CA#3

B11209013　大氣一 甘祐銓

Data Source: 46810-2018072100.edt.txt

1. Virtual Potential Temperature（虛位溫）：$\theta_v = T(1+0.608q_v)\left(\frac{P_0}{P}\right)^{\frac{R_d}{C_p}}$, plot and discuss it.

   Virtual potential temperature combines the effect of pressure and humidity of an air parcel.

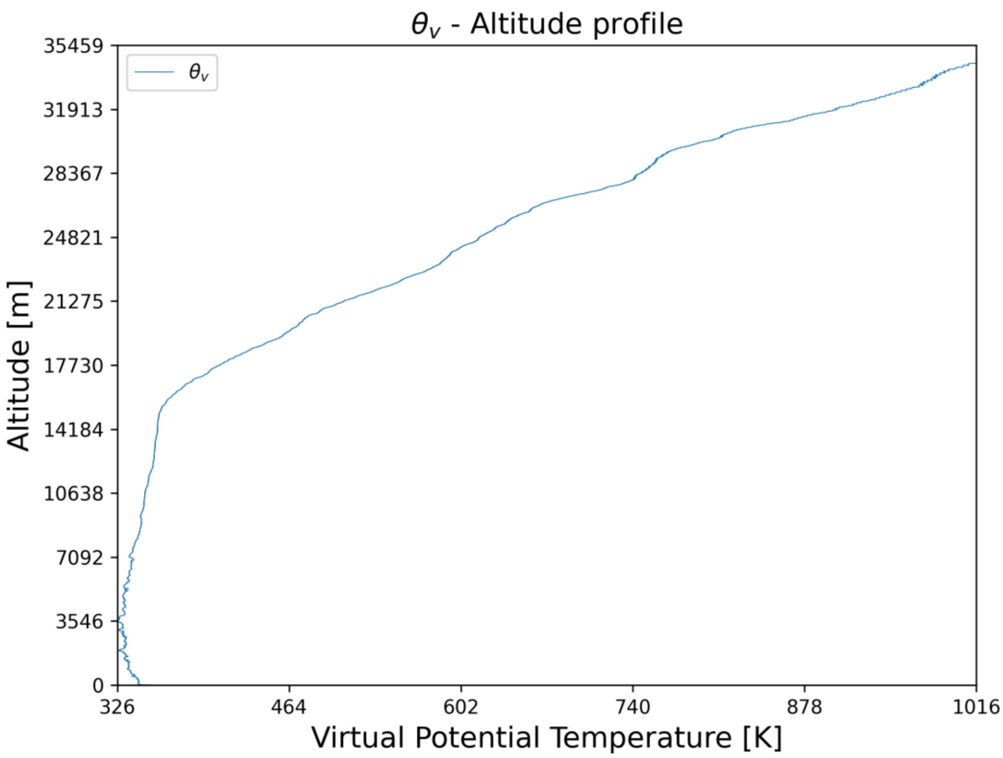
   The constants in the equation have values as follow:

$$R_d = 287\ \frac{J}{kg \cdot K}$$

$$C_p = 1004\frac{J}{kg \cdot K}$$

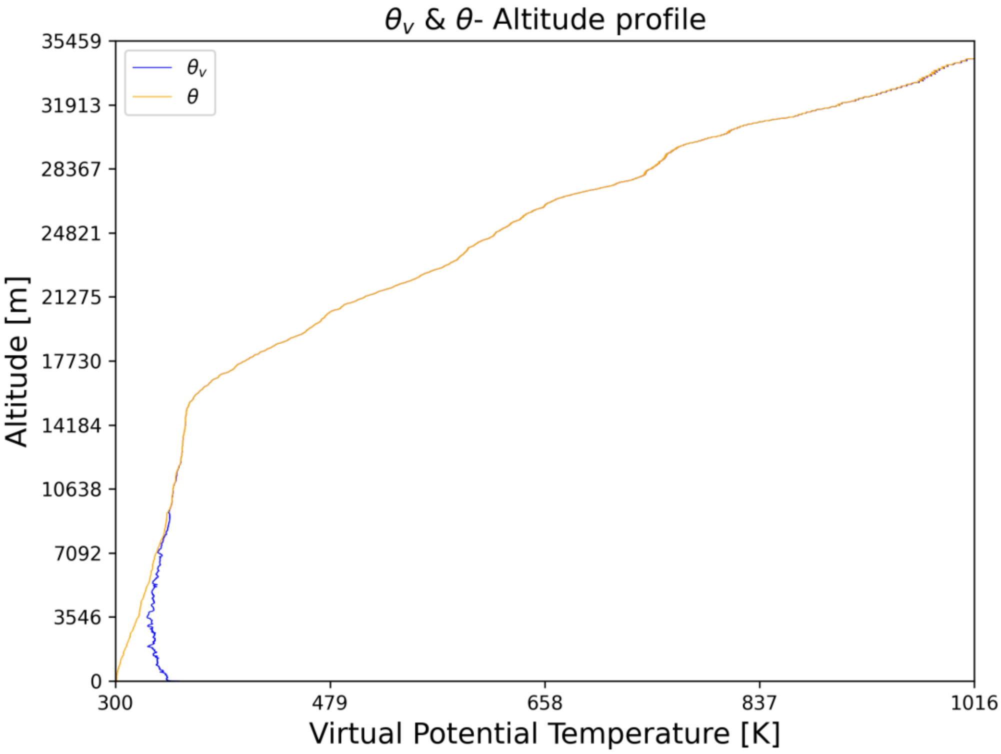
   Suppose P_0 in the equation is $1000\ hPa$.

   The profile of this feature likes below:



   The profile shows that there are two main sections of the distribution. The lower part is governed by both humidity distribution and pressure. Otherwise, the upper part is mainly governed by pressure decreasing.

To show the guessing above, the plot below shows the profile of potential temperature and virtual potential temperature.

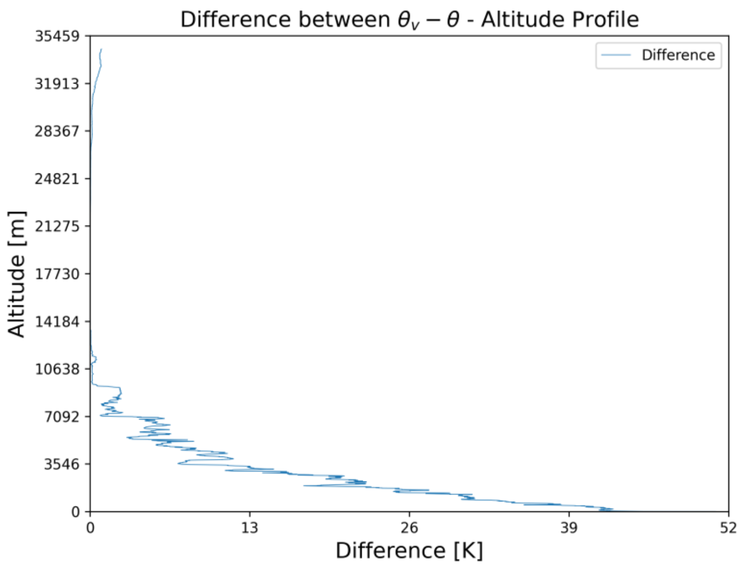


This figure shows that:

In the higher level, potential temperature is almost the same as virtual potential energy. This may because of the humidity is almost close to zero. By the equation of virtual potential temperature:

$$\theta_v = \theta \cdot (1 + 0.602 \cdot q_v)$$

When q_v close to zero, $\theta_v \cong \theta$.



