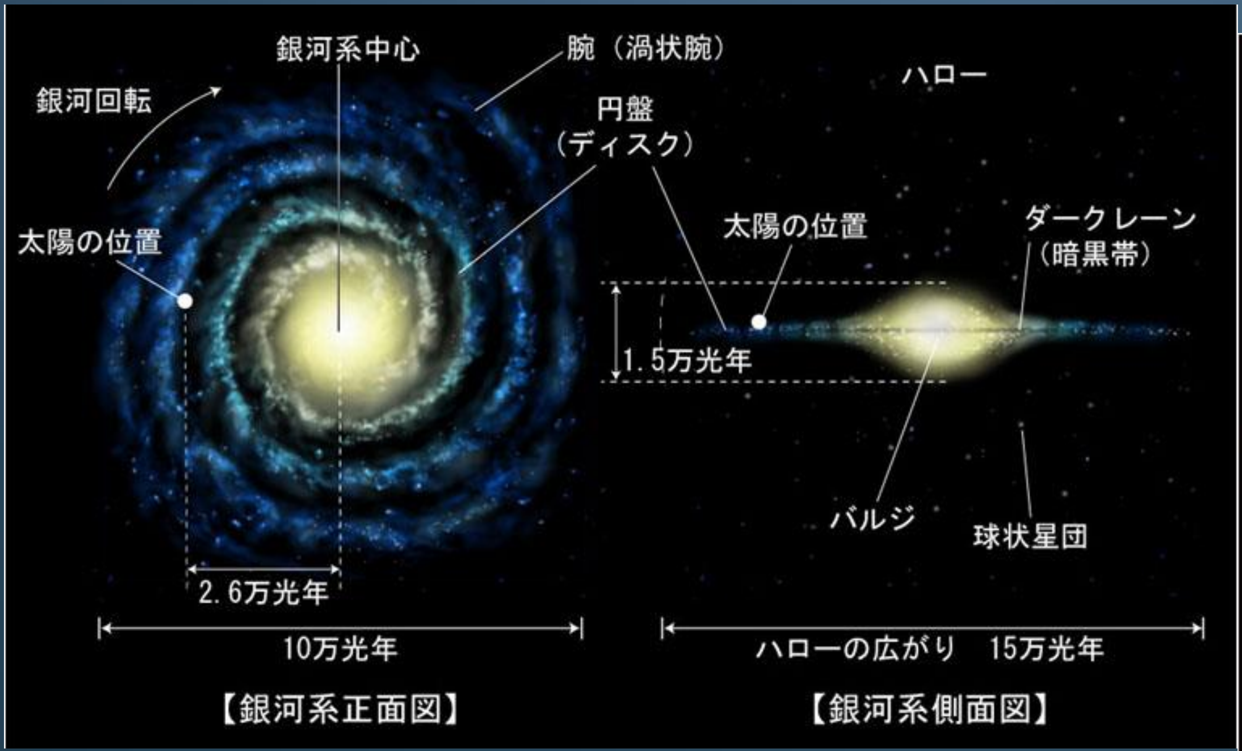
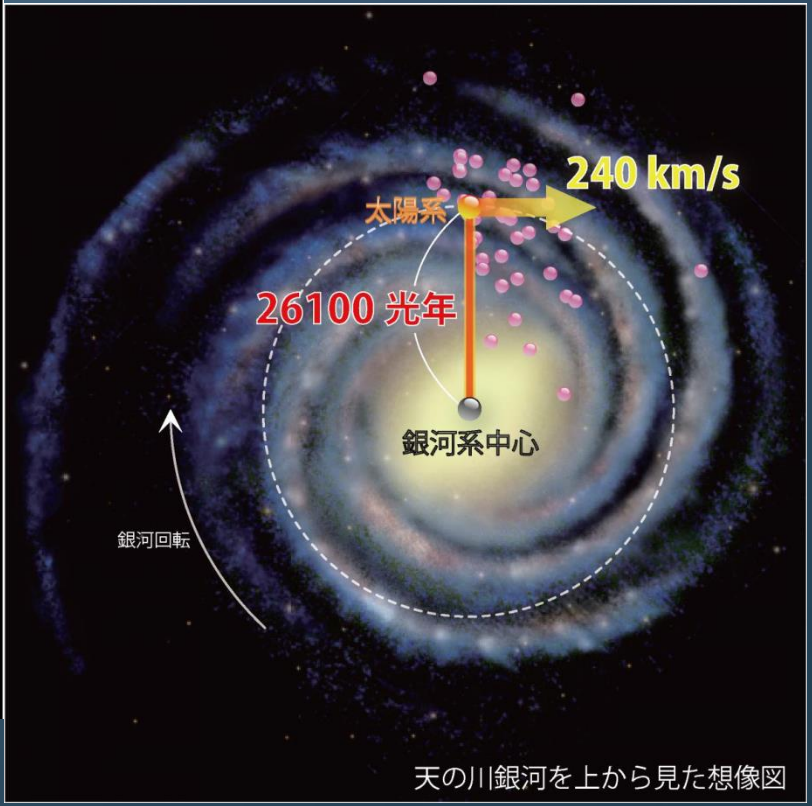
Chapter 13

Gravitation





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13-1 Newton's Law of Gravitation

Learning Objectives

13.01 Apply Newton's law of gravitation to relate the gravitational force between two particles to their masses and their separation.

13.02 Identify that a uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated as a particle at its center.

13.03 Draw a free-body diagram to indicate the gravitational force on a particle due to another particle or a uniform, spherical distribution of matter.

2019年諾貝爾物理學獎》改變人類對宇宙演化認知、發現首顆太陽系外行星 加拿大、瑞士3學者同獲獎

13-1 Newton's Law of Gravitation

- The gravitational force
  - Holds us to the Earth
  - Holds Earth in orbit around the Sun
  - Holds the Sun together with the stars in our galaxy
  - Reaches out across intergalactic space to hold together the Local Group of galaxies
  - Holds together the Local Supercluster of galaxies
  - Pulls the supercluster toward the Great Attractor
  - Attempts to slow the expansion of the universe
  - Is responsible for black holes
- Gravity is far-reaching and very important!

13-1 Newton's Law of Gravitation

- Gravitational attraction depends on the amount of "stuff" an object is made of
- Earth has lots of "stuff" and produces a large attraction
- The force is always attractive, never repulsive
- **Gravitation** **(重力)** is the tendency for bodies to attract each other
- Newton realized this attraction was responsible for maintaining the orbits of celestial bodies
- Newton's **law of gravitation** defines the strength of this attractive force between particles
- For apple & Earth: 0.8 N; for 2 people: $< 1\ \mu N$

Newtonian Gravitation(重力)

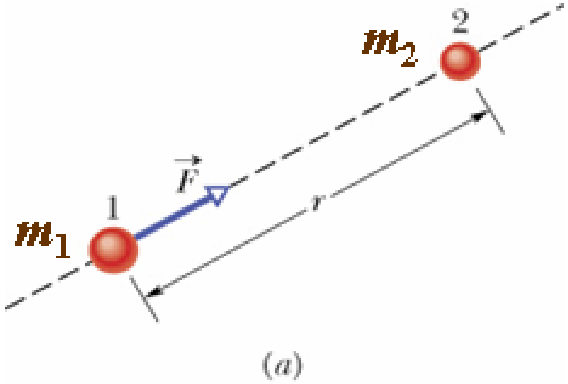
We will explore the following topics:

- Newton’s law of gravitation that describes the attractive force between two point masses and its application to extended objects (萬有引力)
- The acceleration of gravity on the surface of the earth, above it, as well as below it. (重力加速度)

-Gravitational potential energy (重力位能)

- Kepler’s three laws on planetary motion (3個Kepler 行星運動定律)
- Satellites (orbits, energy , escape velocity) (衛星運動)

Figure 13-2



Newton's law of Gravitation

The tendency of objects to move towards each other is known as gravitation:

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

Newton formulated a force law known as Newton's law of gravitation. Every particle attracts any other particle with a gravitational force that has the following characteristics:

1. The force acts along the line that connects the two particles
2. Its magnitude is give by the equation: $F = G\frac{m_1 m_2}{r^2}$

Here m1 and m2 are the masses of the two articles, γ is their separation and *G* is the gravitational constant. Its value is: **$G = 6.67*10^{-11}\ N.m^2/kg^2$**

The gravitational force F_{12} exerted on m1 by m_2 is equal in magnitude to the force F_{21} exerted on m_2 by m1 but opposite in direction. The two forces obey Newton's third law:

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

Newton proved that a uniform shell attracts a particle that is ***outside*** the shell as if the shell's mass were concentrated at the shell center (集中在殼心)

$$F_1 = G\frac{m_1 m_2}{r^2}$$

Note: If the particle is inside the shell, the net force is zero

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

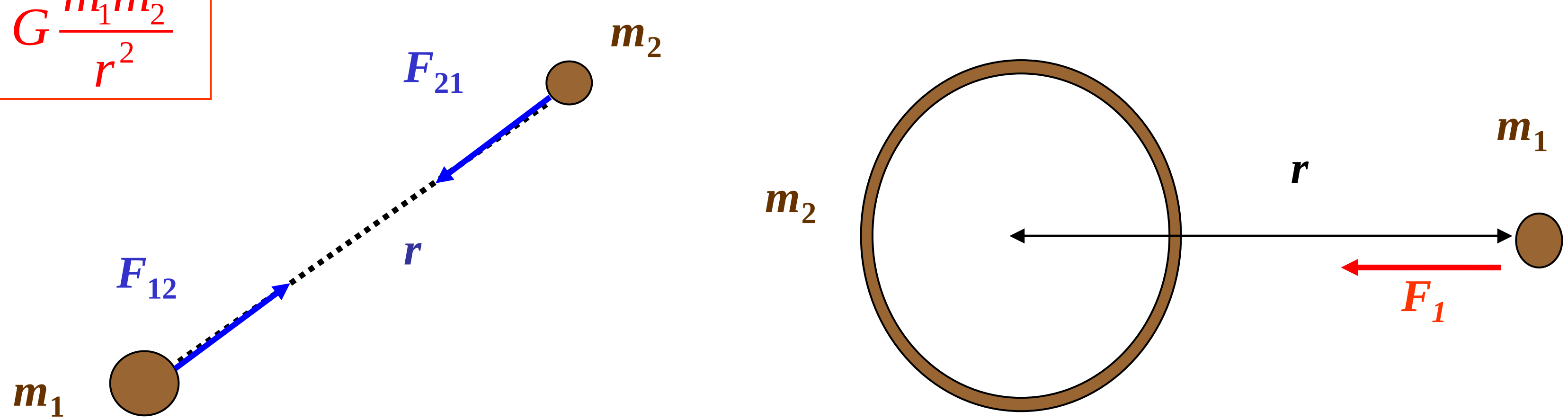
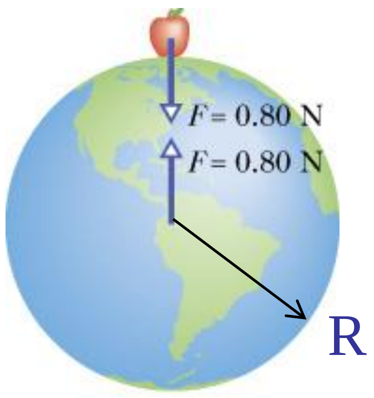
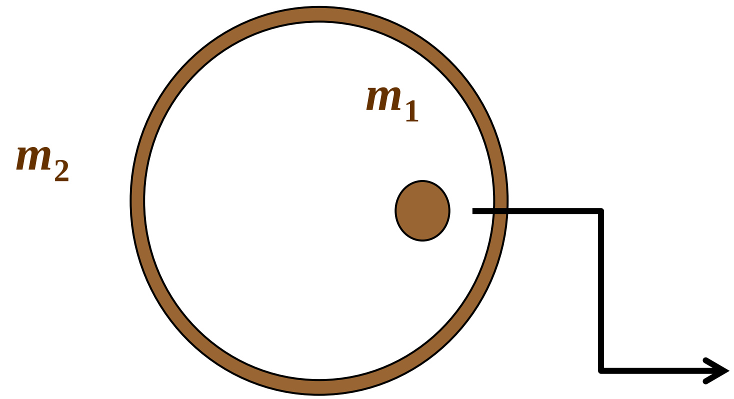


Figure 13-3

Consider the force F the earth (radius **R**, mass **M**) exerts on an apple of mass m. The earth can be thought of as consisting of concentric shells. Thus from the apple's point of view the earth behaves like a point mass at the earth center the magnitude of the force is given by the equation:

$$F = G\frac{mM}{R^2}$$





m_1在殼內:F_{12} = 0 !

