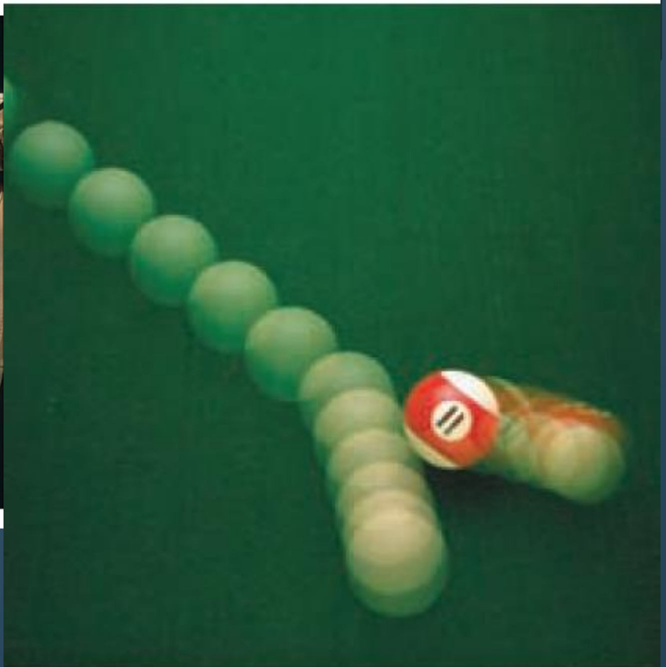
Chapter 9

Center of Mass and Linear Momentum



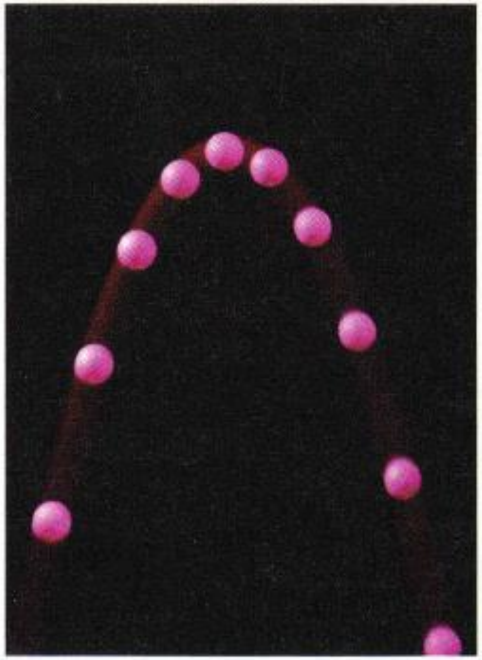
9-1 Center of Mass

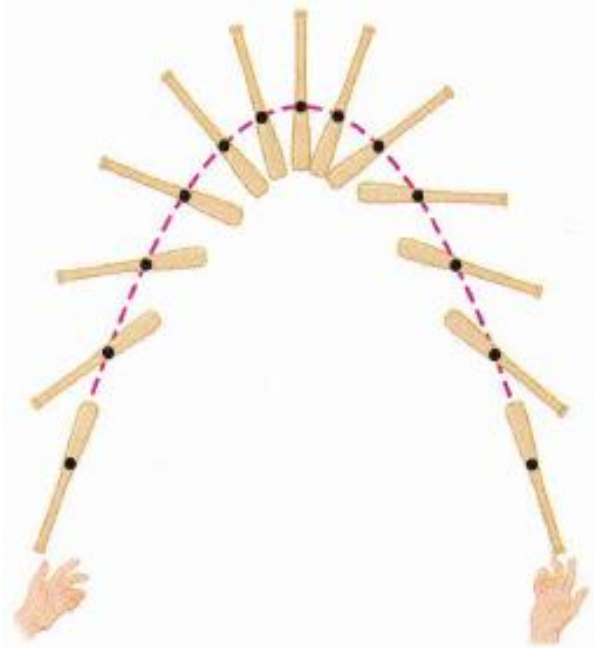
Learning Objectives

9.01 Given the positions of several particles along an axis or a plane, determine the location of their center of mass.

9.02 Locate the center of mass of an extended, symmetric object by using the symmetry.

9.03 For a two-dimensional or three-dimensional extended object with a uniform distribution of mass, determine the center of mass by (a) mentally dividing the object into simple geometric figures, each of which can be replaced by a particle at its center and (b) finding the center of mass of those particles.





9-1 Center of Mass質量中心:

- There is a special point in a system or object, called the ***center of mass (com)***, that moves as if all of the mass of the system is concentrated at that point質量中心:一特殊之點，有若全部質量*M*集中於該點，可用於描述質點受力之運動情況。

The Center of Mass質量中心:

Consider a system of two particles of masses m_1 and m_2 at positions x_1 and x_2, respectively. We define the position of the center of mass (com) as follows:

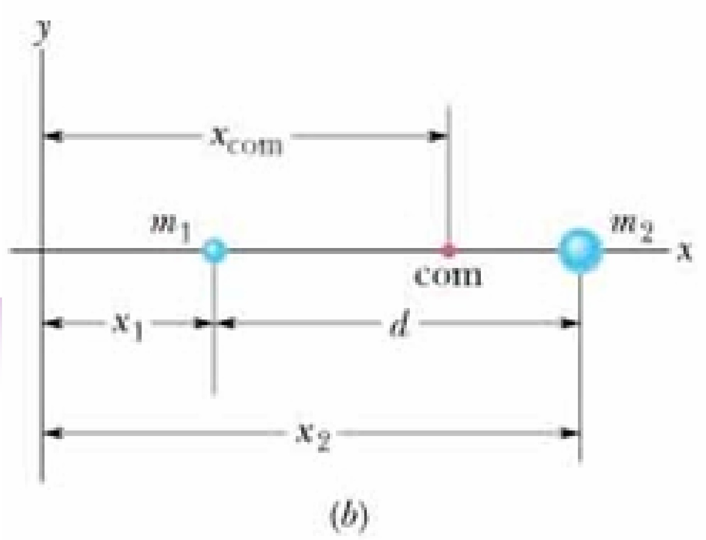


Figure 9-2

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

We can generalize the above definition for a system of n particles as follows:

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots + m_n x_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots + m_n x_n}{M} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i$$

Here M is the total mass of all the particles $M = m_1 + m_2 + m_3 + \ldots + m_n$

質量中心之座標位置: the center of mass of a system of particles in three dimensional space. We assume that the i-th particle (mass m_i) has position vector $\vec{r}_i$

$$\vec{r}_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i$$

The position vector can be written as:

$$\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j} + z_{com}\hat{k}$$

The components of $\vec{r}_{com}$ are given by the equations:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

Figure 9-1

The Center of Mass for Solid Bodies

~ be considered as systems with continuous distribution of matter;

~the calculation of the center of mass of systems become integrals:

具體積質量中心位置之計算:--積分!

$$x_{com} = \frac{1}{M}\int x\,dm \qquad y_{com} = \frac{1}{M}\int y\,dm \qquad z_{com} = \frac{1}{M}\int z\,dm$$

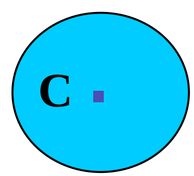
The integrals above are rather complicated. **A** simpler special case is that of uniform objects in which the mass density $\rho = \frac{dm}{dV}$ is constant and equal to $\frac{M}{V}$ $\left(x_{com} = 1/(\cancel{\rho} V) \cdot \int x \cancel{\rho}\, dv\right)$

$$x_{com} = \frac{1}{V}\int x\,dV \qquad y_{com} = \frac{1}{V}\int y\,dV \qquad z_{com} = \frac{1}{V}\int z\,dV$$

In objects with symmetry elements (symmetry point, symmetry line, symmetry plane) it is not necessary to evaluate the integrals.

The center of mass lies on the symmetry element.

For example the com of a uniform sphere coincides with the sphere center in a uniform rectangular object the com lies at the intersection of the diagonals



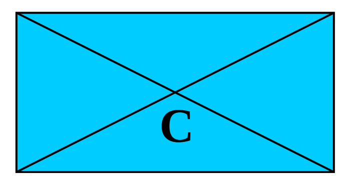
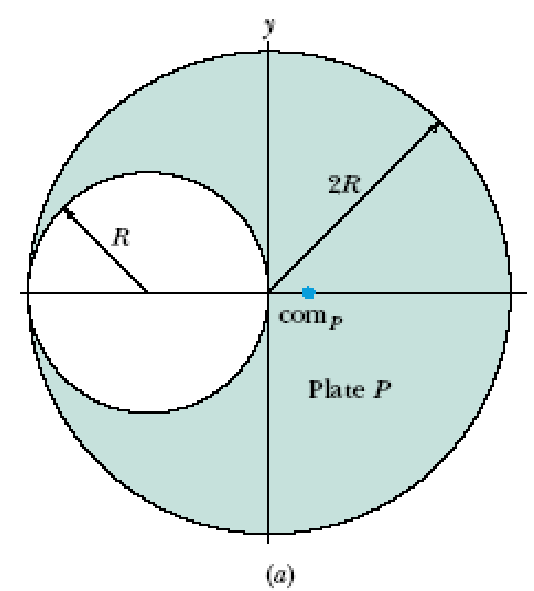
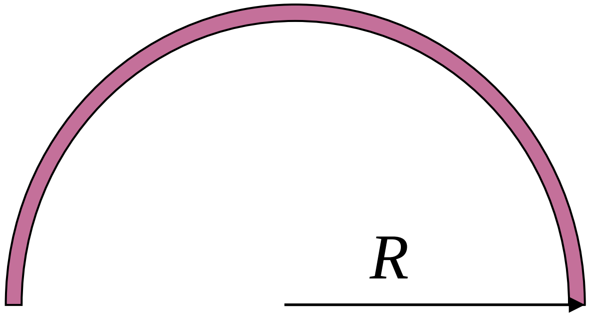
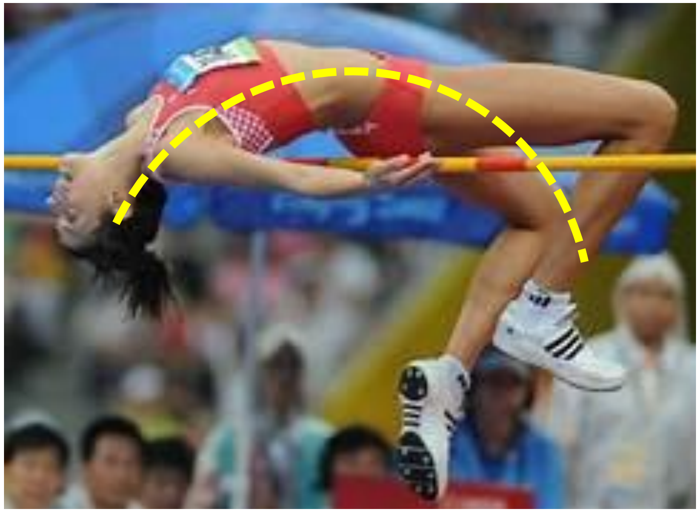


