

# 111-2 Numerical Analysis

## Homework 3

Due Time: 22:00 , Tuesday, 3/14, 2023.

Instructor: Min-Hui Lo

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- **Regulation**

1. **NO PLAGIARISM and NO LATE ASSIGNMENTS.**

- **Submission**

1. Please write down your answers (including discussions and figures) in the same order as the problem sheet in the word/pdf file.
  2. You should upload zip file, including code and pdf (or word) file via NTU COOL.
  3. zip file name: "*hw{hw number}\_g{group id}.zip*" (e.g. *hw01\_g01.zip*)
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1. **Solve the equation** The function  $f(x) = 2\sin(x) - \frac{1}{4}e^x - 1$  is zero for two values near  $x = -5$ . Let the tolerance error be  $TOL = 1 \times 10^{-6}$

1. Use bisection method (initial guess can be  $x = -7$  and  $x = -3$ )
2. Use Newton method (initial guess can be  $x = -7, -5, -3$ )
3. Do they find the same solutions? Otherwise, how much are the differences?
4. How many iterations for those approaches in 1. and 2.?
5. How many iterations will be needed if you change the  $TOL = 1 \times 10^{-10}$  for 1. and 2.?
6. Use the secant method. Start with the two points  $x_1 = 0, x_2 = 1$ , and carry out the first five iterations.

## 2. Applied secant method in meteorology

In some cases, numerical solutions can be easily accessed compared to analytical solutions in meteorology, such as condensation temperature( $T_c$ ).

### **i** Introduction :

About condensation temperature( $T_c$ ): [https://glossary.ametsoc.org/wiki/Condensation\\_temperature](https://glossary.ametsoc.org/wiki/Condensation_temperature)

Given

$$T_c = \frac{B}{\ln \left[ \frac{A\epsilon}{wp_0} \left( \frac{T_0}{T_c} \right)^\kappa \right]}$$

where

- $\epsilon = 0.622$
- $A = 2.53 \times 10^9, B = 5420$
- $\kappa \approx \frac{7}{2}$
- $w$  is mixing ratio (g/g),  $p_0$  is pressure (hPa),  $T_0$  is temperature (K)

1. Write a function Solve\_Tc( $w, p_0, T_0$ ) using Secant method to solve  $T_c$  with given  $w, p_0, T_0$

### **i** Hint

You can let  $f(T_c) = T_c - \frac{B}{\ln \left[ \frac{A\epsilon}{wp_0} \left( \frac{T_0}{T_c} \right)^\kappa \right]}$  and solve  $f(T_c) = 0$  by bisection method

2. Let the tolerance error be  $TOL = 1 \times 10^{-6}$ . When  $w = 10 \text{ g/kg}, P = 1000 \text{ hPa}, T = 300 \text{ K}$ ,  $T_c = ?$
3. Compared with the results using bisection method in HW2, what is the advantage and disadvantage of secant method compared with bisection method?

### **i** Hint

For example: execution time, initial guess or number of iterations...

4. Moreover, is it easy to use Newton method to access  $T_c$ ? Why or Why not?

## 3. Solve a system of nonlinear equations

$$\begin{cases} x^2 + 2x + 2y^2 - 26 = 0 \\ 2x^3 - y^2 + 4y - 19 = 0 \end{cases} \quad \text{Start at } x = 1, y = 1, \text{ and set the tolerance as } 0.0001.$$