13-1 Newton's Law of Gravitation

- The magnitude of the force is given by:

$$\boxed{F = G\frac{m_1 m_2}{r^2} \qquad \text{(Newton's law of gravitation).}}$$ **Eq. (13-1)**

- Where G is the **gravitational constant**:

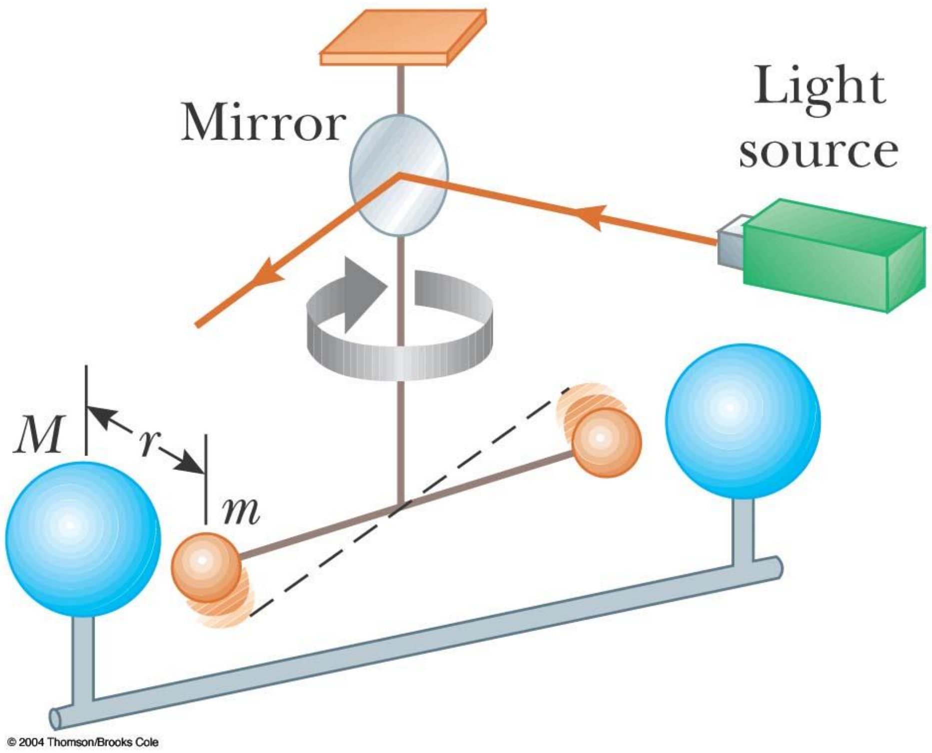
$$\boxed{G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}}$$ **Eq. (13-2)**

- The force always points from one particle to the other, so this equation can be written in vector form:

$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{\mathrm{r}}.$$ **Eq. (13-3)**

Measuring *G*

- *G* was first measured by Henry Cavendish in 1798
- The apparatus shown here allowed the attractive force between two spheres to cause the rod to rotate
- The mirror amplifies the motion
- It was repeated for various masses



13-1 Newton's Law of Gravitation

- The difference in mass causes the difference in the apple:Earth accelerations:

$\sim 10\ \mathrm{m/s^2}$ *vs.* $\sim 10^{-25}\ \mathrm{m/s^2}$

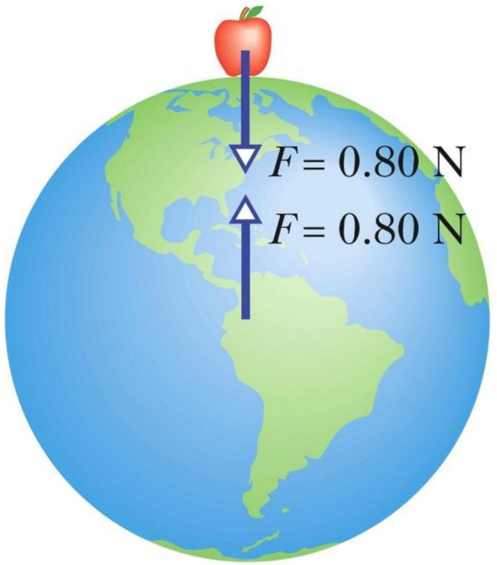


Figure 13-3

Checkpoint 1

A particle is to be placed, in turn, outside four objects, each of mass m: (1) a large uniform solid sphere, (2) a large uniform spherical shell, (3) a small uniform solid sphere, and (4) a small uniform shell. In each situation, the distance between the particle and the center of the object is d. Rank the objects according to the magnitude of the gravitational force they exert on the particle, greatest first.

Answer: All exert equal forces on the particle

13-2 Gravitation and the Principle of Superposition

Learning Objectives

13.04 If more than one gravitational force acts on a particle, draw a free-body diagram showing those forces, with the tails of the force vectors anchored on the particle.

13.05 If more than one gravitational force acts on a particle, find the net force by adding the individual forces as vectors.

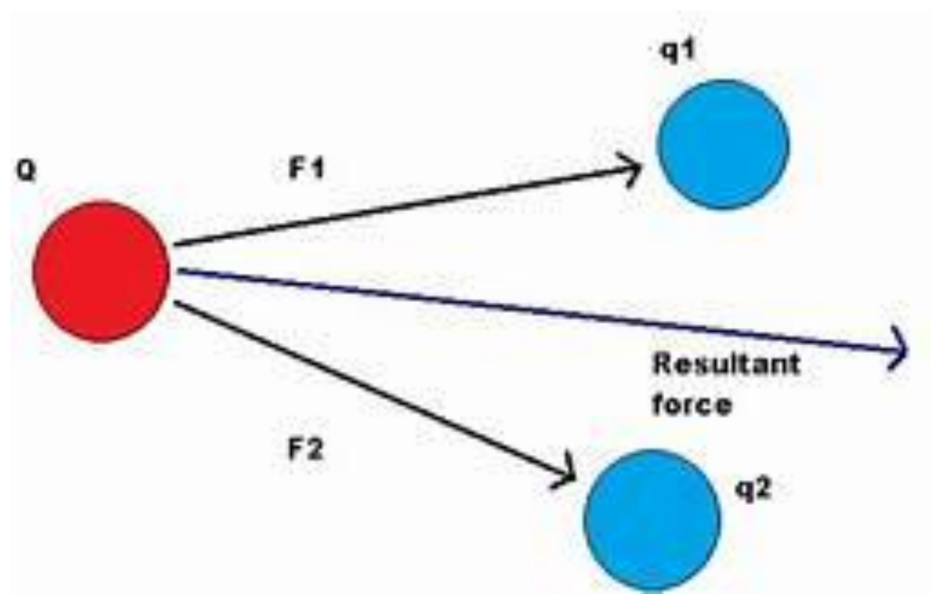
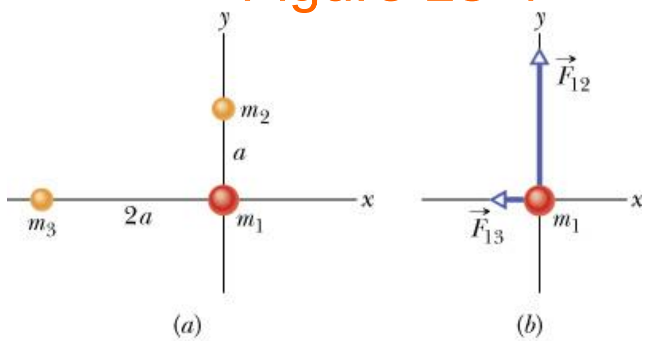


Figure 13-4



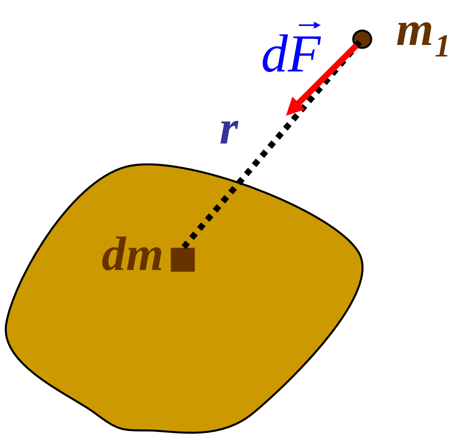
Gravitation and the Principle of Superposition (疊加原理): The net gravitational force exerted by a group of particles is equal to the vector sum of the contribution from each particle.

For example the net force $\vec{F}_1$ exerted on m_1 by m_2 and m_3 is equal to: $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$

Here $\vec{F}_{12}$ and $\vec{F}_{13}$ are the forces exerted on m_1 by m_2 and m_3, respectively.

In general the force exerted on m_1 by n particles is given by the equation:

$$\boxed{\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n} = \sum_{i=2}^{n} \vec{F}_{1i}}$$



The gravitation force exerted by a continuous extended object on a particle of mass m_1 can be calculated using the principle of superposition. The object is divided into elements of mass dm. the net force on m_1 is the vector sum of the forces exerted by each element. The sum takes the form of an integral

$$\vec{F}_1 = \int d\vec{F}$$

$$dF = \frac{Gm_1 dm}{r^2}$$

Here $d\vec{F}$ is the force exerted on m_1 by dm

13-2 Gravitation and the Principle of Superposition

- Find the net gravitational force by the **principle of superposition**: the net is the sum of individual effects
- Add the individual forces as vectors:

$$\vec{F}_{1,\text{net}} = \sum_{i=2}^{n} \vec{F}_{1i}. \qquad \textbf{Eq. (13-5)}$$

- For a real (extended) object, this becomes an integral:

$$\vec{F}_1 = \int d\vec{F}, \qquad \textbf{Eq. (13-6)}$$

- If the object is a uniform sphere or shell we can treat its mass as being at its center instead

13-3 Gravitation Near Earth's Surface

Gravitation near the earth's Surface 自由落體加速度g值~非為常數!

If we assume that the earth is a sphere of mass M, the magnitude F of the force exerted by the earth on an object of mass m placed at a distance r from the center of the earth is: $F = \dfrac{GMm}{r^2}$

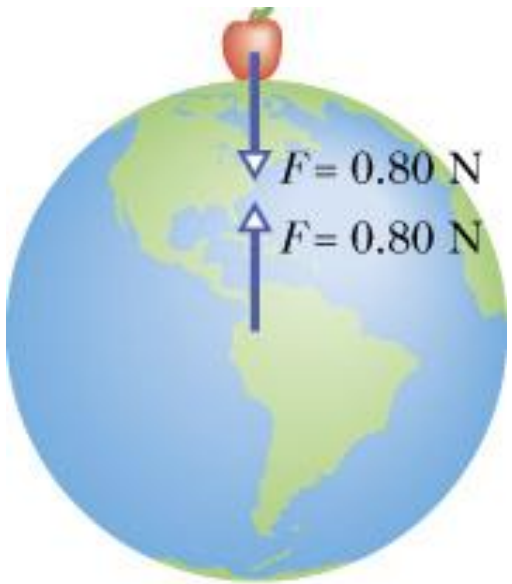


Figure 13-3

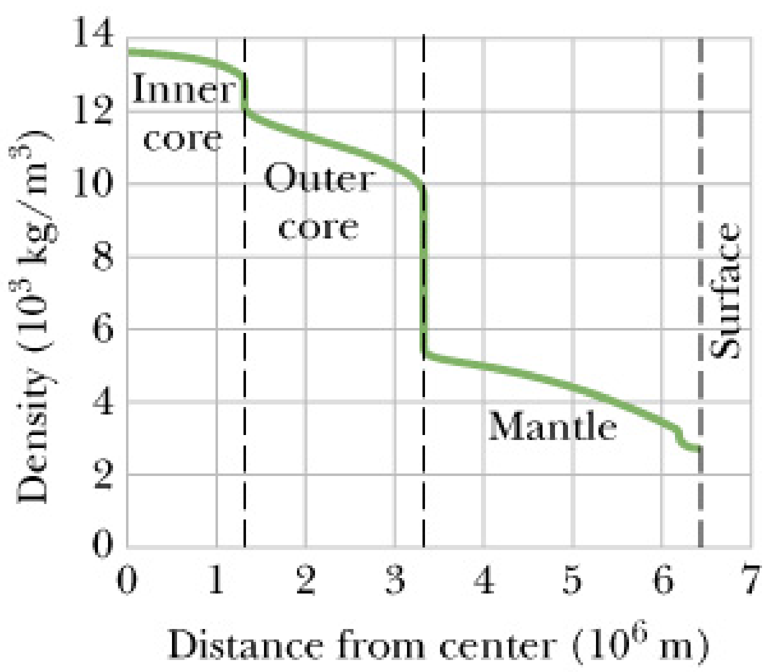


Figure 13-5

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13-3 Gravitation Near Earth's Surface

