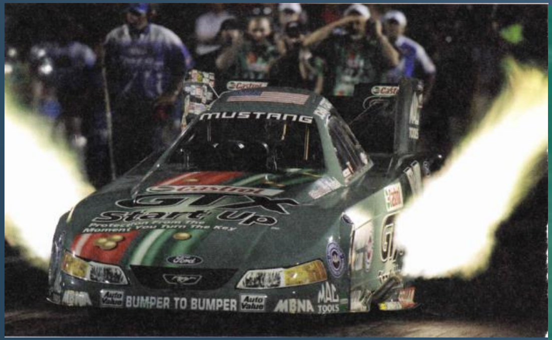
Chapter 7

Kinetic Energy and Work



7-1 Kinetic Energy

Learning Objectives

7.01 Apply the relationship between a particle's kinetic energy, mass, and speed.

7.02 Identify that kinetic energy is a scalar quantity.

7-1 Kinetic Energy

- Energy is required for any sort of motion
- Energy:
  - Is a scalar quantity assigned to an object or a system of objects
  - Can be changed from one form to another
  - Is *conserved* in a closed system, that is the total amount of energy of all types is always the same
- In this chapter we discuss one type of energy (kinetic energy)
- We also discuss one method of transferring energy (work)

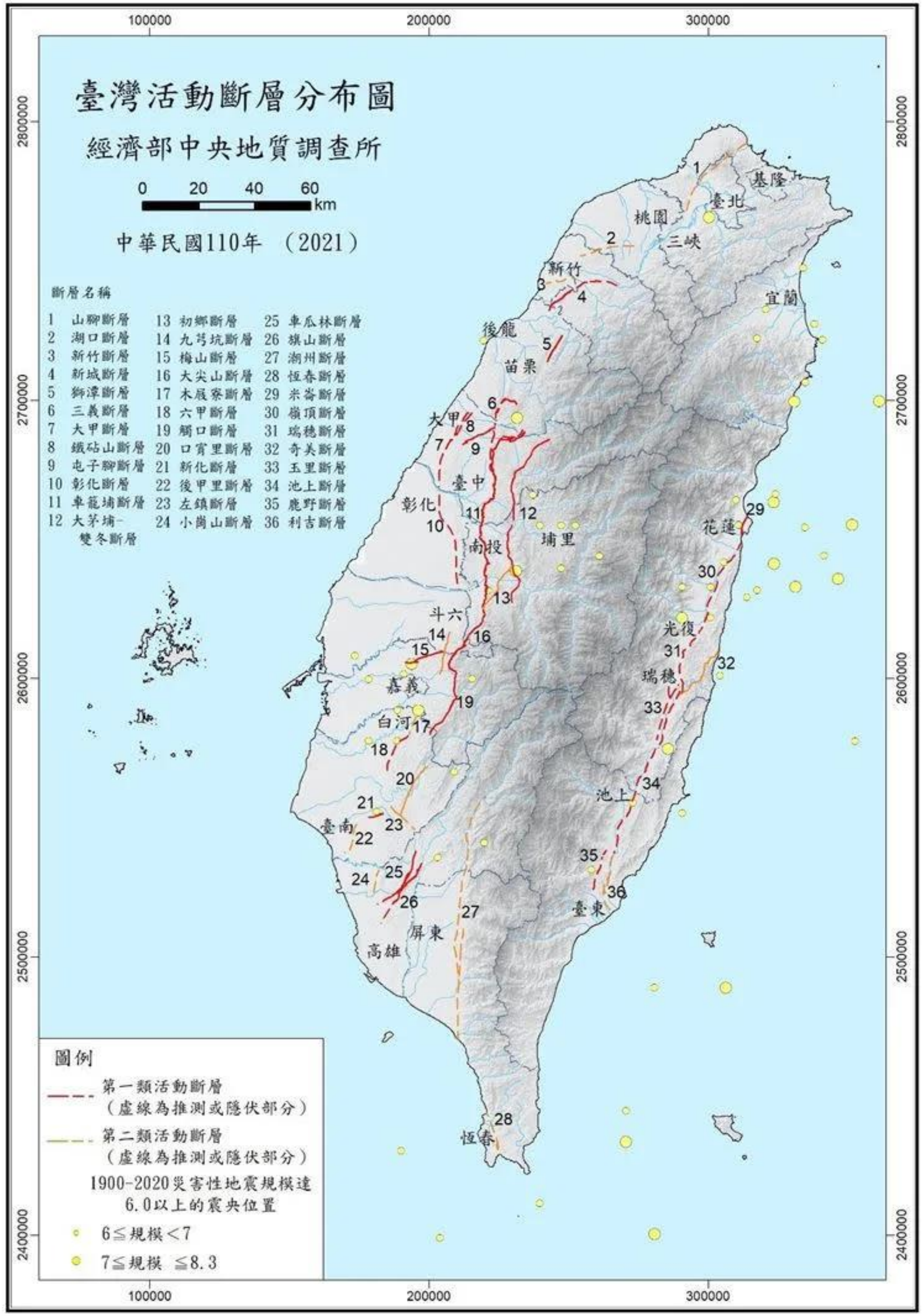
Introduction to Energy

- The concept of energy is one of the most important topics in science
- 自然科學不同領域共同的溝通橋樑:***Energy***

6.20	1.259E+21	1個原子彈
6.30	1.778E+21	
6.40	2.512E+21	2個原子彈
6.50	3.548E+21	
6.60	5.012E+21	4個原子彈
6.70	7.079E+21	
6.80	1.000E+22	8個原子彈
6.90	1.413E+22	
7.00	1.995E+22	16個原子彈
7.10	2.818E+22	
7.20	3.981E+22	32個原子彈
7.30	5.623E+22	(921地震)
7.40	7.943E+22	64個原子彈
7.50	1.122E+23	
7.60	1.585E+23	128個原子彈
7.70	2.239E+23	
7.80	3.162E+23	256個原子彈

36條活動斷層「最危險的在台北」

[36條活動斷層「最危險的在台北」 專家曝2原因：很麻煩 (yahoo.com)]



Kinetic Energy動能:

- Kinetic Energy is the energy of a particle due to its motion
  - $K = 1/2\ mv^2$
    - K is the kinetic energy
    - m is the mass of the particle
    - v is the speed of the particle

$$W = \int_{x_i}^{x_f} \sum F\, dx = \int_{x_i}^{x_f} m\boxed{a}\, dx$$

$\frac{dv}{dt}$

$$W = \int_{v_i}^{v_f} mv\, dv$$

$$\sum W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- A change in kinetic energy is one possible result of doing work to transfer energy into a system:動能之改變來自於對系統作功。

7-1 Kinetic Energy

- **Kinetic energy**:
  - The faster an object moves, the greater its kinetic energy
  - Kinetic energy is zero for a stationary object
- For an object with *v* well below the speed of light:

$$\boxed{K = \tfrac{1}{2}mv^2}$$ **Eq. (7-1)**

- The unit of kinetic energy is a **joule** (J)

$$\boxed{1\ \text{joule} = 1\ \text{J} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2.}$$ **Eq. (7-2)**

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Sample Problem 7-1

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7-1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant $0.26\ m/s^2$, what was the total kinetic energy of the two locomotives just before the collision?

