Chapter 2 基本原理

2-1 基礎

2-1-1 電磁波

$$\lambda = c/\tilde{\nu} \tag{2.1}$$

$$\nu = \tilde{\nu}/c = 1/\lambda \tag{2.2}$$

其中　λ ：波長 (wave length)；$\tilde{\nu}$ ：頻率 (frequency)

　　　ν ：波數 (wave number)；c ：光速 (= 2.99793*10^8m sec^{-1})

單位　μm ：10^{-6} m (公尺) (micro-meter) ； nm：10^{-9} m (nano-meter)

　　　Å (埃)：10^{-10} m

UV：紫外線 ；IR：紅外線

表一

α ：alpha

γ ：gamma

ε ：epsilon

η ：eta

κ ：kappa

μ ：mu /mju/

ξ ：xi /ksi/

σ ：sigma

φ 、ϕ ：phi

ψ ：psi

β ：beta

δ ：delta

ζ ：zeta

θ ：theta

λ ：lambda

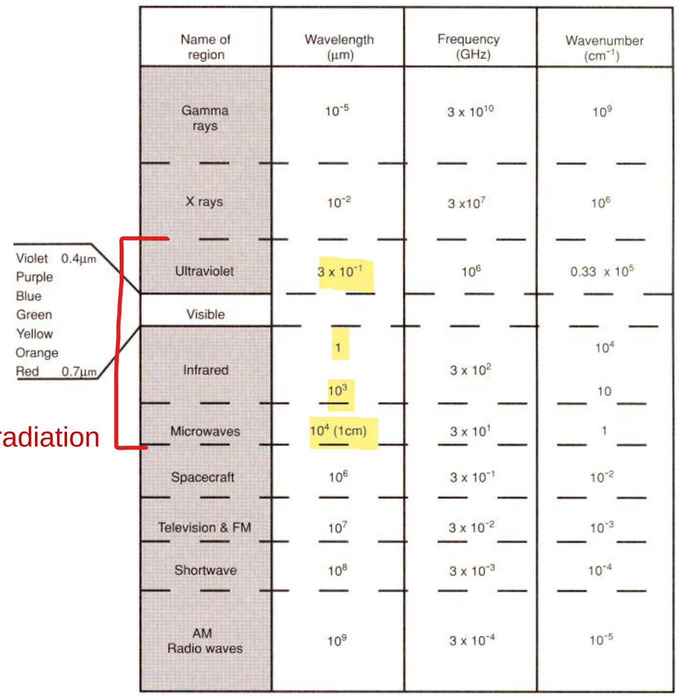
ν ：nu /nu/ or /nju/

ρ ：rho

τ ：tau

χ ：chi

ω ：omega



important in atmospheric radiation

Fig 2.1: The electromagnetic spectrum in terms of wavelength in μm, frequency in GHz, and wavenumber in cm^{-1}. [Liou02, Figure1.1]

4 um+ -> longwave

2-1-2 立體角（solid angle）

$$\Omega = \sigma / r^2 \tag{2.3}$$

$= $ 球表面積 / 半徑的平方

球表面積 $=$ $4\pi r^2$

➔全球立體角 $=$ 4π

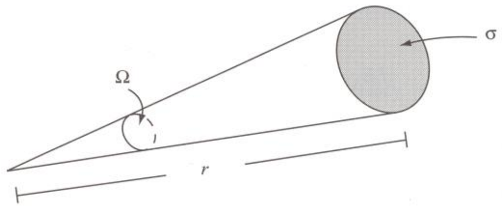


Fig 2.2: Definition of a solid angle Ω, where σ denotes the area and r is the distance. [Liou02, Figure1.2]

所以在 Fig2.3，

$$d\sigma = (r\,d\theta)(r\sin\theta\,d\varphi) \tag{2.4}$$

$$\Rightarrow d\Omega = d\sigma / r^2 = \sin\theta\,d\theta\,d\varphi \tag{2.5}$$