- Combine $F = GMm/r^2$ and $F = ma_g$:

$$\boxed{a_g = \frac{GM}{r^2}.}$$

Eq. (13-11)

- This gives the magnitude of the gravitational acceleration at a given distance from the center of the Earth
- Table 13-1 shows the value for a_g for various altitudes above the Earth's surface

Table 13-1 Variation of a_g with Altitude

Altitude (km)	a_g (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

Table 13-1

13-3 Gravitation Near Earth's Surface

- The calculated a_g will differ slightly from the measured g at any location
- Therefore the calculated gravitational force on an object will not match its weight for the same 3 reasons 自由落體加速度g值~非為常數之3原因:

1. Earth's mass is not uniformly
2. Earth is not a sphere
3. Earth rotates

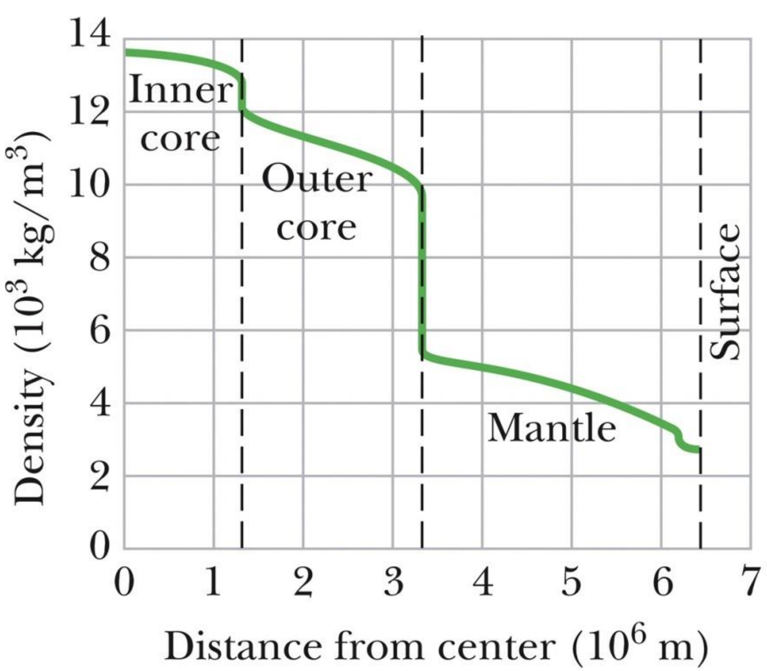

Figure 13-5

The earth is rotating. Consider the crate shown in the figure.
The crate is resting on a scale at a point on the equator
The net force along the y-axis $F_{ynet} = F_g = F_N = ma_g - F_N$
Here $a_g = \frac{GM}{R^2}$ and $\boldsymbol{F_N} = \mathbf{mg}$ is the nor mal force exerted on the crate by the scale. The crate has an acceleration $\boldsymbol{a = \omega^2 R}$ due to the rotation of the earth about its acis every 24 hours. If we apply Newton's second law we get:

$$ma_g - mg = m\omega^2 R \rightarrow mg = ma_g - m\omega^2 R \rightarrow g = a_g - \omega^2 R$$

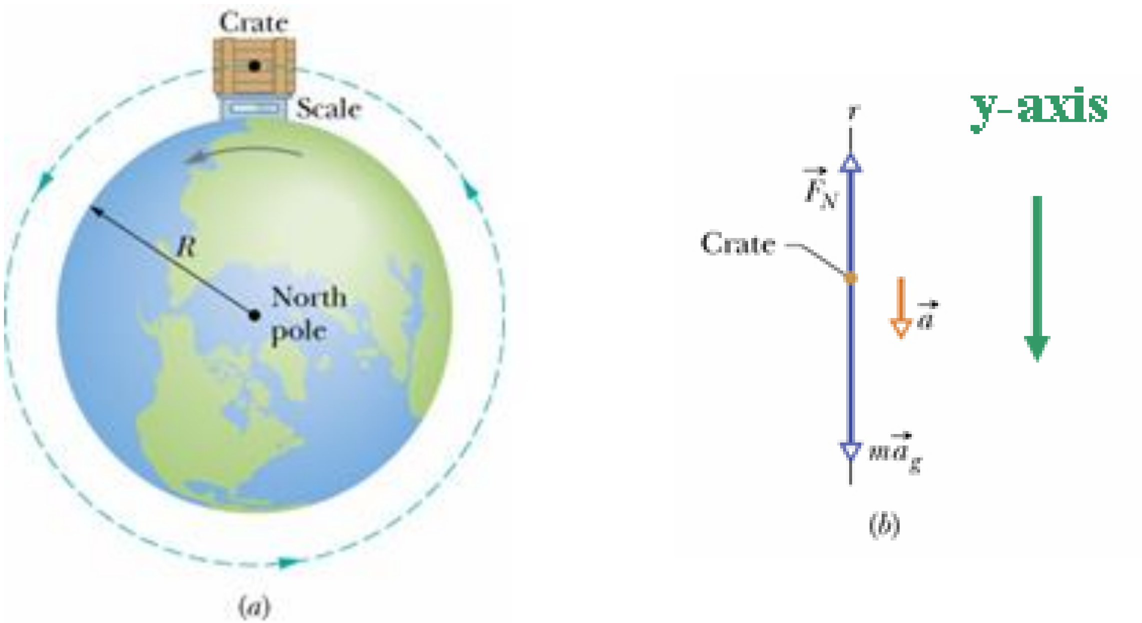

Figure 13-6

Free fall acceleration = gravitational acceleration - centripetal acceleration

The term $\omega^2 R = 0.034$ m/s^2 which is much smaller than 9.8 m/s^2

13-4 Gravitation Inside Earth

- The shell theorem also means that:

> A uniform shell of matter exerts no net gravitational force on a particle located inside it.

- Forces between elements do not disappear, but their vector sum is 0
- Let's find the gravitational force inside a uniform-density Earth
- a solid sphere, not a shell:

$$F = \frac{GmM_{\text{ins}}}{r^2}.$$ **Eq. (13-17)**

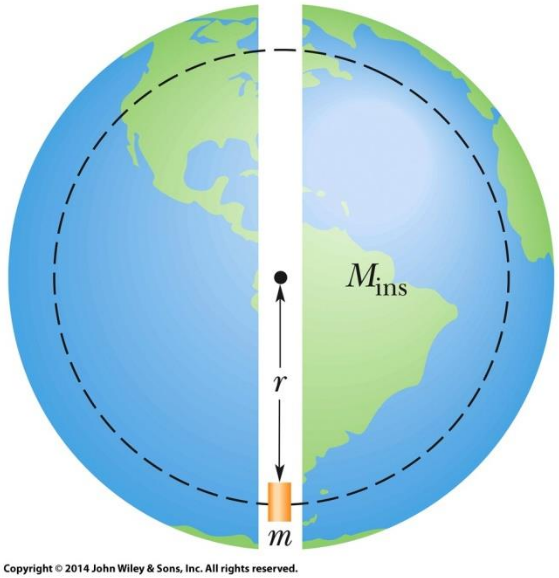

Figure 13-7

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13-4 Gravitation Inside Earth

- The constant density is:

$$\rho = \frac{M_{\text{ins}}}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3}.$$

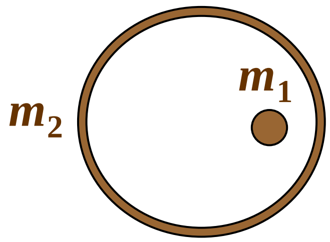
- Substitute in to Eq. 13-17:

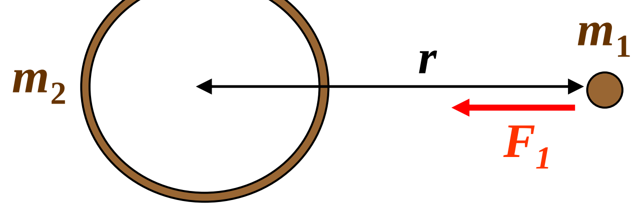
$$\boxed{F = \frac{GmM}{R^3} r.}$$ **Eq. (13-19)**

- If we write this as a vector equation, substituting K for the constants:

$$\vec{F} = -K\vec{r},$$ **Eq. (13-20)**

- Object dropped through Earth oscillates (Hooke's law)





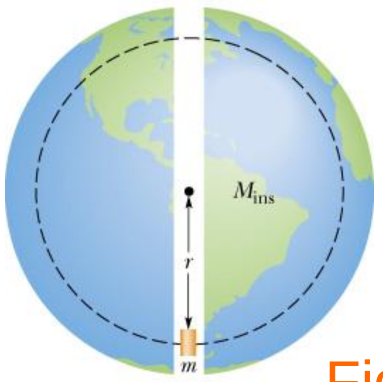


Figure 13-7

Gravitation inside the earth 地球內部之重力:

Newton proved that the net gravitational force on a particle by a shell depends on the postion of the particle with respect to the shell

If the particle is inside the shell, the net force is zero

If the particle is outside the shell the force is given by: $F_1 = G\dfrac{m_1 m_2}{r^2}$

Consider a mass m inside the earth at a distance r from the center of the earth

If we divide the earth in a series of concentric shells, only the shells with radius less than r exert a force on m. The net force on m is: $F = \dfrac{GmM_{ins}}{r^2}$

Here M_{ins} is the mass of the part of the earth inside a sphere os radius r

$$M_{ins} = \rho V_{ins} = \rho\frac{4\pi r^3}{3} \rightarrow \boxed{F = \frac{4\pi G m \rho}{3} r}$$

F is linear with r

~虎克定律!?

~南北極來回震盪! ➔ 週期??

13-5 Gravitational Potential Energy

- Note that gravitational potential energy is a property of a pair of particles
- We cannot divide it up to say how much of it “belongs” to each particle in the pair
- We often speak as of the “gravitational potential energy of an baseball” in the ball-Earth system
- We get away with this because the energy change appears almost entirely as kinetic energy of the ball
- This is only true for systems where one object is much less massive than the other

Gravitational Potential energy (重力位能)

In chapter 8 we derived the potential energy *U* of a mass *m* near the surface of the earth. We will remove this restriction and assume that the mass ***m*** can move away from the surface of the earth, at a distance γ from the center of the earth as shown in the figure. In this case the gravitational potential energy is:

$$U = -\frac{GmM}{r}$$

The negative sign of *U* expresses the fact that the corresponding gravitational force is attractive

Note: The gravitational potential energy is not only associated with the mass ***m*** but with ***M*** as well i.e. with both objects

$$\boxed{U = -\frac{GmM}{r}}$$

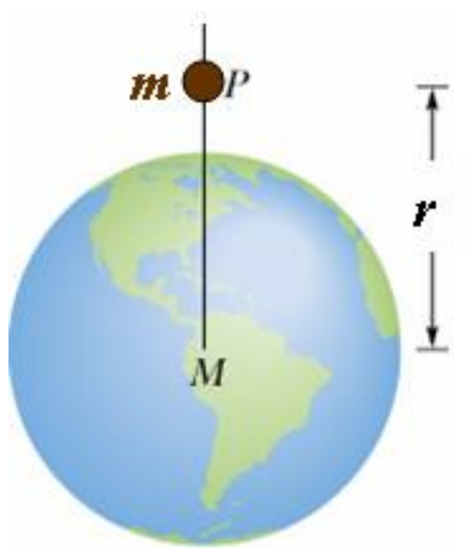
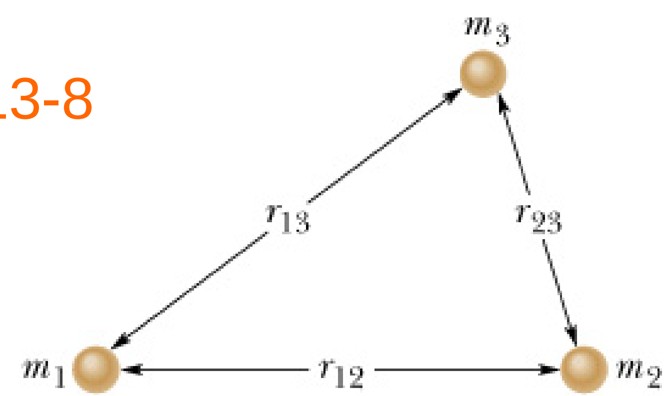