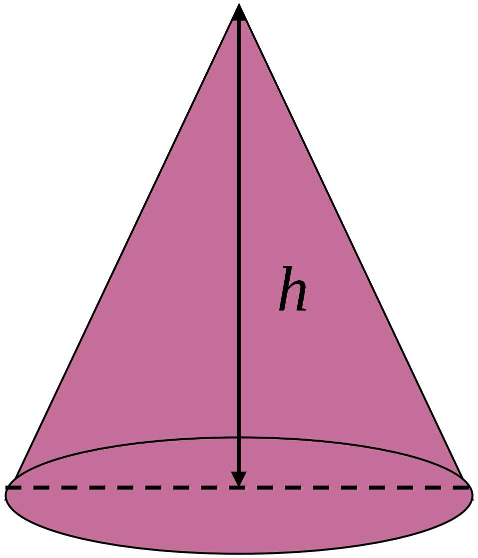
Figure 9-4*a* shows a uniform metal plate P of radius $2R$ from which a disk of radius R has been stamped out (removed) in an assembly line. Using the xy coordinate system shown, locate the center of mass com_P of the plate.



Find the center of mass:







9-2 Newton's Second Law for a System of Particles

Learning Objectives

9.04 Apply Newton's second law to a system of particles by relating the net force (of the forces acting on the particles) to the acceleration of the system's center of mass.

9.05 Apply the constant-acceleration equations to the motion of the individual particles in a system and to the motion of the system's center of mass.

9.06 Given the mass and velocity of the particles in a system, calculate the velocity of the system's center of mass.

9.07 Given the mass and acceleration of the particles in a system, calculate the acceleration of the system's center of mass.

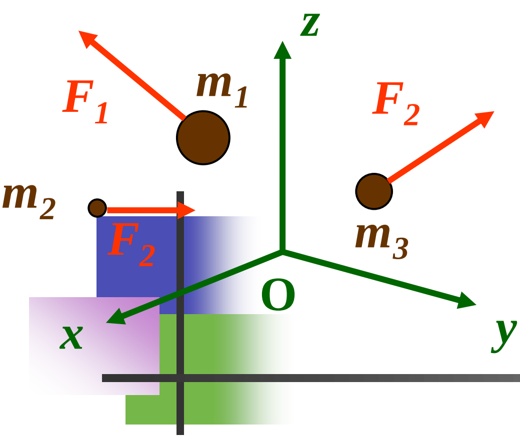
9.08 Given the position of a system's center of mass as a function of time, determine the velocity of the center of mass.

9.09 Given the velocity of a system's center of mass as a function of time, determine the acceleration of the center of mass.

9.10 Calculate the change in the velocity of a com by integrating the com's acceleration function with respect to time.

9.11 Calculate a com's displacement by integrating the com's velocity function with respect to time.

9.12 When the particles in a two-particle system move without the system's com moving, relate the displacements of the particles and the velocities of the particles.



9-2 質點系統之牛頓定律:

Newton's Second Law for a System of Particles

Consider a system of n particles of masses $m_1, m_2, m_3, \ldots, m_n$ and position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_n$, respectively.

The position vector of the center of mass is given by:

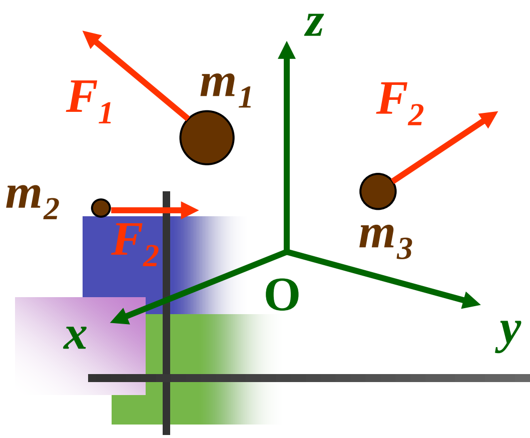
$M\vec{r}_{com} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots + m_n\vec{r}_n$ We take the time derivative of both sides $\rightarrow$

$$M\frac{d}{dt}\vec{r}_{com} = m_1\frac{d}{dt}\vec{r}_1 + m_2\frac{d}{dt}\vec{r}_2 + m_3\frac{d}{dt}\vec{r}_3 + \ldots + m_n\frac{d}{dt}\vec{r}_n \rightarrow$$

$M\vec{v}_{com} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n$ Here $\vec{v}_{com}$ is the velocity of the com and $\vec{v}_i$ is the velocity of the i-th particle. We take the time derivative once more $\rightarrow$

$$M\frac{d}{dt}\vec{v}_{com} = m_1\frac{d}{dt}\vec{v}_1 + m_2\frac{d}{dt}\vec{v}_2 + m_3\frac{d}{dt}\vec{v}_3 + \ldots + m_n\frac{d}{dt}\vec{v}_n \rightarrow$$

$M\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots + m_n\vec{a}_n$ Here $\vec{a}_{com}$ is the acceleration of the com and $\vec{a}_i$ is the acceleration of the i-th particle



$$M\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots + m_n\vec{a}_n$$

We apply Newton's second law for the *i*-th particle:

$m_i\vec{a}_i = \vec{F}_i$ Here $\vec{F}_i$ is the net force on the *i*-th particle

$$M\vec{a}_{com} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n$$

The force $\vec{F}_i$ can be decomposed into two components: applied and internal

$\vec{F}_i = \vec{F}_i^{\,app} + \vec{F}_i^{\,\text{int}}$ The above equation takes the form:

$$M\vec{a}_{com} = \left(\vec{F}_1^{\,app} + \vec{F}_1^{\,\text{int}}\right) + \left(\vec{F}_2^{\,app} + \vec{F}_2^{\,\text{int}}\right) + \left(\vec{F}_3^{\,app} + \vec{F}_3^{\,\text{int}}\right) + \ldots + \left(\vec{F}_n^{\,app} + \vec{F}_n^{\,\text{int}}\right) \rightarrow$$

$$M\vec{a}_{com} = \left(\vec{F}_1^{\,app} + \vec{F}_2^{\,app} + \vec{F}_3^{\,app} + \ldots + \vec{F}_n^{\,app}\right) + \boxed{\left(\vec{F}_1^{\,\text{int}} + \vec{F}_2^{\,\text{int}} + \vec{F}_3^{\,\text{int}} + \ldots + \vec{F}_n^{\,\text{int}}\right)}$$

0

The sum in the first parenthesis on the RHS of the equation above is just $\vec{F}_{net}$

The sum in the second parethesis on the RHS vanishes

by virtue of Newton's third law.

The equation of motion for the center of mass becomes: $M\vec{a}_{com} = \vec{F}_{net}$

In terms of components we have:

$$F_{net,x} = Ma_{com,x} \qquad F_{net,y} = Ma_{com,y} \qquad F_{net,z} = Ma_{com,z}$$

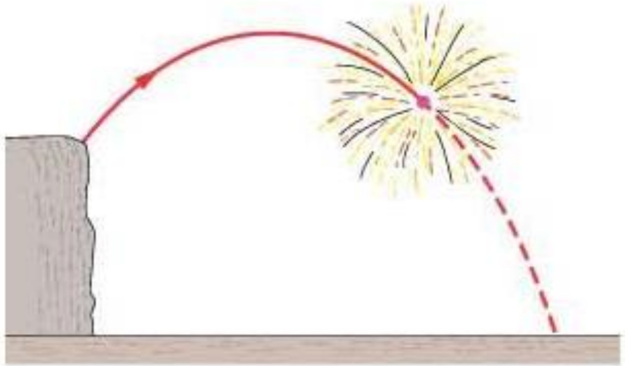


Figure 9-5

$$M\vec{a}_{com} = \vec{F}_{net}$$

$$F_{net,x} = Ma_{com,x}$$
$$F_{net,y} = Ma_{com,y}$$
$$F_{net,z} = Ma_{com,z}$$

The equations above show that the center of mass of a system of particles moves as though all the system's mass were concentrated there, and that the vector sum of all the external forces were applied there.

If the explosion had not occurred, the rocket would have continued to move on the parabolic trajectory (dashed line). The forces of the explosion, even though large, are all internal and as such cancel out.

The only external force is that of gravity and this remains the same before and after the explosion. This means that the center of mass of the fragments follows the same parabolic trajectory that the rocket would have followed had it not exploded

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9-3 Linear Momentum

Learning Objectives

9.13 Identify that momentum is a vector quantity and thus has both magnitude and direction and also components.

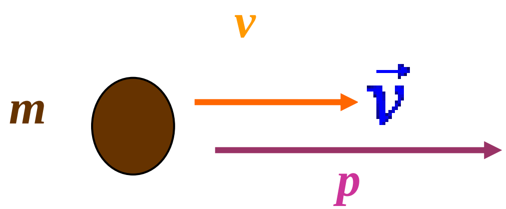
9.14 Calculate the (linear) momentum of a particle as the product of the particle's mass and velocity.

9.15 Calculate the change in momentum (magnitude and direction) when a particle changes its speed and direction of travel.

9.16 Apply the relationship between a particle's momentum and the (net) force acting on the particle.

9.17 Calculate the momentum of a system of particles as the product of the system's total mass and its center-of-mass velocity.