Fig. 7-1 The aftermath of an 1896 crash of two locomotives.

7-1 Kinetic Energy

Example Energy released by 2 colliding trains with given weight and acceleration from rest:

- Find the final velocity of each locomotive:

$$v^2 = v_0^2 + 2a(x - x_0).$$

$$v^2 = 0 + 2(0.26\ \text{m/s}^2)(3.2 \times 10^3\ \text{m}),$$
$$v = 40.8\ \text{m/s} = 147\ \text{km/h}.$$

- Convert weight to mass:

$$m = \frac{1.2 \times 10^6\ \text{N}}{9.8\ \text{m/s}^2} = 1.22 \times 10^5\ \text{kg}.$$

- Find the kinetic energy:

$$K = 2(\tfrac{1}{2} m v^2) = (1.22 \times 10^5\ \text{kg})(40.8\ \text{m/s})^2$$
$$= 2.0 \times 10^8\ \text{J}. \qquad \text{(Answer)}$$

This collision was like an exploding bomb.

**Kinetic Energy at very high speed:

$$K = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$ [Relativistic kinetic energy]

$c = 3.0 \times 10^8$ m/s See CH 37

Using the binominal expansion, and let $n = -1/2$, $x = -(v/c)^2$;

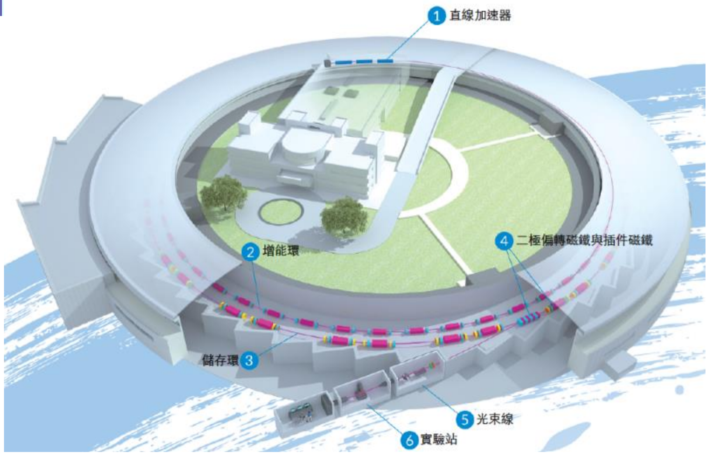
$$(1+x)^n = 1 + nx + n(n-1)x^2/2! + \cdots$$

$$K = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$\boxed{= mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots - 1\right) \approx \frac{1}{2}mv^2}$$

[忽略高次項;$(v/c)^4 \rightarrow 0$.]

$v = ?c$ ←



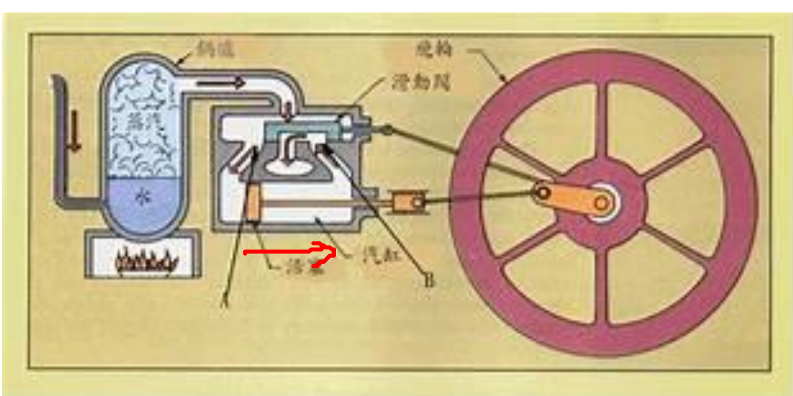
國家同步輻射研究中心台灣光子源(TPS)的加速器系統包括: **注射器:** 含電子能量為1.5億電子伏特之直線加速器與電子能量為30億電子伏特的增能環。**儲存環:** 電子能量為30億電子伏特。

7-2 Work and Kinetic Energy

Learning Objectives

7.03 Apply the relationship between a force (magnitude and direction) and the work done on a particle by the force when the particle undergoes a displacement.

7.04 Calculate work by taking a dot product of the force vector and the displacement vector, in either magnitude-angle or unit-vector notation.



7.05 If multiple forces act on a particle, calculate the net work done by them.

7.06 Apply the work-kinetic energy theorem to relate the work done by a force (or the net work done by multiple forces) and the resulting change in kinetic energy.

7-2 Work and Kinetic Energy

- Account for changes in kinetic energy by saying energy has been transferred *to* or *from* the object
- In a transfer of energy via a force, **work** is:
  - *Done on the object by the force*

> Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

- This is not the common meaning of the word "work"
  - To do work on an object, energy must be transferred
  - Throwing a baseball does work
  - Pushing an immovable wall does not do work

7-2 Work and Kinetic Energy

- Start from force equation and 1-dimensional velocity:

$$F_x = ma_x, \qquad v^2 = v_0^2 + 2a_x d.$$

Eq. (7-3) Eq. (7-4)

- Rearrange into kinetic energies:

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d.$$

Eq. (7-5)

- The left side is now the change in energy
- Therefore work is:

$$\boxed{W = F_x d.}$$

Eq. (7-6)

7-2 Work and Kinetic Energy

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

- For an angle φ between force and displacement:

$$W = Fd\cos\phi \qquad \text{Eq. (7-7)}$$

- As vectors we can write:

$$\boxed{W = \vec{F}\cdot\vec{d}} \qquad \text{Eq. (7-8)}$$

- Notes on these equations:
  - Force is constant
  - Object is particle-like (rigid)
  - Work can be positive or negative