其中 θ：天頂角

ϕ：方位角

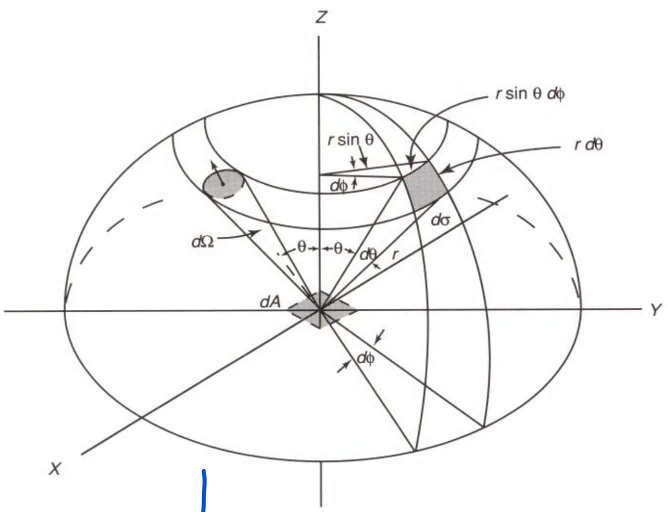
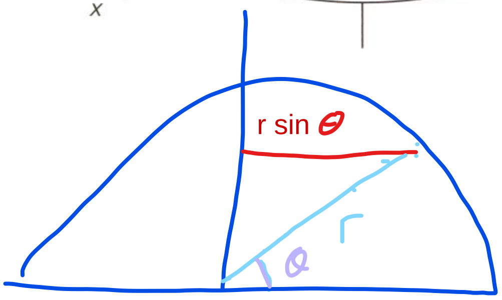
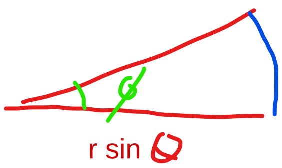


Fig 2.3: Illustration of a differential solid angle and its representation in polar coordinates. Also shown for demonstrative purposes is a pencil of radiation through an element of area dA in directions confined to an element of solid angle $d\Omega$. Other notations defined in the text. [Liou02, Figure1.3]





θ : 天頂角

ϕ : 方位角

2-1-3 輻射強度

對於單位面積(dA)從 dΩ 方向所傳來的電磁波(光)能量，在單位時間(dt)和在波長 λ 與 λ+dλ 之間的值為

$$dE_\lambda = I_\lambda \cos\theta \, d\Omega \, dA \, d\lambda \, dt \qquad (2.6)$$

$$\boxed{\rightarrow I_\lambda(\text{輻射強度}) = \frac{dE_\lambda}{\cos\theta \, d\Omega \, d\lambda \, dt \, dA}} \qquad (2.7)$$

單位：能量 / (面積．時間．立體角．波長)

Defining 輻射通量密度(F_λ) (單位：能量 / (時間．面積．波長))

$$F_\lambda = \int_\Omega I_\lambda \cos\theta \, \underline{d\Omega} \qquad (2.8)$$

integral solid angle

$$\left(= \frac{dE_\lambda}{d\lambda \, dt \, dA} \right)$$

代入(2.5)

$$\rightarrow F_\lambda = \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta,\varphi) \cos\theta \, \underline{\sin\theta \, d\theta \, d\varphi} \qquad (2.9)$$

如果考慮 $I_\lambda(\theta,\phi)$ 不隨 θ 和 ϕ 改變 (isotropic radiation) I is constant

$$\rightarrow F_\lambda = I_\lambda \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\varphi \qquad (2.10)$$

$$= \pi I_\lambda \qquad (2.11)$$

Defining 總輻射通量密度(F) (單位：能量 / (時間．面積))

$$F = \int_0^\infty F_\lambda \, d\lambda \qquad (2.12)$$

Defining 輻射通量(輻射功率)(f) (flux) (單位：能量 / 時間 = w)

$$f = \int_A F dA \qquad (2.13)$$

Symbols, Dimensions, and Units of Various Radiometric Quantities

Symbol	Quantity	Dimension[a]	Unit[b]
E	Energy	ML^2T^{-2}	Joule (J)
f	Flux (luminosity)	ML^2T^{-3}	Joule per second (J $\sec^{-1}$, W)
F	Flux density (irradiance) Emittance	MT^{-3}	Joule per second per square meter (W m^{-2})
I	Intensity (radiance) Brightness (luminance)	MT^{-3}	Joule per second per square meter per steradian (W m^{-2} sr^{-1})

[a] M is mass, L is length, and T is time.
[b] 1 watt (W) = 1 J $\sec^{-1}$.