9.18 Apply the relationship between a system's center-of-mass momentum and the net force acting on the system.



$\boxed{\vec{p} = m\vec{v}}$

9-3 Linear Momentum

Linear momentum $\vec{p}$ of a particle of mass *m* and velocity is defined as: $\vec{p} = m\vec{v}$

The SI unit for lineal momentum is the kg.m/s

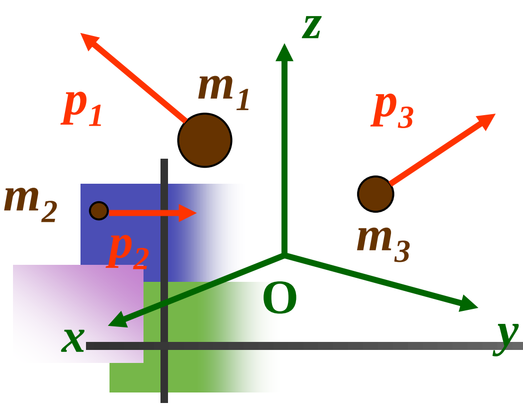
In equation form: $\vec{F}_{net} = \dfrac{d\vec{p}}{dt}$ We will prove that this equation using

Newton's second law $\vec{p} = m\vec{v} \rightarrow \dfrac{d\vec{p}}{dt} = \dfrac{d}{dt}(m\vec{v}) = m\dfrac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{net}$

with constant mass(質量固定時)

牛頓第二運動定律: 力 = 動量之時間變化率

$$\boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}}$$



The Linear Momentum of a System of Particles

In this section we will extedend the definition of linear momentum to a system of particles. The i-th particle has mass m_i, velocity $\vec{v}_i$, and linear momentum $\vec{p}_i$

$$\boxed{\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n = M\vec{v}_{com}}$$

Eq. (9-25)

$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}_{net}}$$

Eq. (9-23)

9-3 Linear Momentum

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

- Taking the time derivative we can write Newton's second law for a system of particles as:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \quad \text{(system of particles)}, \qquad \textbf{Eq. (9-27)}$$

- The net external force on a system changes linear momentum
- Without a net external force, the *tota*l linear momentum of a system of particles cannot change

9-4 Collision and Impulse碰撞與衝量J:

Learning Objectives

9.19 Identify that impulse is a vector quantity and thus has both magnitude and direction and components.

9.20 Apply the relationship between impulse and momentum change.

9.21 Apply the relationship between impulse, average force, and the time interval taken by the impulse.

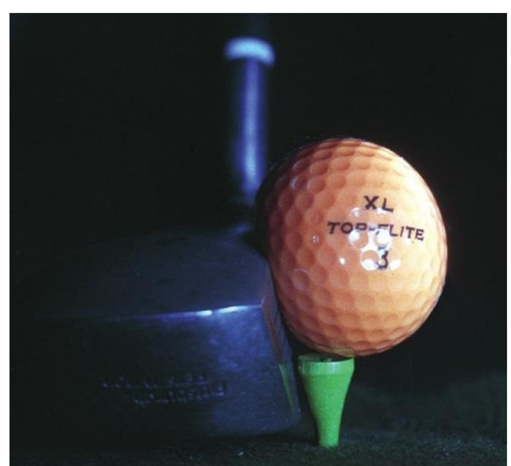
9.22 Apply the constant-acceleration equations to relate impulse to force.

9.23 Given force as a function of time, calculate the impulse (and thus also the momentum change) by integrating the function.

9.24 Given a graph of force versus time, calculate the impulse (and thus also the momentum change) by graphical integration.

9.25 In a continuous series of collisions by projectiles, calculate average force on the target by relating it to the mass collision rate and the velocity change of each projectile.


The collision of a ball with a bat collapses part of the ball.



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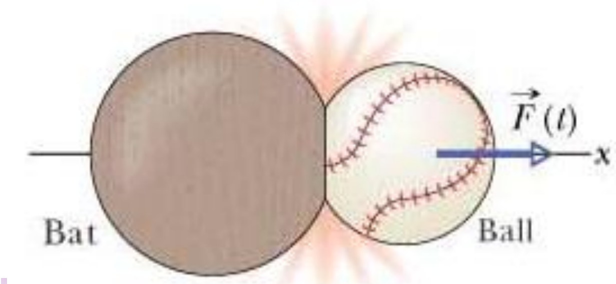
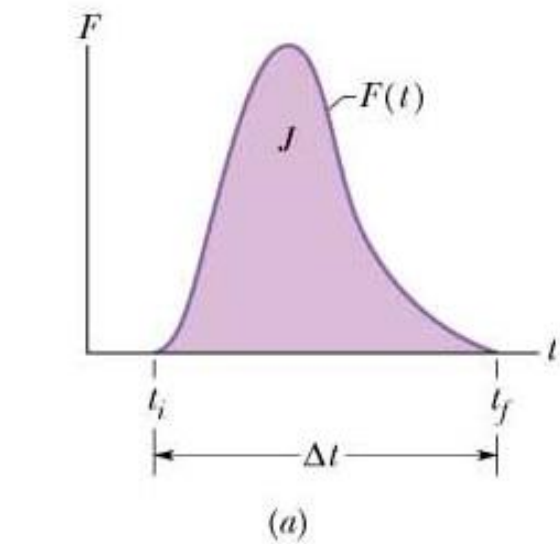


$$\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt \qquad \int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} =$$

change in momentum

Figure 9-8

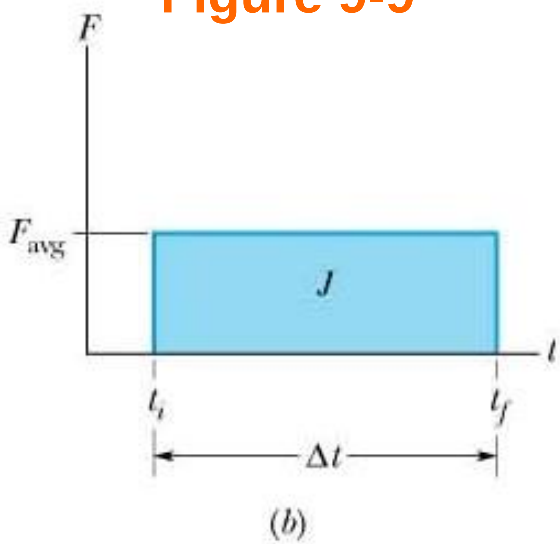
$$\boxed{\int_{t_i}^{t_f} \vec{F}(t)dt}$$ is known as the **impulse** $\vec{J}$ of the collision



$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

The magnitude of $\vec{J}$ is equal to the area under the *F* versus *t* plot of fig.a $\boxed{\rightarrow \Delta\vec{p} = \vec{J}}$

Figure 9-9



In many situations se do not know how the force changes with time but we know the average magnitude F_{ave} of the collision force.
The magnitude of the impulse is given by:

$J = F_{ave}\,\Delta t$ where $\Delta t = t_f - t_i$

Impulse and Momentum衝量與動量:

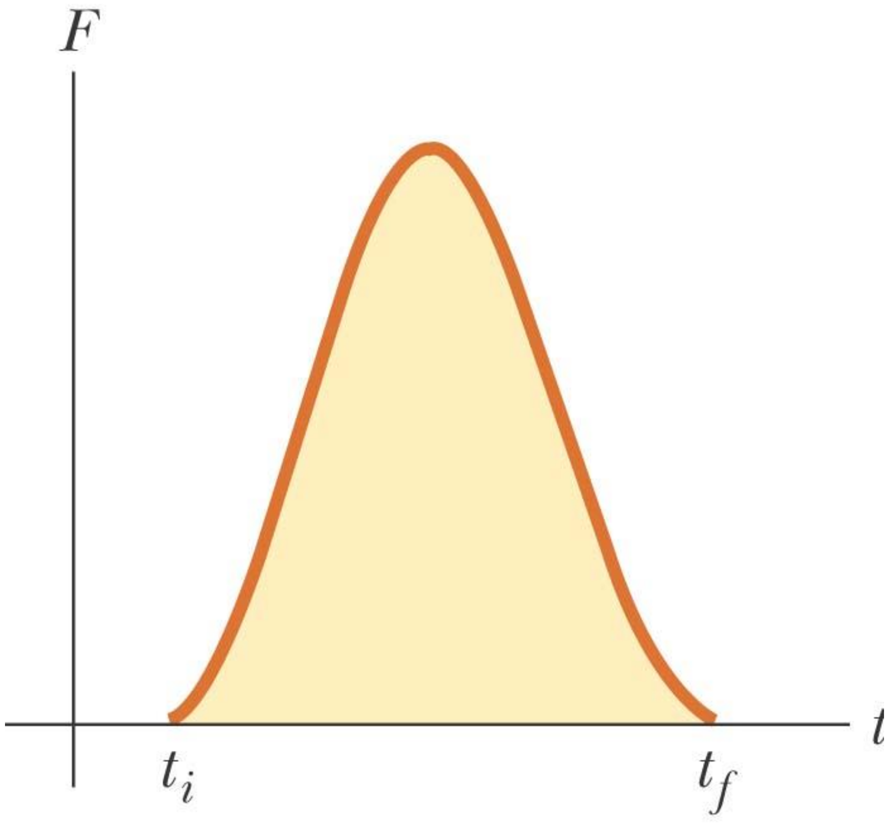
- From Newton's Second Law, $\sum \vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt$
- Solving for *d***p** gives $d\vec{\mathbf{p}} = \sum \vec{\mathbf{F}}\, dt$
- Integrating to find the change in momentum over some time interval

$$\boxed{\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \int_{t_i}^{t_f} \vec{\mathbf{F}}\, dt = \vec{\mathbf{J}}}$$

- The integral is called the *impulse*, $\vec{\mathbf{J}}$, of the force $\vec{\mathbf{F}}$ acting on an object over Δt

More About Impulse

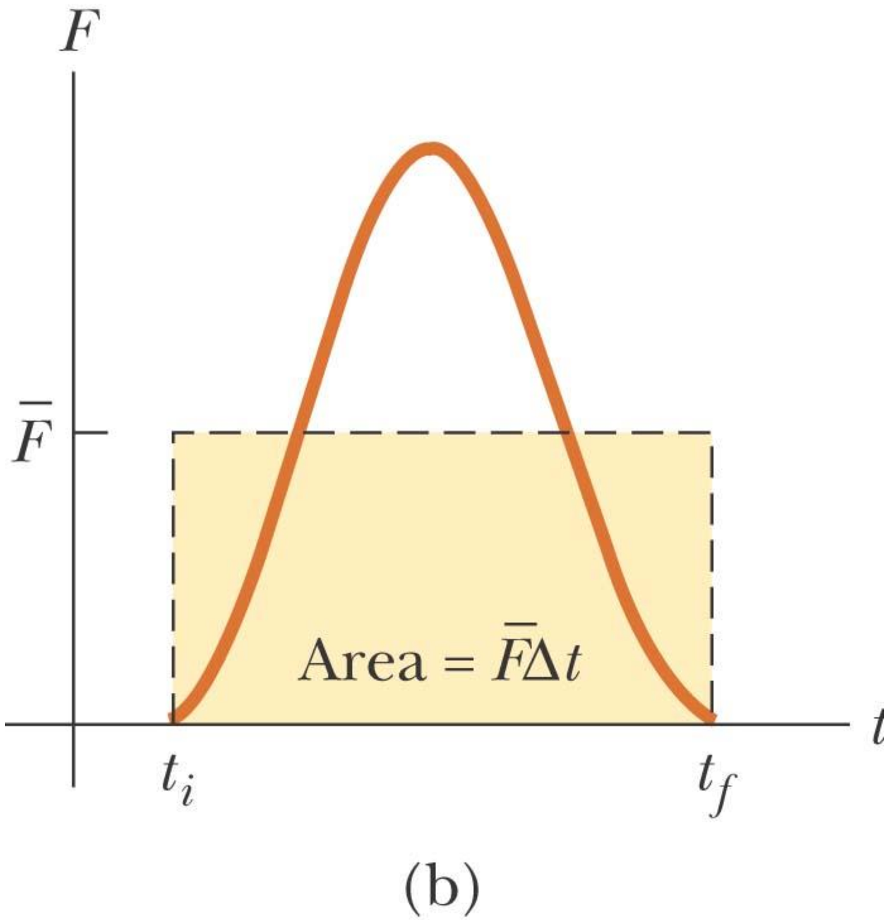
- Impulse is a vector quantity
- The magnitude of the impulse is equal to the area under the force-time curve
- Dimensions of impulse are M L / T
- Impulse is not a property of the particle, but a measure of the change in momentum of the particle

