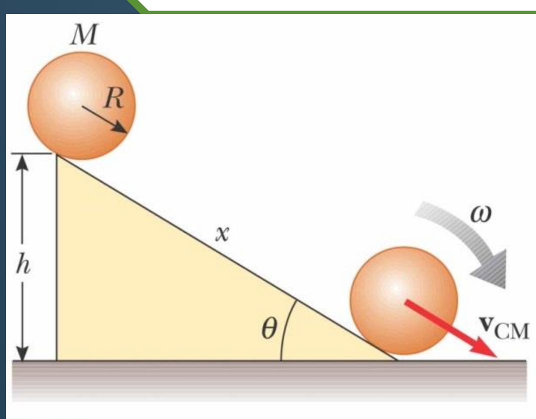
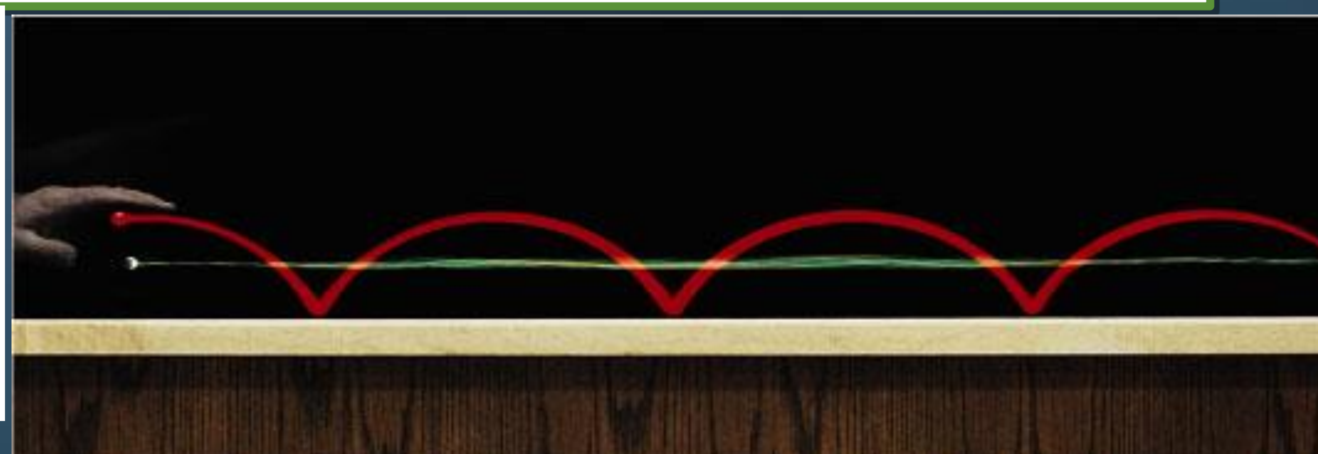
Chapter 11

Rolling, Torque, and Angular Momentum



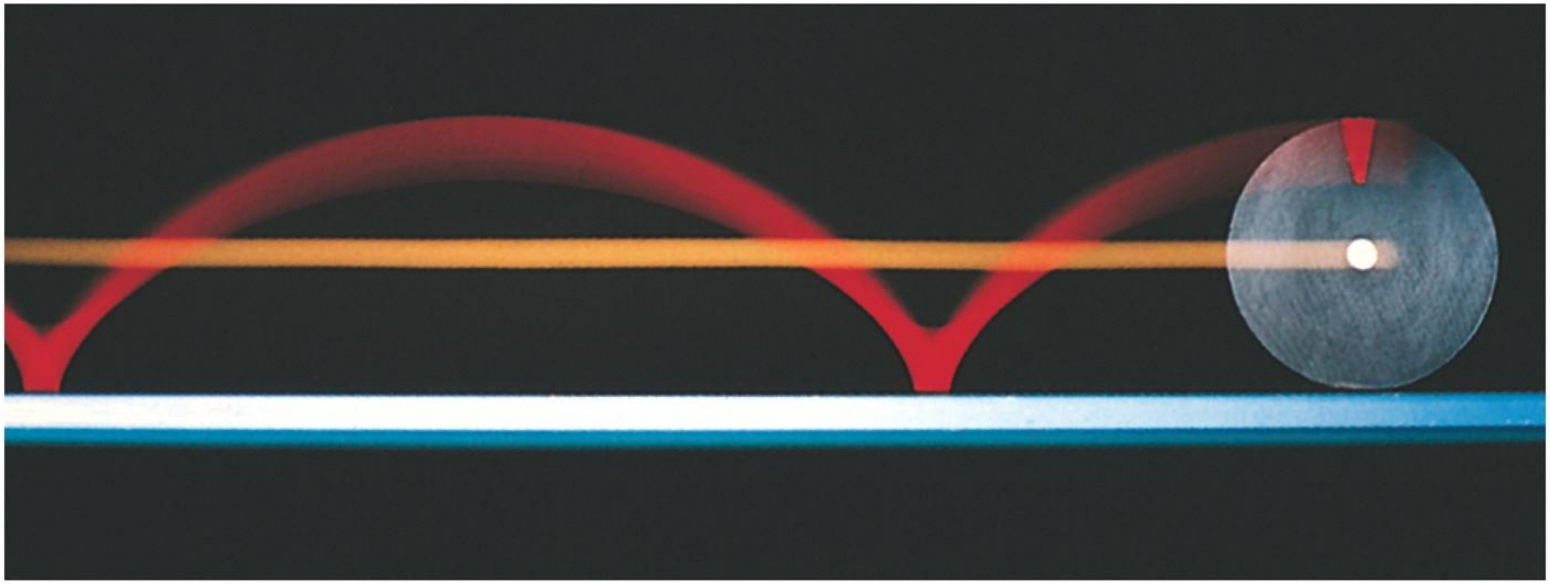


11-1 Rolling as Translation and Rotation Combined

Learning Objectives

11.01 Identify that smooth rolling can be considered as a combination of pure translation and pure rotation.

11.02 Apply the relationship between the center-of-mass speed and the angular speed of a body in smooth rolling.



Richard Megna/Fundamental Photographs

Figure 11-2

11-1 Rolling as Translation and Rotation Combined

- We consider only objects that roll smoothly (no slip)
- The center of mass (com) of the object moves in a straight line parallel to the surface
- The object rotates around the com as it moves
- The rotational motion is defined by:

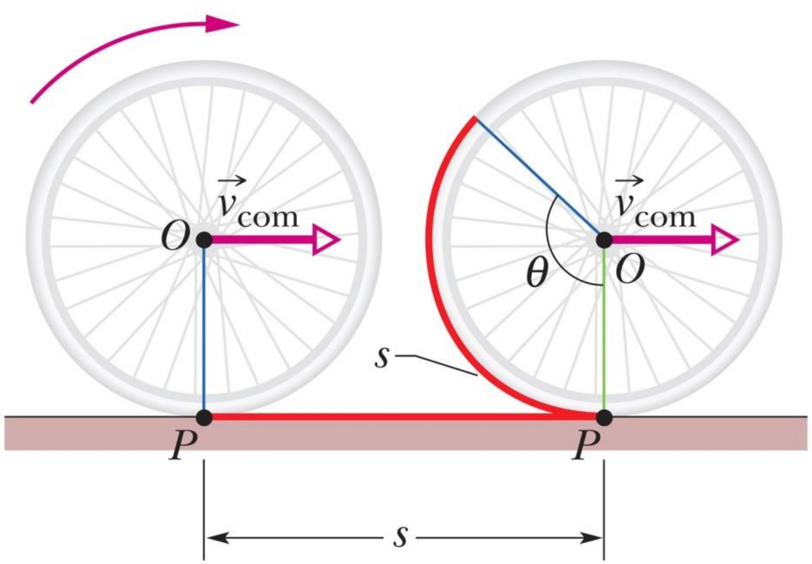
$$\boxed{s = \theta R,}$$

Eq. (11-1)

$$\boxed{v_{com} = \omega R}$$

Eq. (11-2)

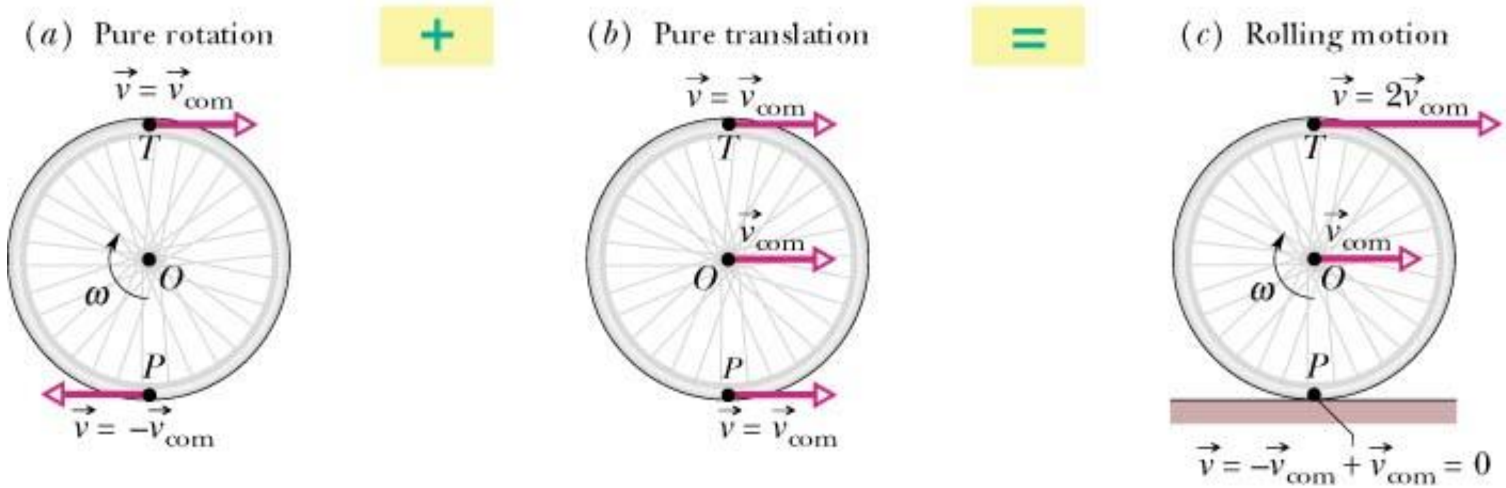
Figure 11-3



without slipping:

~**Rolling (滾動) = Translation (移動) + rotation (轉動)**

- How the velocities of translation and rotation combine at different points on the wheel



$$\boxed{v_{com} = R\omega}$$

Figure 11-4

The velocity of each point is the vector sum of

$$\vec{v} = \vec{v}_{com} + R\vec{\omega}$$

- 1.The velocity of point P is always zero.
- 2. The velocity of the center of mass O is $\vec{v}_{com}$ ($r = 0$).
- 3.The velocity of the top point T is equal to $2\vec{v}_{com}$.

