any turning points and any regions where the particle is not allowed because of energy requirements.

8.12 Explain neutral equilibrium, stable equilibrium, and unstable equilibrium.

$F_x = 0$



8-3 Reading a Potential Energy Curve

- For one dimension, force and potential energy are related (by work) as:

$$F(x) = -\frac{dU(x)}{dx}$$ Eq. (8-22)

- Therefore we can find the force $F(x)$ from a plot of the potential energy $U(x)$, by taking the derivative (slope)
- If we write the mechanical energy out:

$$U(x) + K(x) = E_{\text{mec}}.$$ Eq. (8-23)

- We see how $K(x)$ varies with $U(x)$:

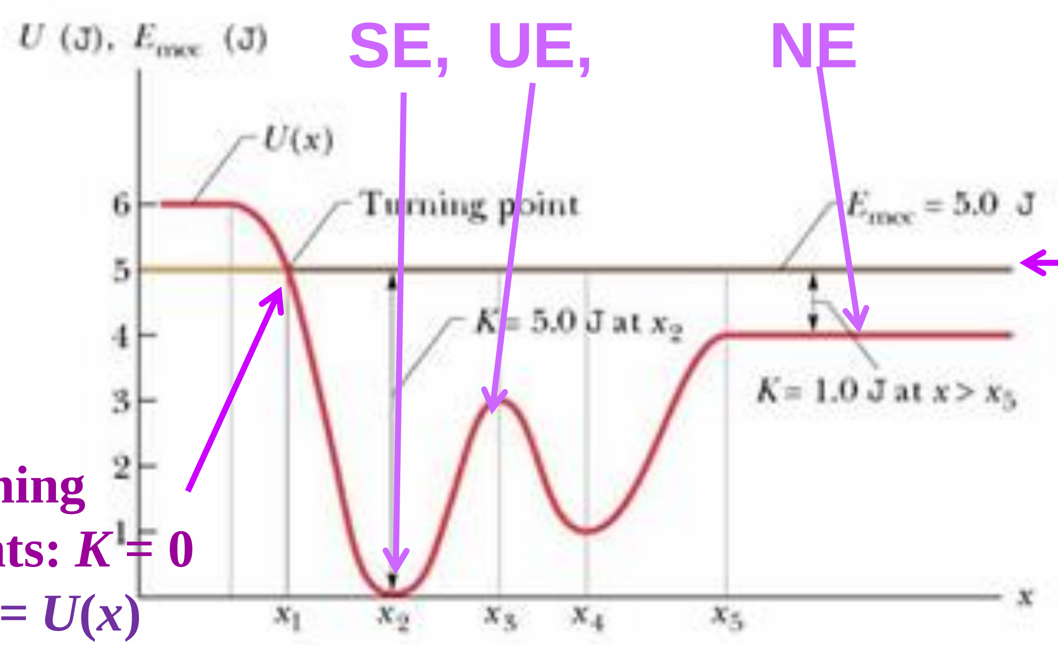
$$K(x) = E_{\text{mec}} - U(x).$$ Eq. (8-24)

8-3 Reading a Potential Energy Curve

- To find $K(x)$ at any place, take the total mechanical energy (constant) and subtract $U(x)$
- Places where $K = 0$ are **turning points**
  - There, the particle changes direction (K cannot be negative)
- At equilibrium points, the slope of $U(x)$ is 0
- A particle in **neutral equilibrium** [自然(永久)平衡]. (NE) is stationary, with potential energy only, net force = 0
  - If displaced to one side slightly, it would remain in its new position
  - Example: a marble on a flat tabletop(平面桌面上的大理石)

8-3 Reading a Potential Energy Curve

- A particle in **unstable equilibrium** (UE) is stationary, with potential energy only, and net force = 0
  - If displaced slightly to one direction, it will feel a force propelling it in that direction
  - Example: a marble balanced on a bowling ball
- A particle in **stable equilibrium** (SE) is stationary, with potential energy only, and net force = 0
  - If displaced to one side slightly, it will feel a force returning it to its original position
  - Example: a marble placed at the bottom of a bowl



Turning Points: $K = 0$

$E_{mec} = U(x)$

Figure 8-9

The total mechanical energy

$E_{mec}=K(x)+U(x)$

From the definition of $K = \dfrac{mv^2}{2}$ the kinetic energy cannot be negative.