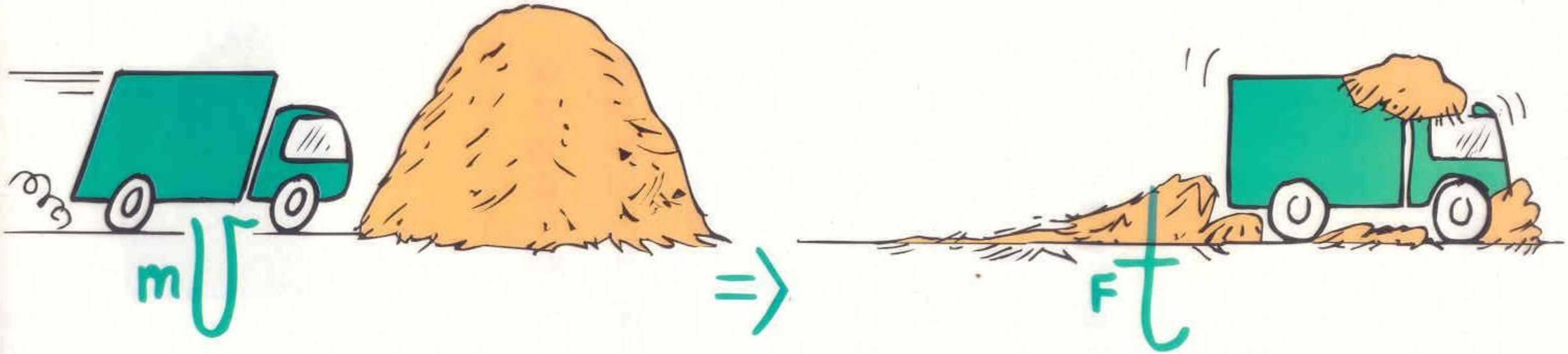


(a)

Impulse, Final

- The impulse can also be found by using the time averaged force F_{avg}(平均作用力)
- $\vec{\mathbf{J}} = \sum \vec{\mathbf{F}}_{avg} \Delta t$
- This would give the same impulse as the time-varying force does




mv
⇒
Ft

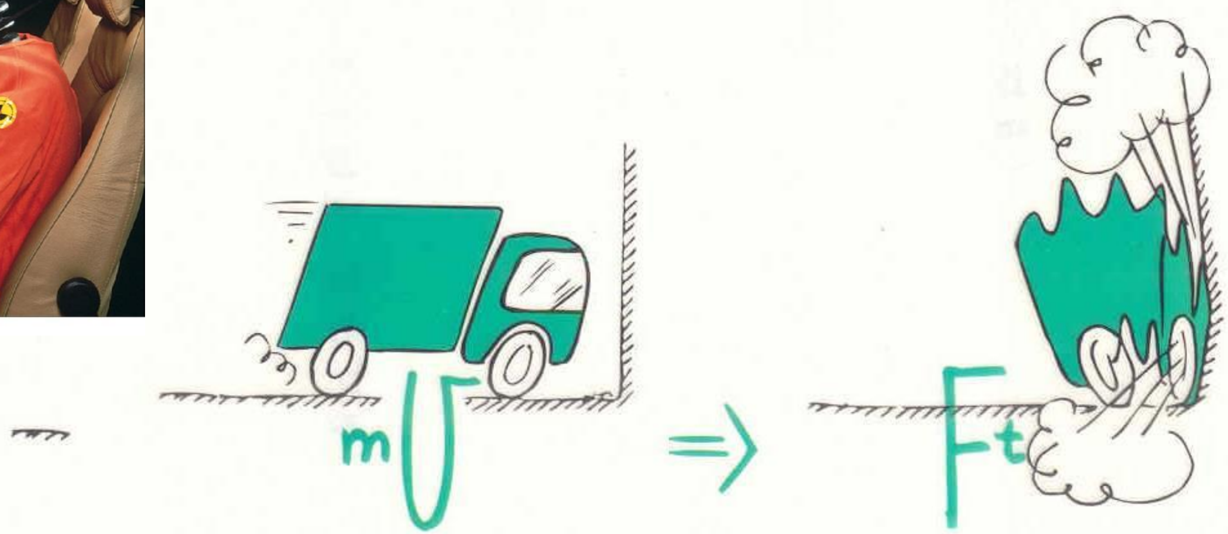

mv
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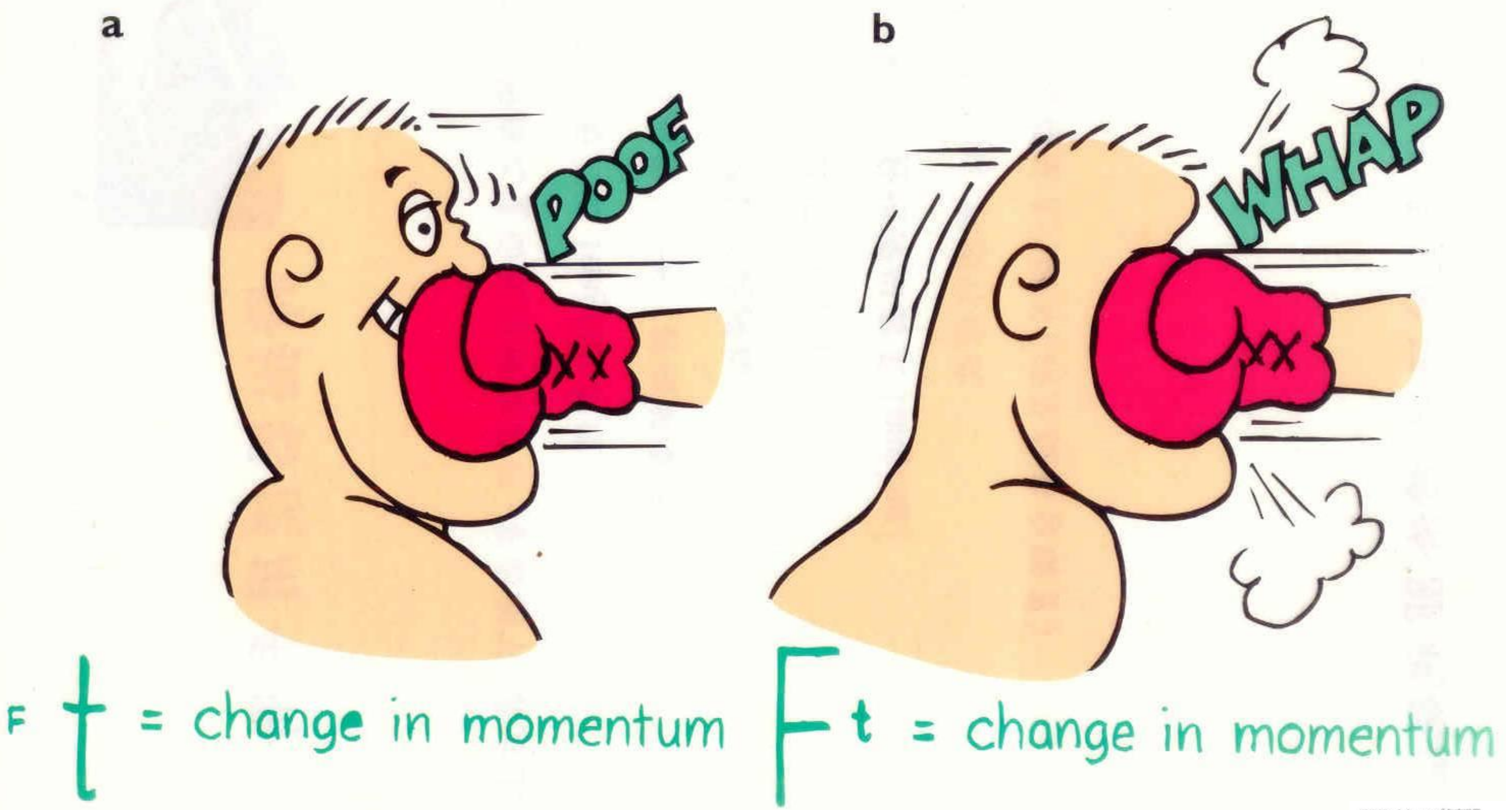
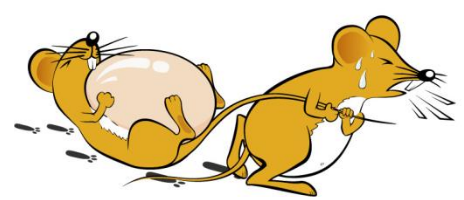
Fig. 5.6





Series of Collisions

Consider a target which collides with a steady stream of identical particles of mass *m* and velocity $\vec{v}$ along the *x*-axis