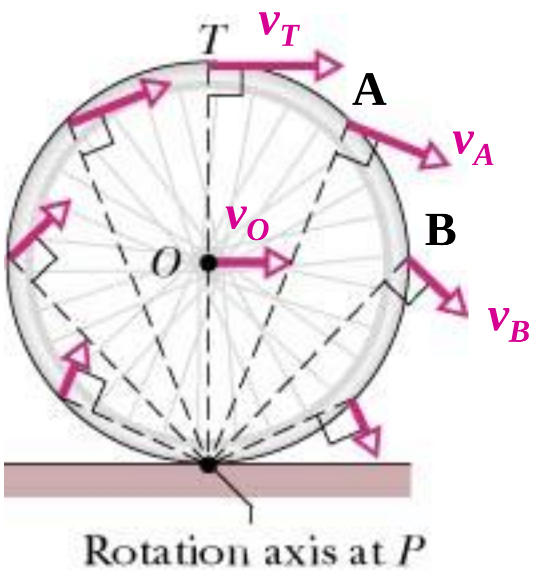


Figure 11-6

Rolling as Pure Rotation

Another way of looking at rolling is shown in the figure We consider rolling as a pure rotation about an axis of rotation that passes through the contact point ***P*** **(轉軸在P點)** between the wheel and the road. The angular velocity of the rotation is

$$\omega = \frac{v_{com}}{R}$$

- 1.At point **A** the velocity vector $\vec{v}_A$ is perpendicular to the dotted line that connects point **A** with point **P**. The speed of each point is given by: $v=\omega r$. Here r is the distance between a particular point and the contact point P.
- 2. At point **T :** $r=2R$. Thus $v_T = 2R\omega = 2v_{com}$.
- 3. For point ***O:*** $r = \boldsymbol{R}$ thus $v_0 = \omega R = v_{com}$
- 4. For point ***P*** $r = 0$ thus $v_p = 0$

摩擦力 f_k or f_s ?

11-2 Forces and Kinetic Energy of Rolling

Learning Objectives

11.03 Calculate the kinetic energy of a body in smooth rolling as the sum of the translational kinetic energy of the center of mass and the rotational kinetic energy around the center of mass.

11.04 Apply the relationship between the work done on a smoothly rolling object and its kinetic energy change.

11.05 For smooth rolling (and thus no sliding), conserve mechanical energy to relate initial energy values to the values at a later point.

11.06 Draw a free-body diagram of an accelerating body that is smoothly rolling on a horizontal surface or up or down on a ramp

(more…).

11-2 Forces and Kinetic Energy of Rolling

- Combine translational and rotational kinetic energy:

$$K = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2.$$

Eq. (11-5)

A rolling object has two types of kinetic energy: a rotational kinetic energy ($\frac{1}{2} I_{com}\omega^2$) due to its rotation about its center of mass and a translational kinetic energy ($\frac{1}{2} M v_{com}^2$) due to translation of its center of mass.

- If a wheel accelerates, its angular speed changes
- A force must act to prevent slip

$$a_{com} = \alpha R$$

Eq. (11-6)

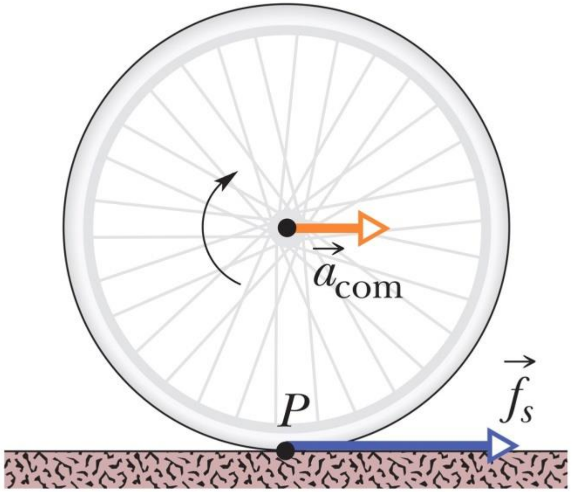
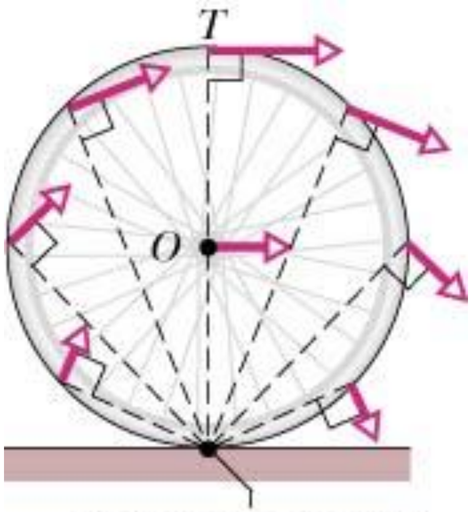


Figure 11-7



The Kinetic energy of rolling

Consider the rolling object shown in the figure : (轉軸在P點)

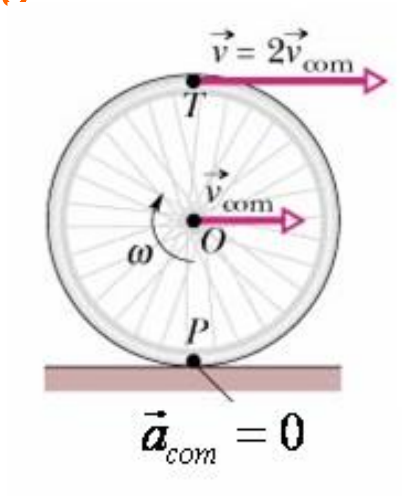
The kinetic energy K is then given by the equation: $K = \frac{1}{2} I_P \omega^2$. Here I_P is the rotational inertia of the rolling body about point P. We can determine I_P using $\boxed{\text{the parallel axis theorem.}}$ $I_P = I_{com} + MR^2 \rightarrow K = \frac{1}{2}\left(I_{com} + MR^2\right)\omega^2$

$$K = \frac{1}{2}\left(I_{com} + MR^2\right)\omega^2 = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} MR^2\omega^2$$

$$\boxed{K = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2}$$

Kinetic energy of rolling $\boldsymbol{K = K_{rotation\ about\ com} + K_{translation\ of\ com}}$

Figure 11-4



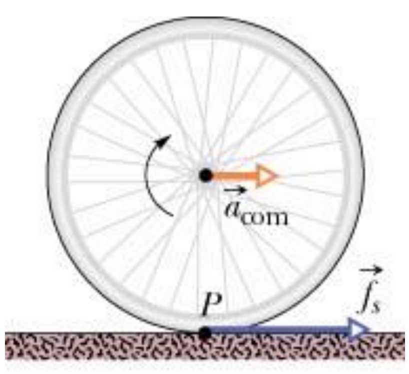
The Forces of rolling:

Friction and Rolling:

At $f_s < f_{s,\,max}$;

⇒Smooth Rolling (without f_s, no rolling)

Figure 11-7



⇒P點與地面無滑動，摩擦力=靜摩擦力 = 外力(例如:引擎使轉動加快或剎車減慢)

*本章只討論此情形!

The rolling condition results in a connection beteen the magnitude of the acceleration a_{com} of the center of mass and its angular acceleration α

$v_{com} = \omega R$ We take time derivatives of both sides $\rightarrow$ $a_{com} = \dfrac{dv_{com}}{dt} = R\dfrac{d\omega}{dt} = R\alpha$

$$\boxed{a_{com} = R\alpha} \qquad (11\text{-}6)$$

加速度a = 1000/36/3.4, 角加速度α = a/0.35
$I \sim 40(0.35)^2$,
每一輪胎承受之normal force = 1430/4 ≈ 360 kgw ≈ 3600 N
➔ $3600\mu_s \times 0.35 = 40*(0.35)^2 \times \alpha$, →可得$\mu_{s,min}$

法拉利458 Spider規格表

車型	雙門雙座位開篷跑車
引擎	V8
最大馬力	570hp/9,000rpm
最大扭力	55.1kgm/6,000rpm
驅動	中置引擎，後輪驅動
傳動	7前速F1加減自動波
車身體積	4,527x1,937x1,211毫米（長x闊x高）
車身重量	1,430公斤
0-100km/h	3.4秒
極速	時速320公里
售價	4,398,000港元

$\mu_{s,\,min} \approx$?

輪胎~ 40 Kg, 直徑~ 70 cm
→ “抓地力”？
→ **防鎖死煞車系統**（Anti-lock Braking System，**ABS**），一種在摩托車和汽車中使用，能夠避免車輛失控，並一般能減少制動距離，以提高車輛安全性的技術。

11-2 Forces and Kinetic Energy of Rolling

- If slip occurs, then the motion is *not* smooth rolling!
- For smooth rolling down a ramp:
  1. The gravitational force is vertically down;
  2. The normal force is perpendicular to the ramp
  3. The force of friction points up the slope
- We can use this equation to find the acceleration of such a body

$$a_{\mathrm{com},x} = -\frac{g \sin \theta}{1 + I_{\mathrm{com}}/MR^2}.$$

- Note that the frictional force produces the rotation
- Without friction, the object will simply slide