Table 2.1 [Liou02, Table1.1]

2-1-4 散射和吸收 (削弱過程)

◎散射(Scattering)：光在傳播時，在其經過的粒子會吸收入射光的能量，然後將所吸收的能量再傳播到每個方向。

➔反射、折射都是散射的一種

◎散射的方式取決於粒子和入射光波長的比($x = 2\pi a/\lambda$)

<i>Rayleigh scattering (雷氏)： $x << 1$ (molecules ➔blue sky)

depends on wavelength ($\propto \lambda^{-4}$)

<ii>Lorenz-Mie scattering (米氏)： $x \geq 1$ (white clouds)

depends on particle size, leas depends on wavelength

◎輻射過程中包含了吸收(absorption)和再輻射(re-radiate)

◎當 $x >> 1$ 時，一般所使用的方法為幾何光學(geometric optics)

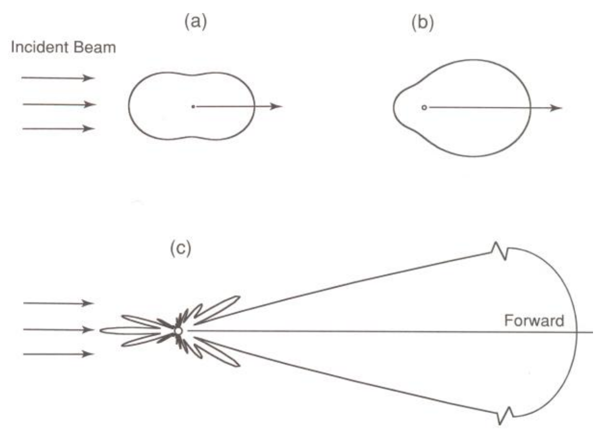
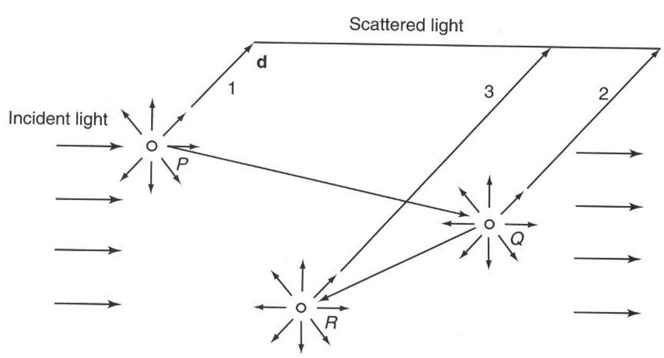


Fig 2.4: Demonstrative angular patterns of the scattered intensity from spherical aerosols of three sizes illuminated by the visible light of 0.5 μm, (a) 10^{-4} μm, (b) 0.1 μm, and (c) 1 μm. The forward scattering pattern for the 1 μm aerosol is extremely large and is scaled for presentation purposes. [Liou02, Figure1.4]



多重散射

Figure 2.5: Multiple scattering process involving first (P), second (Q), and third (R) order scattering in the direction denote by **d**. [Liou02 ,Figure1.5]

more aerosol
Multiple scattering is more important

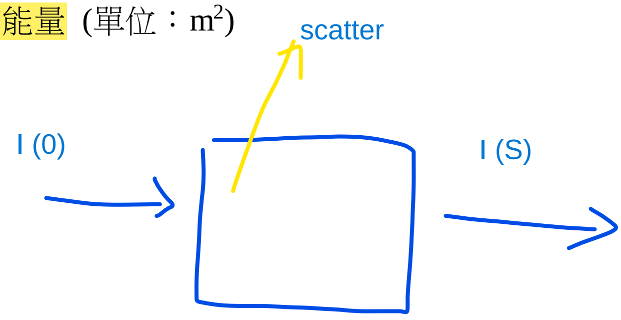
2-1-5 削弱截面積(extinction cross section)
和削弱係數(extinction coefficient)

◎削弱截面積：粒子從入射光所移去的能量 (單位：m^2)

$$\sigma = f / F_0 \qquad (\text{ see } (2.13))$$

其中 F_0 ：入射能量

f ：所移去的能量



◎削弱係數 $\beta = \sigma\, n$ (單位：m^{-1})

其中 n：粒子數密度

2-2 黑體輻射

黑體輻射(blackbody radiation):在熱力平衡下，在所有波長及所以方向的輻射都被完全的吸收或放射(吸收和放射是輻射的一體兩面)，所以黑體輻射是均質的(homogeneous)，無偏極化的(unpolarized)和無方向性的(isotropic)。

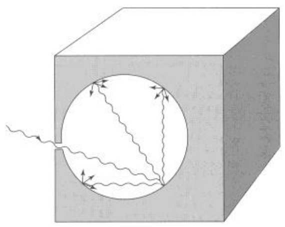


Figure 2.6: A blackbody radiation cavity to illustrate that absorption is complete. [Liou02, Figure1.6]