F_{ave}

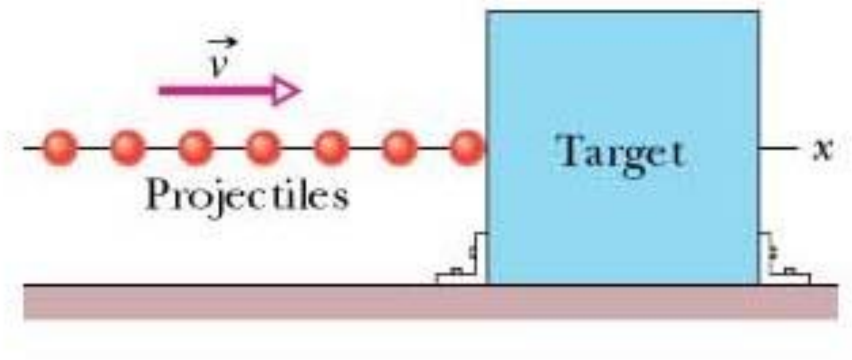


Figure 9-10

9-4 Collision and Impulse

- For a steady stream of *n* projectiles, each undergoes a momentum change *Δp*

$$J = -n\,\Delta p,$$ Eq. (9-36)

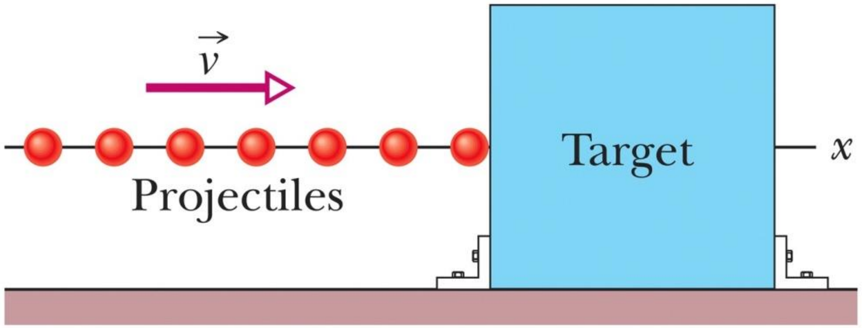


Figure 9-10

- The average force is: $$F_{\text{avg}} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\,\Delta p = -\frac{n}{\Delta t}\,m\,\Delta v.$$ Eq. (9-37)
  - If the particles stop:

$$\Delta v = v_f - v_i = 0 - v = -v,$$ Eq. (9-38)

  - If the particles bounce back with equal speed:

$$\Delta v = v_f - v_i = -v - v = -2v.$$ Eq. (9-39)

9-4 Collision and Impulse

- The product *nm* is the total mass for *n* collisions so we can write:

$$\boxed{F_{avg} = -\frac{\Delta m}{\Delta t}\Delta v.}$$

Eq. (9-40)

9-5 Conservation of Linear Momentum

Learning Objectives

9.26 For an isolated system of particles, apply the conservation of linear momenta to relate the initial momenta of the particles to their momenta at a later instant.

9.27 Identify that the conservation of linear momentum can be done along an individual axis by using components along that axis, *provided* there is no net external force component along that axis.

9-5 Conservation of Linear Momentum

- For an impulse of zero we find:

$$\vec{P} = \text{constant} \qquad \text{(closed, isolated system).}$$

Eq. (9-42)

- Which says that:

> If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

- This is called the **law of conservation of linear momentum**

- Check the components of the net external force to know if you should apply this

> If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

Conservation of Linear Momentum

Consider a system of particles for which $\vec{F}_{net} = 0$

$$\frac{d\vec{P}}{dt} = \vec{F}_{net} = 0 \rightarrow \vec{P} = \text{ Constant}$$

$$\vec{\mathbf{p}}_{tot} = M\vec{\mathbf{v}}_{CM} = \text{constant}\left(\text{when} \sum \mathbf{F}_{ext} = 0\right)$$

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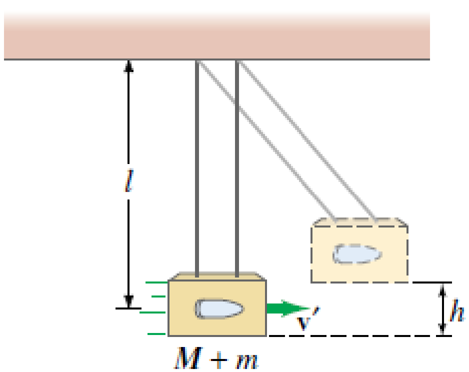
9-6 Momentum and Kinetic Energy in Collisions

Learning Objectives

9.28 Distinguish between elastic collisions, inelastic collisions, and completely inelastic collisions.

9.29 Identify a one-dimensional collision as one where the objects move along a single axis, both before and after the collision.



9.30 Apply the conservation of momentum for an isolated one-dimensional collision to relate the initial momenta of the objects to their momenta after the collision.

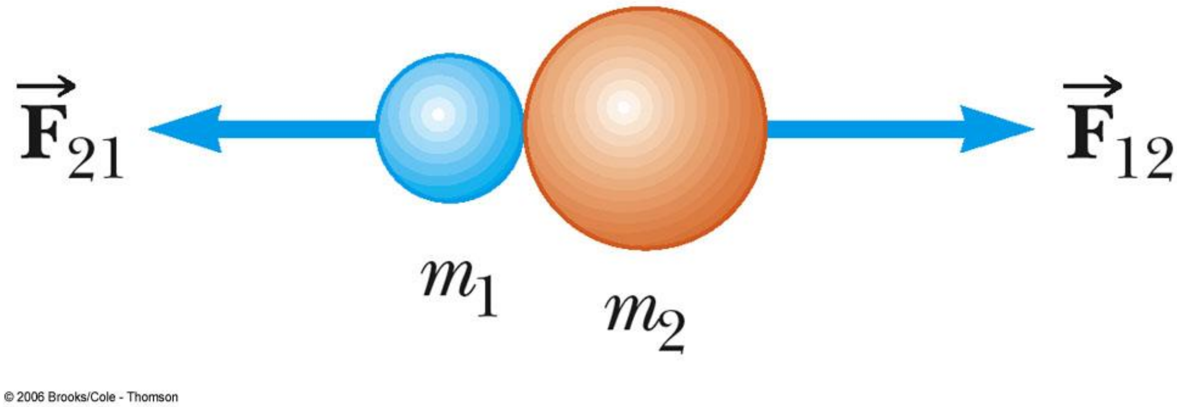
9.31 Identify that in an isolated system, the momentum and velocity of the center of mass are not changed even if the objects collide.

9-6 Momentum and Kinetic Energy in Collisions

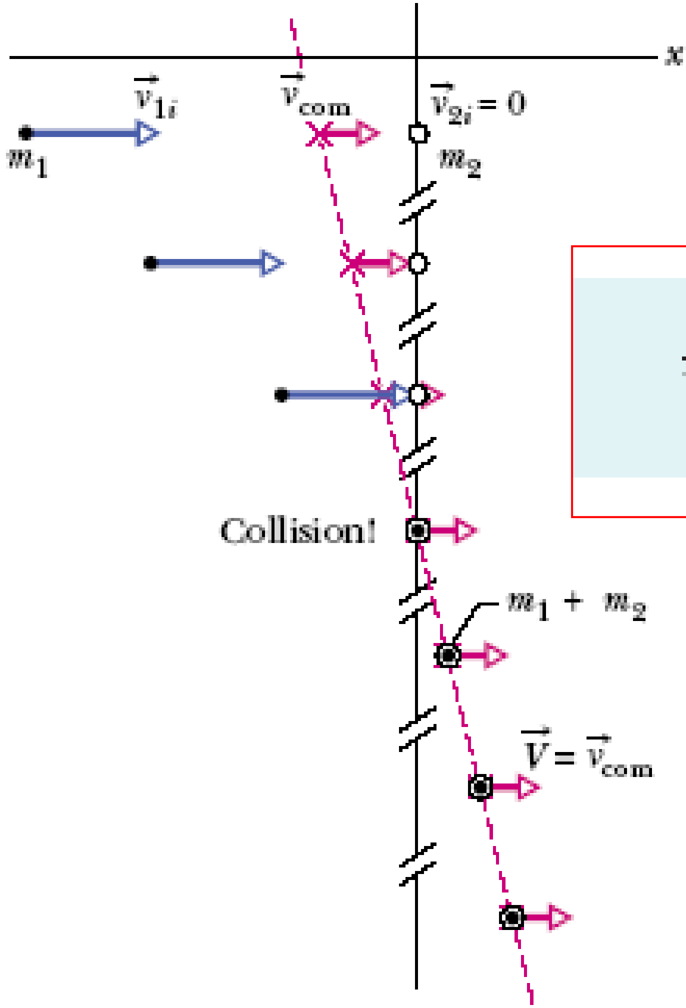
- Types of collisions(碰撞之型式):
- **Elastic collisions**(彈性碰撞):
    - Total kinetic energy is unchanged (conserved)
    - A useful approximation for common situations
    - In real collisions, some energy is always transferred
- **Inelastic collisions**(非彈性碰撞): some energy is transferred
- **Completely inelastic collisions**(完全非彈性碰撞):
    - The objects stick together
    - Greatest loss of kinetic energy

Collisions 碰撞:

- Collisions may be the result of direct contact
- The impulsive forces may vary in time in complicated ways
  - This force is internal to the system
- Momentum is conserved 碰撞前後總動量不變!