7-2 Work and Kinetic Energy

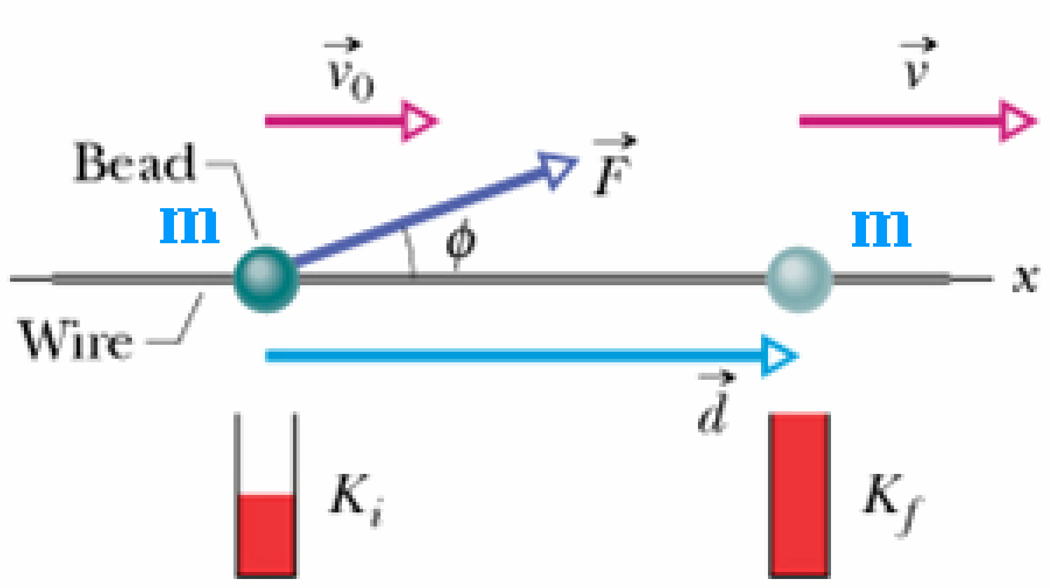
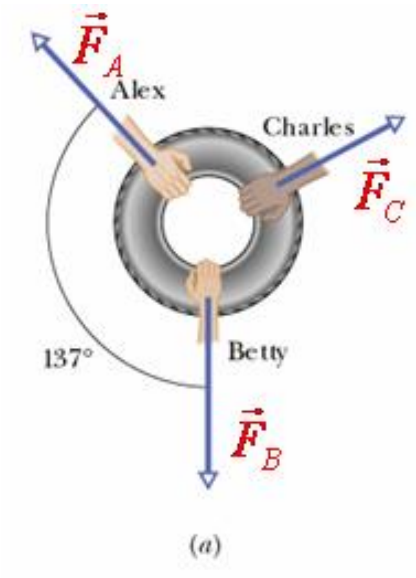


Figure 7-2

- Work has the SI unit of joules (J), the same as energy
- In the British system, the unit is foot-pound (ft lb)



$$\boxed{W = Fd\cos\varphi}$$

$$\boxed{W = \vec{F} \cdot \vec{d}}$$

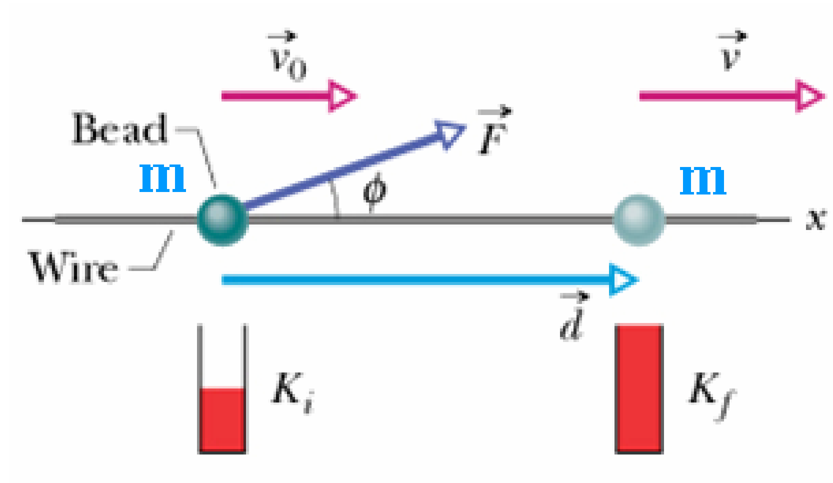


Figure 7-2

The unit of *W* is the same as the of *K* i.e. joules

Note 1:

The expressions for work we have developed apply when F is constant

Note 2:

We have made the implicit assumption that the moving object is point-like

Note 3:

$W > 0$ if $0 < \phi < 90°$, $W < 0$ if $90° < \phi < 180°$

Net Work:

If we have several forces acting on a body (say three as in the picture)

There are two methods that can be used to calculate the net work W_{net}

Method 1:

First calculate the work done by each force: W_A by force $\vec{F}_A$, W_B by force $\vec{F}_B$, and W_C by force $\vec{F}_C$.

Then determine $W_{net} = W_A + W_B + W_C$

Method 2:

Calculate first $\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_C$; Then determine $W_{net} = \vec{F} \cdot \vec{d}$

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7-2 Work and Kinetic Energy

- The **work-kinetic energy theorem** states功-能定理:

$$\Delta K = K_f - K_i = W, \qquad \textbf{Eq. (7-10)}$$

- *(change in kinetic energy) = (the net work done)*
- Or we can write it as:

$$K_f = K_i + W, \qquad \textbf{Eq. (7-11)}$$

- (*final KE*) = (*initial KE*) + (*net work*),

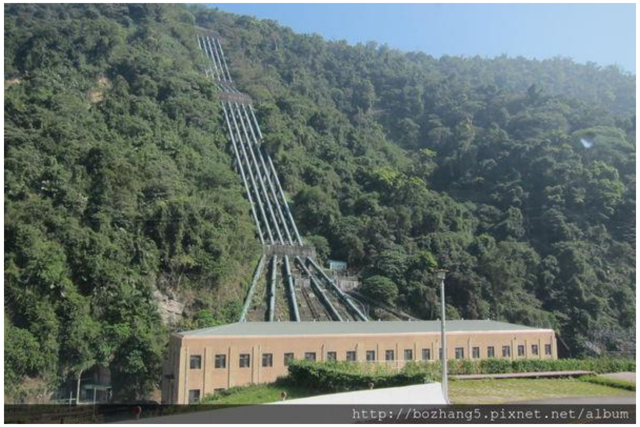
or: $\Sigma W = K_f - K_i = \Delta K$: 動能之改變 = 對系統所作之功。

7-3 Work Done by the Gravitational Force

Learning Objectives

7.07 Calculate the work done by the gravitational force when an object is lifted or lowered.

7.08 Apply the work-kinetic energy theorem to situations where an object is lifted or lowered.



7-3 Work Done by the Gravitational Force

- We calculate the work as we would for any force
- Our equation is:

$$W_g = mgd \cos \phi$$ Eq. (7-12)