圖 2.6 可用來說明黑體輻射。所有從小洞進入箱子的輻射均無法逃出來，直到所有的輻射都被箱子的內牆所吸收。以下是關於黑體輻射的四大定律：

2-2-1 Planck's 定律 (對於特定波長的輻射強度)

從輻射能量 $$E = nh\tilde{\nu} \tag{2.14}$$

$$B_\lambda = \frac{2hc^2}{\lambda^5 (e^{hc/K\lambda T} - 1)} \tag{2.15}$$

其中 B_λ ：Planck's 函數 (和 I_λ 相同的單位)

h ：Planck's 常數 = 6.626810^{-34} Jsec

K ：Boltzmann 常數 = 1.3806810^{-23} Jdeg^{-1}

$\tilde{\nu}$ ：頻率

n ：量子數

➔黑體輻射強度 depends on T(溫度) 和波長(λ)，且其強度符合(2.15)的要求。

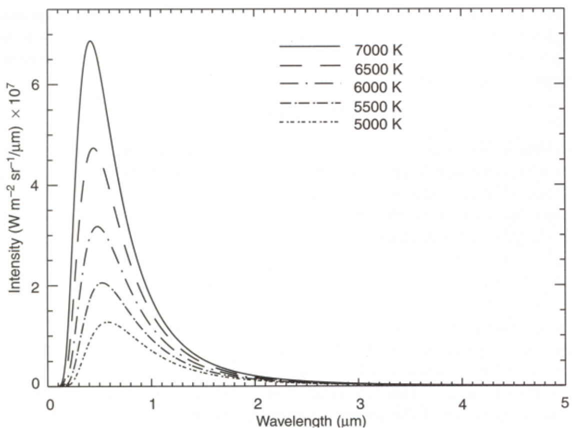


Fig 2.7: Blackbody intensity (Planck function) as a function of wavelength for a number of emitting temperatures. [Liou02, Figure1.7]

2-2-2 Stefan-Boltzmann 定律

-對於所有波長的輻射強度

$$B(T) = \int_0^\infty B_\lambda(T)d\lambda = bT^4 \tag{2.16}$$

其中 $b = 2\pi^4 k^4 / 15c^2 h^3$

因為黑體輻射是各向同性 (isotropic) (see (2.11))

$$\rightarrow \mathrm{F}(\text{輻射總通量密度}) = \pi B(T) = \boxed{\sigma T^4} \tag{2.17}$$

其中 σ (Stefan-Boltamann 常數) = $\boxed{5.67*10^{-8}\ \mathrm{Jm}^{-2}}$

➔黑體輻射的總輻射通量密度和溫度(T)的四次方成正比

2-2-3 Wien's 位移定律

-計算最大輻射所在的波長

$$\frac{\partial B_\lambda(T)}{\partial \lambda} = 0 \tag{2.18}$$

differential with radiation

$$\rightarrow \quad \lambda_m = \frac{2.897*10^{-3}}{T} \tag{2.19}$$

[m]

➔黑體輻射最大強度的波長和溫度成反比

2-2-4 Kirchhoff's 定律

在熱力平衡狀態下，即在均勻溫度分佈和無方向性的情況下，對任一波長而言，熱輻射的吸收(absorption)和放射(emission)必須相等。

$$\varepsilon_\lambda(\mathit{emissivity}\text{放射率}) = A_\lambda(\mathit{absorptivity}\text{吸收率}) \tag{2.20}$$

其中 $\varepsilon_\lambda = \dfrac{\text{放射強度}}{\text{黑體輻射強度}}$

$$A_\lambda = \frac{\text{吸收強度}}{\text{黑體輻射強度}}$$

➔對於一個黑體輻射

$$\varepsilon_\lambda = A_\lambda = 1 \tag{2.21}$$

如果$\varepsilon_\lambda = A_\lambda < 1$，稱之為灰體輻射 (2.22)

2-3 輻射傳遞方程 (RTF)

2-3-1 輻射傳遞方程式

Defining 質量削弱截面積(mass extinction cross section)：$k_\lambda(m^2 kg^{-1})$

對單位質量所削弱的截面積

$$k_\lambda = -\frac{dI_\lambda}{mI_\lambda}A \qquad (m\text{：質量 kg})$$

$$= -\frac{dI}{I_\lambda}\frac{Ads}{mds}$$

$$= -\frac{dI}{I_\lambda}\boxed{\frac{V}{m}}\frac{1}{ds}$$

ρ

$$= -\frac{dI}{I_\lambda}\frac{1}{\rho ds}$$

$$\rightarrow dI_\lambda = -k_\lambda \rho I_\lambda ds \tag{2.23}$$

lost depends on :
medium density / medium type (k)
/ path (ds) / incoming energy

k_λ ：includes effects of absorption and scattering depends on 介質

ρ ：介質密度 (kg m-3)