Velocity of the center of mass:

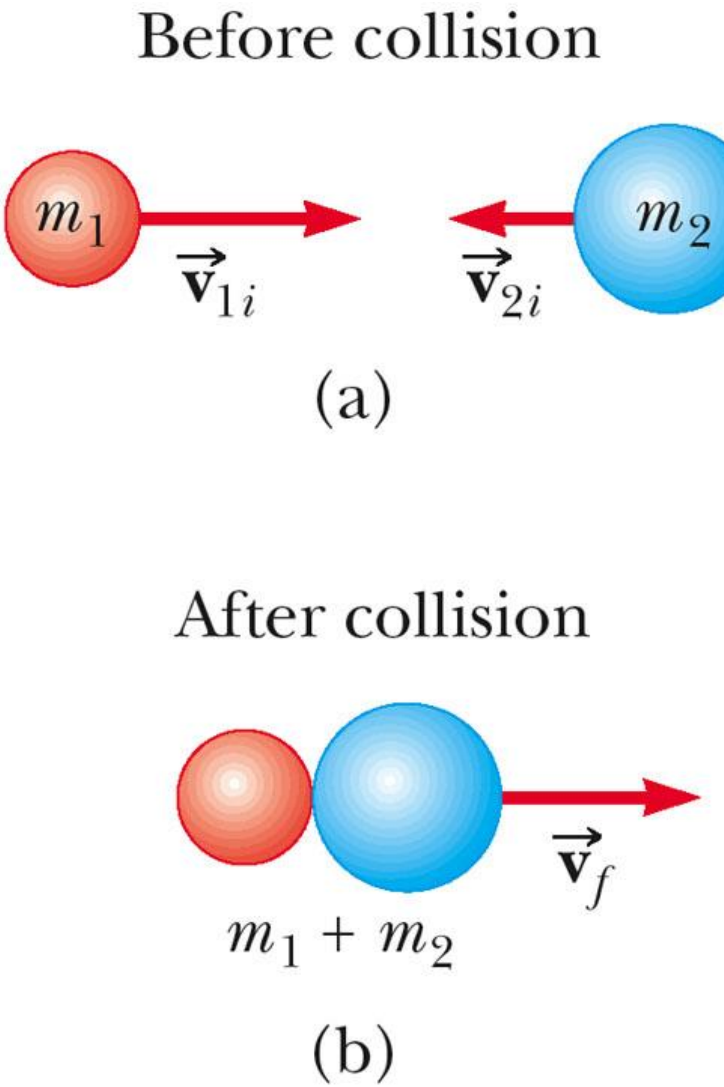


$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}.$$

Completely Inelastic Collisions完全非彈性碰撞:

- Since the objects stick together, they share the same velocity after the collision
- $m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)\, v_f$,

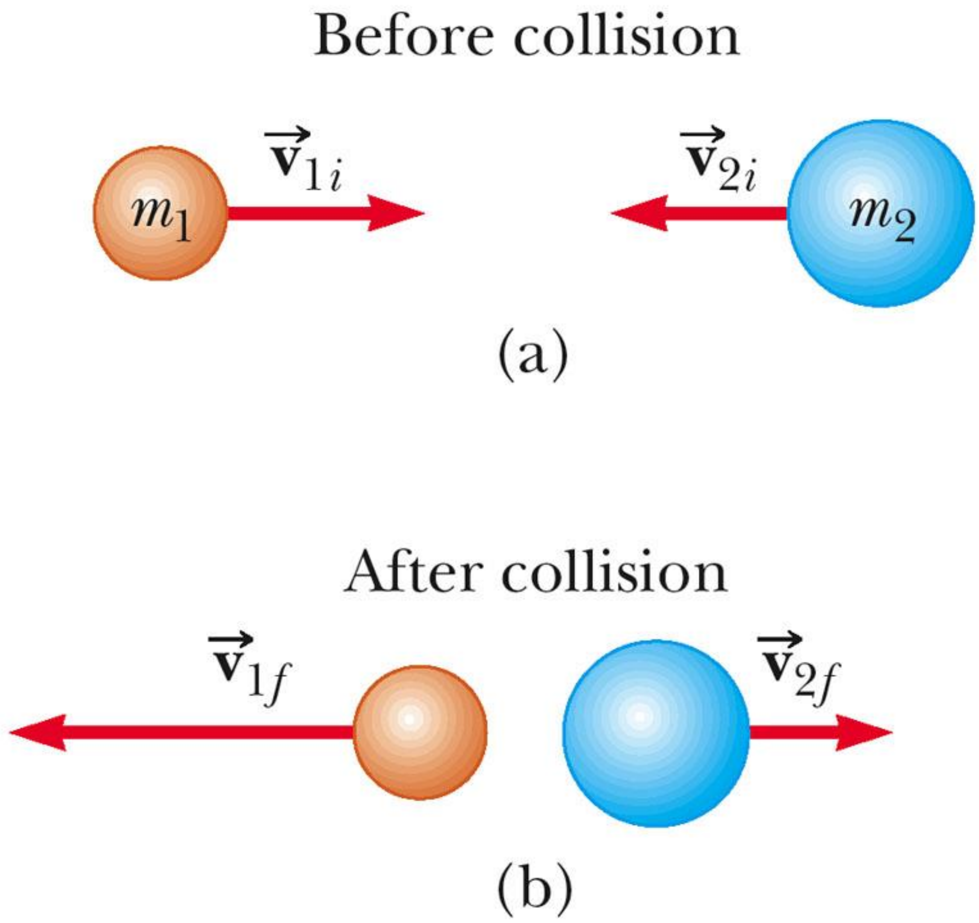
$v_f = v_{com}$



Elastic Collisions彈性碰撞:

- Both momentum and kinetic energy are conserved

$$\boxed{\begin{gathered} m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = \\ m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} \\ \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \\ \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \end{gathered}}$$



If the system is isolated i.e. the net force $\vec{F}_{net} = 0$ linear momentum is conserved

The conervation of linear momentum is true regardless of the the collision type

This is a powerful rule thet allows us to determine the results of a collision without knowing the details. Collisions are divided into two broad classes: elastic and inelastic.

A collision is elastic if there is no loss of kinetic energy i.e. $K_i = K_f$

A collision is inelastic if kinetic energy is lost during the collision due to conversion into other forms of energy. In this case we have: $K_f < K_i$

A special case of inelastic collisions are known as completely inlelastic. In these collisions the two colliding objects stick together and they move as a single body. In these collisions the loss of kinetic energy is maximum (完全非彈性碰撞:**動能損失最大**)

One-Dimensional Completely Inelastic Collision

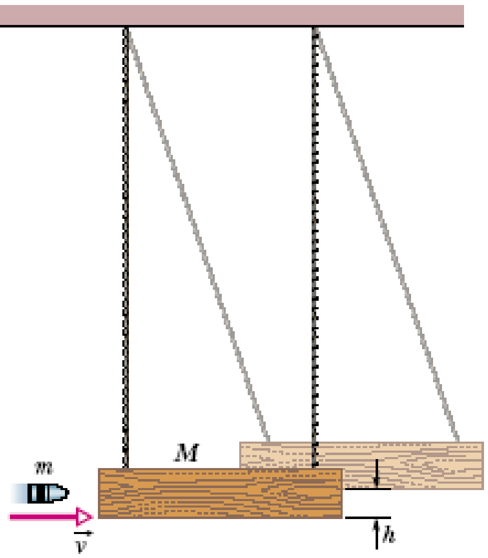
Ballistic pendulum:

$$m_1 v_{1i} = (m_1 + m_2)V \quad (9\text{-}52)$$

$$V = \frac{m_1}{m_1 + m_2} v_{1i}. \quad (9\text{-}53)$$

The *ballistic pendulum* was used to measure the speeds of bullets before electronic timing devices were developed. The version shown in Fig. 9-17 consists of a large block of wood of mass $M = 5.4$ kg, hanging from two long cords. A bullet of mass $m = 9.5$ g is fired into the block, coming quickly to rest. The *block* + *bullet* then swing upward, their center of mass rising a vertical distance $h = 6.3$ cm before the pendulum comes momentarily to rest at the end of its arc. What is the speed of the bullet just prior to the collision?

Solution: We can see that the bullet's speed v must determine the rise height h. However, a **Key Idea** is that we cannot



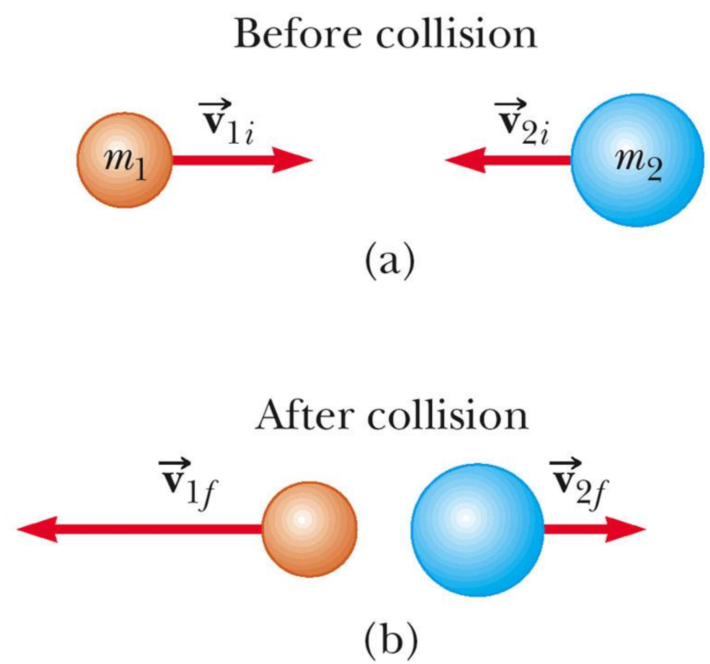
9-7 Elastic Collisions in One Dimension

Learning Objectives

9.32 For isolated elastic collisions in one dimension, apply the conservation laws for both the total energy and the net momentum of the colliding bodies to relate the initial values to the values after the collision.

9.33 For a projectile hitting a stationary target, identify the resulting motion for the three general cases: equal masses, target more massive than projectile, projectile more massive than target.

$$\boldsymbol{v}_{1f}, \boldsymbol{v}_{2f} = ?$$



9-7 Elastic Collisions in One Dimension

- Total kinetic energy is conserved in elastic collisions

For a stationary target (v_{2i} = 0), conservation laws give:

$$\boxed{m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad \text{(linear momentum).}}$$

Eq. (9-63)

$$\boxed{\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \qquad \text{(kinetic energy).}}$$

Eq. (9-64)

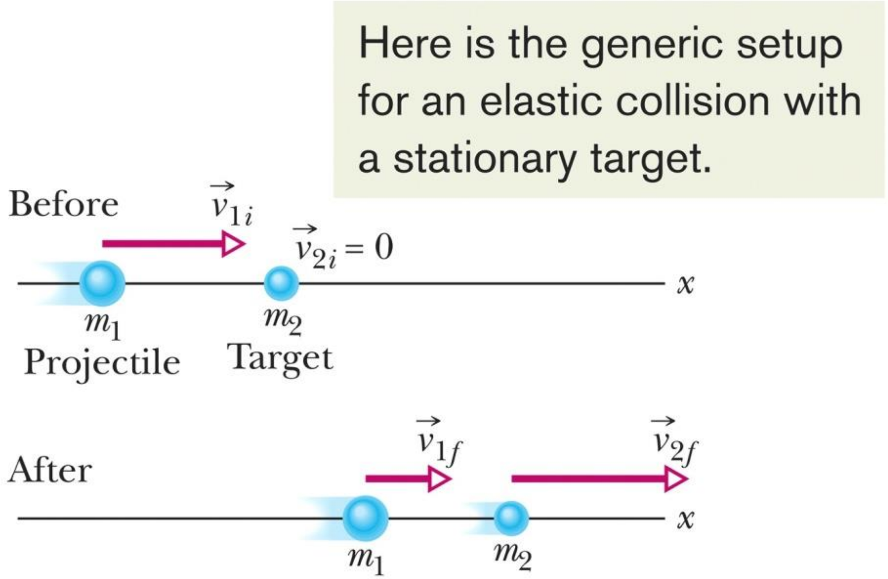


Figure 9-18

9-7 Elastic Collisions in One Dimension

- With some algebra we get:

$$\boxed{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}} \quad \text{Eq. (9-67)}$$

$$\boxed{v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.} \quad \text{Eq. (9-68)}$$

