CHAPTER TEN

Beyond this book

The previous chapters have presented the basics in atmospheric thermodynamics. As we know, in atmospheric sciences the ultimate goal is to predict as accurately as possible the changes in weather and climate. Thermodynamic processes are crucial in predicting changes in weather patterns. For example, during cloud and precipitation formation vast amounts of heat are exchanged with the environment that affect the atmosphere at many different spatial scales. In this final chapter we will present the basic concepts behind predicting weather changes. This chapter is not meant to treat the issue thoroughly, but only to offer a glimpse of what comes next.

10.1 Basic predictive equations in the atmosphere

In the Newtonian framework the state of the system is described exactly by the position and velocity of all its constituents. In the thermodynamical framework the state is defined by the temperature, pressure, and density of all its constituents. In a dynamical system such as the climate system both frameworks apply. Accordingly, a starting point in describing such a system will be to seek a set of equations that combine both the mechanical motion and thermodynamical evolution of the system.

The fundamental equations that govern the motion and evolution of the atmosphere (and for that matter of the oceans and sea ice) are derived from the three basic conservation laws: the conservation of momentum, the conservation of mass, and the conservation of energy. For the atmosphere the equation of state relates temperature, density, and pressure. In summary, from the conservation of momentum we derive the following set of predictive equations of motion (Washington and Parkinson (1986))

$$\frac{du}{dt} - \left(f + u\frac{\tan\phi}{a}\right)v = -\frac{1}{a\cos\phi}\frac{1}{\rho}\frac{\partial p}{\partial\lambda} + F_\lambda \qquad (10.1)$$

$$\frac{dv}{dt} + \left(f + u\frac{\tan\phi}{a}\right)u = -\frac{1}{\rho a}\frac{\partial p}{\partial\phi} + F_\phi \qquad (10.2)$$

$$\frac{d\omega}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z \qquad (10.3)$$

with

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi} + \omega\frac{\partial}{\partial z}$$

$$u = a\cos\phi\frac{d\lambda}{dt}$$

$$v = a\,\frac{d\phi}{dt}$$

$$\omega = \frac{dz}{dt}$$

where u, v, and ω are the horizontal and vertical components of the motion, ϕ is the latitude, λ is the longitude, a is the radius of the earth, f is the Coriolis force, and F is the friction force. In the above equations the forces that drive the motion are the local pressure gradients, gravity, Coriolis, and friction.

Equation (10.3) can be approximated by the hydrostatic equation (by assuming that $d\omega/dt = 0$ and $F_z = 0$)

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad (10.4)$$

which relates density to pressure.

From the law of conservation of mass we can derive the continuity equation

$$\frac{\partial\rho}{\partial t} = -\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial_\lambda}(\rho u) + \frac{\partial}{\partial\phi}(\rho v\cos\phi)\right] - \frac{\partial}{\partial z}(\rho\omega) \qquad (10.5)$$

which provides a predictive equation for density. The last two equations in this basic formulation come from thermodynamics. The first law of thermodynamics (which expresses the law of conservation of energy)

$$C_p\,\frac{dT}{dt} - \frac{1}{\rho}\,\frac{dp}{dt} = \frac{dQ}{dt}, \qquad (10.6)$$

where dQ/dt is the net heat gain, provides a predictive equation for temperature. The above equations (10.1, 10.2, 10.3 (or 10.4), 10.5, and 10.6) make a system of five equations with six unknowns $(u, v, \omega, p, \rho, T)$. The equation of state

$$p = \rho RT \tag{10.7}$$

provides the additional equation that connects pressure, density, and temperature, thus resulting in a system of six equations (called the *primitive equations system*), with six unknowns (assuming of course that $dQ/dt, F_\lambda$, and F_ϕ are constants and known). Note, however, that this system is not a closed one because $dQ/dt, F_\lambda$, and F_ϕ must be determined from the other variables. These terms are very important for climate simulations but for short-term weather prediction they are often ignored.

10.2 Moisture

The above system of predictive equations was derived without the inclusion of moisture. Even though a model can be derived without moisture in it, including moisture can dramatically improve modeling and prediction. In a manner analogous to the continuity of mass, the changes of moisture must be balanced by the moisture's sources and sinks. An equation of continuity for water vapor mixing ratio can be written as

$$\frac{dw}{dt} = \frac{1}{\rho}M + E \tag{10.8}$$

where M is the time rate of change of water vapor per unit volume due to condensation or freezing, E is the time rate of change of water vapor per unit mass due to evaporation from the surface and to horizontal and vertical diffusion of moisture occurring on scales below the resolution of the model, and w is the mixing ratio. Often the above equation is combined with equation (10.5) to obtain

$$\frac{\partial(\rho\omega)}{\partial t} + \frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}(\rho\omega u) + \frac{\partial}{\partial\phi}(\rho\omega v\cos\phi)\right] + \frac{\partial}{\partial z}(\rho\omega w) = M + \rho E. \tag{10.9}$$

When water vapor changes to water (or ice) and vice versa, heat is added to or removed from the atmospheric system. In this case $dQ = d(lw_{\mathrm{s}}/T)$.

The above general and simple equations constitute only the starting point for studying and investigating the fascinating dynamics of atmospheric motion. The equations have to be modified and adjusted in order to include the effect of many other factors (for example, radiative processes) and to be applied over a specific range of scales (mesoscale models, for example). Such studies are crucial in the effort to understand the climate and ecology of our planet.