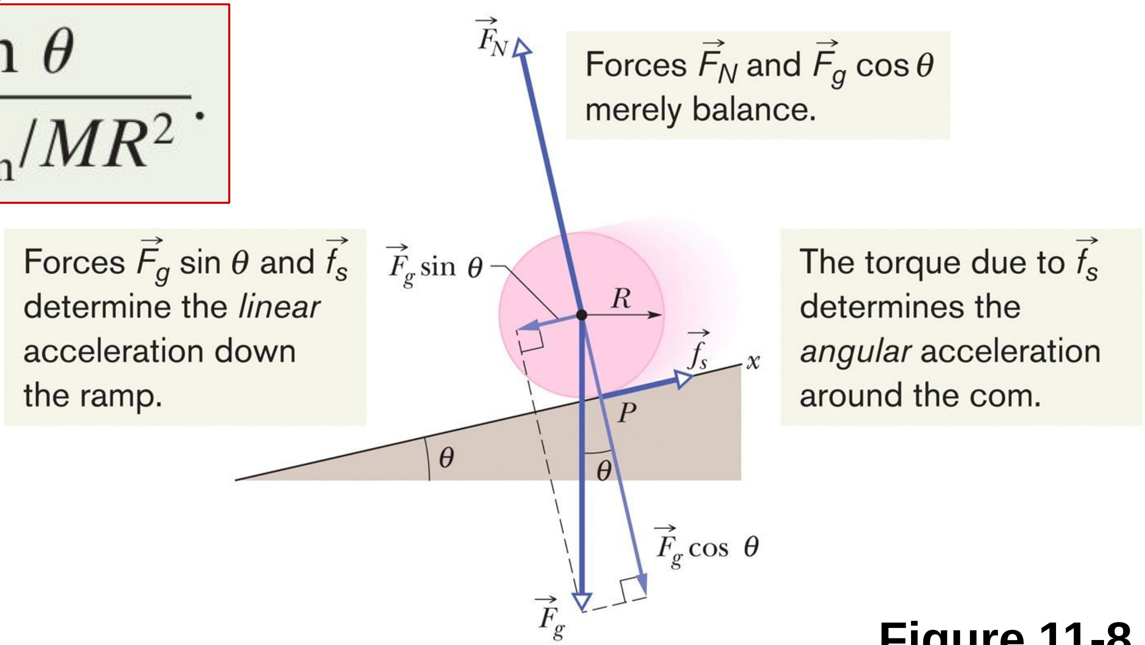
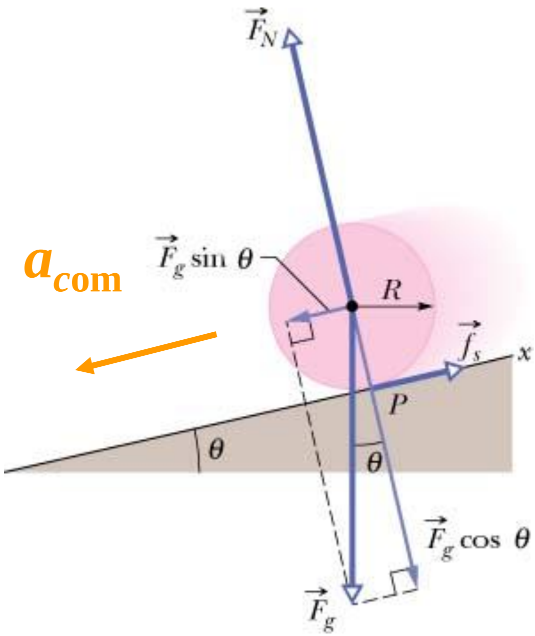


Figure 11-8

Figure 11-8



Rolling Down a Ramp

Consider a round uniform body of mass M and radius R rolling down an inclined plane of angle θ. We will calculate the acceleration a_{com} of the center of mass along the x-axis using Newton's second law for the translational and rotational motion

Newton's second law for motion along the x-axis: $f_s - Mg\sin\theta = Ma_{com}$ (eqs.1)

Newton's second law for rotation about the center of mass: $\tau = Rf_s = I_{com}\alpha$

$\alpha = -\dfrac{a_{com}}{R}$ We substitute α in the second equation and get: $Rf_s = -I_{com}\dfrac{a_{com}}{R} \rightarrow$

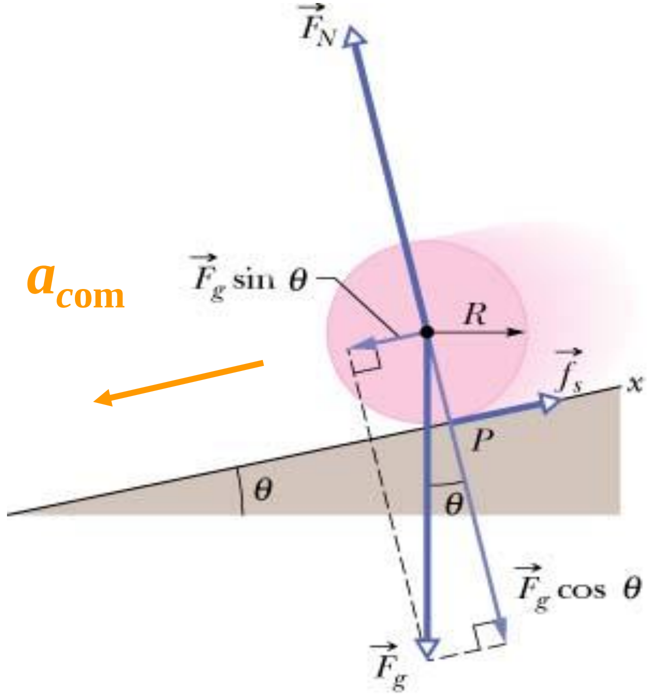
$f_s = -I_{com}\dfrac{a_{com}}{R^2}$ (eqs.2) We substitute f_s from equation 2 into equation 1 $\rightarrow$

$-I_{com}\dfrac{a_{com}}{R^2} - Mg\sin\theta = Ma_{com}$

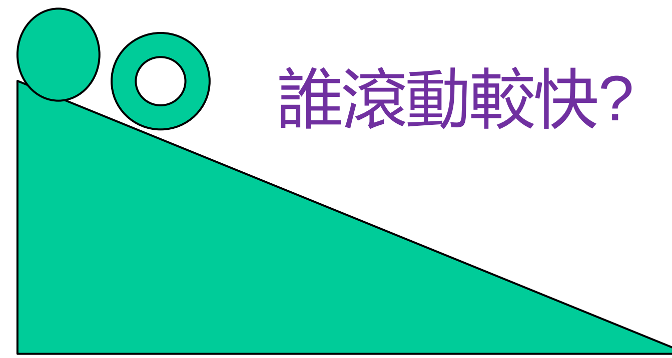
$$\boxed{a_{com} = -\frac{g\sin\theta}{1+\dfrac{I_{com}}{MR^2}}}$$

➔思考: $\mu_s = ?$
➔ If $\mu_{s,\,max} = 0.5$, θ之限制 for smooth rolling motion?

Figure 11-8



$$|a_{com}| = \frac{g\sin\theta}{1+\dfrac{I_{com}}{MR^2}}$$



誰滾動較快?

Cylinder

$$I_1 = \frac{MR^2}{2}$$

$$a_1 = \frac{g\sin\theta}{1+I_1/MR^2}$$

$$a_1 = \frac{g\sin\theta}{1+MR^2/2MR^2}$$

$$a_1 = \frac{g\sin\theta}{1+1/2}$$

$$a_1 = \frac{2g\sin\theta}{3} = (0.67)g\sin\theta$$

Hoop

$$I_2 = MR^2$$

$$a_2 = \frac{g\sin\theta}{1+I_2/MR^2}$$

$$a_2 = \frac{g\sin\theta}{1+MR^2/MR^2}$$

$$a_2 = \frac{g\sin\theta}{1+1}$$

$$a_2 = \frac{g\sin\theta}{2} = (0.5)g\sin\theta$$

11-3 The Yo-Yo (溜溜球)

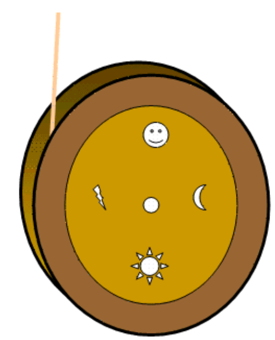
Learning Objectives

11.09 Draw a free-body diagram of a yo-yo moving up or down its string.

11.10 Identify that a yo-yo is effectively an object that rolls smoothly up or down a ramp with an incline angle of 90°.

11.11 For a yo-yo moving up or down its string, apply the relationship between the yo-yo's acceleration and its rotational inertia.

11.12 Determine the tension in a yo-yo's string as the yo-yo moves up or down the string.



11-3 The Yo-Yo

- As a yo-yo moves down a string, it loses potential energy *mgh* but gains rotational and translational kinetic energy
- To find the linear acceleration of a yo-yo accelerating down its string:
  1. Rolls down a "ramp" of angle 90°
  2. Rolls on an axle instead of its outer surface
  3. Slowed by tension *T* rather than friction

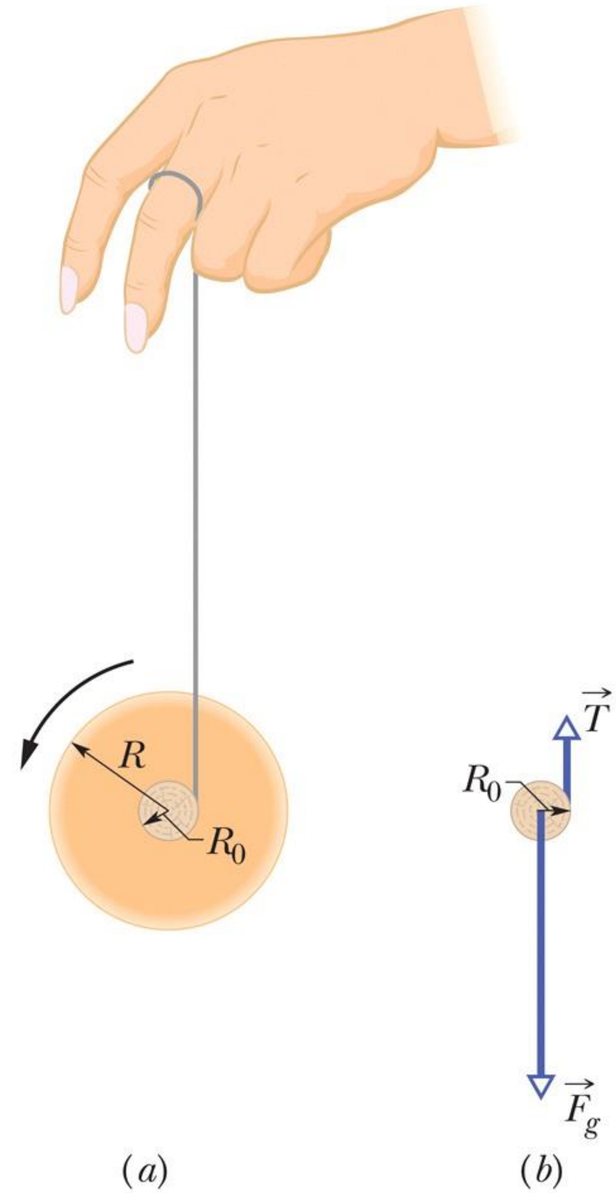


Figure 11-9

The Yo-Yo (溜溜球)