- For a rising object:

$$W_g = mgd \cos 180° = mgd(-1) = -mgd.$$ Eq. (7-13)

- For a falling object:

$$W_g = mgd \cos 0° = mgd(+1) = +mgd.$$ Eq. (7-14)

7-3 Work Done by the Gravitational Force

- Work done in lifting or lowering an object, applying an upwards force: $$\Delta K = K_f - K_i = W_a + W_g,$$ **Eq. (7-15)**

- For a stationary object:
  - Kinetic energies are zero
  - We find:

$$W_a + W_g = 0$$

$$W_a = -W_g.$$ **Eq. (7-16)**

- In other words, for an applied lifting force:

$$W_a = -mgd\cos\phi \quad \text{(work done in lifting and lowering; } K_f = K_i\text{)},$$

Eq. (7-17)

- Applies regardless of path

7-3 Work Done by the Gravitational Force

- Figure 7-7 shows the orientations of forces and their associated works for upward and downward displacement
- Note that the works (in 7-16) need not be equal, they are only equal if the initial and final kinetic energies are equal
- If the works are unequal, you will need to know the difference between initial and final kinetic energy to solve for the work

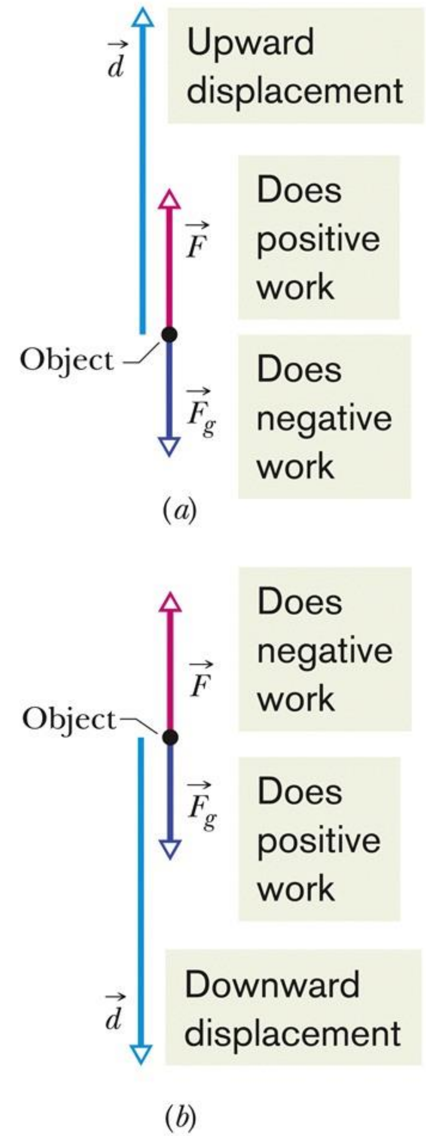


Figure 7-7

7-3 Work Done by the Gravitational Force

Examples You are a passenger: (in a 200-kg sleigh, θ = 30º, d = 20 m)

- Being pulled up a ski-slope
  - Tension does positive work, gravity does negative work

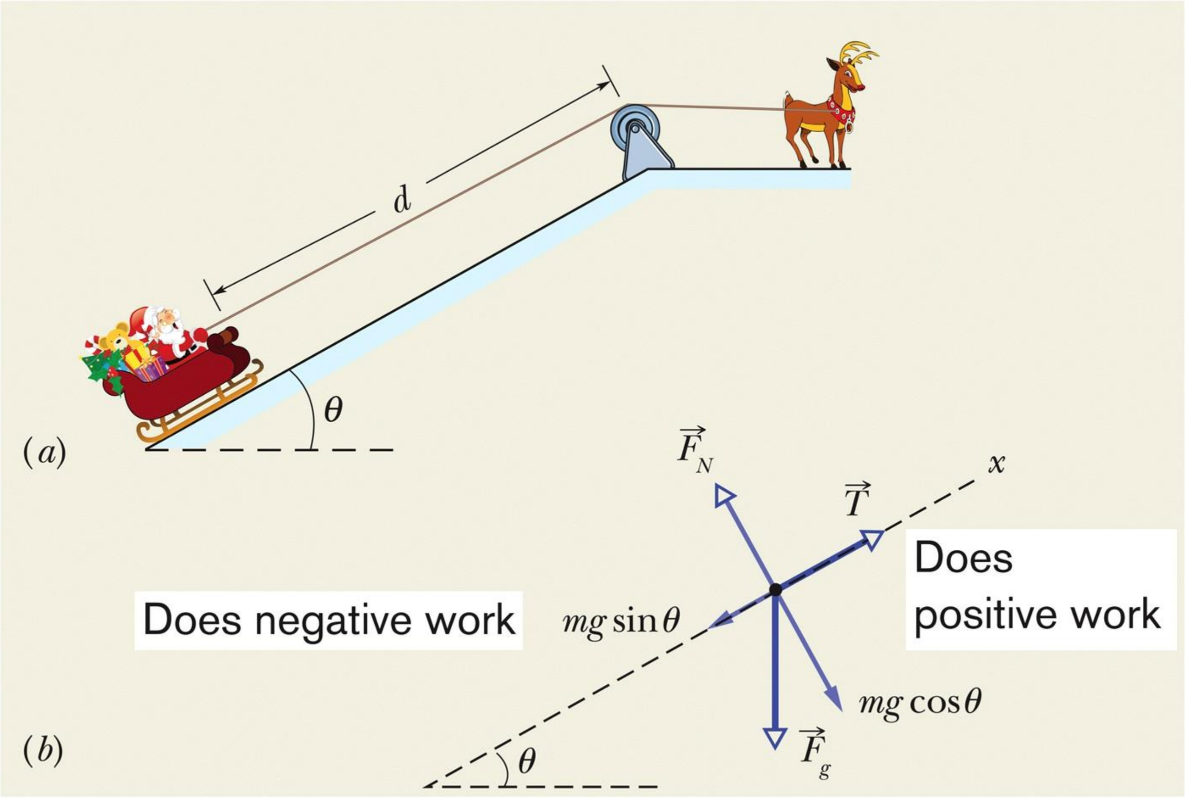


Figure 7-8

7-4 Work Done by a Spring Force

Learning Objectives

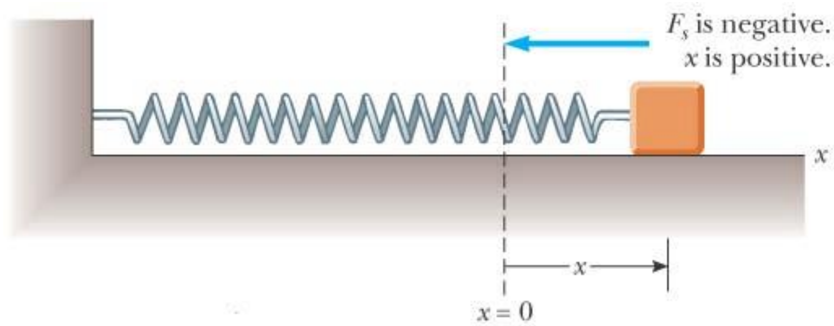
7.09 Apply the relationship (Hooke's law) between spring force, the stretch or compression of the spring, and the spring constant.