Defining 輻射源函數係數(source function coefficient)：

j_λ(單位質量的輻射通量) includes emission (放射) and multiple scattering (多次散射)

$$j_\lambda = \frac{A\,dI_\lambda}{m}$$

$$\rightarrow dI_\lambda = j_\lambda m/A$$

$$= j_\lambda \rho ds \tag{2.24}$$

如果將輻射源定義成和輻射強度(I_λ)相同的形式

$$J_\lambda \equiv j_\lambda / k_\lambda \tag{2.25}$$

def

(輻射通量/面積 = 輻射通量密度 = 輻射強度)

$\rightarrow dI_\lambda = k_\lambda \rho\, J_\lambda ds$

Combining (2.23), (2.24) and (2.25)

$$dI_\lambda = -k_\lambda \rho I_\lambda ds + j_\lambda \rho ds \tag{2.26}$$

lost (extinct) gain(source)

change of I

$$= -k_\lambda \rho I_\lambda ds + k_\lambda \rho J_\lambda ds \tag{2.27}$$

$$\rightarrow \boxed{\frac{dI_\lambda}{k_\lambda \rho ds} = -I_\lambda + J_\lambda} \tag{2.28}$$

Radiative Transfer Function (RTF)

Note：When radiation passes through a medium, the change of radiation is determined by：

<i> $-I_\lambda$：extinction of incident radiation, including absorption and scattering by medium

decrease

<ii> J_λ：radiation sources in medium, including emission and multiple scattering

increase

can be ignored when air is very clear

◎輻射傳遞方程式(2.28) 的物理意義：

輻射強度通過某介質(ρ)經過距離 ds 所變化的程度和介質的特性(k_λ)和密度(ρ)及所經過的距離(ds)有關，同時正比於入射輻射的強度(I_λ)和介質本身的輻射的強度(J_λ)。

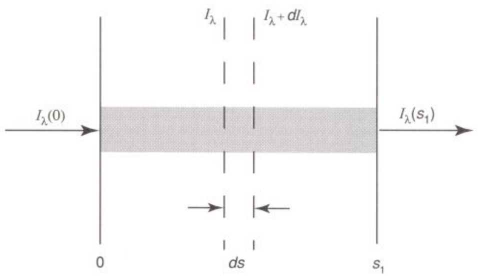


Fig 2.8: Depletion of the radiant intensity in traversing an extinction medium.[Liou02, Figure1.13]

2-3-2 Beer-Bouguer-Lambert (Beer's)定律

good for short wave
(only care about extinction)

Assuming $I_\lambda(0)$ at s = 0, $I_\lambda(s_1)$ at s = s1

surface SR flux is fewer then TOA
because atmosphere absorbs the radiation
and medium will not radiate SR.

From (2.23)

0 is Top Of Atmosphere

Beer's law calculate different height's short wave flux

$$\frac{dI_\lambda}{I_\lambda} = -k_\lambda \rho ds$$

$$\Rightarrow \int_{I_\lambda(0)}^{I_\lambda(s_1)} \frac{1}{I_\lambda} dI_\lambda = \int_0^{s_1} -k_\lambda \rho ds$$

$$\Rightarrow \ln I_\lambda \Big|_{I_\lambda(0)}^{I_\lambda(s_1)} = \int_0^{s_1} -k_\lambda \rho ds$$

$$\Rightarrow \frac{I_\lambda(s_1)}{I_\lambda(0)} = \exp\left(\int_0^{s_1} -k_\lambda \rho ds\right)$$

$$\Rightarrow I_\lambda(s_1) = I_\lambda(0)\exp\left(-\int_0^{s_1} k_\lambda \rho ds\right) \tag{2.29}$$

If the medium is homogeneous, defining 吸收體含數(path length)

$$u = \int_0^{s_1} \rho\, ds$$

$$\Rightarrow I_\lambda(s_1) = I_\lambda(0)\, e^{-k_\lambda u}$$

➔the change of the radiation exponentially depends on the path length of the medium.