This property of K allows us to determine which regions of the x-axis motion is allowed. $K(x) = K(x) = E_{mec} - U(x)$

If $K > 0 \rightarrow E_{mech} - U(x) > 0 \rightarrow U(x) < E_{mec}$ Motion is allowed!

If $K < 0 \rightarrow E_{mech} - U(x) < 0 \rightarrow U(x) > E_{mec}$ Motion is forbidden!

The points at which: $E_{mec} = U(x)$ are known as turning points for the motion. For example x_1 is the turning point for the U versus x plot above. At the turning point $K = 0$ (折返點)

8-4 Work Done on a System by an External Force

Learning Objectives

8.13 When work is done on a system by an external force with no friction involved, determine the changes in kinetic energy and potential energy.

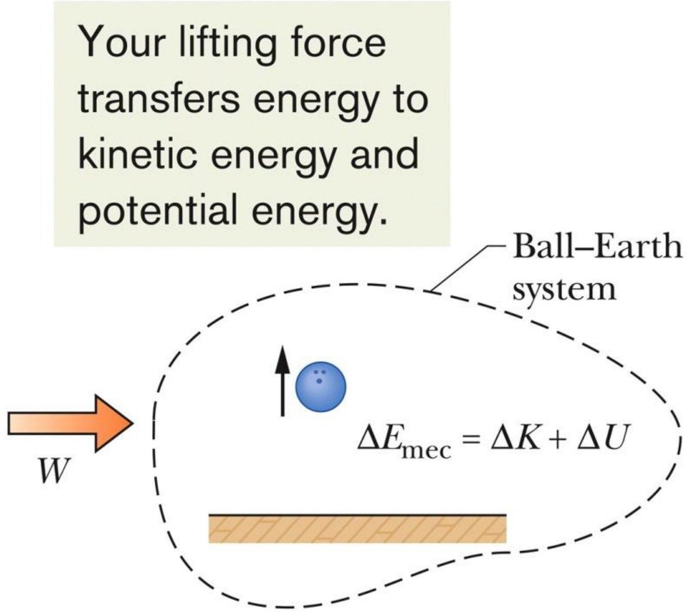
8.14 When work is done on a system by an external force with friction involved, relate that work to the changes in kinetic energy, potential energy, and thermal energy.

8-4 Work Done on a System by an External Force

- We can extend work on an object to work on a system:

> Work is energy transferred to or from a system by means of an external force acting on that system.

- For a system of more than 1 particle, work can change both *K* and *U*, or other forms of energy of the system
- For a frictionless system:



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$$W = \Delta K + \Delta U, \quad \textbf{Eq. (8-25)}$$

$$\boxed{W = \Delta E_{mec}} \quad \textbf{Eq. (8-26)}$$

Figure 8-12

8-4 Work Done on a System by an External Force

- For a system with friction:

$$\Delta E_{th} = f_k d \quad \text{(increase in thermal energy by sliding).}$$ Eq. (8-31)

$$\boxed{W_{ext} = \Delta E_{mec} + \Delta E_{th}}$$ Eq. (8-33)

- The thermal energy comes from the forming and breaking of the welds between the sliding surfaces