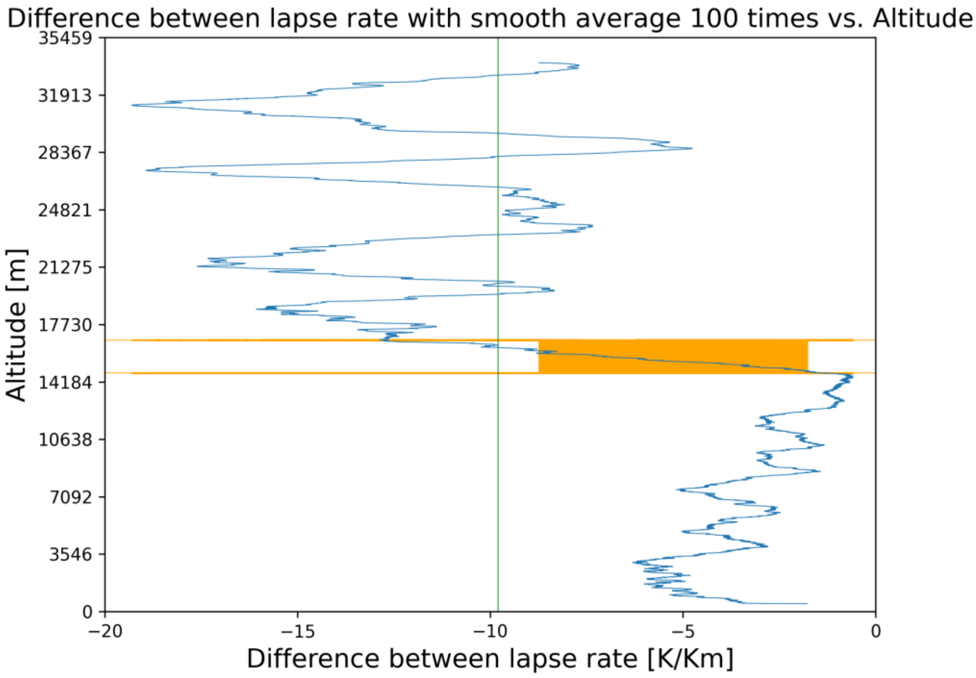
The figure shows that: the difference between potential temperature and virtual temperature almost the same in higher altitude, which supports our guessing above.

By the definition of virtual potential temperature:

$$\theta_v - \theta = 0.602 * qv * \theta$$

So the plot above looks similar to the distribution of relative humidity.

2. Please plot $\Gamma_d - \Gamma$ in height coordinate, try to determine tropopause with it and discuss what you see. (You can smoothen the profile by using moving average.)



This is the graph shows that $\Gamma_d - \Gamma$ when using the moving average with 100 items forward and backward.

The green line in the plot represents the lapse rate of $-9.8\ K/km$, the orange region is possible region of tropopause. The reason is that: lapse rate difference transfers from greater than -6 to less than -13.

The most possible altitude for tropopause is 16330 m. By the profile of temperature as altitude of atmosphere, there is a section upper than tropopause that has no temperature change. Thus, the lapse rate of that section is 0, which is the intersection of the blue and green line on the plot above.

This feature will show the stability of atmosphere. When the lapse rate of air parcel is less than dry air lapse rate, the atmosphere is unstable.

As discussing the stability of atmosphere, the potential temperature is also related to the stability. By the derivation from the first law of thermodynamics and ideal gas law:

$$\Gamma_d - \Gamma = \frac{T}{\theta}\frac{\partial \theta}{\partial z}$$

The equation shows that the features discuss above is related to vertical distribution of potential temperature. By the concept of potential temperature, if the $\frac{\partial\theta}{\partial z}$ greater, the air parcel is relatively hard to move upward, in another words, the atmosphere is much stable.

3. The hypsometric equation describes the relationship between pressure and height. Please use the hypsometric equation to finish the question below. Notice that the ideal gas law is P= $\rho R_d T_v$, so you should use T_v in the hypsometric equation. Calculate the physical depth (in meter) of a 10-hPa-thick air column for every 50hPa (1000 hPa – 990 hPa, 950 hPa – 940 hPa, ..., 150 hPa – 140hPa). Also, calculate the result with the virtual temperature profile 10 K warmer. Plot the profile of the depth difference (pressure for vertical, Δz(warm)−Δz for horizontal coordinate) and make a brief discussion. (Hint: You may choose the closest data point to calculate, or interpolate the data.)