Consider a yo-yo of mass M, radius R, and axle radius R_o rolling down a string. We will calculate the acceleration a_{com} of the center of its mass along the y-axis using Newton's second law for the tralational and rotational motion as we did in the previous problem

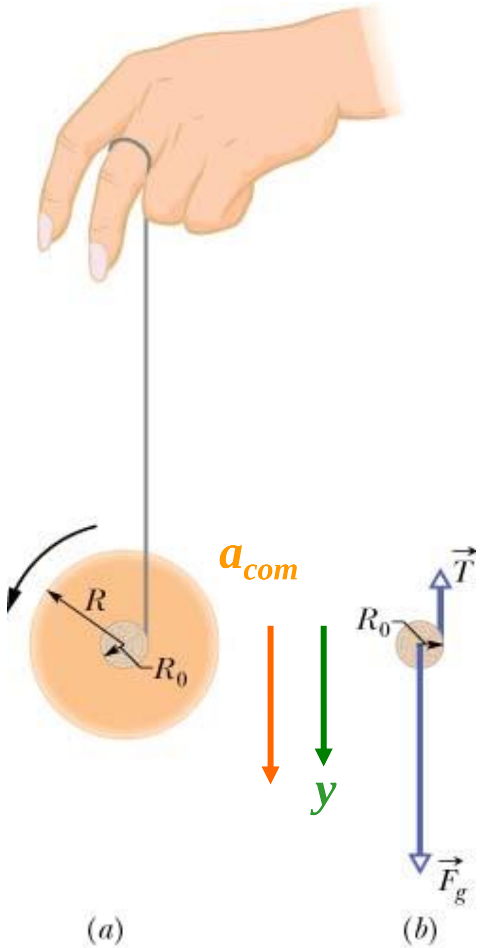


Figure 11-9

Newton's second law for motion along the y-axis:

$Mg - T = Ma_{com}$ (eqs.1)

Newton's second law for rotation about the center of mass:

$\tau = R_o T = I_{com}\alpha.$ Angular acceleration $\alpha = \dfrac{a_{com}}{R_o}$

We substitute α in the second equation and get:

$T = I_{com}\dfrac{a_{com}}{R_o^{\,2}}$ (eqs.2) We substitute T from equation 2 into equation 1$\rightarrow$

$$Mg - I_{com}\frac{a_{com}}{R_o^{\,2}} = Ma_{com} \rightarrow \quad a_{com} = \frac{g}{1 + \dfrac{I_{com}}{MR_o^{\,2}}}$$

(取向下為+方向)

11-3 The Yo-Yo

- Replacing the values in 11-10 leads us to:

$$\boxed{a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2},} \quad \textbf{Eq. (11-13)}$$

Example Calculate the acceleration of the yo-yo

- M = 150 grams, R_0 = 3 mm, I_{com} = $Mr^2/2$ = 3E-5 kg m^2
- Therefore a_{com} = -9.8 m/s^2 / (1 + 3E-5 / (0.15 * 0.003^2))
  = - .4 m/s^2

11-4 Torque Revisited

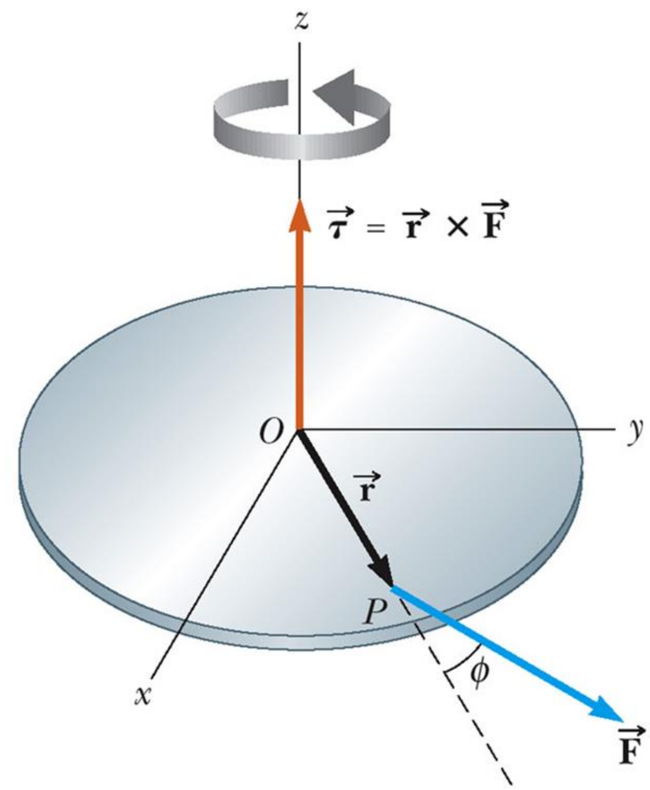
Learning Objectives

11.13 Identify that torque is a vector quantity.

11.14 Identify that the point about which a torque is calculated must always be specified.

11.15 Calculate the torque due to a force on a particle by taking the cross product of the particle's position vector and the force vector, in either unit-vector notation or magnitude-angle notation.

11.16 Use the right-hand rule for cross products to find the direction of a torque vector.



11-4 Torque Revisited

- Previously, torque was defined only for a rotating body and a fixed axis
- Now we redefine it for an individual particle that moves along any path relative to a fixed point
- The path need not be a circle; torque is now a vector
- Direction determined with right-hand-rule

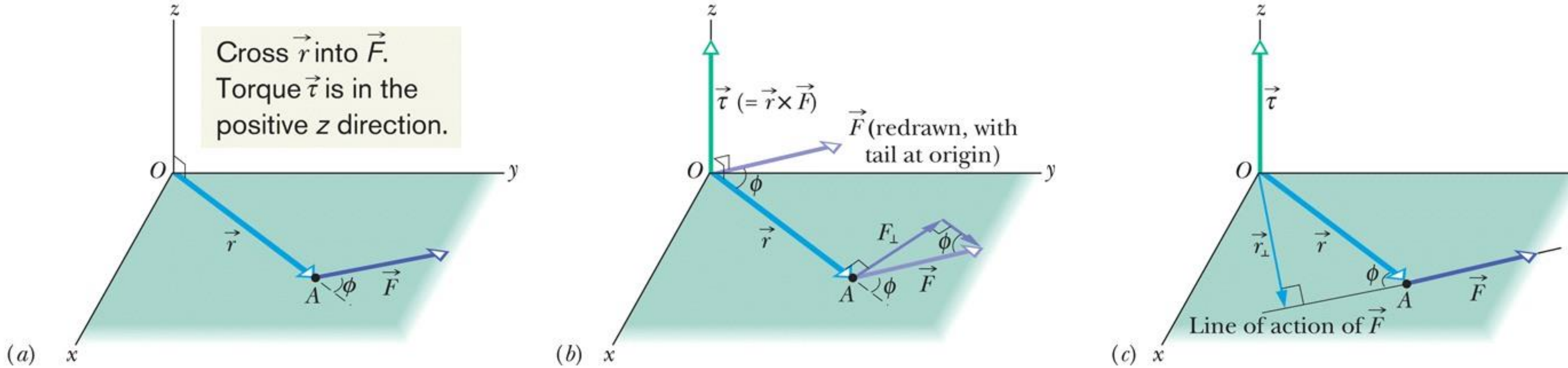


Figure 11-10

11-4 Torque Revisited

- The general equation for torque is:

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

Eq. (11-14)

- We can also write the magnitude as:

$$\boxed{\tau = rF\sin\phi,}$$

Eq. (11-15)

Unit: N·m

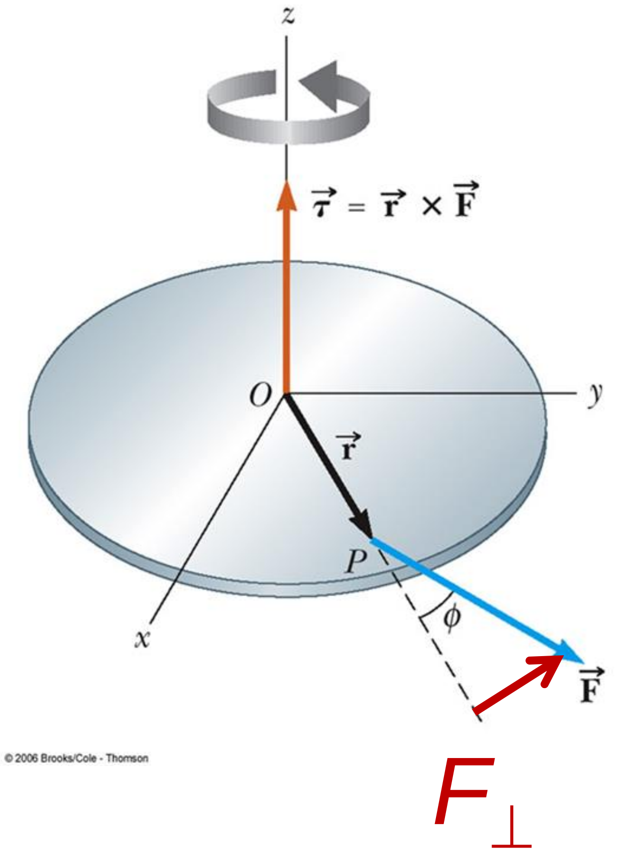
- Or, using the perpendicular component of force or the moment arm of *F*:

$$\tau = rF_{\perp},$$

Eq. (11-16)

$$\tau = r_{\perp}F,$$

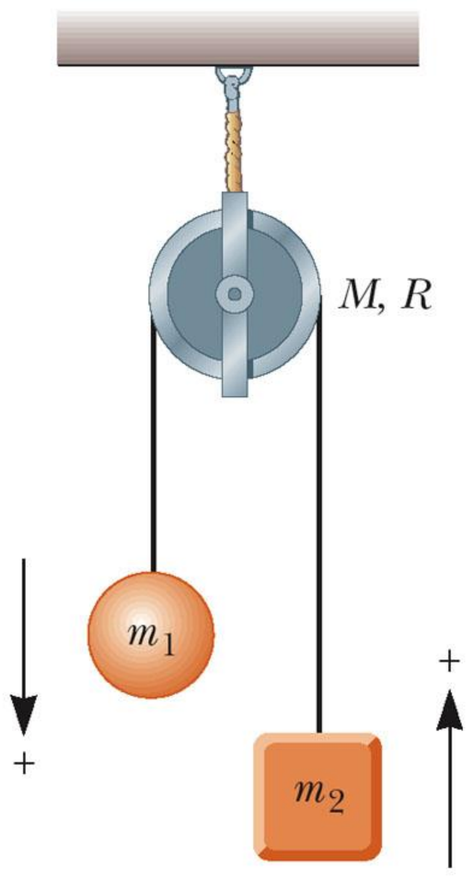
Eq. (11-17)