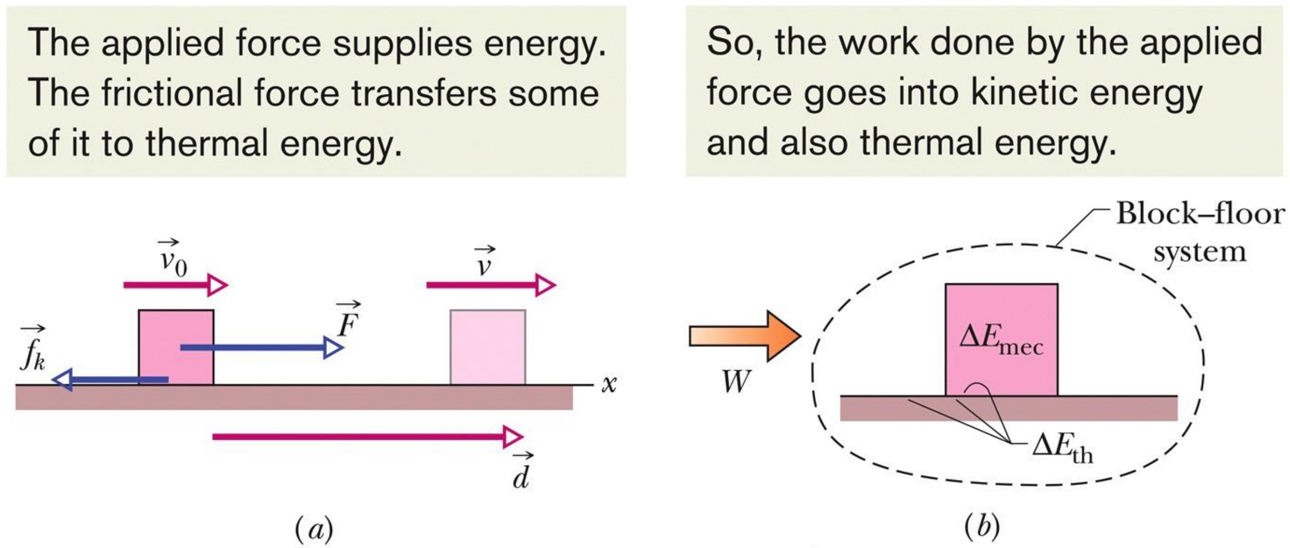

Figure 8-13

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8-5 Conservation of Energy

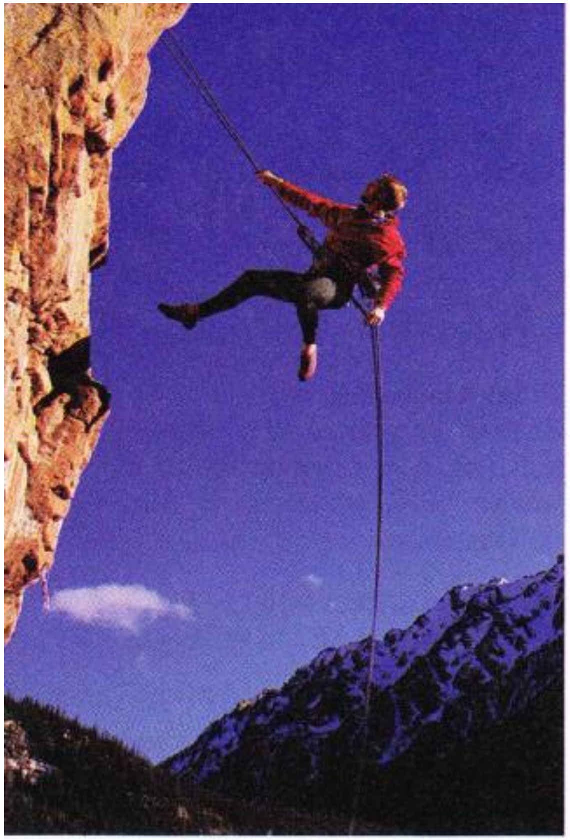
Learning Objectives

8.15 For an isolated system (no net external force), apply the conservation of energy to relate the initial total energy (energies of all kinds) to the total energy at a later instant.

8.16 For a nonisolated system, relate the work done on the system by a net external force to the changes in the various types of energies within the system.

8.17 Apply the relationship between average power, the associated energy transfer, and the time interval in which that transfer is made.

8.18 Given an energy transfer as a function of time (either as an equation or graph), determine the instantaneous power (the transfer at any given instant).



8-5 Conservation of Energy

- Energy transferred between systems can always be accounted for
- The **law of conservation of energy** concerns
  - The **total energy** *E* of a system
  - Which includes mechanical, thermal, and other internal energy

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

- Considering only energy transfer through work:

$$\boxed{W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}},}$$ **Eq. (8-35)**

8-5 Conservation of Energy

- An isolated system is one for which there can be ***no*** *external* energy transfer

The total energy E of an isolated system cannot change.

- Energy transfers may happen internal to the system
- We can write: $$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0$$ **Eq. (8-36)**

- Or, for two instants of time:

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}.$$ **Eq. (8-37)**

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times.*

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能量守恆:
~系統總能E只在有能量轉移出去或轉移進來(外力做功)才會改變!

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}, \quad (8\text{-}35)$$

ΔE_{mec}:力學能變化；ΔE_{th}:系統熱能變化；ΔE_{int}:系統內能(e.g., 化學能) 變化；

孤立系統:
~ 總能量E是不變的!