Figure 13-8



13-5 Gravitational Potential Energy

- Gravitational potential energy for a two-particle system is written:

$$U = -\frac{GMm}{r} \quad \text{(gravitational potential energy).}$$

Eq. (13-21)

- Note this value is negative and approaches 0 for $r \rightarrow \infty$
- The gravitational potential energy of a system is the sum of potential energies for all pairs of particles

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right).$$

Eq. (13-22)

- We take into account each pair once

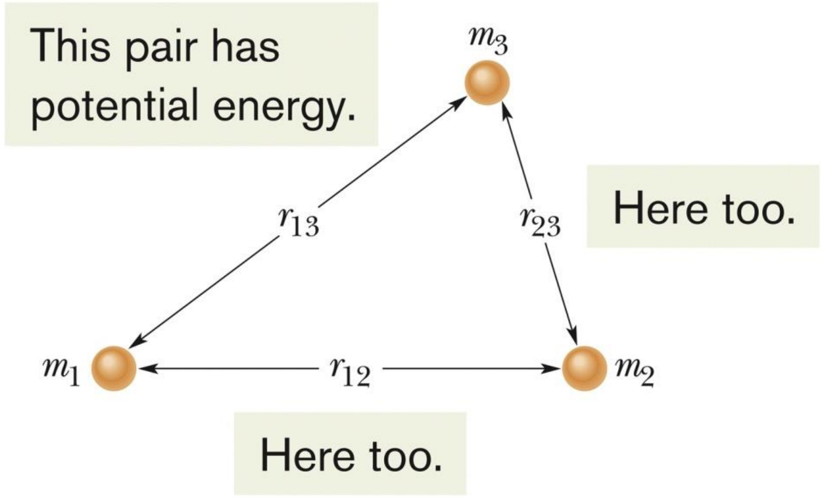


Figure 13-8

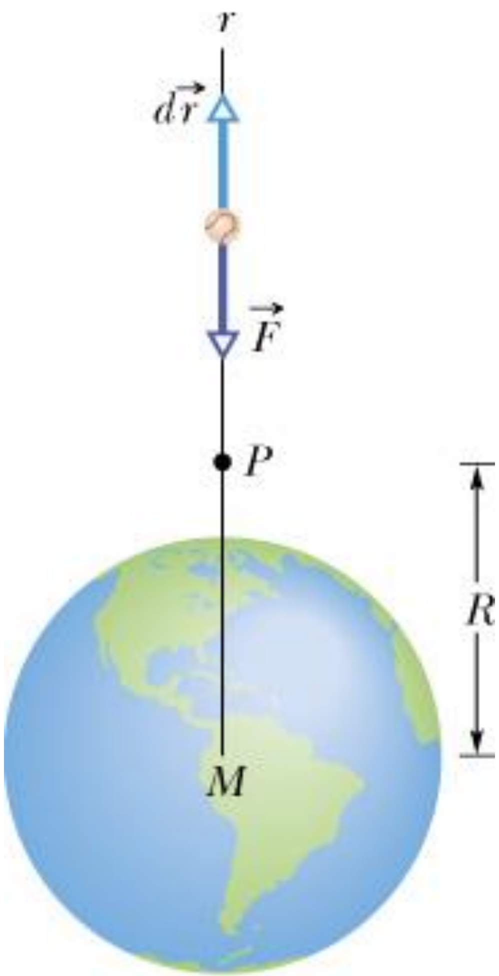


Figure 13-9

公式推導:

Gravitational potential energy ***U*** of the earth and a mass ***m*** placed at a distance **R** from the center of the earth.

We assume that we move the mass m upwards so that it reaches a great distance (practically infinite,~到無窮遠處所需之功) from the earth.

The potential energy ***U*** is equal to the work ***W*** that the gravitational force does on ***m***.

$$U = W = \int_R^\infty \vec{F}(r)\cdot d\vec{r} = \int_R^\infty -\frac{GMm}{r^2}dr = GMm\left[\frac{1}{r}\right]_R^\infty = -\frac{GMm}{R}$$

(Define $U = 0$ at $R = \infty$, not at ground as discussed in previous chapters)

13-5 Gravitational Potential Energy

- The gravitational force is conservative
- The work done by this force does not depend on the path followed by the particles, only the difference in the initial and final positions of the particles(只積分徑向路逕即可)

$$\int_{R}^{\infty} -\frac{GMm}{r^2}dr$$

- Since the work done is independent of path, so is the gravitational potential energy change

$$\Delta U = U_f - U_i = -W.$$ Eq. (13-26)

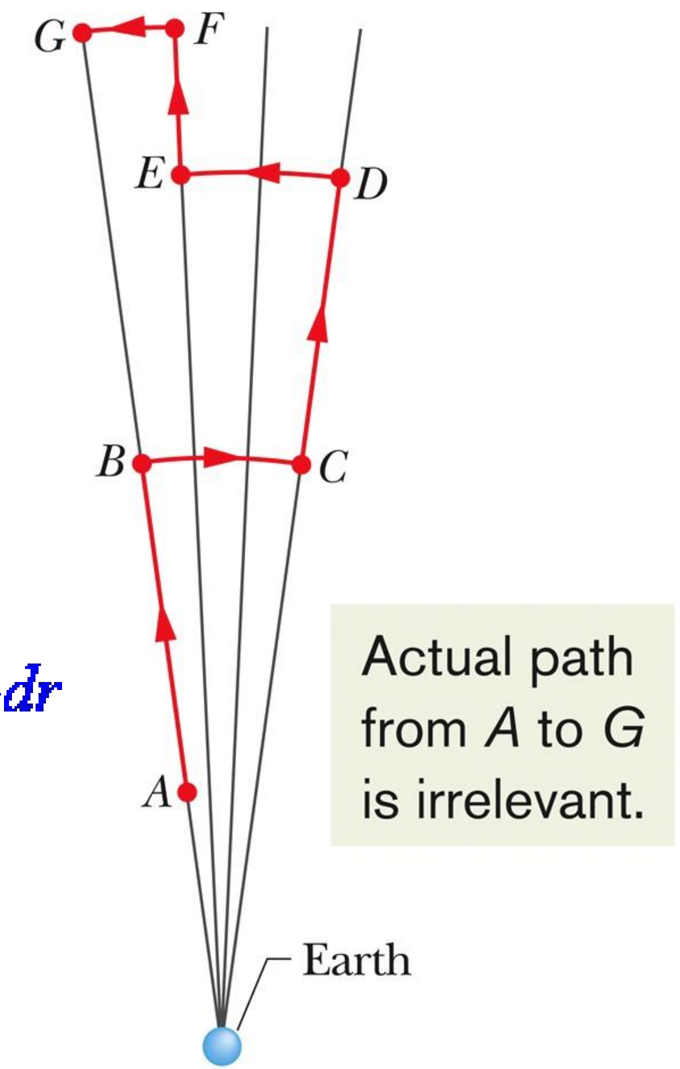


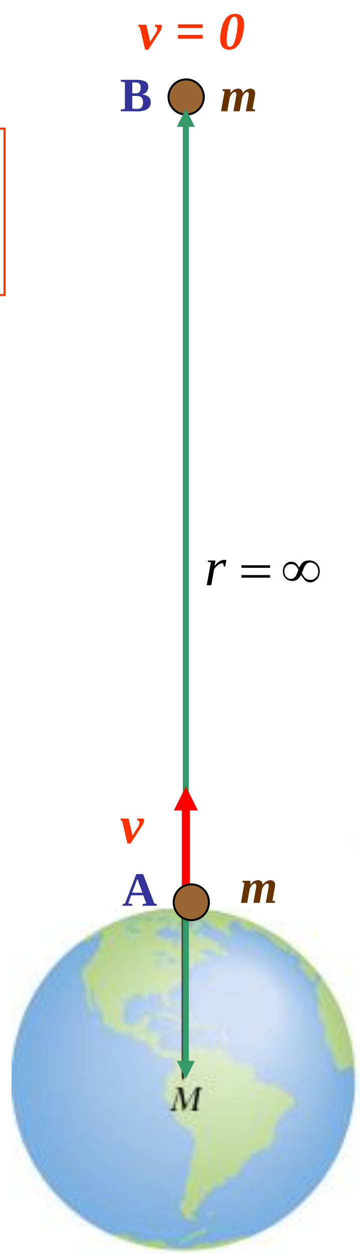
Figure 13-10

Escape Speed

If a projectile of mass *m* is fired upward at point **A** as shown in the figure, the projectile will stop momentarily and return to the earth.

$$v = \sqrt{\frac{2GM}{R}}$$

There is however a minimum initial speed for which the projectile will escape from the gravitational pull of the earth and will stop at infinity (point **B** in the figure). This minimum speed is known as escape velocity. We can determine the escape velocity using energy conservation between point **A** and point **B**.



$$E_A = K + U = \frac{mv^2}{2} - \frac{GMm}{R} \qquad E_B = K + U = 0$$

$$E_A = E_B \rightarrow \frac{mv^2}{2} - \frac{GMm}{R} = 0 \rightarrow v = \sqrt{\frac{2GM}{R}}$$ Eq. (13-28)

The escape speed form the earth is 11.2 km/s

Note: The escape speed does not depend on *m*

13-5 Gravitational Potential Energy

Table 13-2 Some Escape Speeds

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a] (穀神星)	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

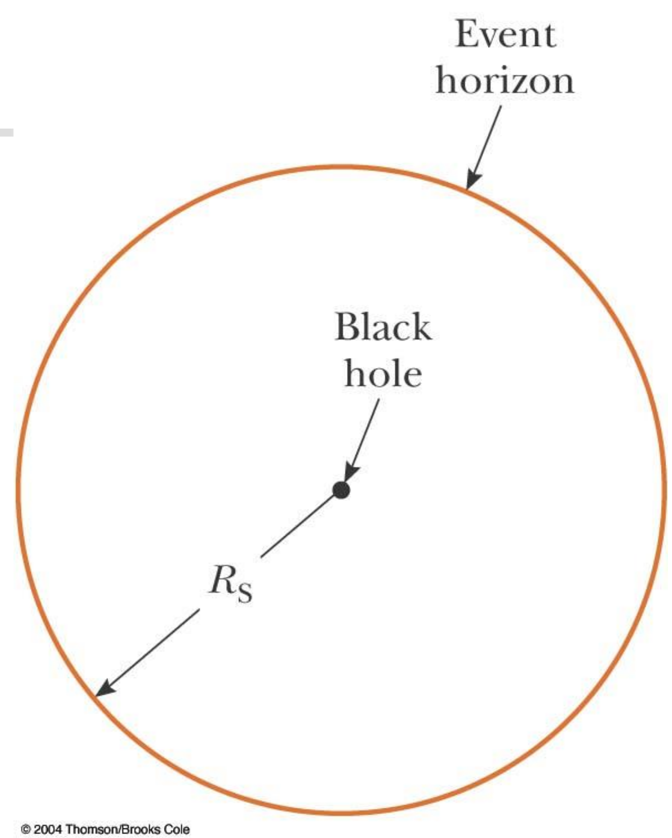
- This explains why some planets have atmospheres and others do not
  - Lighter molecules have higher average speeds and are more likely to reach escape speeds
  - This also explains the composition of the atmosphere

Black Holes

- A ***black hole*** is the remains of a star that has collapsed under its own gravitational force
- The escape speed for a black hole is very large due to the concentration of a large mass into a sphere of very small radius
  - If the escape speed exceeds the speed of light, radiation cannot escape and it appears black

Black Holes, cont

- The critical radius at which the escape speed equals *c* is called the **Schwarzschild radius**, R_S
- The imaginary surface of a sphere with this radius is called the **event horizon**
  - This is the limit of how close you can approach the black hole and still escape



