111-2 Numerical Analysis
Homework 2

Due Time: 22:00 , Tuesday, 3/7, 2023.

Instructor: Min-Hui Lo

- **Regulation**

  1. **NO PLAGIARISM and NO LATE ASSIGNMENTS.**

- **Submission**

  1. Please write down your answers (including discussions and figures) in the same order as the problem sheet in the word/pdf file.
  2. You should upload zip file,including code and pdf (or word) file via NTU COOL.
  3. zip file name: "*hw{hw number}_g{group id}.zip" (e.g. hw01_g01.zip)*

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1. **Error estimation practice**

   1. Consider the function
   $$f(x) = \frac{e^x - 1}{x}$$
      (a) Calculate $f(x)$ for $x = 0.00275$ and round to 5 decimal places (apply rounding at each term).

      (b) Use half, single, double and long double precision to calculate $f(x)$ for $x = 0.00275$ (apply at each term). Consider the value calculated by long double precision to be the true value, and calculate the true relative error due to rounding in the value of $f(x)$ that was obtained on part (a), half, single and double precision.

      Introduction :
      Python's built-in float type has double precision. We can use Numpy package to change precision.
      About Numpy Precision Type: `https://numpy.org/doc/stable/user/basics.types.html`

   2. The Taylor series expansion of $cos(x)$ is given by:
   $$cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$$
      Use the first three terms in Eq. (1) to calculate the value of $cos(\frac{\pi}{3})$ and round to 6 decimal places (apply rounding at each term). Calculate the truncation error.

2. **Bisection method practice**

   The function $f(x) = 8 - 4.5[x - sin(x)]$; $f(x)$ is zero near $x = 2$, following below rules

   - Let the tolerance error be $TOL = 1 \times 10^{-6}$
   - Use the bisection method, (initial guess will be 1 and 3)

   1. Please use "for loop + if" to find its root.
   2. Please use "while loop + if" to find its root.

3. **Applied bisection method in meteorology**

   In some cases, numerical solutions can be easily accessed compared to analytical solutions in meteorology, such as condensation temperature(T_c).

   Introduction :

   About condensation temperature(T_c): `https://glossary.ametsoc.org/wiki/Condensation_temperature`

   Given

   $$T_c = \frac{B}{ln\left[\frac{A\epsilon}{wp_0}\left(\frac{T_0}{T_c}\right)^{\kappa}\right]}$$

   where

   - $\epsilon = 0.622$
   - $A = 2.53 \times 10^9, B = 5420$
   - $\kappa \approx \frac{7}{2}$
   - w is mixing ratio (g/g), p_0 is pressure (hPa), T_0 is temperature (K)

   1. Write a function Solve_Tc(w,p_0,T_0) using bisection method to solve T_c with given $w,\ p_0,\ T_0$

      Hint

      You can let $f(T_c) = T_c - \frac{B}{ln\left[\frac{A\epsilon}{wp_0}\left(\frac{T_0}{T_c}\right)^{\kappa}\right]}$ and solve $f(T_c) = 0$ by bisection method

   2. Let the tolerance error be $TOL = 1 \times 10^{-6}$. When $w = 10\ g/kg, P = 1000\ hPa, T = 300\ K$, T_c =?