Defining 光程 (optical depth) (沒有單位) dimensionless

$$\tau(s_1, 0) \equiv \int_0^{s_1} k_\lambda \rho ds \tag{2.30}$$

very clear atmosphere = 0

(note $d\tau(s_1, s) = -k_\lambda \rho ds$)

max: infinity

$$\Rightarrow I_\lambda(s_1) = I_\lambda(0)\, e^{-\tau_\lambda(s_1,0)} \tag{2.31}$$

➔物理意義(τ)：介質的有效厚度

$\tau \nearrow$ ➔輻射減弱越多

➔介質的透光率愈低

Note： $\tau \nearrow$ ➔ $s \searrow$ (愈接近 0)

$$\Rightarrow d\tau(s_1, s) = -k_\lambda \rho\, ds$$

Defining 透射係數 (transmissivity) $\Im_\lambda$

$$\Im_\lambda \equiv \frac{\text{透射輻射強度}}{\text{入射輻射強度}}$$

$$= \frac{I_\lambda(s_1)}{I_\lambda(0)}$$

$$= e^{-\tau_\lambda(s_1,0)} \qquad (2.32)$$

0~1

For 非散射的介質 (只有吸收)

$$A_\lambda = 1 - \Im_\lambda = 1 - e^{-\tau_\lambda(s_1,0)} \qquad (2.33)$$

For 反射的介質

從能量守衡

$$\rightarrow \Im_\lambda + A_\lambda + R_\lambda = 1 \qquad (2.34)$$

其中 R_λ：反射率 (reflectivity)

Note：反射是散射的一種

2-3-3 Schwarzschild 方程 (2.38)

use in long wave

absorption
emission

physical meaning
assume we have incoming radiation
???

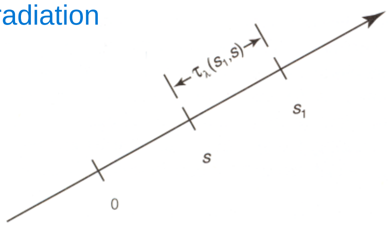


Fig 2.9: Configuration of the optical thickness τ_λ defined in Eq. (1.4.15). [Liou02, Figure1.14]

Considering 非散射介質在熱力平衡下(e.g.地球長波輻射)，如果用 Planck 函數 $(B_\lambda(T))$代替 J_λ in(2.28) 和 using(2.30)

$$\frac{dI_\lambda(s)}{-d\tau_\lambda(s_1,s)} = -I_\lambda(s) + B_\lambda(T(s)) \qquad (2.35)$$

將(2.35)乘上$e^{-\tau_\lambda(s_1,s)}$

$$\rightarrow e^{-\tau_\lambda(s_1,s)} dI_\lambda(s) = -I_\lambda(s) e^{-\tau_\lambda(s_1,s)} d(-\tau_\lambda(s_1,s)) - B_\lambda(T(s)) e^{-\tau_\lambda(s_1,s)} d\tau_\lambda(s_1,s)$$

$$= -I_\lambda(s) de^{-\tau_\lambda(s_1,s)} - B_\lambda(T(s)) e^{-\tau_\lambda(s_1,s)} d\tau_\lambda(s_1,s)$$

vdu + udv

$$\rightarrow e^{-\tau_\lambda(s_1,s)} dI_\lambda(s) + I_\lambda(s) de^{-\tau_\lambda(s_1,s)} = -B_\lambda(T(s)) e^{-\tau_\lambda(s_1,s)} d\tau_\lambda(s_1,s)$$

d(uv)

$$\rightarrow d\left(e^{-\tau_\lambda(s_1,s)} I_\lambda(s)\right) = -B_\lambda(T(s)) e^{-\tau_\lambda(s_1,s)} d\tau_\lambda(s_1,s) \qquad (2.36)$$

積分(2.36) from 0 to s_1

$$\rightarrow -\int_0^{s_1} d\left(e^{-\tau_\lambda(s_1,s)} I_\lambda(s)\right) = \int_0^{s_1} B_\lambda(T(s)) e^{-\tau_\lambda(s_1,s)} d\tau_\lambda(s_1,s) \qquad (2.37)$$

$$\rightarrow -e^{-\tau_\lambda(s_1,s)} I_\lambda(s)\Big|_0^{s_1} = \int_0^{s_1} B_\lambda(T(s)) e^{-\tau_\lambda(s_1,s)} d\tau_\lambda(s_1,s)$$

$$-I_\lambda(s_1)+I_\lambda(0)e^{-\tau_\lambda(s_1,0)}=\int_0^{s_1}B_\lambda(T(s))e^{-\tau_\lambda(s_1,s)}d\tau_\lambda(s_1,s)$$

$$\rightarrow I_\lambda(s_1)=I_\lambda(0)e^{-\tau_\lambda(s_1,0)}-\int_0^{s_1}B_\lambda(T(s))e^{-\tau_\lambda(s_1,s)}d\tau_\lambda(s_1,s) \qquad (2.38)$$