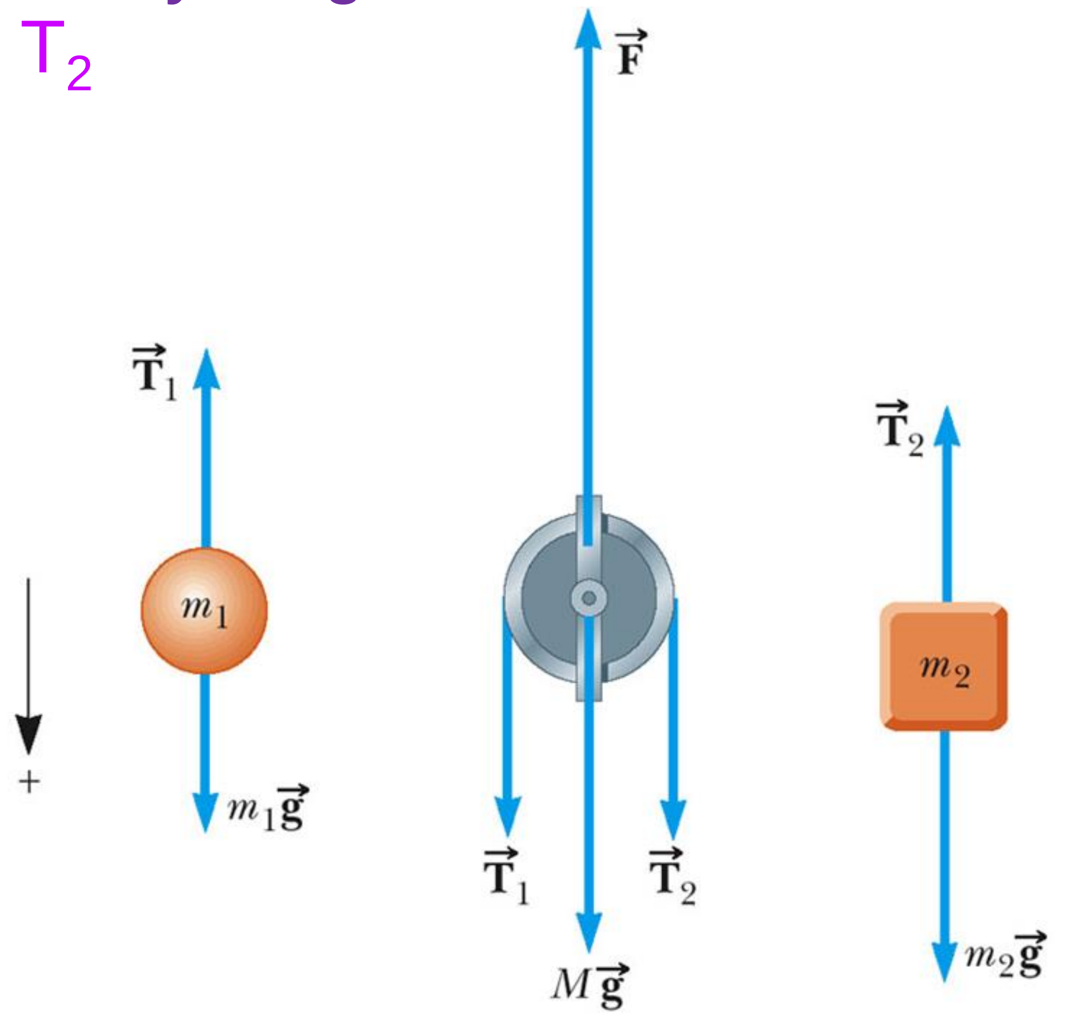


11-4 Torque Revisited

Free body diagram:

$T_1 \neq T_2$





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11-5 Angular Momentum角動量

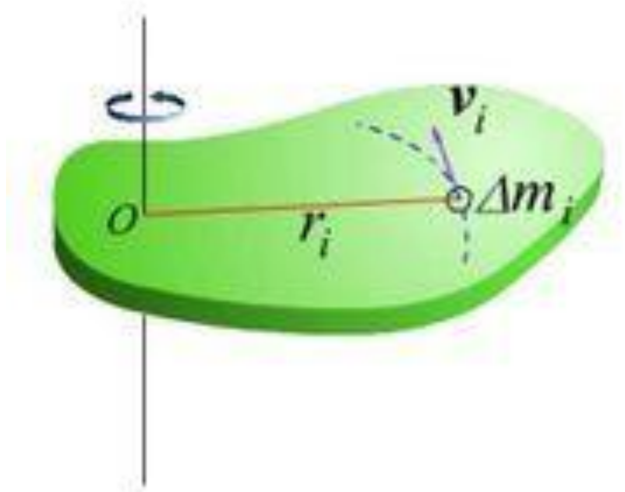
Learning Objectives

11.17 Identify that angular momentum is a vector quantity.

11.18 Identify that the fixed point about which an angular momentum is calculated must always be specified.

11.19 Calculate the angular momentum of a particle by taking the cross product of the particle's position vector and its momentum vector, in either unit-vector notation or magnitude-angle notation.

11.20 Use the right-hand rule for cross products to find the direction of an angular momentum vector.



11-5 Angular Momentum

- Here we investigate the angular counterpart to linear momentum
- We write:

$$\boxed{\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})} \qquad \boxed{\ell = r_{\perp} mv}$$

Eq. (11-18)

- The unit of angular momentum is kg m^2/s, or J s

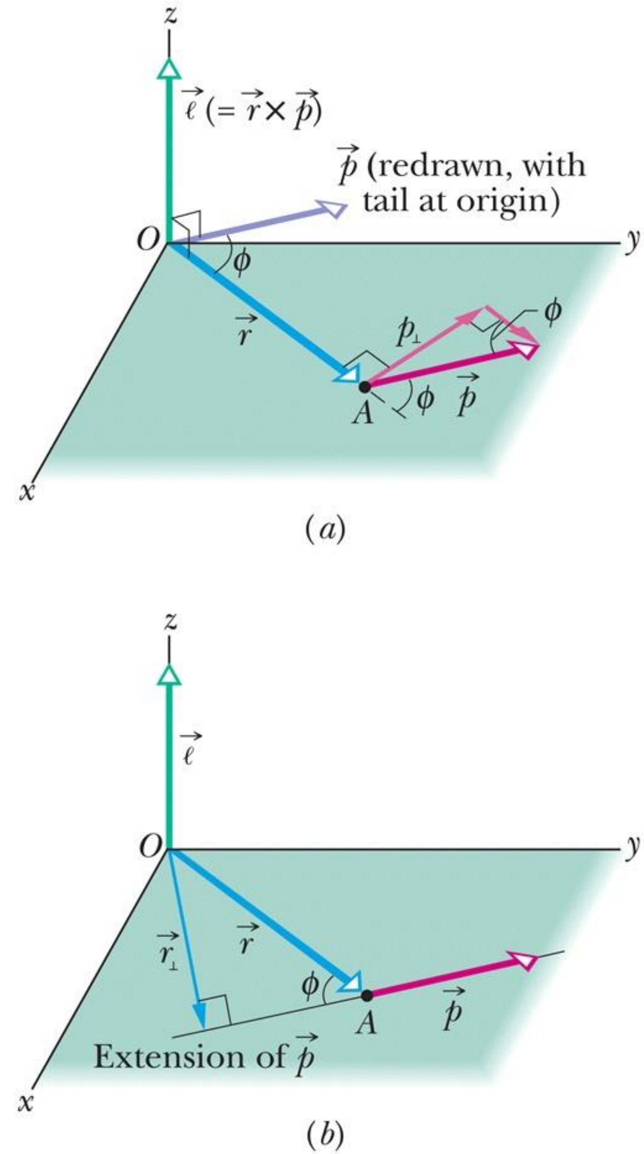


Figure 11-12

11-5 Angular Momentum

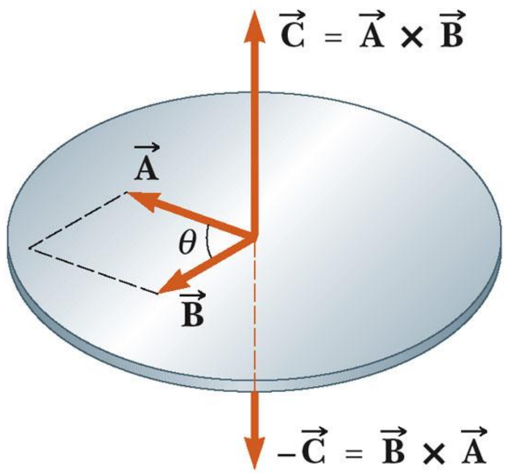
- To find the direction of angular momentum, use the right-hand rule to relate *r* and *v* to the result
- To find the magnitude, use the equation for the magnitude of a cross product:

$$\ell = rmv \sin \phi, \qquad \text{Eq. (11-19)}$$

- Which can also be written as:

$$\ell = rp_\perp = rmv_\perp, \qquad \text{Eq. (11-20)}$$

$$\ell = r_\perp p = r_\perp mv, \qquad \text{Eq. (11-21)}$$

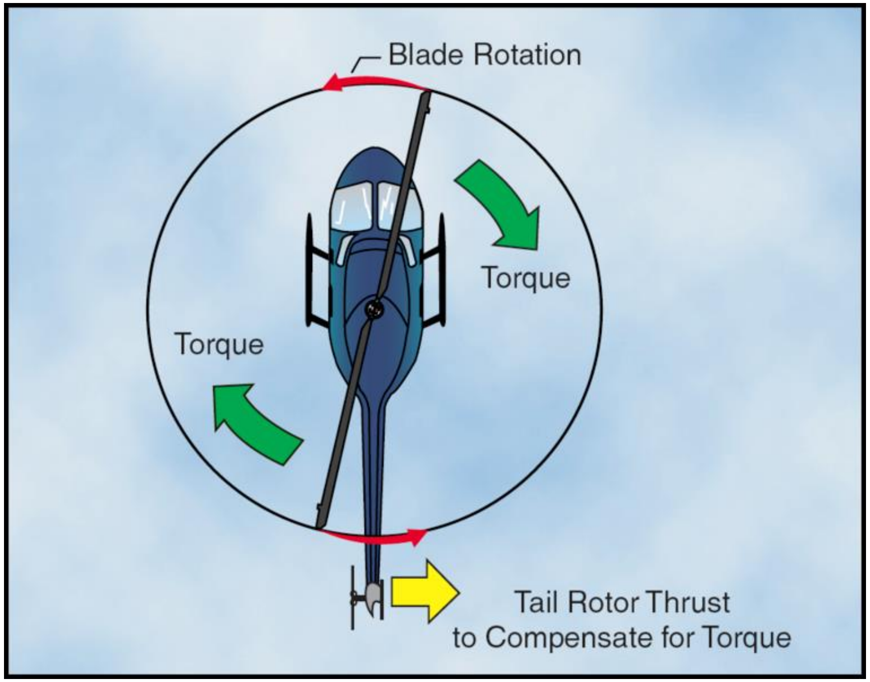


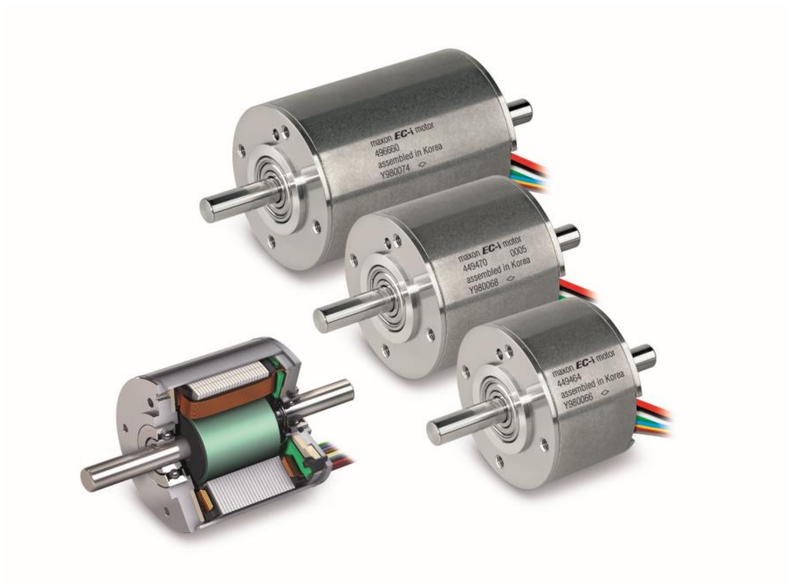
Right-hand rule

11-6 Newton's Second Law in Angular Form

Learning Objectives

11.21 Apply Newton's second law in angular form to relate the torque acting on a particle to the resulting rate of change of the particle's angular momentum, all relative to a specified point.





11-6 Newton's Second Law in Angular Form

Newton's Second Law in Angular Form

Newton's second law for linear motion has the form: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$. Below we will derive the angular form of Newton's second law for a particle.

$$\vec{\ell} = m(\vec{r}\times\vec{v}) \rightarrow \frac{d\vec{\ell}}{dt} = m\frac{d}{dt}(\vec{r}\times\vec{v}) = m\left(\vec{r}\times\frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt}\times\vec{v}\right) = m(\vec{r}\times\vec{a} + \vec{v}\times\vec{v})$$

$$\vec{v}\times\vec{v} = 0 \rightarrow \frac{d\vec{\ell}}{dt} = m(\vec{r}\times\vec{a}) = (\vec{r}\times m\vec{a}) = (\vec{r}\times\vec{F}_{net}) = \vec{\tau}_{net}$$

Thus: $\vec{\tau}_{net} = \frac{d\vec{\ell}}{dt}$ Compare with: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$ **Eq. (11-22)**

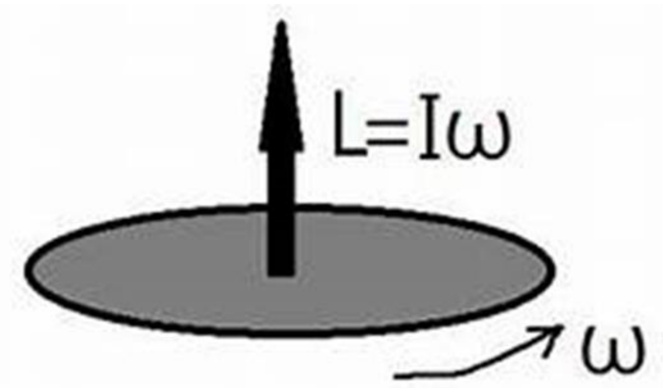
$$\boxed{\vec{\tau}_{net} = \frac{d\vec{\ell}}{dt}}$$

Eq. (11-23)

11-7 Angular Momentum of a Rigid Body

Learning Objectives

11.22 For a system of particles, apply Newton's second law in angular form to relate the net torque acting on the system to the rate of the resulting change in the system's angular momentum.



11.23 Apply the relationship between the angular momentum of a rigid body rotating around a fixed axis and the body's rotational inertia and angular speed around that axis.

11.24 If two rigid bodies rotate about the same axis, calculate their total angular momentum.

11-7 Angular Momentum of a Rigid Body

- We sum the angular momenta of the particles to find the angular momentum of a system of particles:

$$\vec{L} = \vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3 + \cdots + \vec{\ell}_n = \sum_{i=1}^{n} \vec{\ell}_i.$$

Eq. (11-26)

- The rate of change of the net angular momentum is:

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \vec{\tau}_{\text{net},i}.$$

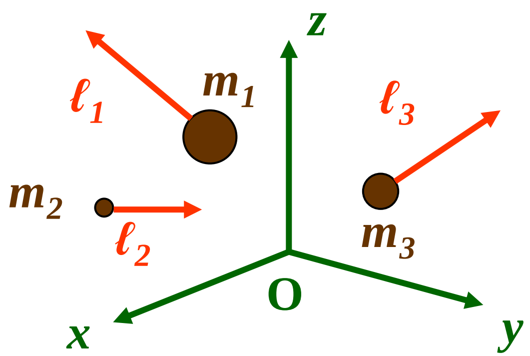
Eq. (11-28)

- In other words, the net torque is defined by this change:

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \qquad \text{(system of particles)},$$

Eq. (11-29)

The net external torque $\vec{\tau}_{\text{net}}$ acting on a system of particles is equal to the time rate of change of the system's total angular momentum $\vec{L}$.



The Angular Momentum of a System of Particles

We will now explore Newton's second law in angular form for a system of n particles that have angular momentum $\vec{\ell}_1, \vec{\ell}_2, \vec{\ell}_3, \ldots, \vec{\ell}_n$

The angular momentum $\vec{L}$ of the system is $\vec{L} = \vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3 + \ldots + \vec{\ell}_n = \sum_{i=1}^{n} \vec{\ell}_i$

The time derivative of the angular momentum is $\dfrac{d\vec{L}}{dt} = \sum_{i=1}^{n} \dfrac{d\vec{\ell}_i}{dt}$

The time derivative for the angular momentum of the i-th particle $\dfrac{d\vec{\ell}_i}{dt} = \vec{\tau}_{net,i}$

Where $\vec{\tau}_{net,i}$ is the net torque on the particle. This torque has contributions from external as well as internal forces between the particles of the system. Thus

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \vec{\tau}_{net,i} = \vec{\tau}_{net}$$

Here $\vec{\tau}_{net}$ is the net torque due to all the external forces.

內力矩向量和 = 0 (third law)

$$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau}_{net}}$$

(System of particles, (11-29))

Angular Momentum of a Rigid Body Rotating about a Fixed Axis

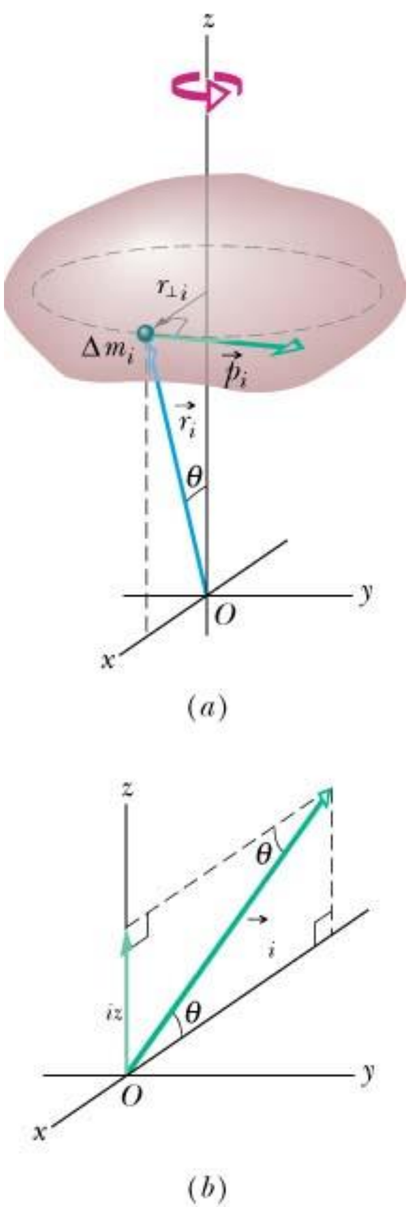


Figure 11-15

以z axis為轉軸:

The angular momentum $\vec{\ell}_i$ of the i-the element is: $\vec{\ell}_i = \vec{r}_i \times \vec{p}_i$

Its magnitude $\ell_i = r_i p_i \left(\sin 90°\right) = r_i \Delta m_i v_i$ The z-compoment ℓ_{iz} of ℓ_i is: $\ell_{iz} = \ell_i \sin\theta = \left(r_i \sin\theta\right)\left(\Delta m_i v_i\right) = r_{\perp i}\Delta m_i v_i$

The z-component of the angular momentum L_z is the sum:

$$\boxed{L_z = \sum_{i=1}^{n} \ell_{iz} = \sum_{i=1}^{n} r_{\perp i}\Delta m_i v_i = \sum_{i=1}^{n} r_{\perp i}\Delta m_i\left(\omega r_{i\perp}\right) = \omega\left(\sum_{i=1}^{n}\Delta m_i r_{i\perp}^2\right)}$$

The sum $\sum_{i=1}^{n}\Delta m_i r_{i\perp}^2$ is the rotational inertia I of the rigid body

Thus: $L_z = I\omega$

$$\boxed{L_z = I\omega}$$ Or : $L = I\omega$ (11-31)

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11-7 Angular Momentum of a Rigid Body

Table 11-1

TABLE 11-1

More Corresponding Variables and Relations for Translational and Rotational Motion[a]

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\ (= \vec{r} \times \vec{F})$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\ (= \vec{r} \times \vec{p})$
Linear momentum[b]	$\vec{P}\ (= \Sigma \vec{p}_i)$	Angular momentum[b]	$\vec{L}\ (= \Sigma \vec{\ell}_i)$
Linear momentum[b]	$\vec{P} = M\vec{v}_{\text{com}}$	Angular momentum[c]	$L = I\omega$
Newton's second law[b]	$\vec{F}_{\text{net}} = \dfrac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{\text{net}} = \dfrac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}$ = a constant	Conservation law[d]	$\vec{L}$ = a constant

[a]See also Table 10-3.