7.10 Identify that a spring force is a variable force.

7.11 Calculate the work done on an object by a spring force by integrating the force from the initial position to the final position of the object or by using the known generic result of the integration.

7.12 Calculate work by graphically integrating on a graph of force versus position of the object.

7.13 Apply the work-kinetic energy theorem to situations in which an object is moved by a spring force.



7-4 Work Done by a Spring Force

- A **spring force** is *the variable force* from a spring
  - A spring force has a particular mathematical form
  - Many forces in nature have this form

- Figure (a) shows the spring in its **relaxed state**: since it is neither compressed nor extended, no force is applied

- If we stretch or extend the spring it resists, and exerts a *restoring force* that attempts to return the spring to its relaxed state

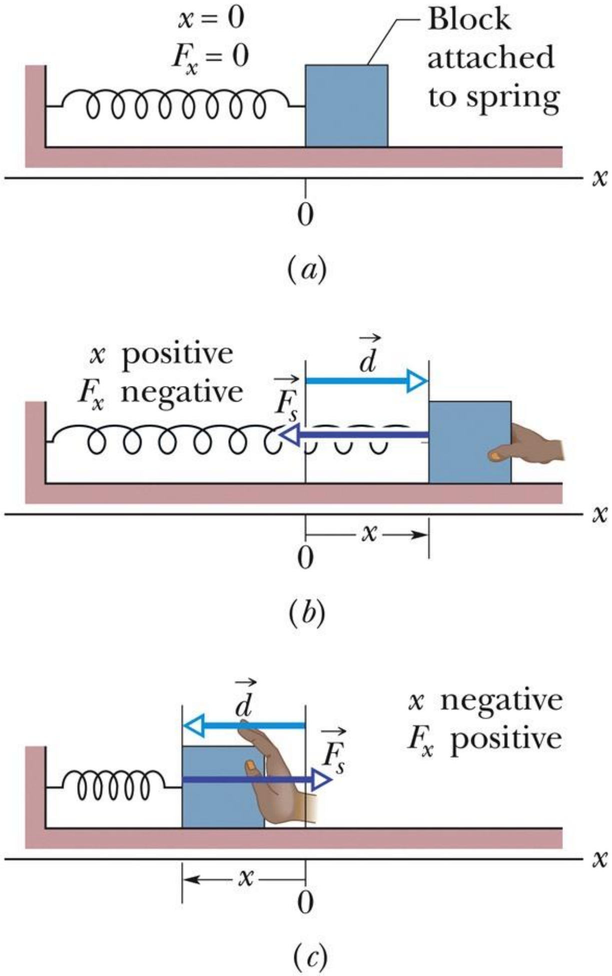


Figure 7-10

7-4 Work Done by a Spring Force

- The spring force is given by **Hooke's law**虎克定律:

$$\vec{F}_s = -k\vec{d} \qquad \text{Eq. (7-20)}$$

- The negative sign represents that the force always opposes the displacement (形變量)
- The **spring constant** *k* (彈性係數) is a is a measure of the stiffness of the spring
- This is a variable force (function of position) and it exhibits a linear relationship between *F* and *d*
- For a spring along the *x*-axis we can write:

$$\boxed{F_x = -kx} \qquad \text{Eq. (7-21)}$$

7-4 Work Done by a Spring Force

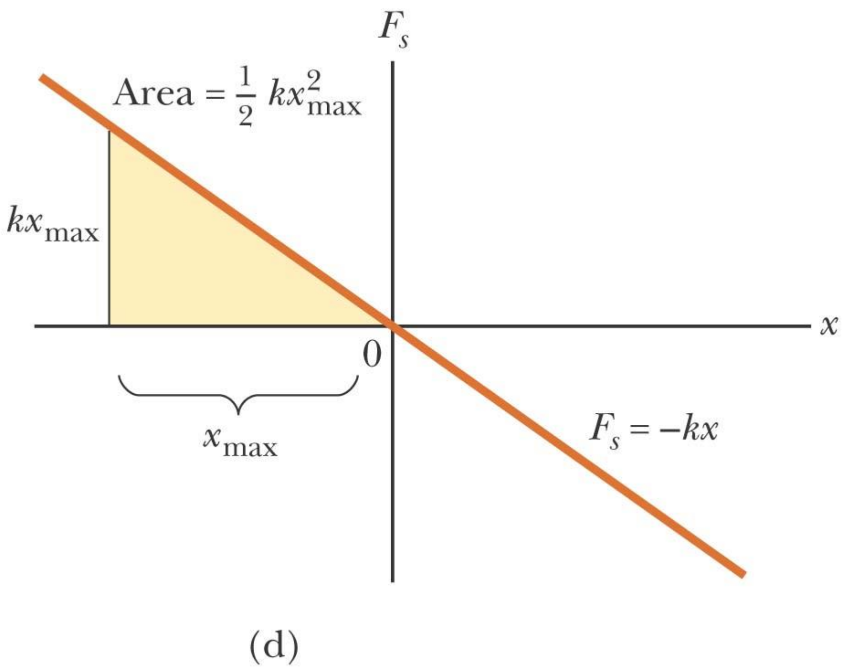
- We can find the work by integrating:

$$W_s = \int_{x_i}^{x_f} -F_x \, dx. \qquad \text{Eq. (7-23)}$$

- Plug *kx* in for F_x:

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{Eq. (7-25)}$$

- The work:
  - Can be positive or negative
  - Depends on the *net* energy transfer



7-5 Work Done by a General Variable Force

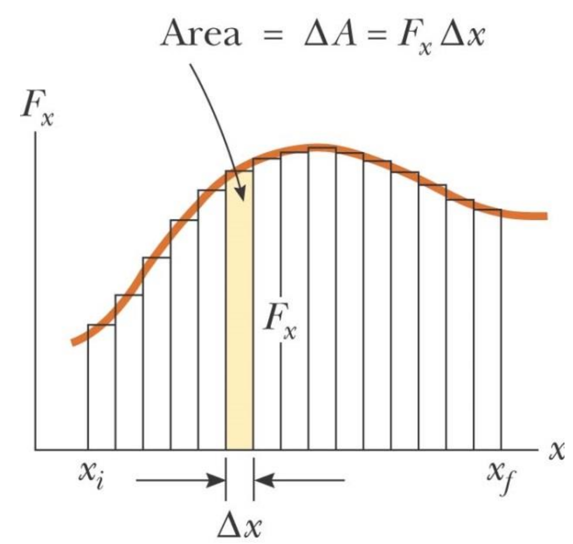
Learning Objectives

7.14 Given a variable force as a function of position, calculate the work done by it on an object by integrating the function from the initial to the final position of the object in one or more dimensions.