$R_s = 2\,G\,M/\,c^{\,2}$

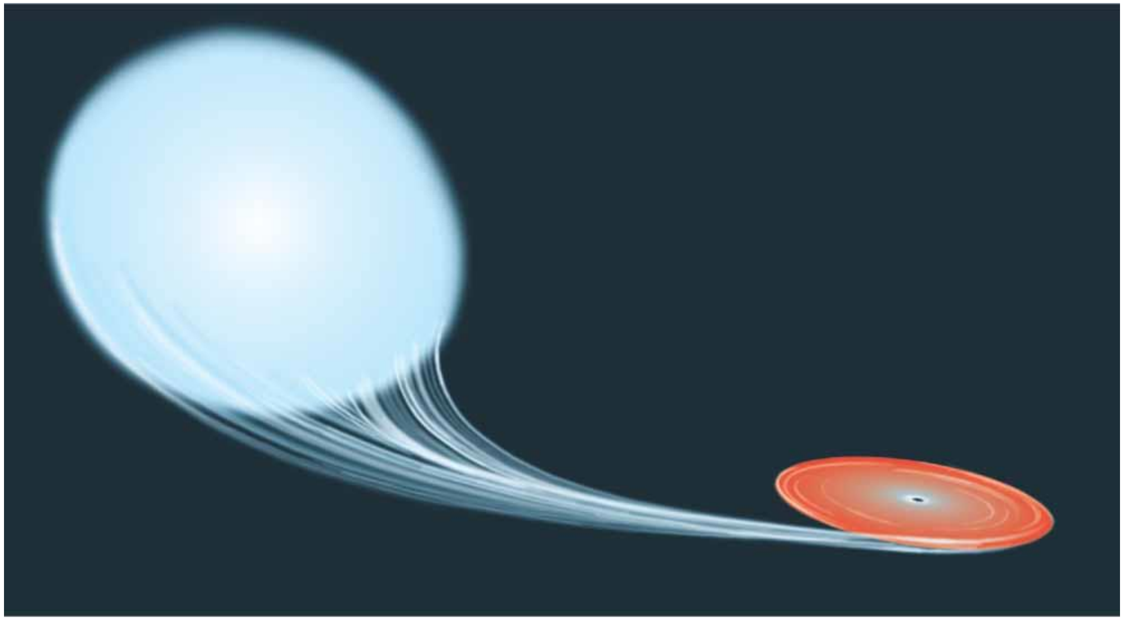
*M*代表天體質量

Black Holes and Accretion Disks

- Although light from a black hole cannot escape, light from events taking place near the black hole should be visible
- If a binary star system has a black hole and a normal star, the material from the normal star can be pulled into the black hole

Black Holes and Accretion Disks, cont

- This material forms an ***accretion disk***(吸積盤) around the black hole
- Friction among the particles in the disk transforms mechanical energy into internal energy



Black Holes and Accretion Disks, final

- The orbital height of the material above the event horizon decreases and the temperature rises
- The high-temperature material emits radiation, extending well into the x-ray region
- These x-rays are characteristics of black holes

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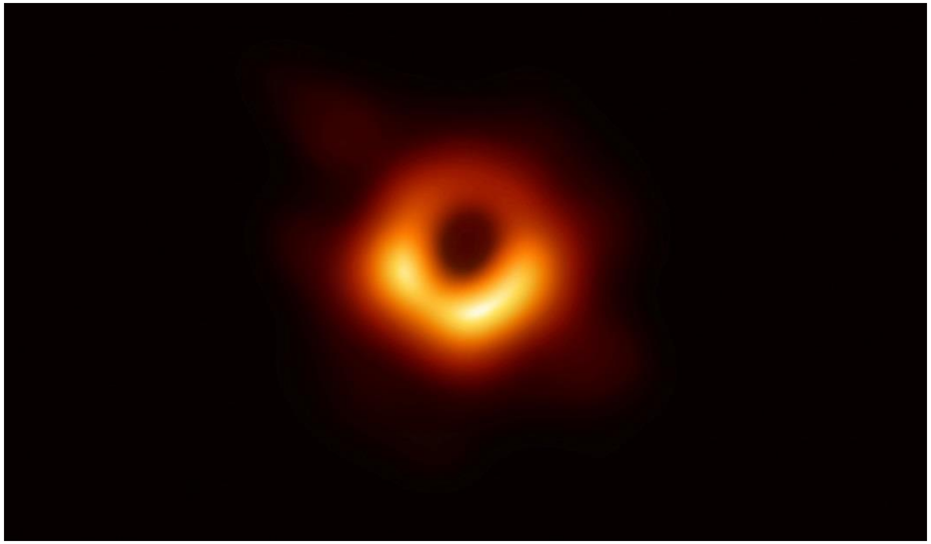
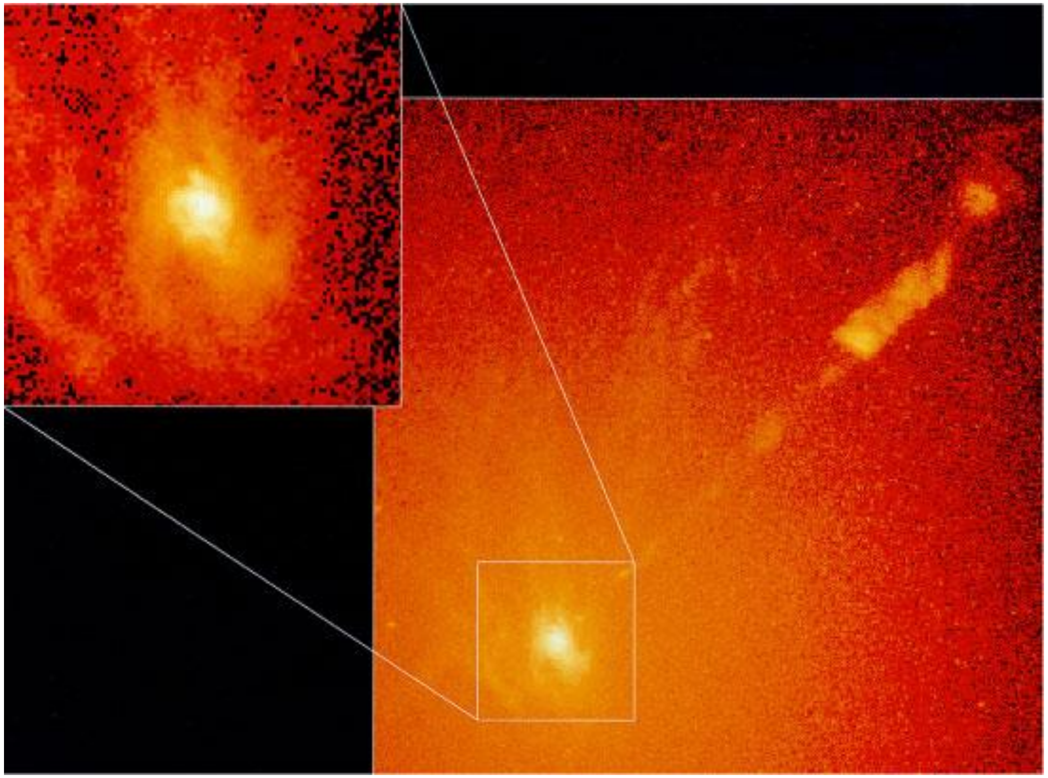
2019/04/10
https://www.msn.com/zh-tw/news/techandscience/%E9%97%9C%E6%96%BC%E5%8F%B2%E4%B8%8A%E9%A6%96%E5%BC%B5%E9%BB%91%E6%B4%9E%E7%85%A7%E7%89%87%EF%BC%8C%E9%80%99%E4%BA%9B%E6%98%AF%E4%BD%A0%E9%9C%80%E8%A6%81%E7%9A%84%E7%A7%91%E6%99%AE%E8%B3%87%E6%96%99/ar-BBVMzcp



Black Holes at Centers of Galaxies

- There is evidence that supermassive black holes exist at the centers of galaxies
- Theory predicts jets of materials should be evident along the rotational axis of the black hole



- *An HST image of the galaxy M87. The jet of material in the right frame is thought to be evidence of a supermassive black hole at the galaxy's center.*

"See" a black hole:

Let's return to the story that opens this chapter. Figure 13-16 shows the observed orbit of the star S2 as the star moves around a mysterious and unobserved object called Sagittarius A* (pronounced "A star"), which is at the center of the Milky Way galaxy. S2 orbits Sagittarius A* with a period of $T = 15.2$ y and with a semimajor axis of $a = 5.50$ light-days ($= 1.42 \times 10^{14}$ m). What is the mass M of Sagittarius A*? What is Sagittarius A*?

KEY IDEA The period T and the semimajor axis a of the orbit are related to the mass M of Sagittarius A* according to Kepler's law of periods. From Eq. 13-34, with a replacing the radius r of a circular orbit, we have

$(2\pi/T)^2 = \omega^2 = GM/R^3)$

$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3. \qquad (13\text{-}36)$$

Calculations: Solving Eq. 13-36 for M and substituting the given data lead us to

$$M = \frac{4\pi^2 a^3}{GT^2}$$

$$= \frac{4\pi^2(1.42 \times 10^{14}\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)[(15.2\ \text{y})(3.16 \times 10^7\ \text{s/y})]^2}$$

$$= 7.35 \times 10^{36}\ \text{kg}. \qquad \text{(Answer)}$$

To figure out what Sagittarius A* might be, let's divide this mass by the mass of our Sun ($M_{\text{Sun}} = 1.99 \times 10^{30}$ kg) to find that

$$M = (3.7 \times 10^6)M_{\text{Sun}}.$$

Sagittarius A* has a mass of 3.7 million Suns! However, it cannot be seen. Thus, it is an extremely compact object. Such a huge mass in such a small object leads to the reasonable conclusion that this object is a *supermassive* black hole. In fact, evidence is mounting that a supermassive black hole lurks at the center of most galaxies. (Movies of the stars orbiting Sagittarius A* are available on the Web; search under "black hole galactic center.")

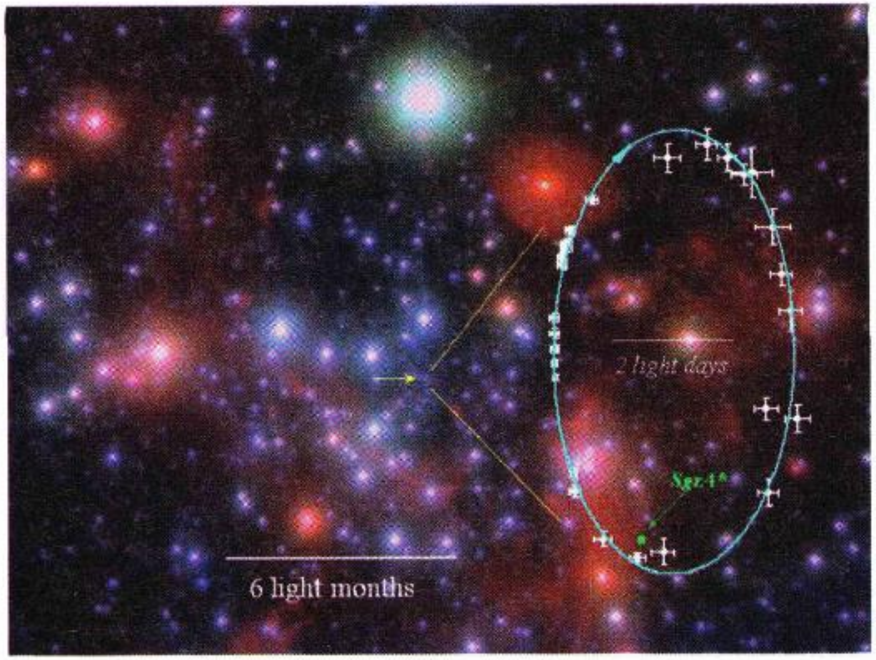


FIG. 13-16 The orbit of star S2 about Sagittarius A* (Sgr A*). The elliptical orbit appears skewed because we do not see it from directly above the orbital plane. Uncertainties in the location of S2 are indicated by the crossbars. *(Courtesy Reinhard Genzel)*

13-6 Planets and Satellites: Kepler's Laws

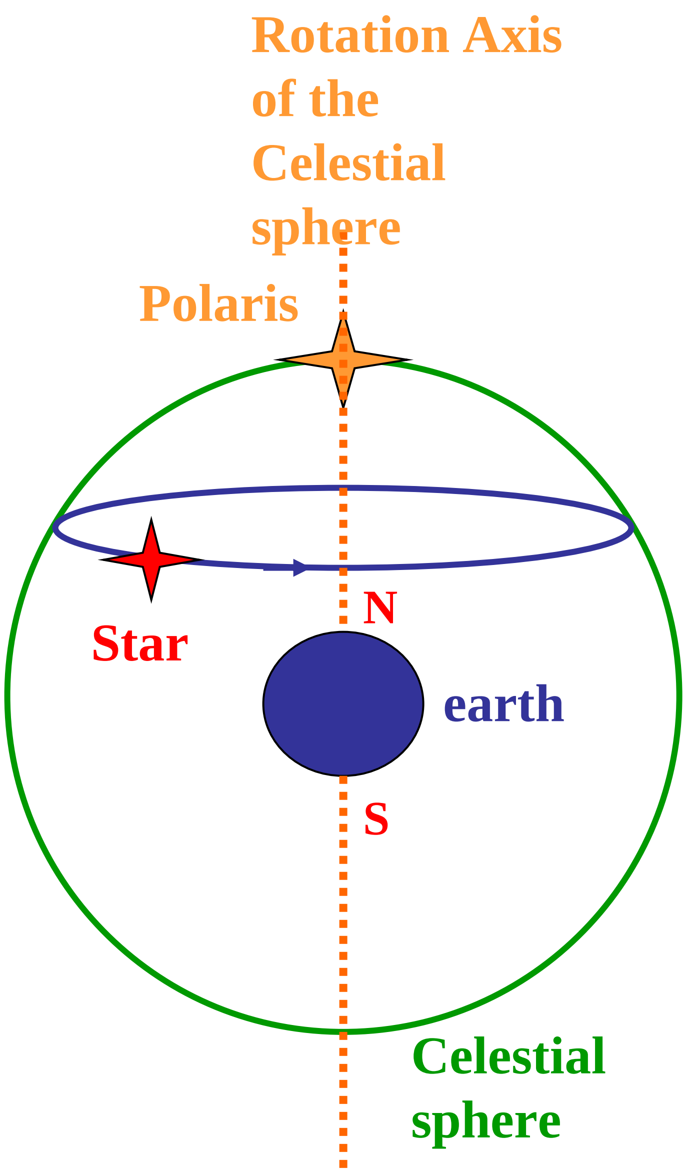
Planets and Satellites: Kepler's Laws:

Stars follow regular paths in the evening sky.