- Results
  - Equal masses: $v_{1f} = 0$, $v_{2f} = v_{1i}$: the first object stops, The two colliding objects have exchanged velocities
  - Massive target, $m_2 >> m_1$: the first object just bounces back, speed mostly unchanged
  - Massive projectile, $m_1 >> m_2$: $v_{1f} \approx v_{1i}$, $v_{2f} \approx 2v_{1i}$: the first object keeps going, the target flies forward at about twice its speed

1. A massive target

$m_2 >> m_1 \rightarrow m_1/m_2 << 1$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} v_{1i} \approx -v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2\left(\frac{m_1}{m_2}\right)}{\frac{m_1}{m_2} + 1} v_{1i} \approx 2\left(\frac{m_1}{m_2}\right) v_{1i}$$

2. A massive projectile

$m_1 >> m_2 \rightarrow m_2/m_1 << 1$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} v_{1i} \approx v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2}{1 + \frac{m_2}{m_1}} v_{1i} \approx 2v_{1i}$$

9-7 Elastic Collisions in One Dimension

- For a target that is also moving ($v_{2i} \neq 0$), we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$ Eq. (9-75)

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$ Eq. (9-76)

Here is the generic setup for an elastic collision with a moving target.

Figure 9-19

9-8 Collisions in Two Dimensions

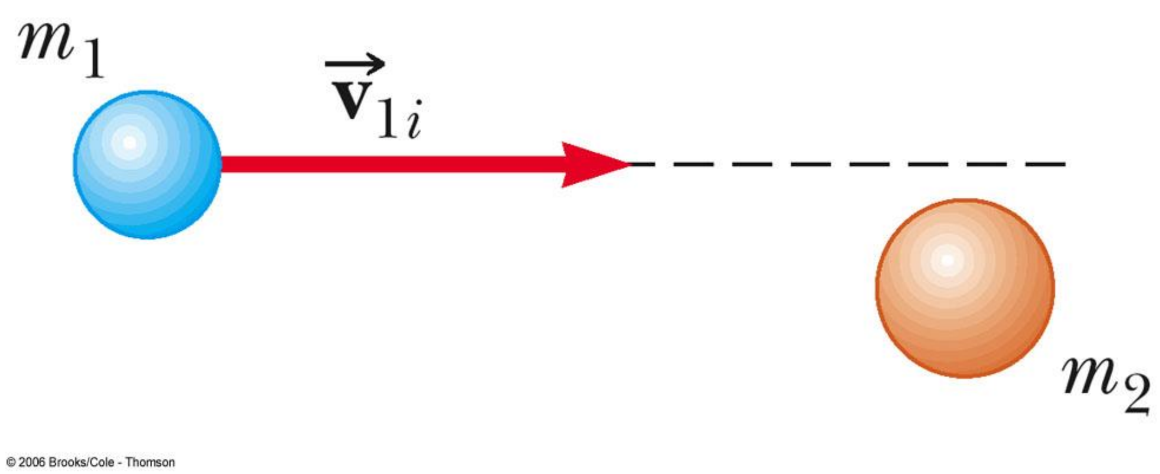
Learning Objectives

9.34 For an isolated system in which a two-dimensional collision occurs, apply the conservation of momentum along each axis of a coordinate system to relate the momentum components along an axis before the collision to the momentum components *along the same axis* after the collision.

9.35 For an isolated system in which a two-dimensional *elastic* collision occurs, (a) apply the conservation of momentum along each axis to relate the momentum components along an axis before the collision to the momentum components *along the same axis* after the collision and (b) apply the conservation of total kinetic energy to relate the kinetic energies before and after the collision.

Two-Dimensional Collisions二維碰撞:

- Particle 1 is moving at velocity $\boldsymbol{v}_{1i}$ and particle 2 is at rest
- 碰撞前:In the *x*-direction, the initial momentum is m_1v_{1i}
- 碰撞前:In the *y*-direction, the initial momentum is 0



9-8 Collisions in Two Dimensions

- Apply the conservation of momentum along each axis
- Apply conservation of energy for elastic collisions

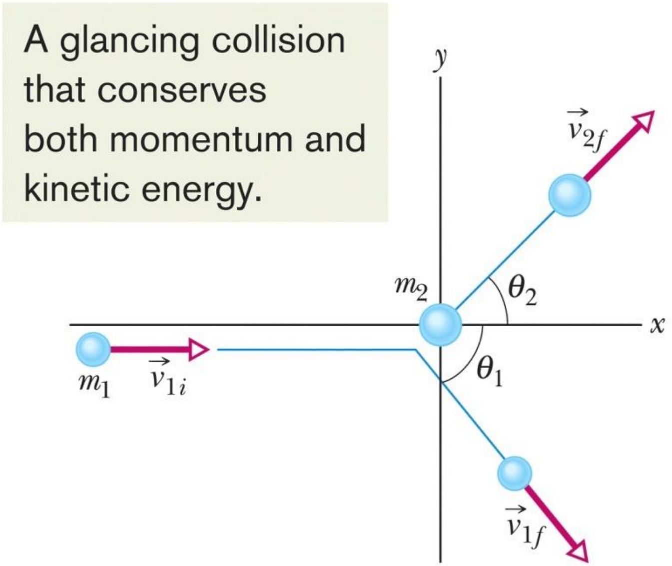


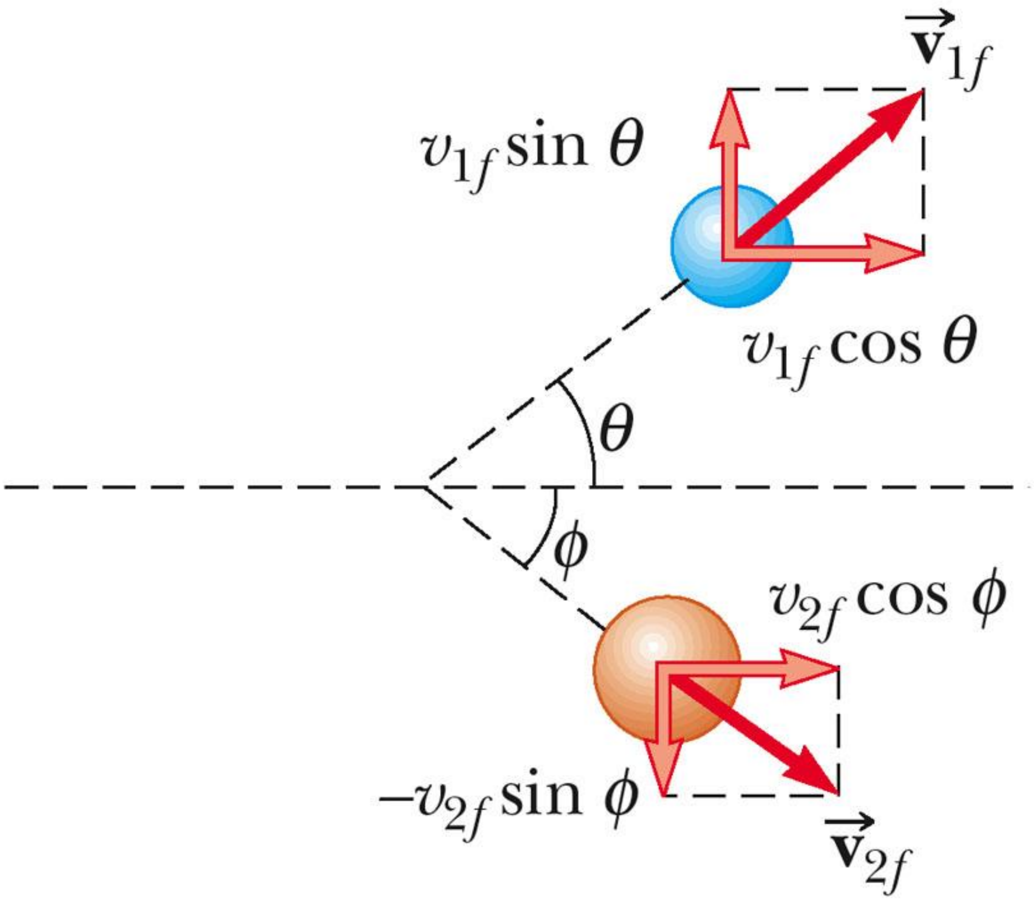
Figure 9-21

Example For Figure 9-21 for a stationary target:

- Along x: $$\boxed{m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2,}$$ **Eq. (9-79)**
- Along y: $$\boxed{0 = -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2.}$$ **Eq. (9-80)**
- Energy: $$\boxed{\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2}$$ **Eq. (9-81)**

Two-Dimensional Collision, example cont

- 碰撞後:After the collision, the momentum in the *x*-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$
- 碰撞後:After the collision, the momentum in the *y*-direction is $m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$
- *若質量相同且為彈性碰撞: $\theta+\phi = 90^o$ ––Why?*



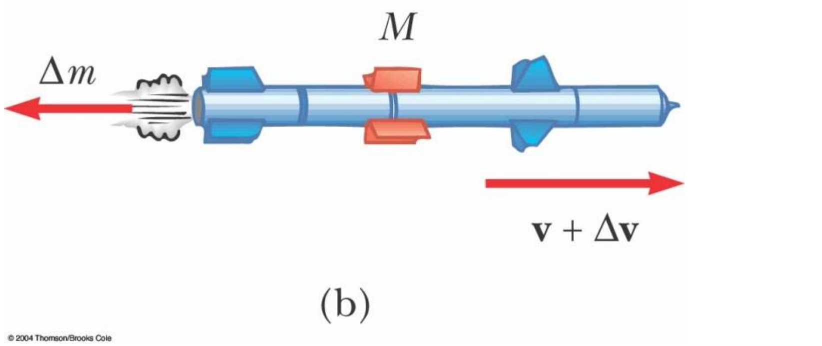
9-9 Systems with Varying Mass: A Rocket

Learning Objectives

9.36 Apply the first rocket equation to relate the rate at which the rocket loses mass, the speed of the exhaust products relative to the rocket, the mass of the rocket, and the acceleration of the rocket.