7.15 Given a graph of force versus position, calculate the work done by graphically integrating from the initial position to the final position of the object.

7.16 Convert a graph of acceleration versus position to a graph of force versus position.

7.17 Apply the work-kinetic energy theorem to situations where an object is moved by a variable force.



(a)

7-5 Work Done by a General Variable Force

- We take a one-dimensional example
- We need to integrate the work equation (which normally applies only for a constant force) over the change in position
- We can show this process by an approximation with rectangles under the curve

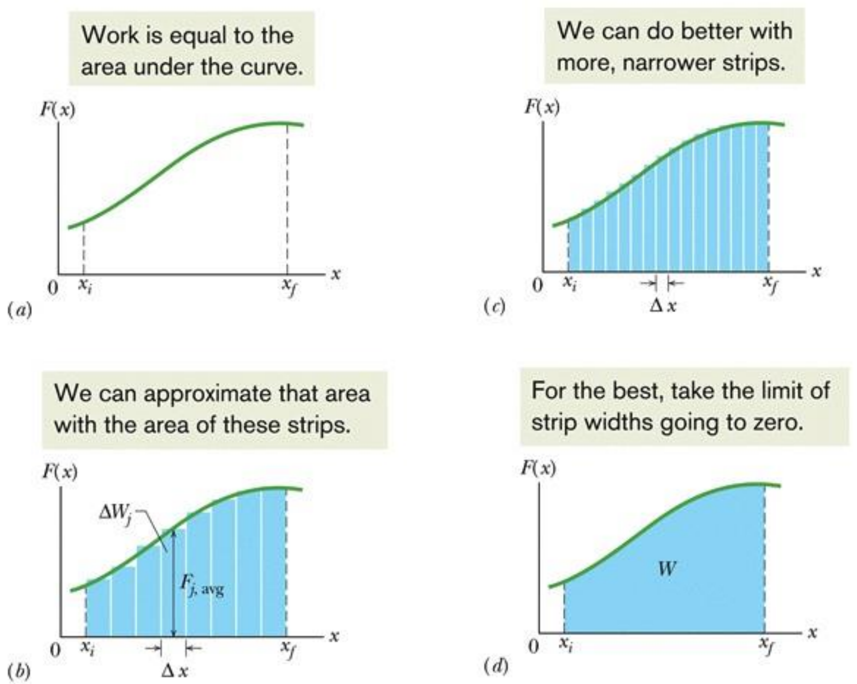


Figure 7-12

7-5 Work Done by a General Variable Force

- Our sum of rectangles would be:

$$W = \lim_{\Delta x \to 0} \sum F_{j,\text{avg}} \Delta x. \qquad \textbf{Eq. (7-31)}$$

- As an integral this is:

$$\boxed{W = \int_{x_i}^{x_f} F(x)\, dx} \qquad \textbf{Eq. (7-32)}$$

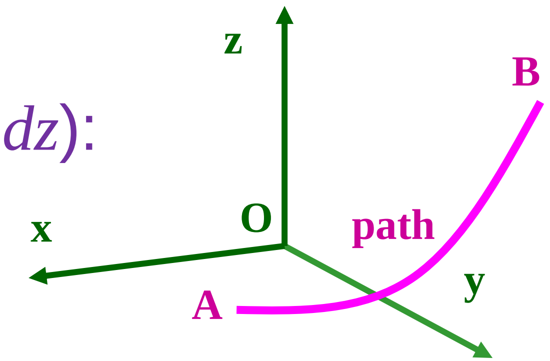
- In three dimensions, we integrate each separately:

$$\boxed{W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy + \int_{z_i}^{z_f} F_z\, dz.} \qquad \textbf{Eq. (7-36)}$$

- The work-kinetic energy theorem still applies!

~非直角座標系統之微量長度積分(i. e., dx, dy, dz):

注意“尺規係數”(metric coefficient, see CH3)



7-6 Power

Learning Objectives

7.18 Apply the relationship between average power, the work done by a force, and the time interval in which that work is done.

7.19 Given the work as a function of time, find the instantaneous power.

7.20 Determine the instantaneous power by taking a dot product of the force vector and an object's velocity vector, in magnitude-angle and unit-vector notations.

目前太陽能模組安裝成本: ~ 2.3 USD/W

7-6 Power

- **Power** is the time rate at which a force does work
- A force does *W* work in a time *Δt*; the **average power** due to the force is:

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad \textbf{Eq. (7-42)}$$

- The **instantaneous power** at a particular time is:

$$\boxed{P = \frac{dW}{dt}} \quad \textbf{Eq. (7-43)}$$

- The SI unit for power is the watt (W): 1 W = 1 J/s
- Therefore work-energy can be written as (*power*) x (*time*) e.g. kWh, the kilowatt-hour

7-6 Power

- Solve for the instantaneous power using the definition of work:

$$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right),$$

$$P = Fv\cos\phi.$$

Eq. (7-47)

- Or:

$$\boxed{P = \vec{F}\cdot\vec{v}}$$

Eq. (7-48)

Unit of *P*:

The SI unit of power is the watt. It is defined as the power of an engine that does work $W = 1\ J$ in a time $t = 1$ second

A commonly used non-SI power unit is the horsepower (hp) defined as:

1 hp = 746 W

The kilowatt-hour ≡ 1度電

The kilowatt-hour ($kW\,h$) is a unit of work. It is defined as the work performed by an engine of power

$P = 1000$ W in a time $t=1$ hour $W = Pt = 1000 * 3600 = \boxed{3.60 * 10^6\ J}$ The $kW\,h$ is used by electrical utility companies (check your latest electric bill)

Background/Motivation:

台東推太陽光電 公有房舍學校起跑

中廣新聞網 – 2013年5月20日 下午5:18
台東縣將利用日照優勢，從公有房舍、學校開始，啟動太陽光電綠能政策，投資三點五億元，在二年內完成太陽光電系統的設置，每年為縣庫入帳至少一千萬元。
台東縣長黃健庭表示，台東縣日照天數多，但過去在替代能源的推動上，卻只有零星的計畫與成果，*這次運用經濟部能源局推動太陽光電優惠電能躉購機制，由廠商來承租縣內所轄的公有房舍屋頂投資賣電，*合作契約長達二十年，將售電所得百分之十五點八作為權利金，百分之十作為回饋金，初估縣府每年權利金、回饋金收入至少一千萬元。
台東縣政府表示，包括台東縣所屬機關、學校約一百四十處、總面積十公頃的屋頂，未來將陸續設置太陽光電系統；縣府將規劃部份回饋金供學校或管理單位運用。另外，得標廠商李長榮集團承諾，將協助台東縣至少四所學校設置太陽光電教育與展示平台，作為節能減碳教育用。（照片：吳鳳雪攝）

假設發電平均:每年300日，每日6 hr，使用20年:
~每度電成本= 每瓦成本/20年總發電度數
=(2.3 USD)/[(1 W x 6 x 300 x 20)/(1000 W hr)] ≈ 0.07 USD
土地租金: 2坪/ kW ~ 1/15 USD/度 (每月每坪~5 USD)
➔ Total: ~0.14 USD /度= 4.2 NTD /度; +系統維護 : > 4.5 NTD /度

永續能源 高市打造全國首座陽光社區

自立晚報 – 2013年5月5日 上午12:00

【記者林香織高雄報導】高雄市政府為響應陽光屋頂百萬座計畫太陽光電應用之願景，推動太陽光電陽光社區建置，塑造太陽光電輔助供電之群聚應用示範，提出杉林大愛陽光社區發展計畫，*率先全國獲得「經濟部推動陽光社區補助要點」經費補助*，成為全國首案。

高雄市政府經濟發展局曾文生局長3日表示，該局結合市府重建會、都發局正積極推動大愛陽光社區建置，藉由重建區大愛園區永久屋基地做為實驗基地，於屋頂設置太陽光電系統，建立太陽光電示範社區，初步規劃設置容量1650千瓦，希冀藉由推動杉林大愛陽光社區發展計畫以塑造綠能減碳環境氛圍，引領全民落實節約能源並達成國家節能目標，並擴大太陽能發電的市場內需，以提升綠能廠商投資高雄意願，並創造在地居民就業機會，希望大愛陽光社區成為全國首座大型光電永續能源生活概念區域。

目前高雄市政府對於住宅裝置太陽光電提供多項的優惠補助及綠色融資措施，如經發局率先全國於102年1月3日推出高雄市專有之住宅型綠色融資，民眾於自家屋頂裝置太陽光電設備可享有「零出資」全額貸款(每案最高新台幣60萬元)、優惠利率(目前為2.825%)、免保證人、免抵押品、長期貸款(年限10年)、本金寬限期(年限1年)等優惠，貸款手續簡便，目前已審核通過案件2件，融資金額73萬；另經發局已成立市府太陽光電單一受理窗口，提供民眾與業者有關市府太陽光電政策諮詢服務、裝設問題協助、融資補助等協助，並規劃今年經濟部正式委辦太陽光電發電設備之同意備案、查驗、設備登記、撤銷、廢止及其他相關業務後，提供民眾太陽光電之設備認定服務。

曾文生指出，發展再生能源與節能減碳乃是目前綠能發展的主流，已有廠商看好高雄日照條件，正積極籌設民營太陽能發電廠，總投資額約新台幣4億元，因設置地點位於台灣最具代表性之高速鐵路及都會捷運機場屋頂*建置太陽光電系統，裝置規模高達6百萬瓦，完工後年發電量可望超過1000萬度，年減4,400頓二氧化碳排放量，*是繼高雄世運主場館之後，最具代表性之太陽光電系統示範建設，加上後續發電系統之營運維護管理，對本市除環境及太陽光電發展具有重大正面影響外，亦能有效提升本市就業機會，相關推動計畫正由經發局日光屋頂專案推動辦公室進行中。

市民朋友如有裝設太陽光電相關問題，歡迎電洽經發局日光屋頂專案推動辦公室，將有專人為您服務。（電話：3368333轉3165）。2013/5/4

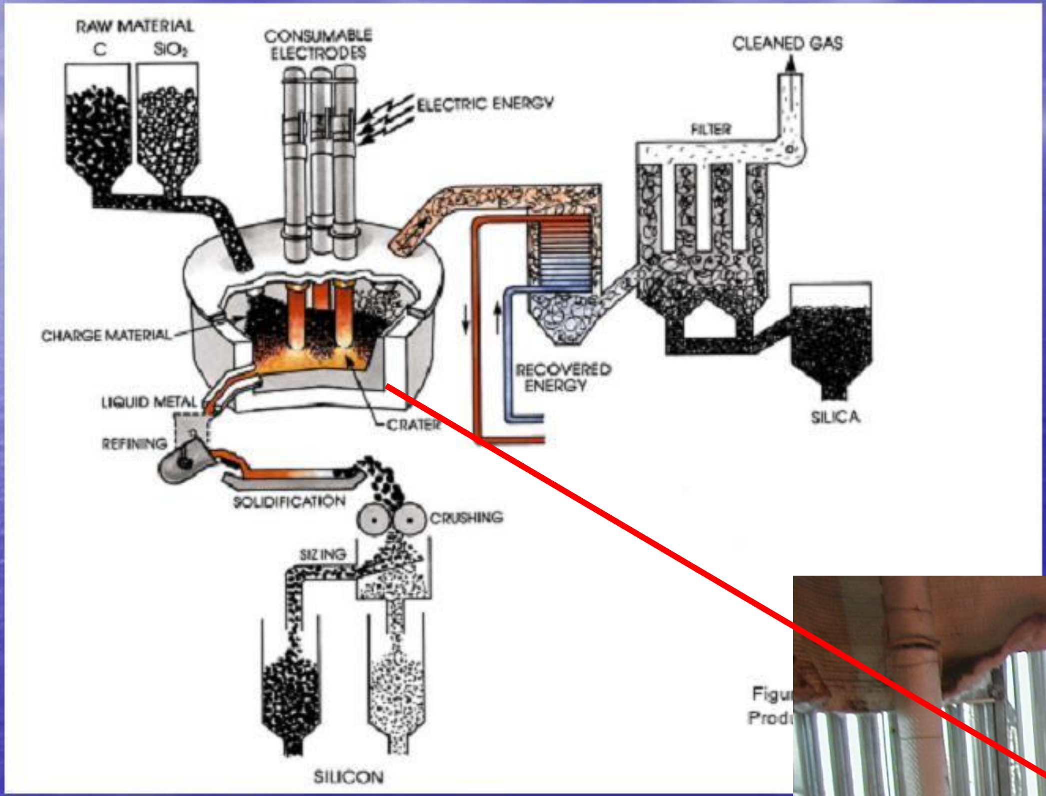
~The problems:???