They rotate once every 24 hours about an axis that passes through the star polaris. Polaris is the only star that does not move in the sky.

The stars have fixed spatial spatial relationships among them.

Humans have classified them in groups known as "constellations" ~星座:圍繞北極星轉動!



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13-6 Planets and Satellites: Kepler's Laws

- The motion of planets in the solar system was a puzzle for astronomers, especially curious motions such as in Figure 13-11(火星路徑~行星運動軌跡似乎複雜許多!)
- Johannes Kepler (1571-1630) derived laws of motion using Tycho Brahe's (1546-1601) measurements; Kepler 整理出3個經驗公式，再由牛頓證明之:

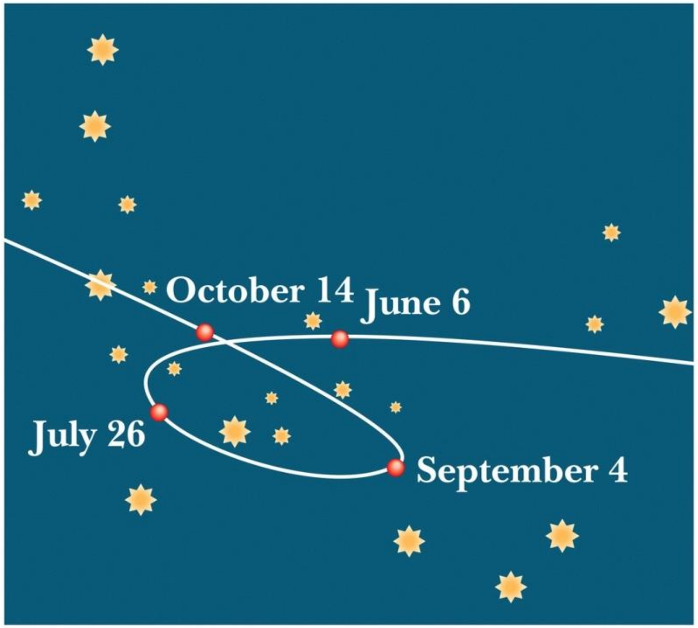


Figure 13-11

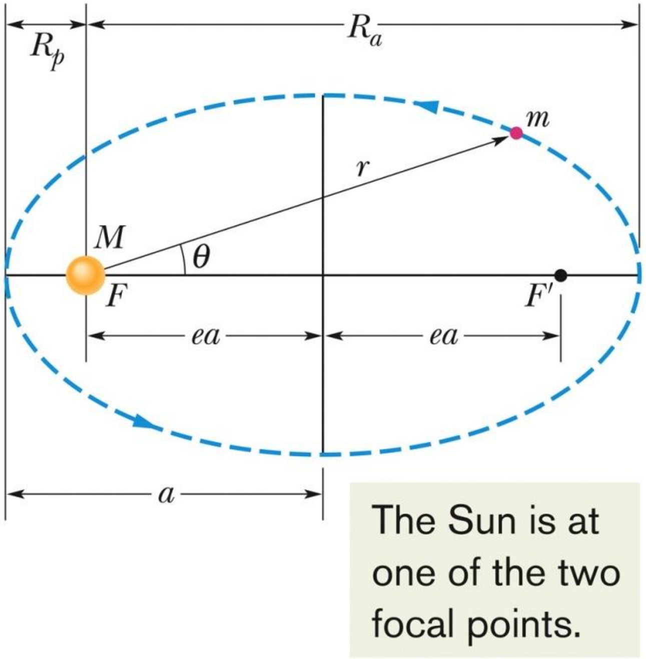


Figure 13-12

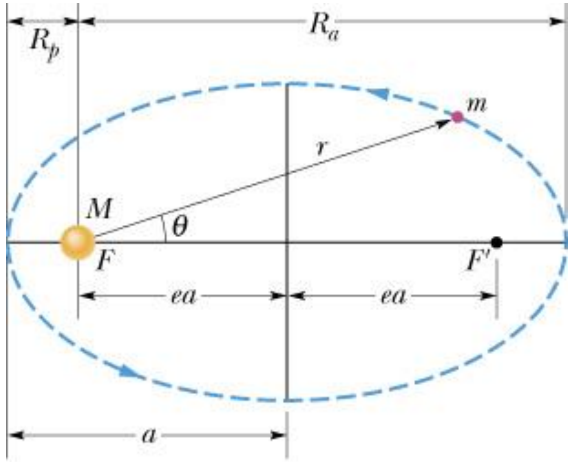


Figure 13-12

Kepler's First Law (Law of orbits). All planets move on elliptical orbits with the sun at one focus. The orbits are described by two parameters: The semimajor axis a and the eccentricity e

The orbit in the figure has $e = 0.74$. The actual eccentricity of the earth's orbit is only 0.0167

Kepler's Second Law (Law of Areas)

$$\boxed{\frac{dA}{dt} = \text{constant}}$$

The line that connects a planet to the sun sweeps out equal areas ΔA in the plane of the orbit in equal time intervals Δt . $\frac{dA}{dt} = \text{constant}$

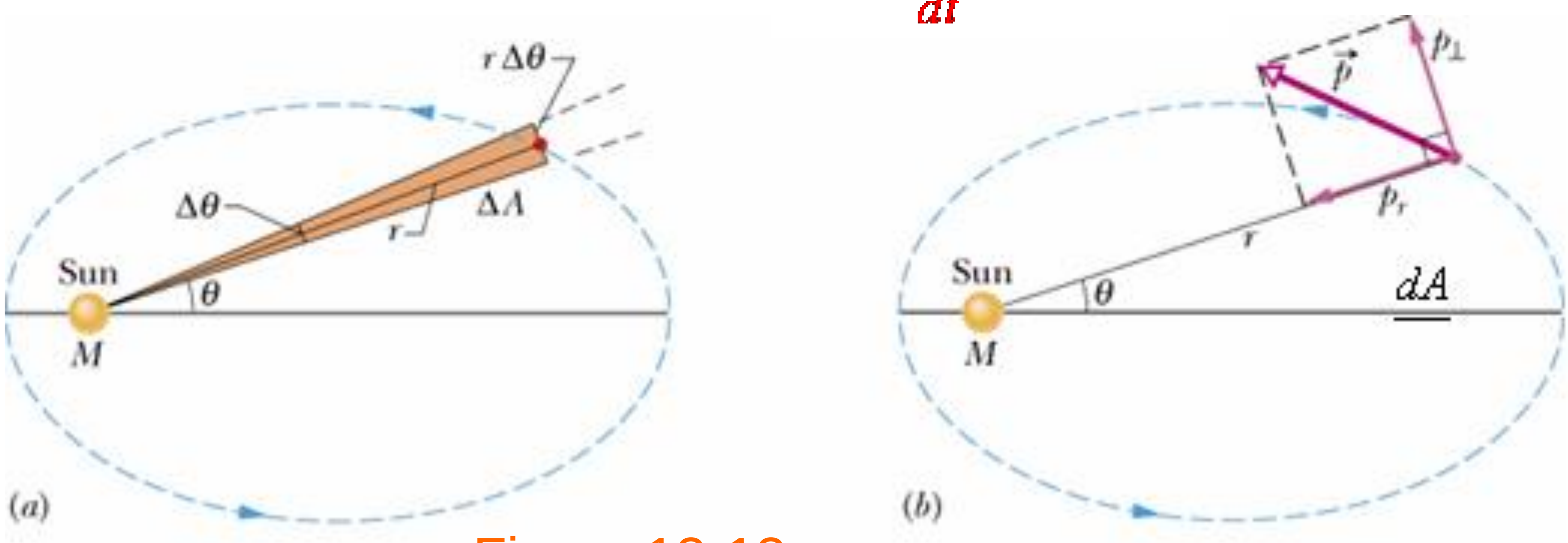


Figure 13-13

The area ΔA swept out by the planet (fig.a) is given by the equation: $\Delta A \approx \frac{1}{2} r^2 \Delta\theta$

$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega$ where ω is the planet's angular speed. The planet's angular momentum $L = rp_{\perp} = rmv_{\perp} = rm\omega r = mr^2\omega \rightarrow \frac{dA}{dt} = \frac{L}{2m}$ Kepler's second law is equivalent to the law of conservation of angular momentum

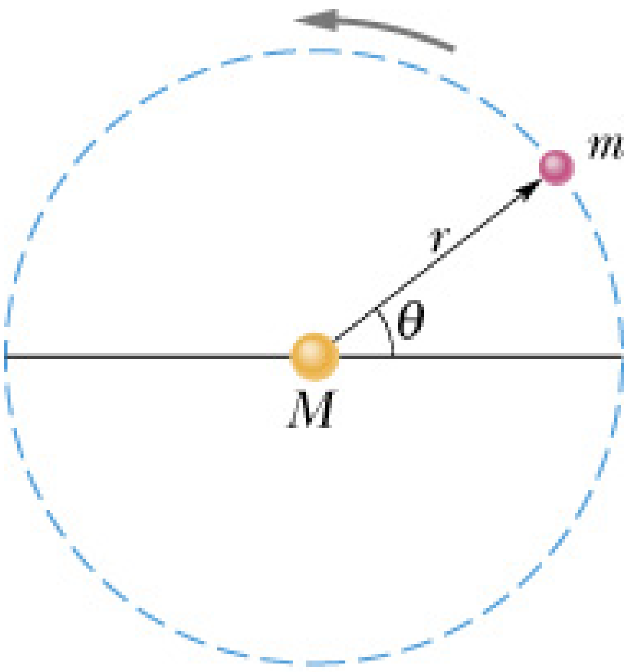


Figure 13-14

Kepler's Third law (law of periods)

The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit. For the sake of simplicity we will consider the circular orbit shown in the figure.

$$\boxed{\frac{T^2}{r^3} = \frac{4\pi^2}{MG}}$$

A planet of mass m moves on a circular orbit of radius r around a star of mass M.

We apply Newton's second law to the motion:

$F_g = \frac{GMm}{r^2} = ma = (m)(\omega^2 r) = m\omega^2 r \rightarrow \frac{GM}{r^3} = \omega^2$ (eqs.1) The period T can be expressed in terms of the angular speed ω. $T = \frac{2\pi}{\omega} \rightarrow T^2 = \frac{4\pi^2}{\omega^2}$ (eqs.2)

If we substitute ω^2 form eqs.1 into eqs.2 we get: $\frac{T^2}{r^3} = \frac{4\pi^2}{MG}$

Note 1: The ratio $\frac{T^2}{r^3}$ does not depend on the mass m of the planet but only on the mass M of the central star.

Note 2: For elliptical orbits the ratio $\frac{T^2}{a^3}$ remains constant ~證明!??