The hypsometric equation can be derived from pressure difference and ideal gas law.

By observing a little section of air column:

$$(p + dp) \cdot A = \rho g A \cdot dz + p \cdot A$$

$$\Rightarrow \frac{dp}{dz} = -\rho g$$

$$\Rightarrow \frac{dp}{dz} = -\frac{p}{R_d T_v} g$$

$$\Rightarrow \frac{1}{p} dp = -\frac{g}{R_d} \cdot \frac{1}{T_v} dz$$

$$\Rightarrow \int_{p(z_1)}^{p(z_2)} \frac{1}{p}\, dp = -\frac{g}{R_d} \int_{z_1}^{z_2} \frac{1}{T_v}\, dz$$

$$\Rightarrow \ln\left(\frac{p(z_2)}{p(z_1)}\right) = -\frac{g}{R_d} \int_{z_1}^{z_2} \frac{1}{T_v}\, dz$$

$$\Rightarrow p(z_2) = p(z_1) \cdot \exp\left(-\frac{g}{R_d} \int_{z_1}^{z_2} \frac{1}{T_v}\, dz\right)$$

By the discussion above, setting $z_1 = 0$, then the pressure decreasing tendency is shown as exponentially.

To simplify calculating process, the equation above can be written in:

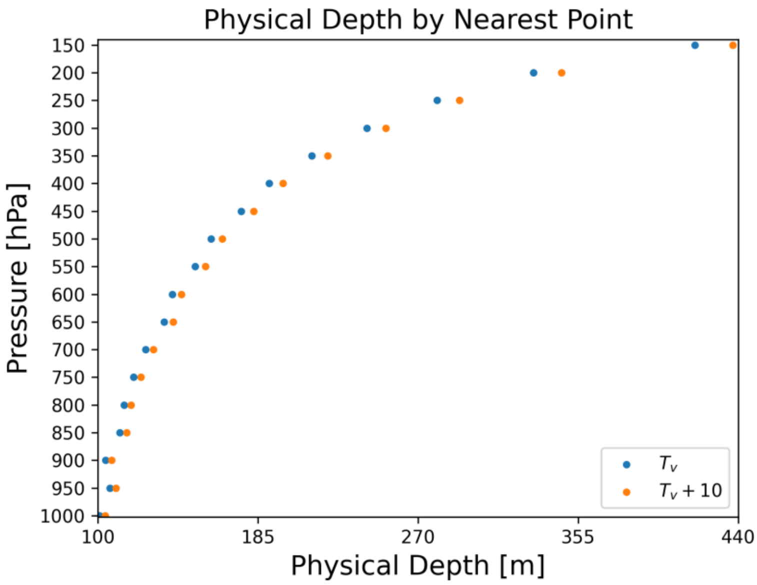
$$p(z_2) = p(z_1) \cdot \exp\left(-\frac{g}{R_d \overline{T_v}} \int_{z_1}^{z_2} 1\, dz\right)$$

The $\overline{T_v}$ in the equation is average value of virtual temperature, I use 'np.mean' in the program to deal with this problem. Calculating the integration above, the equation can be written in:

$$p(z_2) = p(z_1) \cdot \exp\left(-\frac{g}{R_d \overline{T_v}} \Delta z\right)$$

$$\Rightarrow \Delta z = \ln\left(\frac{p(z_1)}{p(z_2)}\right) \cdot \frac{R_d}{g} \overline{T_v}$$

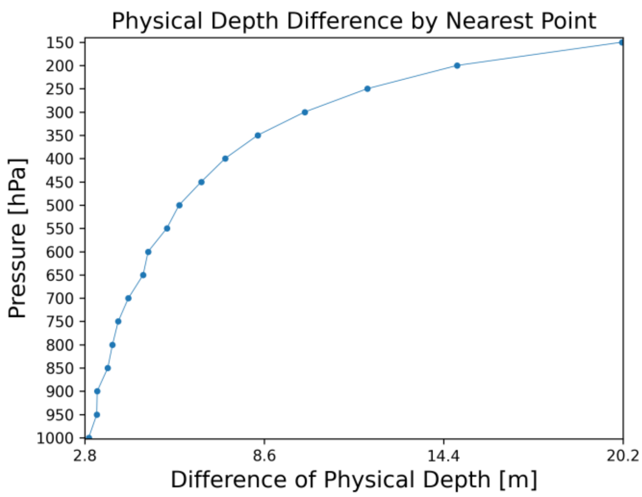
By the equation above, the profile looks like:



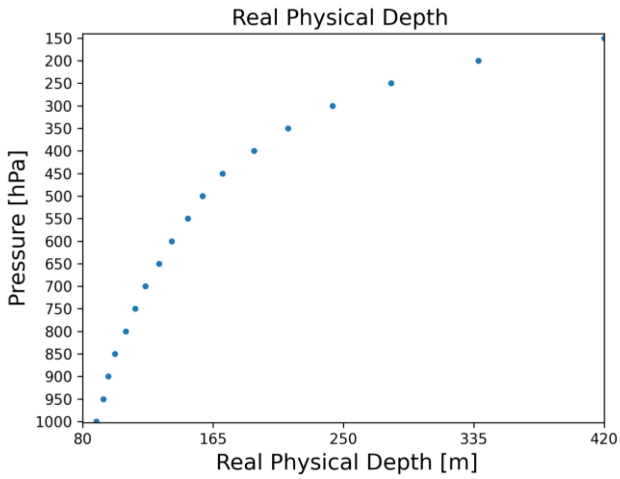
By the graph, as the pressure decreasing, the physical depth of $10\ hPa$ per $50\ hPa$ is increasing. The maximum physical depth can reach $440\ m$.

By hypsometric equation, the pressure lapse rate becomes lower when the altitude increase. In another words, for $10\ hPa$, the physical depth will thinner in the lower altitude.

The orange point represents the condition of virtual temperature adding $10\ K$, by the equation, the $\overline{T_v}$ would shift $10\ K$ more than the original point. And the difference between the two ways of calculation seems to be larger as the pressure decreasing. The difference shows like below:

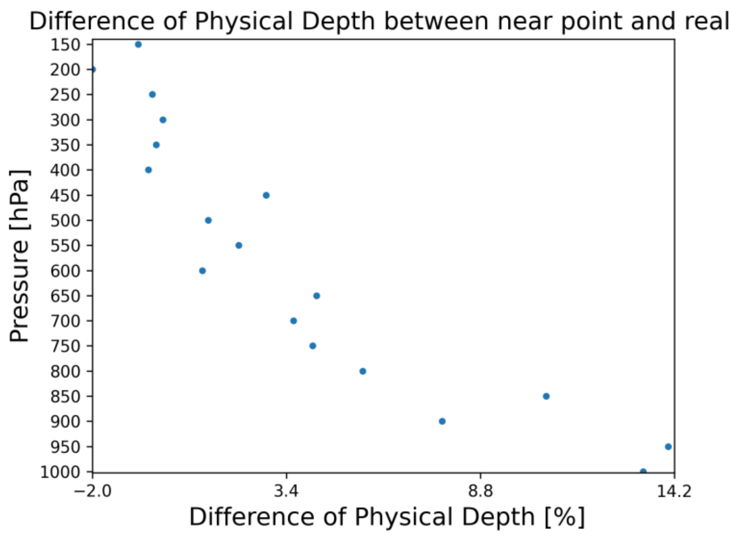


If using interpolation method to ask for the real physical depth, the plot would be shown as below:



The tendency of physical depth increasing is as the equation describe. Thus, the tendency may seem to be correct.

Because using numerical method, the compare of numerical data and real data is needed. The relative error ($(numerical - real)/real * 100\%$)is like below:



This figure shows that the relative error decreases as the pressure increasing. The near-ground level error is up to 14%, this may because the variation of the humidity near ground is complex and usually large.

However, the upper atmosphere(lower stratosphere) is has smaller error because of the change of temperature and humidity is close to 0, which means the difference between points would not be so obvious, thus the error will also be less obvious.