Atmospheric Thermodynamics 2023

Homework 1

Online due time: 2023/04/19 23:59 TPE

1. Specific humidity ($\boldsymbol{q_v}$) and mixing ratio ($\boldsymbol{r_v}$) are both an expression of humidity. What is the definition and relationship of the two variables?

2. Derive the virtual temperature $\boldsymbol{T_v}$ in terms of specific humidity.

3. Derive the hypsometric equation in terms of virtual temperature.

4. Both potential temperature, dry static energy, and entropy describe the energy in the atmosphere, you will find that there are some similarities after some derivation. **You should describe what assumptions or laws are used in the derivation.**

   (a) Poisson's equation describes the temperature at which a parcel moves adiabatically to the reference pressure level, we call it potential temperature if the reference pressure level is set to 1000 hPa. Please derive potential temperature from the first law of thermodynamics.

   $$\theta = T\left(\frac{P_0}{P}\right)^{\frac{R_d}{C_p}}$$

   (b) Dry static energy is the summation of internal energy and geopotential energy. Derive dry static energy from the first law of thermodynamics, and show the adiabatic lapse rate for dry air parcel.

   (c) Derive the relationship between potential temperature and dry static energy from Poisson's equation. (Hint: $\mathrm{T}/\theta \approx 1$ in lower troposphere)

   $$dS_d \approx C_p d\theta$$

   (d) Relate entropy and potential temperature from Poisson's equation and the first law of thermodynamics.

   $$dS \approx C_p d\ln\theta$$

5. Derive the efficiency of the Carnot heat engine in term of temperature from the first law of thermodynamics.

$$\eta = \frac{q_{in} - q_{out}}{q_{in}} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}, \qquad \text{where } T_1 > T_2$$