13-6 Planets and Satellites: Kepler's Laws

Table 13-3 Kepler's Law of Periods for the Solar System

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Table 13-3

13-7 Satellites: Orbits and Energy

- Relating the centripetal acceleration of a satellite to the gravitational force, we can rewrite as energies:

$$\boxed{K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r},}$$ **Eq. (13-38)**

- Meaning that:

$$K = -\frac{U}{2} \quad \text{(circular orbit).}$$ **Eq. (13-39)**

- Therefore the total mechanical energy is:

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$\boxed{E = -\frac{GMm}{2r} \quad \text{(circular orbit).}}$$ **Eq. (13-40)**

13-7 Satellites: Orbits and Energy

- Total energy E is the negative of the kinetic energy
- For an ellipse, we substitute *a* for *r*
- Therefore the energy of an orbit depends only on its semimajor axis, not its eccentricity
- All orbits in Figure 13-15 have the same energy

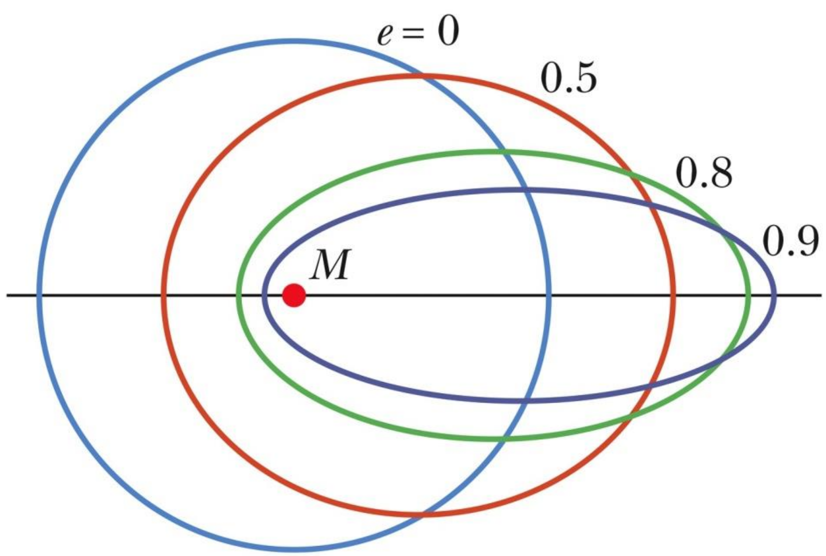
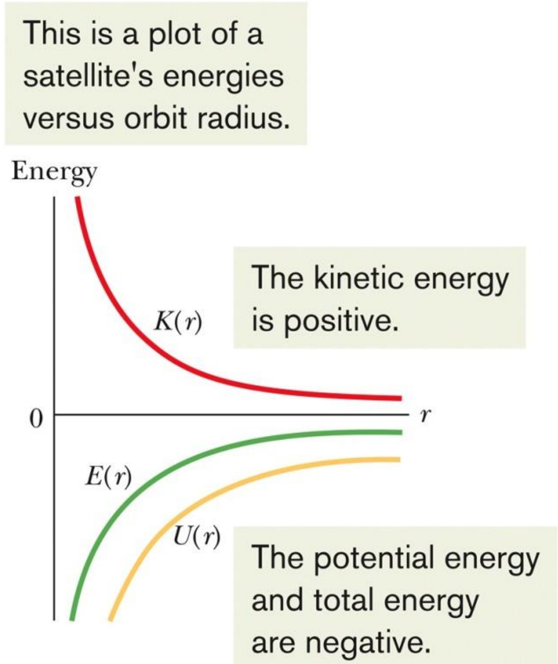
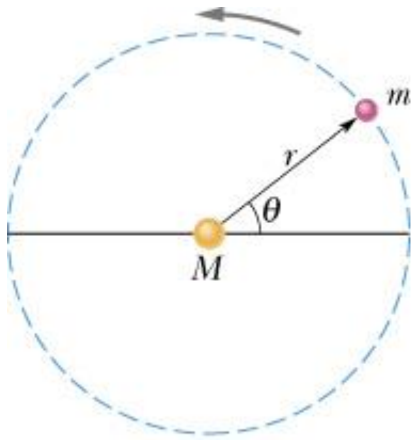
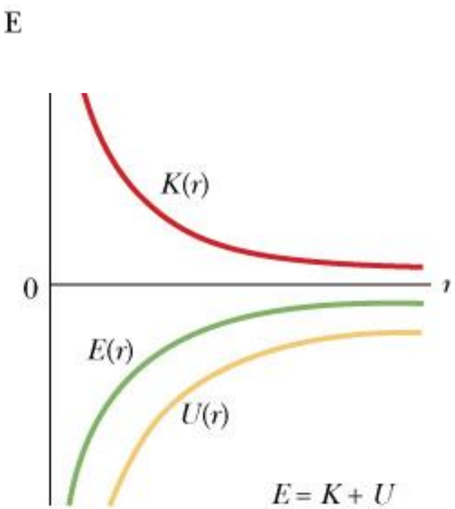


Figure 13-15

Figure 13-16







$$\boxed{E = -\frac{GMm}{2r}}$$

Satellites: Orbits and Energy 衛星之軌道與能量:

Consider a satellite that follows a circular orbit of radius r around a planet of mass M. We apply Newton's second law and have: $\dfrac{GMm}{r^2} = ma = m\dfrac{v^2}{r} \to v^2 = \dfrac{GM}{r}$

The kinetic energy $\quad K = \dfrac{mv^2}{2} = \dfrac{GMm}{2r} \qquad$ (eqs.1)

The potential energy $\quad U = -\dfrac{GMm}{r} \qquad$ (eqs.2)

If we compare eqs.1 with eqs.2 we have: $K = -\dfrac{U}{2} \qquad$ (eqs.3)

The total energy $E = K + U = \dfrac{GMm}{2r} - \dfrac{GMm}{r} = -\dfrac{GMm}{2r} = -K$

The energies $E, K,$ and U are plotted as function of r in the figure to the left.

Note: For elliptical orbits $E = -\dfrac{GMm}{2a}$ ***~check it?***

克卜勒定律推導:https://zh.wikipedia.org/zh-tw/%E5%BC%80%E6%99%AE%E5%8B%92%E5%AE%9A%E5%BE%8B

克普勒三大運動定律的發現:

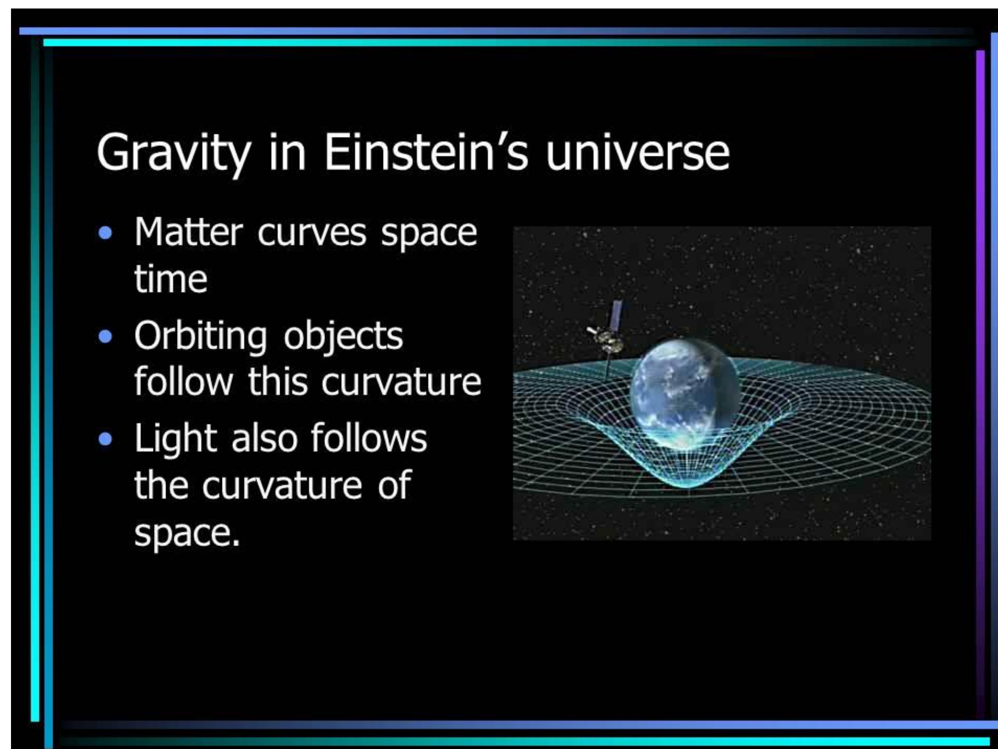
https://sites.google.com/site/keplerslawsha/home

13-8 Einstein and Gravitation

Learning Objectives

13.24 Explain Einstein's principle of equivalence.

13.25 Identify Einstein's model for gravitation as being due to the curvature of spacetime.



13-8 Einstein and Gravitation

- The **general theory of relativity** describes gravitation
- Its fundamental postulate is the **principle of equivalence**
- Gravitation and acceleration are equivalent
- The experimenter inside this box is unable to tell whether he is on Earth experiencing $g = 9.8$ m/s^2, or in free space accelerating at 9.8 m/s^2.

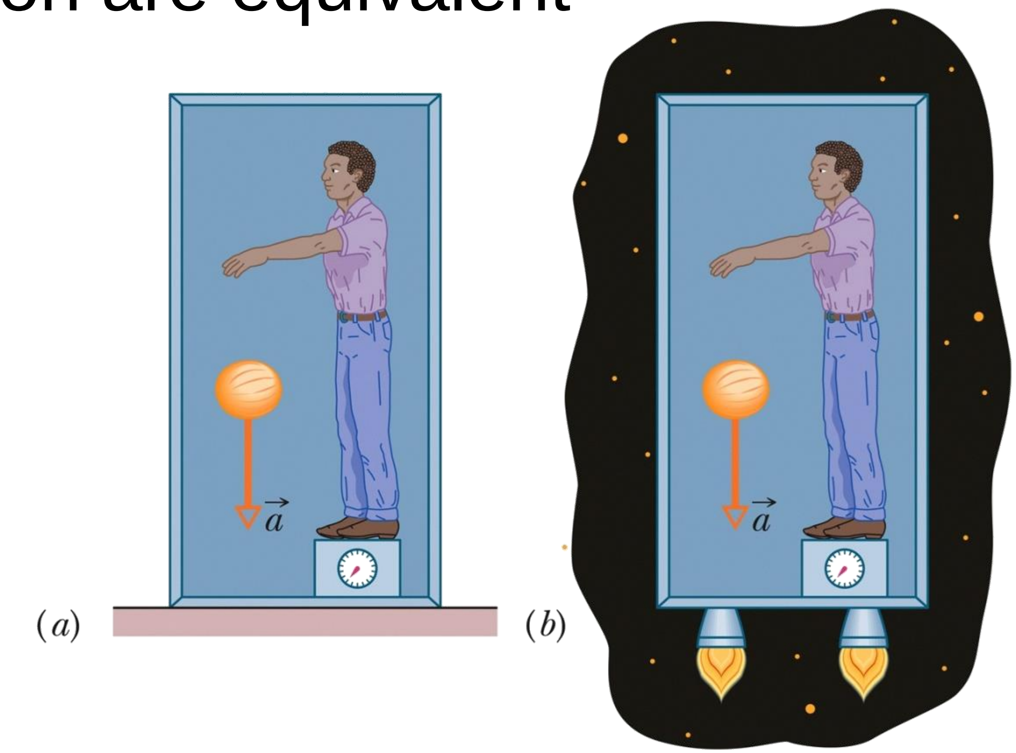


Figure 13-18

13-8 Einstein and Gravitation

- Space (spacetime) is curved
- Analogies: In (a):兩船平行往南開，最後在南極相撞~有額外之力? and (b):兩蘋果從外太空平行下落~~在地心“相撞” ~有額外之力?, paths that appear to be parallel, along the surface of the Earth or falling toward the Earth's center, actually converge
- We can see why by stepping “outside” the curved Earth ~(a)可以地球表面曲率理解, but we can‘t step “outside” of curved space ~以(c)圖理解:

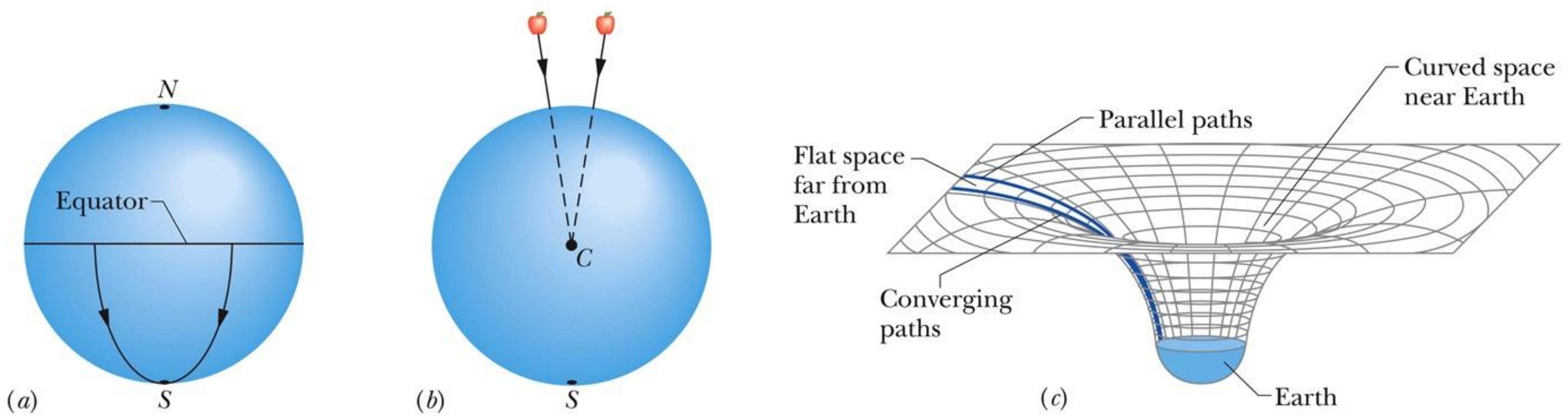


Figure 13-19

13-8 Einstein and Gravitation

- But we can observe the curvature of space~在廣義相對論中，重力被描述為時空的一種幾何屬性（曲率）；
- Light bends as it passes by massive objects: an effect called *gravitational lensing* (重力透鏡)
- In extreme cases we observe the light coming from multiple places, or bent into an *Einstein ring*

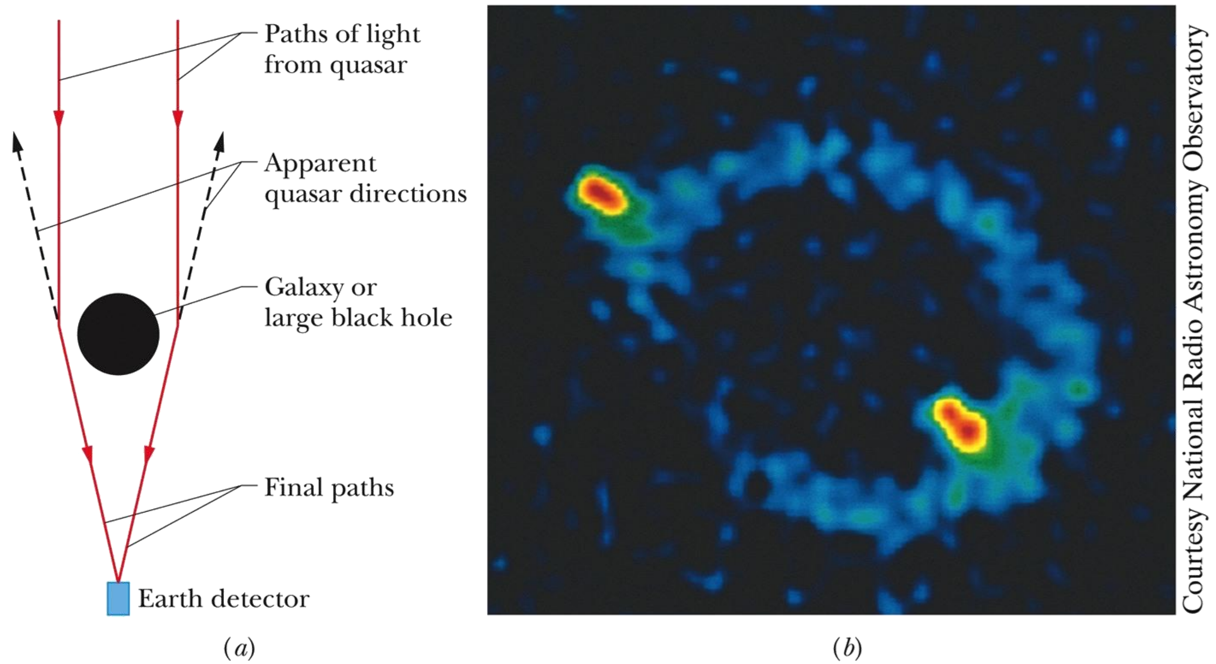
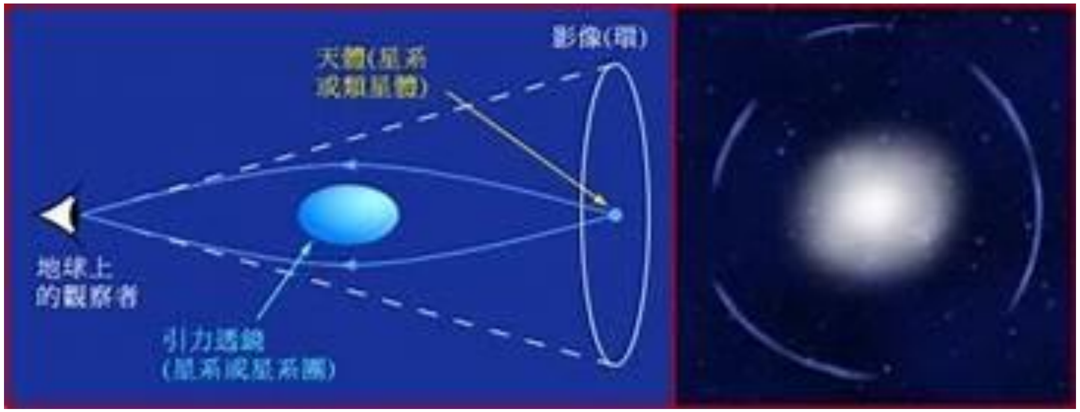
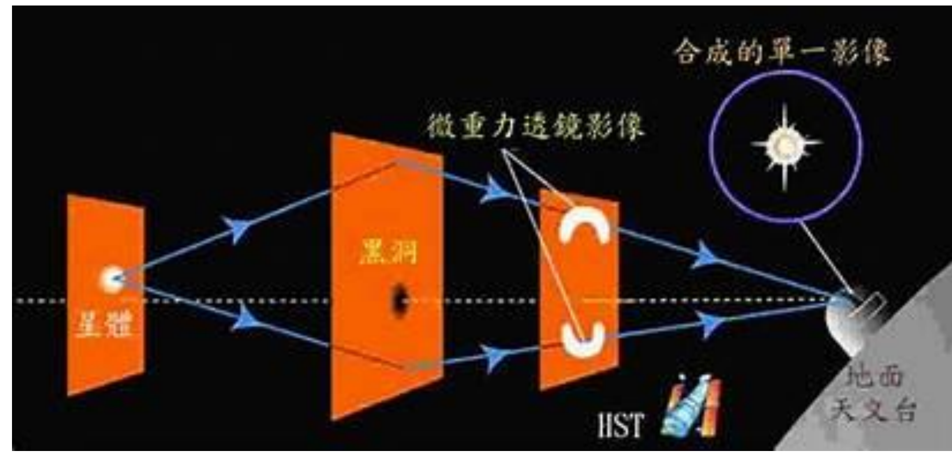
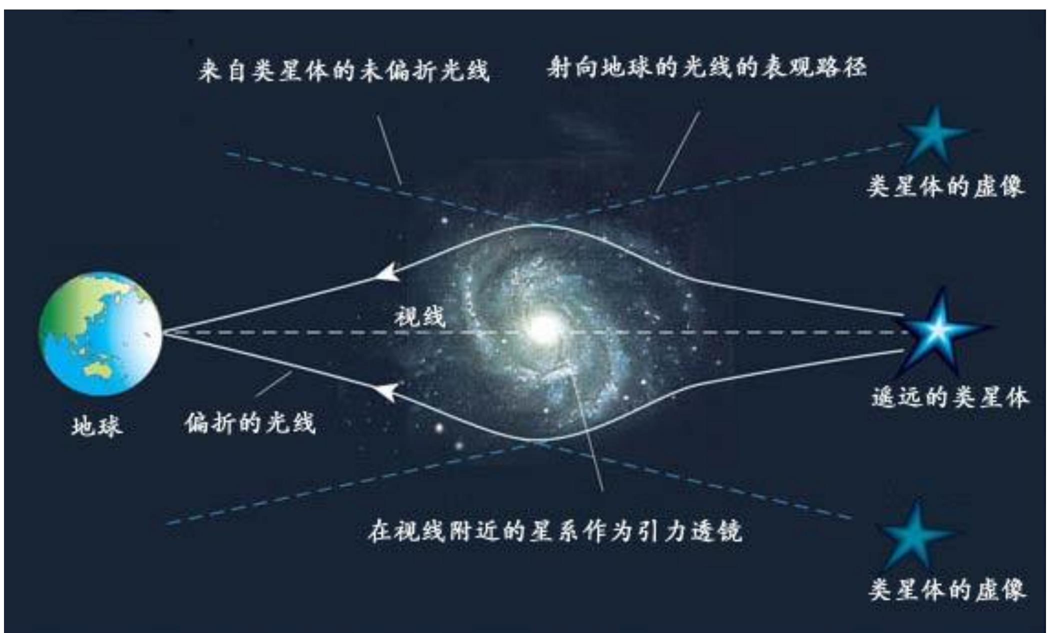


Figure 13-20

重力透鏡:


影像(環)
天體(星系
或類星體)
地球上
的觀察者
引力透鏡
(星系或星系團)


合成的單一影像
微重力透鏡影像
黑洞
星體
HST
地面
天文台


来自类星体的未偏折光线
射向地球的光线的表观路径
类星体的虚像
视线
地球
偏折的光线
遥远的类星体
在视线附近的星系作为引力透镜
类星体的虚像

13-8 Einstein and Gravitation

- The *source* of gravitation, however, is still unclear
- Is it purely an effect of curved spacetime?
- Is it a force between masses?
- Is it due to a fundamental particle, the *graviton*, which would carry the gravitational force?
- These questions are not yet settled
- **重力子（graviton，又稱引力子):**一種基於量子場論的架構，提出的假設基本粒子，這種量子的交換，可產生重力。但目前仍未知是否真正存在。重力子被設想為一個自旋為2、質量為零、不帶電荷的玻色子。為了傳遞重力，重力子必須永遠相吸、作用範圍無限遠及以無限多的型態出現。
- 量子物理學的標準模型，認為基本交互作用都是由量子交換產生，並提出規範玻色子理論，如電磁力由光子交換產生，弱作用力由W及Z玻色子交換產生，強核力由膠子交換產生。這個理論預測，重力也應該是由某種玻色子的交換而產生，這種玻色子被稱為重力子。或許，重力子是跟希格斯玻色子有關（因為重力跟質量成正比）
- See: https://zh.wikipedia.org/wiki/%E5%BC%95%E5%8A%9B%E5%AD%90

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13 Summary

The Law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Eq. (13-1)

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

Eq. (13-2)

Superposition

$$\vec{F}_{1,\mathrm{net}} = \sum_{i=2}^{n} \vec{F}_{1i}.$$

Eq. (13-5)

Gravitational Behavior of Uniform Spherical Shells

- The net force on an *external* object: calculate as if all the mass were concentrated at the center of the shell

Gravitational Acceleration

$$a_g = \frac{GM}{r^2}.$$

Eq. (13-11)

13 Summary

Free-Fall Acceleration and Weight

- Earth's mass is not uniformly distributed, the planet is not spherical, and it rotates: the calculated and measured values of acceleration differ

Gravitation within a Spherical Shell

- A uniform shell exerts no net force on a particle inside
- Inside a solid sphere:

$$F = \frac{GmM}{R^3}r. \quad \textbf{Eq. (13-19)}$$

Gravitational Potential Energy

$$U = -\frac{GMm}{r} \quad \textbf{Eq. (13-21)}$$

Potential Energy of a System

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right).$$

Eq. (13-22)

13 Summary

Escape Speed

$$v = \sqrt{\frac{2GM}{R}}.$$

Eq. (13-28)

Energy in Planetary Motion

$$E = -\frac{GMm}{2r}$$

Eq. (13-42)

Kepler's Laws

- The law of orbits: ellipses
- The law of areas: equal areas in equal times
- The law of periods:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

Eq. (13-34)

Kepler's Laws

- Gravitation and acceleration are equivalent
- **General theory of relativity** explains gravity in terms of curved space

CH13習題:

6, 24, 31, 37, 48, 61, 64 and 65

補充_克卜勒定律:
https://zh.wikipedia.org/zh-tw/%E5%BC%80%E6%99%AE%E5%8B%92%E5%AE%9A%E5%BE%8B