◎物理意義相似於(2.28)

2-3-4 平行大氣的輻射傳遞方程

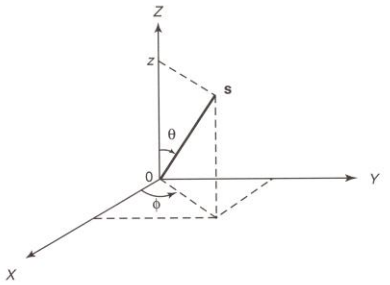


Fig 2.10: Geometry for plane-parallel atmospheres where θ and ϕ denote the zenith and azimuthal angles, respectively, and **s** represents the position vector. [Liou02, Figure1.15]

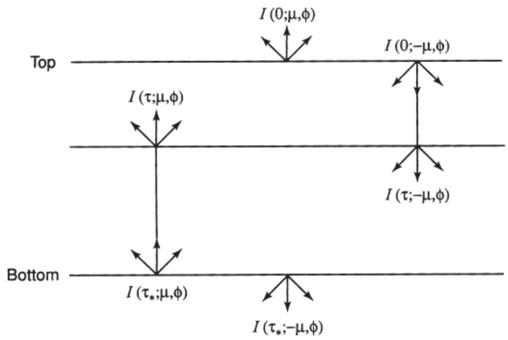


Fig 2.11: Upward (μ) and downward ($-\mu$) intensities at a given level τ and at top ($\tau=0$) and bottom ($\tau=\tau_*$) levels in a finite, plane-parallel atmosphere. [Liou02, Figure1.16]

對於 point $(z;\theta,\phi)$，(省略下標 λ)

$$(2.28)\rightarrow \frac{dI(z;\theta,\varphi)}{k\rho ds}=-I(z;\theta,\varphi)+J(z;\theta,\varphi)$$

$$\rightarrow \frac{dI(z;\theta,\varphi)}{k\rho\sec\theta dz}=-I(z;\theta,\varphi)+J(z;\theta,\varphi)$$

$$\rightarrow \cos\theta\frac{dI(z;\theta,\varphi)}{k\rho dz}=-I(z;\theta,\varphi)+J(z;\theta,\varphi) \qquad (2.39)$$

Defining 法線光程 (normal optical depth)

$$\tau=\int_z^\infty k\rho dz' \qquad (\rightarrow d\tau=-k\rho dz)$$

→從無限遠$(\tau=0)$到 z 的光程在 $\vec{z}$ 方向

$$(2.39)\rightarrow \mu\frac{dI(\tau;\mu,\varphi)}{d\tau}=I(\tau;\mu,\varphi)-J(\tau;\mu,\varphi) \qquad (2.40)$$

其中 $\mu=\cos\theta$

Following section 2-3-3，將 (2.40) 乘上$e^{-\tau/\mu}$

對於向上輻射$(\mu>0)$，(積分 from τ to τ_*)

longwave is not only from surface

boundary terms　　atmospheric terms

$$\rightarrow I(\tau;\mu,\varphi)=I(\tau_*;\mu,\varphi)e^{-(\tau_*-\tau)/\mu}+\int_{\tau}^{\tau_*}J(\tau';\mu,\varphi)e^{-(\tau'-\tau)/\mu}\frac{d\tau'}{\mu} \tag{2.41}$$

對於向下輻射$(-\mu>0)$　，(積分　from 0 to τ)

$$\rightarrow I(\tau;-\mu,\varphi)=I(0;-\mu,\varphi)e^{-\tau/\mu}+\int_{0}^{\tau}J(\tau';-\mu,\varphi)e^{-(\tau-\tau')/\mu}\frac{d\tau'}{\mu} \tag{2.42}$$

◎ 物理意義相似於(2.28)

<i>1st term of r.h.s.：外來的輻射 (outside of τ to $\tau*$)