[b]For systems of particles, including rigid bodies.

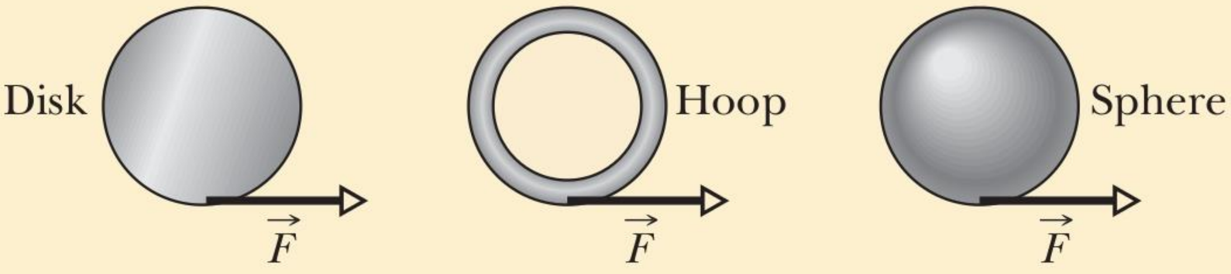
[c]For a rigid body about a fixed axis, with L being the component along that axis.

[d]For a closed, isolated system.

11-7 Angular Momentum of a Rigid Body

Checkpoint 6

In the figure, a disk, a hoop, and a solid sphere are made to spin about fixed central axes (like a top) by means of strings wrapped around them, with the strings producing the same constant tangential force $\vec{F}$ on all three objects. The three objects have the same mass and radius, and they are initially stationary. Rank the objects according to (a) their angular momentum about their central axes and (b) their angular speed, greatest first, when the strings have been pulled for a certain time t.



Answer: (a) All angular momenta will be the same, because the torque is the same in each case (b) sphere, disk, hoop

11-8 Conservation of Angular Momentum

Learning Objectives

11.25 When no external net torque acts on a system along a specified axis, apply the conservation of angular momentum to relate the initial angular momentum value along *that axis* to the value at a later instant.

11-8 Conservation of Angular Momentum

- Since we have a new version of Newton's second law, we also have a new conservation law:

$$\vec{L} = \text{a constant} \qquad \text{(isolated system).}$$

Eq. (11-32)

- The **law of conservation of angular momentum** states that, for an isolated system,

(*net initial angular momentum*) = (*net final angular momentum*)

$$\boxed{If\ \vec{\tau}_{net} = 0, \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = a\ \ cons\tan t}$$

$$\vec{L}_i = \vec{L}_f \qquad \text{(isolated system).}$$

Eq. (11-33)

11-8 Conservation of Angular Momentum

If the net external torque acting on a system is zero, the angular momentum $\vec{L}$ of the system remains constant, no matter what changes take place within the system.

- Since these are vector equations, they are equivalent to the three corresponding scalar equations
- This means we can separate axes and write:

If the component of the net *external* torque on a system along a certain axis is zero, then the component of the angular momentum of the system along that axis cannot change, no matter what changes take place within the system.

- If the distribution of mass changes with no external torque, we have:

$$\boxed{I_i\omega_i = I_f\omega_f.}$$

Eq. (11-34)

11-8 Conservation of Angular Momentum

Example Angular momentum conservation

- A student spinning on a stool: rotation speeds up when arms are brought in, slows down when arms are extended
- A springboard diver: rotational speed is controlled by tucking her arms and legs in, which reduces rotational inertia and increases rotational speed
- A long jumper: the angular momentum caused by the torque during the initial jump can be transferred to the rotation of the arms, by windmilling them, keeping the jumper upright

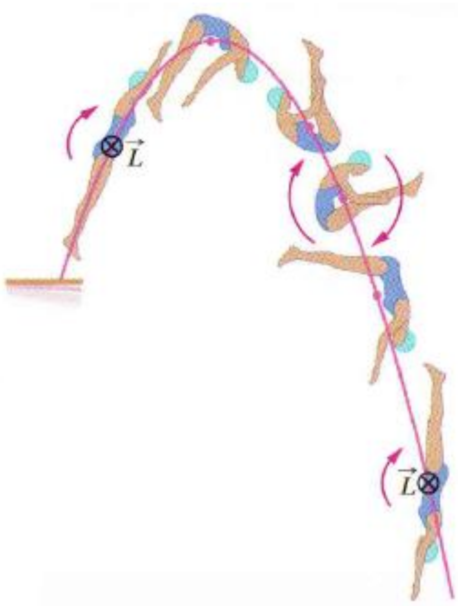


Figure 11-17

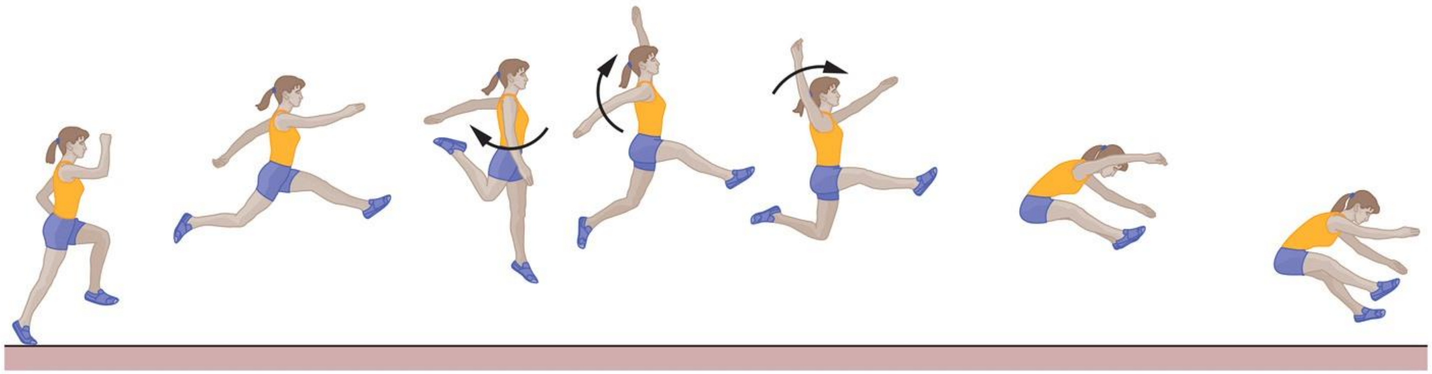


Figure 11-18

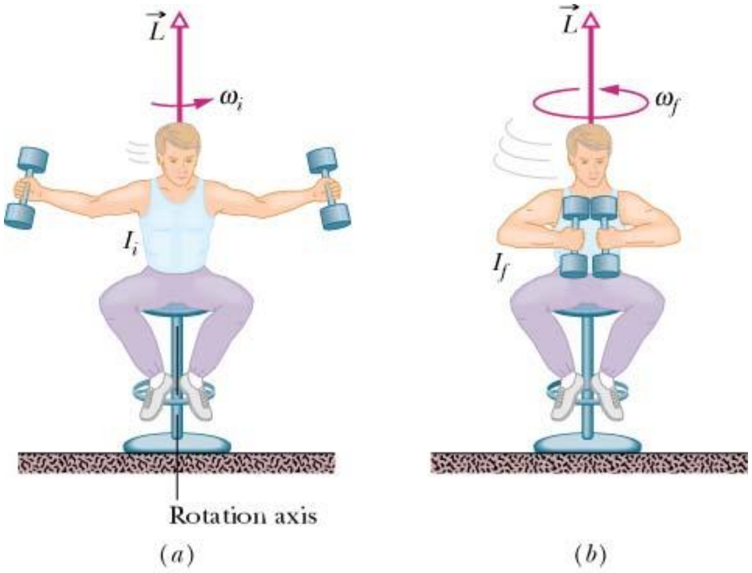


Figure 11-16

Example: The figure shows a student seated on a stool that can rotate freely about a vertical axis. The student who has been set into rotation at an initial angular speed ω_i , holds two dumbbells in his outstretched hands. His angular momentum vector $\vec{L}$ lies along the rotation axis, pointing upward.

The student then pulls in his hands as shown in fig.b. This action reduces the rotational inertia from an initial value I_i to a smaller final value I_f.

No net external torque acts on the student-stool system. Thus the angular momentum of the system remains unchanged.