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0 \quad \text{(isolated system).} \quad (8\text{-}36)$$

8-5 Conservation of Energy

- External forces can act on a system without doing work:

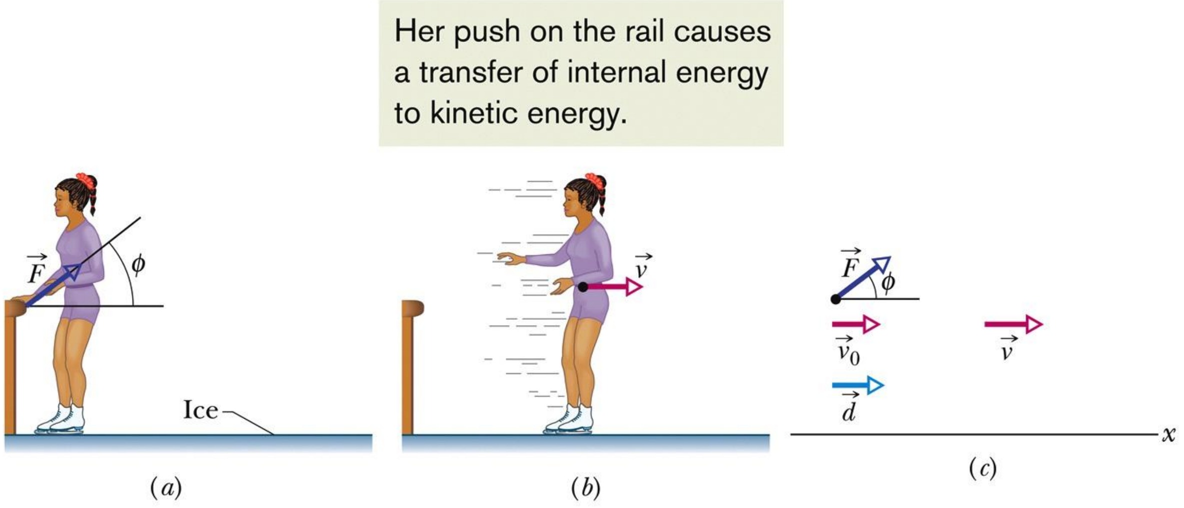

Figure 8-15

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- The skater pushes herself away from the wall
- She turns internal chemical energy in her muscles into kinetic energy
- Her *K* change is caused by the force from the wall, but the wall does *not* provide her any energy

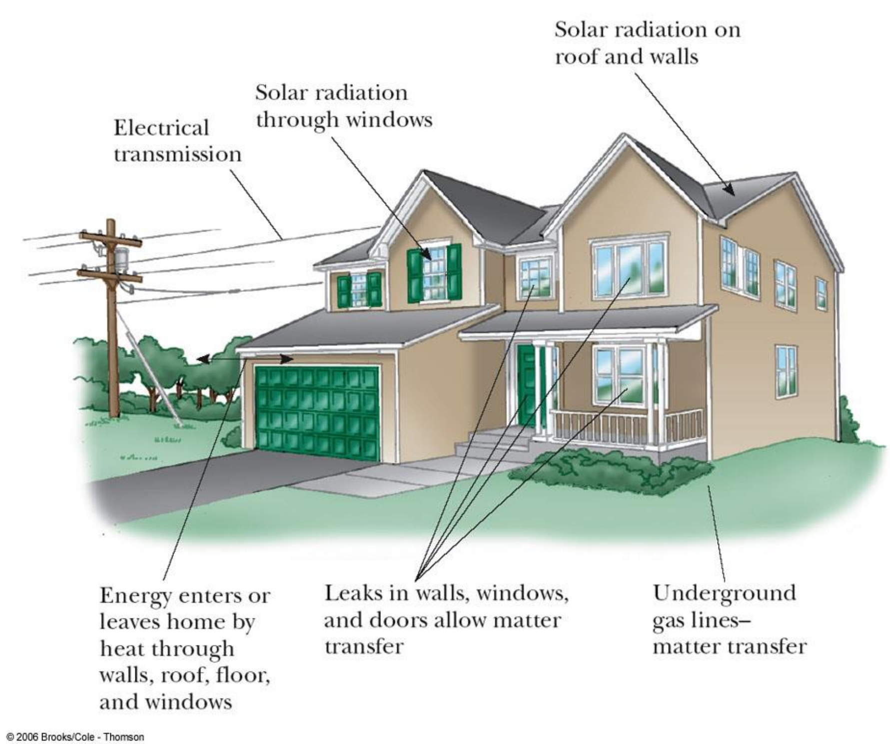
8-5 Conservation of Energy

- We can expand the definition of power
- In general, power is the rate at which energy is transferred by a force from one type to another
- If energy ΔE is transferred in time Δt, the **average power** is:

$$P_{avg} = \frac{\Delta E}{\Delta t}. \quad \text{Eq. (8-40)}$$

- And the **instantaneous power** is:

$$P = \frac{dE}{dt}. \quad \text{Eq. (8-41)}$$



Nonisolated System in Steady State, House Example

8 Summary

Conservative Forces

- Net work on a particle over a closed path is 0

Potential Energy

- Energy associated with the configuration of a system and a conservative force

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx.$$ **Eq. (8-6)**

Gravitational Potential Energy

- Energy associated with Earth + a nearby particle

$$U(y) = mgy$$ **Eq. (8-9)**

Elastic Potential Energy

- Energy associated with compression or extension of a spring

$$U(x) = \tfrac{1}{2}kx^2$$ **Eq. (8-11)**

8 Summary

Mechanical Energy

$$E_{\text{mec}} = K + U$$ **Eq. (8-12)**

- For only conservative forces within an isolated system, mechanical energy is conserved

Potential Energy Curves

$$F(x) = -\frac{dU(x)}{dx}$$ **Eq. (8-22)**

- At turning points a particle reverses direction
- At equilibrium, slope of $U(x)$ is 0

Work Done on a System by an External Force

- Without/with friction:

$$W = \Delta E_{\text{mec}}$$ **Eq. (8-26)**

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$$ **Eq. (8-33)**

Conservation of Energy

- The **total energy** can change only by amounts transferred in or out of the system

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}},$$

Eq. (8-35)

8 Summary

Power

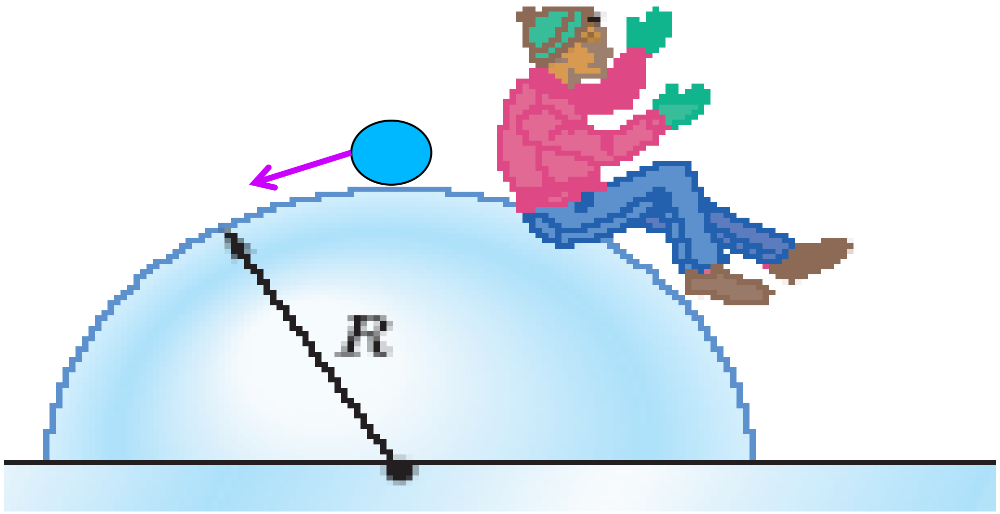
- The rate at which a force transfers energy
- **Average power**:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}.$$ **Eq. (8-40)**

- **Instantaneous power**:

$$P = \frac{dE}{dt}.$$ **Eq. (8-41)**

P75 A boy is initially seated on the top of a hemispherical ice mound of radius $R = 13.8$ m. He begins to slide down the ice, with a negligible initial speed (Fig. 8-47). Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?



ex. 撐竿跳高度之極限?

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CH 8 習題:

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