<ii>2end term of r.h.s：在 level of $\tau-\tau*$中，介質所放射的輻射。

◎ 比較於實際大氣：

在上述公式中，水平均質(homogenous)特性是個必要條件，但在實際大氣中，此一假設不一定成立。例如有雲的時候，雲內外並不均質。

2-4 大氣輻射

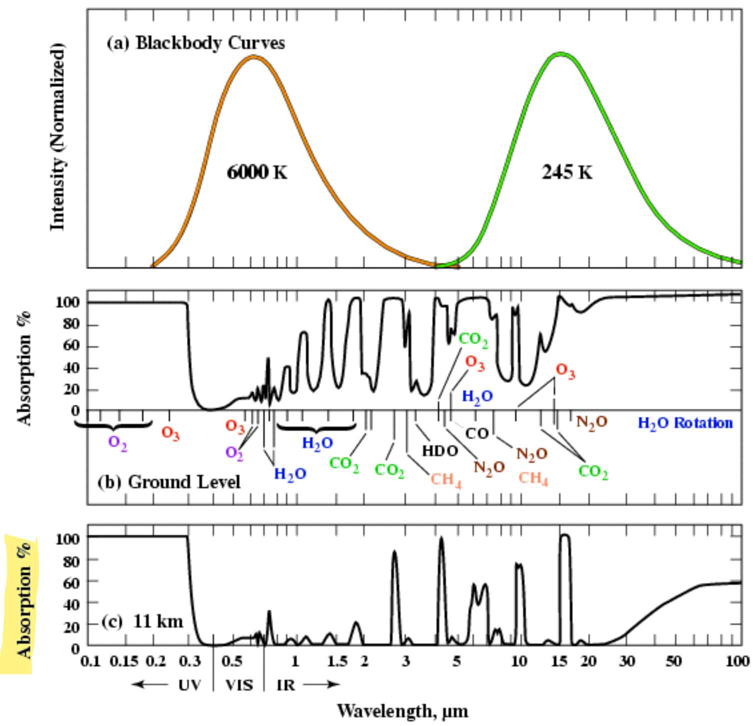


Fig 2.12 (a) Blackbody radiation curves for temperature characteristic of the Sun' s surface and upper levels on Earth' s atmosphere from which infrared radiation escape to space. (b) Absorption of radiation at each wavelength if the beam passes through the entire atmosphere from top to ground level or vice versa. (c) As in b, but for radiation passing from the top of the atmosphere to 11 km or vice versa. After Trenberth (1992) and Goody and Yung (1989).

2.4-1 太陽短波輻射(可見光)

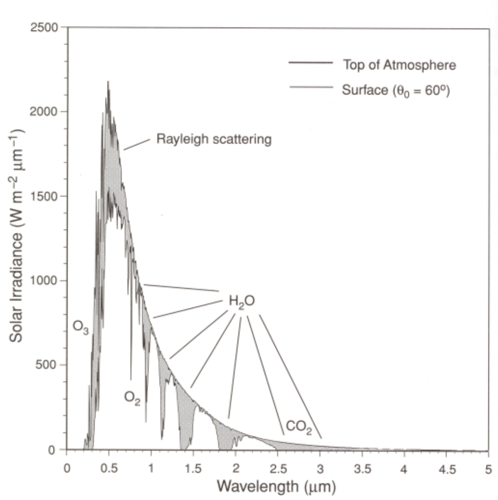


Fig 2.13: Solar irradiance curve for a 50 cm^{-1} spectral interval at the top of the atmosphere (see Fig.2.9) and at the surface for a solar zenith angle of 60° in an atmosphere without aerosols or clouds. Absorption and scattering regions are indicated. See also Table 3.3 for the absorption of N_2O, CH_4, CO, and NO_2. [Liou02, Figure3.9]

2.4-2 地球長波輻射(紅外線)

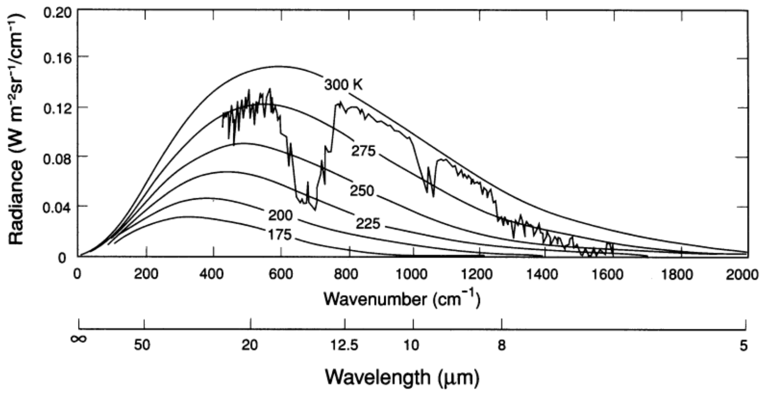


Fig 2.14: Theoretical Planck radiance curves for a number of the earth's atmospheric temperatures as a function of wavenumber and wavelength. Also shown is a thermal infrared emission spectrum observed from the Nimbus 4 satellite based on an infrared interferometer spectrometer. [Liou02, Figure4.1]