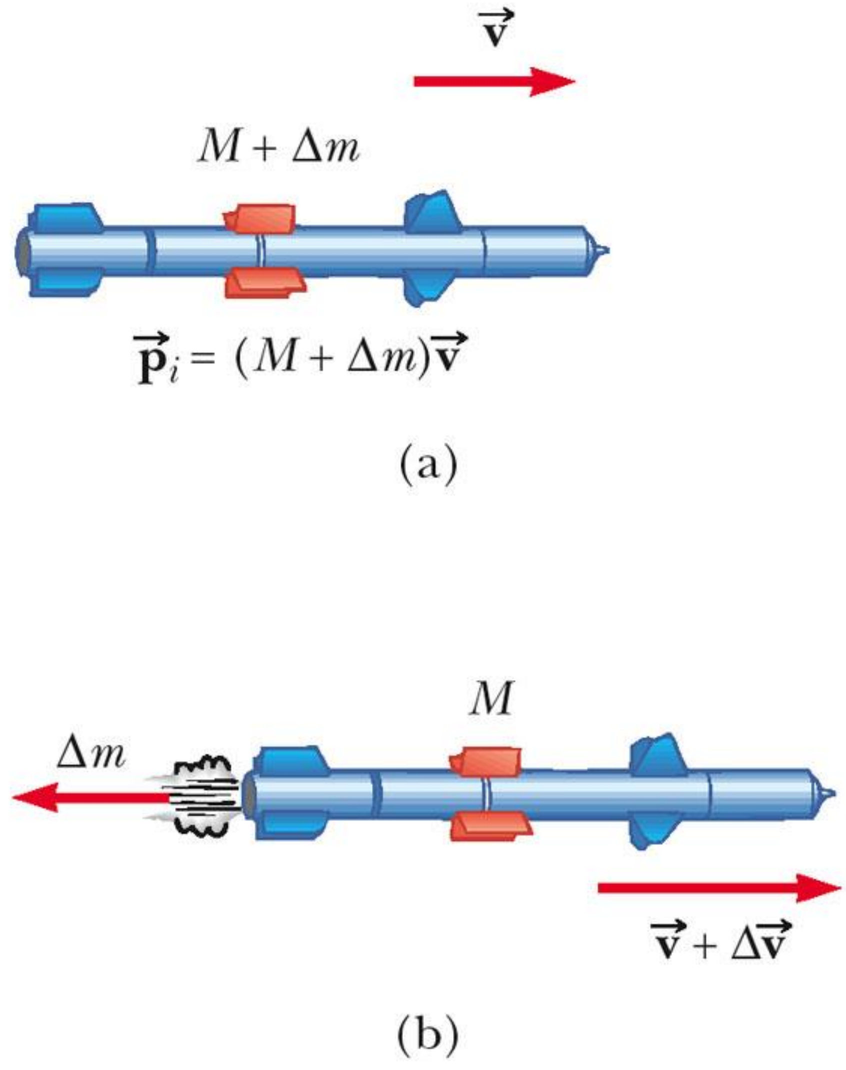
9.37 Apply the second rocket equation to relate the change in the rocket's speed to the relative speed of the exhaust products and the initial and final mass of the rocket.

9.38 For a moving system undergoing a change in mass at a given rate, relate that rate to the change in momentum.



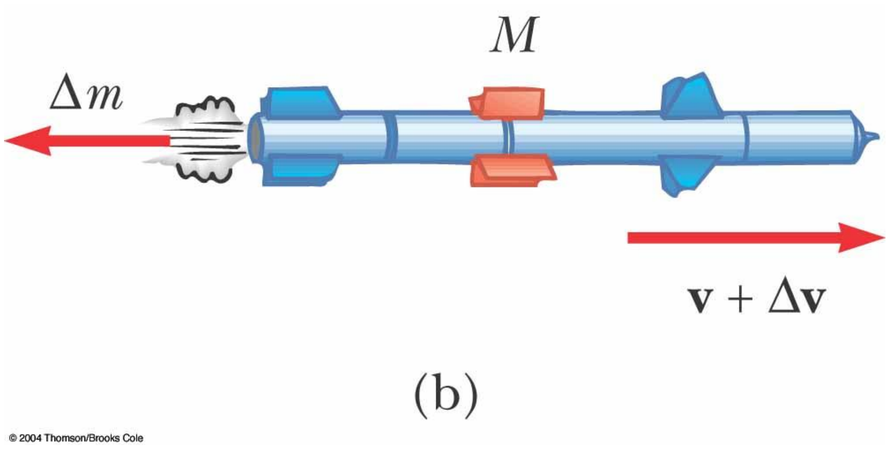
Rocket Propulsion火箭推進:

- The initial mass of the rocket plus all its fuel is $M + \Delta m$ at time t_i and velocity $\vec{\mathbf{v}}$
- The initial momentum of the system is $(M + \Delta m)v$



Rocket Propulsion, 3

- At some time $t + \Delta t$, the rocket's mass has been reduced to M and an amount of fuel, Δm has been ejected
- The rocket's speed has increased by $\Delta \mathbf{v}$



求火箭速度：

Conserve momentum $P_i = P_f$:

v_{ex} = 氣體(燃料)相對於火箭之噴發速率(固定)

$$(M+\Delta m)v = M(v+\Delta v) + \Delta m(v - v_{ex})$$

$$\Rightarrow M\Delta v = v_{ex}\Delta m = -v_{ex}\Delta M \qquad (\Delta M = -\Delta m)$$

$$\therefore M\frac{\Delta v}{\Delta t} = -v_{ex}\frac{\Delta M}{\Delta t}$$

$$or \quad M\frac{dv}{dt} = -v_{ex}\frac{dM}{dt}$$

$$\Rightarrow dv = -v_{ex}\frac{dM}{M}$$

$$\int_{v_i}^{v_f} dv = -v_{ex}\int_{M_i}^{M_f}\frac{dM}{M}$$

$$v_f - v_i = -v_{ex}\left[\ln(M_f) - \ln(M_i)\right]$$

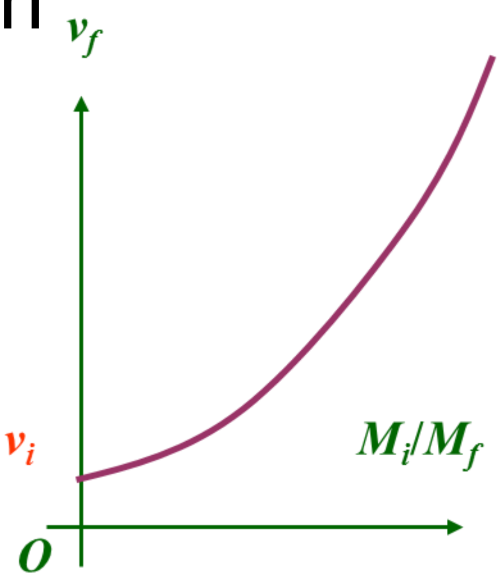
$$\Rightarrow v_f = v_i + v_{ex}\ln\left(\frac{M_i}{M_f}\right)$$

Rocket Propulsion, 5

- The basic equation for rocket propulsion is

$$v_f - v_i = v_{rel} \ln\left(\frac{M_i}{M_f}\right)$$

- The increase in rocket speed is proportional to the speed of the escape gases (v_{rel}: *氣體噴射速度*)



9-9 Systems with Varying Mass: A Rocket

- The first rocket equation:

$$\boxed{Rv_{\text{rel}} = Ma}$$ Eq. (9-87)

- *R* is the mass rate of fuel consumption
- The left side of the equation is **thrust**, *T*
- Derive the velocity change for a given consumption of fuel as the second rocket equation:

$$\boxed{v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f}}$$ Eq. (9-88)

Sample 9.09

A rocket whose initial mass M_i is 850 kg consumes fuel at the rate $R = 2.3$ kg/s. The speed v_{rel} of the exhaust gases relative to the rocket engine is 2800 m/s. What thrust does the rocket engine provide?

KEY IDEA Thrust T is equal to the product of the fuel consumption rate R and the relative speed v_{rel} at which exhaust gases are expelled, as given by Eq. 9-87.

Calculation: Here we find

$$T = Rv_{rel} = (2.3\text{ kg/s})(2800\text{ m/s}) = 6440\text{ N} \approx 6400\text{ N}. \quad \text{(Answer)}$$

(b) What is the initial acceleration of the rocket?

KEY IDEA We can relate the thrust T of a rocket to the magnitude a of the resulting acceleration with $T = Ma$, where M is the rocket's mass. However, M decreases and a increases as fuel is consumed. Because we want the initial value of a here, we must use the intial value M_i of the mass.

Calculation: We find

$$a = \frac{T}{M_i} = \frac{6440\text{ N}}{850\text{ kg}} = 7.6\text{ m/s}^2. \quad \text{(Answer)}$$

To be launched from Earth's surface, a rocket must have an initial acceleration greater than $g = 9.8\text{ m/s}^2$. Put another way, the thrust T of the rocket engine must exceed the initial gravitational force on the rocket, which here has the magnitude M_ig, which gives us $(850\text{ kg})(9.8\text{ m/s}^2)$, or 8330 N. Because the acceleration or thrust requirement is not met (here $T = 6400$ N), our rocket could not be launched from Earth's surface by itself; it would require another, more powerful, rocket.

~若其中攜帶50-kg之衛星，末速度=?

9 Summary

Linear Momentum & Newton's 2nd Law

- Linear momentum defined as:

$$\vec{P} = M\vec{v}_{com}$$

Eq. (9-25)

- Write Newton's 2nd law:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

Eq. (9-27)

Conservation of Linear Momentum

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system).}$$

Eq. (9-42)

Collision and Impulse

- Defined as:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$

Eq. (9-30)

- Impulse causes changes in linear momentum

Inelastic Collision in 1D

- Momentum conserved along that dimension

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

Eq. (9-51)

9 Summary

Motion of the Center of Mass

- Unaffected by collisions/internal forces

Elastic Collisions in One Dimension

- K is also conserved

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$ **Eq. (9-67)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$ **Eq. (9-68)**

Collisions in Two Dimensions

- Apply conservation of momentum along each axis individually
- Conserve K if elastic

Variable-Mass Systems

$$Rv_{\text{rel}} = Ma \quad \text{(first rocket equation).}$$

Eq. (9-87)

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \quad \text{(second rocket equation)}$$

Eq. (9-88)

CH 9習題:

24, 39, 49, 52, 53, and 59

真的向上滾?

水往上流
花東海岸公路欣賞地理「奇觀」的最佳景點