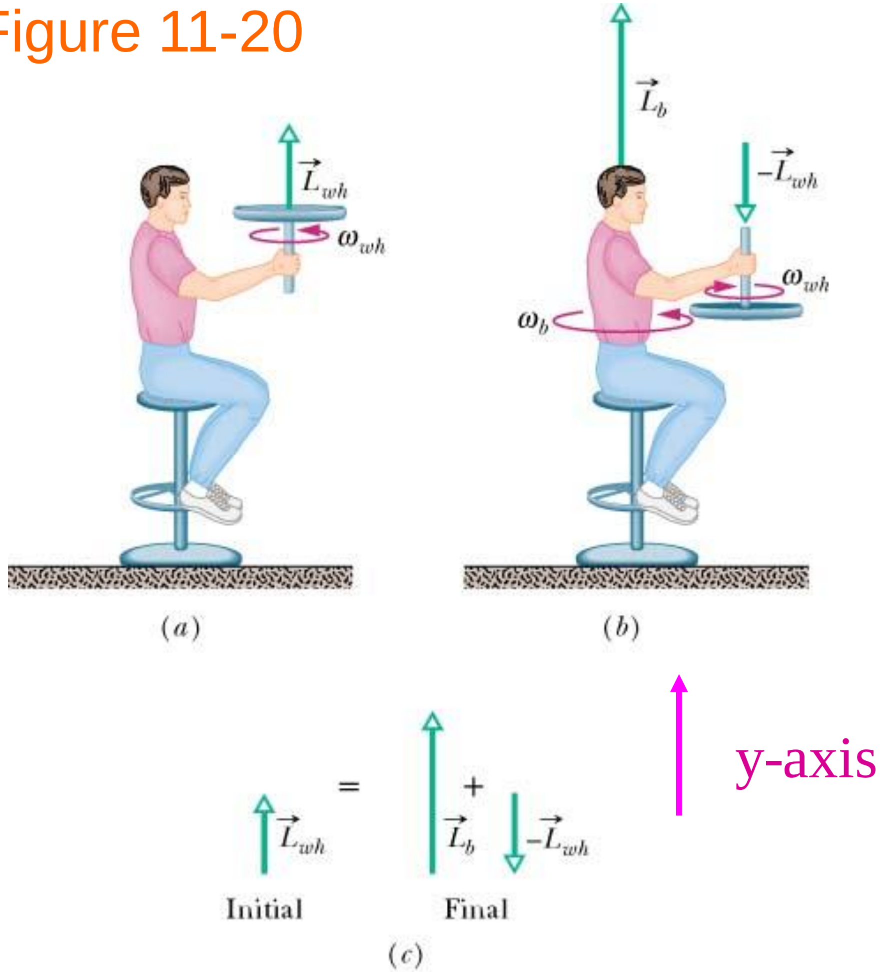
Angular momentum at t_i: $L_i = I_i\omega_i$ Angular momentum at t_f: $L_f = I_f\omega_f$

$$L_i = L_f \rightarrow I_i\omega_i = I_f\omega_f \rightarrow \omega_f = \frac{I_i}{I_f}\omega_i$$

Since $I_f < I_i \rightarrow \frac{I_i}{I_f} > 1 \rightarrow \quad \omega_f > \omega_i$

The rotation rate of the student in fig.b is faster

Figure 11-20



Sample Problem 11.05:

$$I_{wh} = 1.2 \text{ kg.m}^2$$

$$\omega_{wh} = 2\pi \times 3.9 \text{ rad/s}$$

$$I_b = 6.8 \;\; \text{kg m}^2$$

$$\omega_b = ?$$

$$L_i = L_f \rightarrow L_{wh} = -L_{wh} + L_b \rightarrow L_b = 2L_{wh}$$

$$I_b\omega_b = 2I_{wh}\omega_{wh} \rightarrow \omega_b = \frac{2I_{wh}\omega_{wh}}{I_b} = \frac{2\times 1.2\times 2\pi\times 3.9}{6.8} = 2\pi\times 1.4 \text{ rad/s}$$

Analogies between translational and rotational Motion

Translational Motion		Rotational Motion
x	$\leftrightarrow$	θ
v	$\leftrightarrow$	ω
a	$\leftrightarrow$	α
p	$\leftrightarrow$	ℓ
$K = \dfrac{mv^2}{2}$	$\leftrightarrow$	$K = \dfrac{I\omega^2}{2}$
m	$\leftrightarrow$	I
$F = ma$	$\leftrightarrow$	$\tau = I\alpha$
F	$\leftrightarrow$	τ
$P = Fv$	$\leftrightarrow$	$P = \tau\omega$
$\vec{F}_{net} = \dfrac{d\vec{p}}{dt}$	$\leftrightarrow$	$\vec{\tau}_{net} = \dfrac{d\vec{\ell}}{dt}$
$p = mv$	$\leftrightarrow$	$L = I\omega$

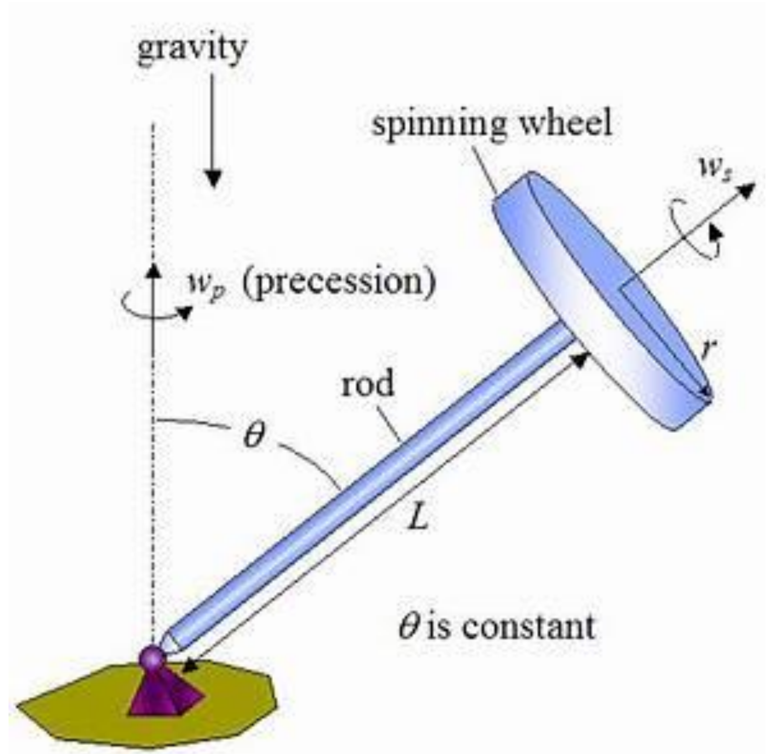
11-9 Precession of a Gyroscope (陀螺儀；迴轉儀)

Learning Objectives

11.26 Identify that the gravitational force acting on a spinning gyroscope causes the spin angular momentum vector (and thus the gyroscope) to rotate about the vertical axis in a motion called precession.

11.27 Calculate the precession rate of a gyroscope.

11.28 Identify that a gyroscope's precession rate is independent of the gyroscope's mass.



11-9 Precession of a Gyroscope

$d\mathbf{L}/dt = \boldsymbol{\tau}$ (11-41)

$\tau = Mgr\sin 90^o = Mgr$ (11-42)

If $L = 0$ in Gyroscope, it falls…

If there is spin ($L \neq 0$), it goes into precession (進動)

~*See Fig. 11-22(b):*

$L = I\omega$, (11-43); $d\mathbf{L} = \boldsymbol{\tau}\, dt$ (11-44);

However, $\mathbf{L} \,//\, \mathbf{r}$, $\boldsymbol{\tau} \perp \mathbf{r}$ *(*τ *因重力Mg引起),*

$\Rightarrow \tau$ *只改變L之方向，不改變其大小；L維持* $= I\omega$ => *r軸在xy平面掃(轉)動!*

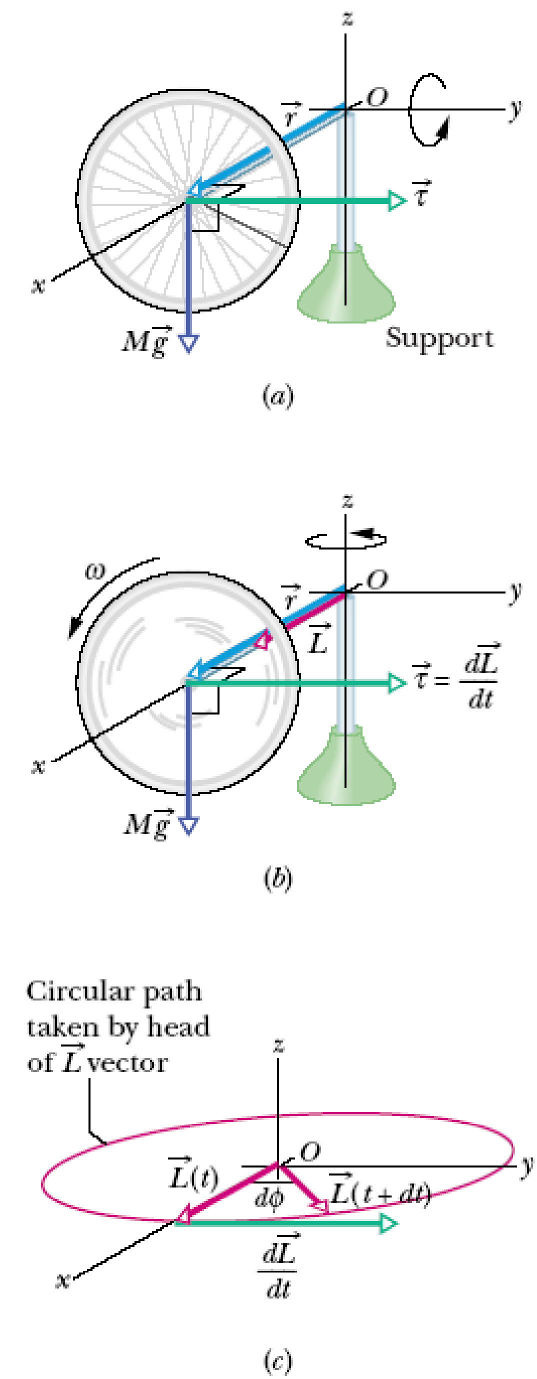


Figure 11-22

● *Precession rate* Ω:

$dL = \tau dt = Mgrdt$, $\quad d\phi = dL/L = Mgrdt/I\omega$

$\Omega \equiv d\phi/dt = = Mgr/I\omega$ (11-46), Ω*與質量無關!*

11 Summary

Rolling Bodies

$$v_{com} = \omega R$$ Eq. (11-2)

$$K = \tfrac{1}{2}I_{com}\omega^2 + \tfrac{1}{2}Mv^2_{com}.$$ Eq. (11-5)

$$a_{com} = \alpha R$$ Eq. (11-6)

Torque as a Vector

- Direction given by the right-hand rule

$$\vec{\tau} = \vec{r} \times \vec{F}$$ Eq. (11-14)

Angular Momentum of a Particle

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

Eq. (11-18)

Newton's Second Law in Angular Form

$$\vec{\tau}_{net} = \frac{d\vec{\ell}}{dt}$$ Eq. (11-23)

11 Summary

Angular Momentum of a System of Particles

$$\vec{L} = \vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3 + \cdots + \vec{\ell}_n = \sum_{i=1}^{n} \vec{\ell}_i.$$

Eq. (11-26)

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

Eq. (11-29)

Angular Momentum of a Rigid Body

$$L = I\omega$$

Eq. (11-31)

Conservation of Angular Momentum

$$\vec{L} = \text{a constant}$$

Eq. (11-32)

$$\vec{L}_i = \vec{L}_f$$

Eq. (11-33)

Precession of a Gyroscope

$$\Omega = \frac{Mgr}{I\omega}$$

Eq. (11-46)

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CH 11習題:

20, 22, 44, 46, 55, and 58