- 2012年12/12日，政府未來四年(102-105)綠能方案:~希望到2020年太陽光電發電成本每度可降至3.5~4元目標…
- 2012年12/10日工商時報:現階段太陽光電產業之生存，係依賴"台電以約每度8.3971元收購太陽能發電“…
- 太陽光電只是一個依賴政府(全民)扶持的*“包袱產業“!?*即使，太陽光電發電每度降至2020年3.5~4元目標，這仍然只算是及格邊緣!
- 當太陽光電發電成本(~ 7.177元，2012年12/10日工商時報大於使用台電之購電成本情形下，*目前裝設太陽光電越多，耗費地球資源越多!?*…

~新政策之新思維:---研發經費與補助購電(陽光電)經費之重新分配!

核能: ~0.8 NTD/度； 燃煤: ~1.6 NTD/度；天然氣: ~3.0 NTD/度

火法冶金法多晶矽計畫



RAW MATERIAL
C
SiO2
CONSUMABLE ELECTRODES
ELECTRIC ENERGY
CLEANED GAS
FILTER
CHARGE MATERIAL
RECOVERED ENERGY
LIQUID METAL
CRATER
SILICA
REFINING
SOLIDIFICATION
CRUSHING
SIZING
SILICON

7 Summary

Kinetic Energy

- The energy associated with motion

$$K = \frac{1}{2}mv^2$$ **Eq. (7-1)**

Work

- Energy transferred to or from an object via a force
- Can be positive or negative

Work Done by a Constant Force

$$W = Fd\cos\phi$$ **Eq. (7-7)**

$$W = \vec{F}\cdot\vec{d}$$ **Eq. (7-8)**

- The **net work** is the sum of individual works

Work and Kinetic Energy

$$\Delta K = K_f - K_i = W,$$ **Eq. (7-10)**

$$K_f = K_i + W,$$ **Eq. (7-11)**

7 Summary

Work Done by the Gravitational Force

$$W_g = mgd\cos\phi$$ **Eq. (7-12)**

Work Done in Lifting and Lowering an Object

$$W_a + W_g = 0$$

$$W_a = -W_g.$$ **Eq. (7-16)**

Spring Force

- Relaxed state: applies no force
- Spring constant *k* measures stiffness

$$\vec{F}_s = -k\vec{d}$$ **Eq. (7-20)**

Spring Force

- For an initial position *x* = 0:

$$W_s = -\tfrac{1}{2}kx^2$$ **Eq. (7-26)**

7 Summary

Work Done by a Variable Force

- Found by integrating the constant-force work equation

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

Eq. (7-32)

Power

- The rate at which a force does work on an object
- Average power:

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Eq. (7-42)

- Instantaneous power:

$$P = \frac{dW}{dt}$$

Eq. (7-43)

- For a force acting on a moving object:

$$P = Fv\cos\phi.$$

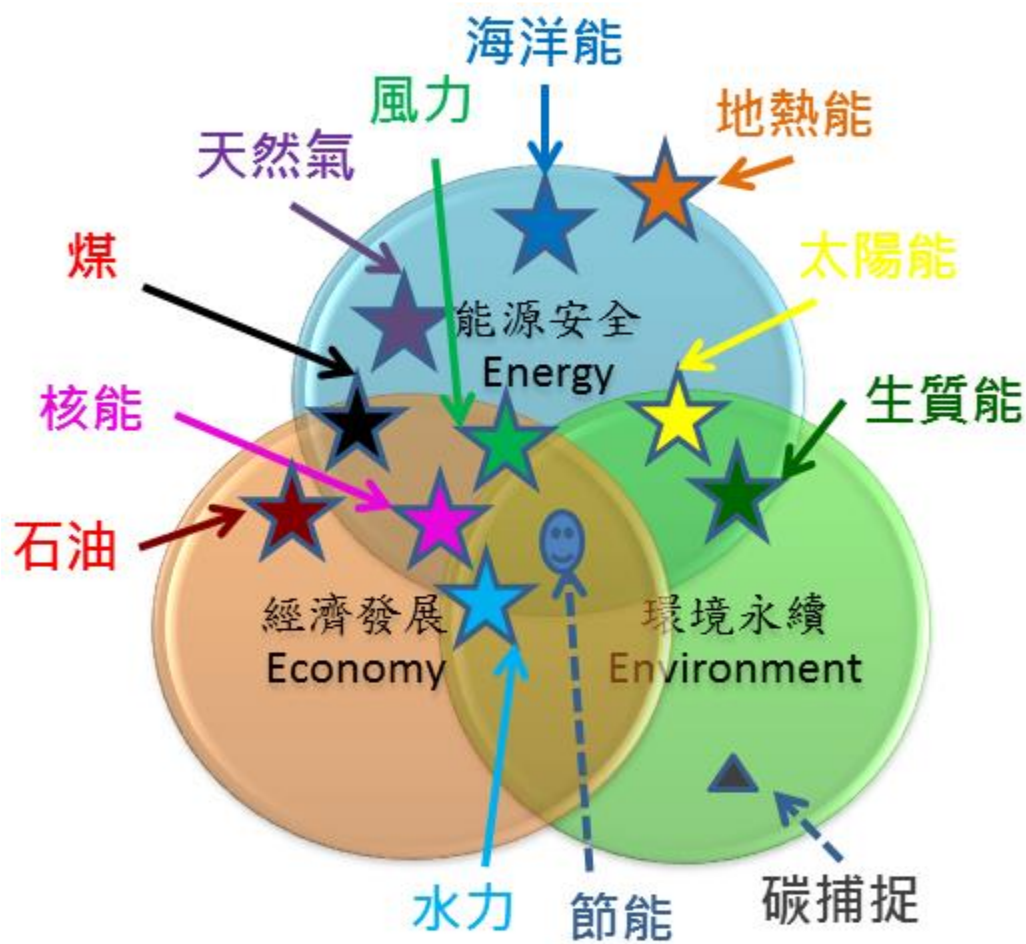
Eq. (7-47)

$$P = \vec{F}\cdot\vec{v}$$

Eq. (7-48)

CH 7 習題:

9, 10, 14, 24, 32, and 40



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