111-2 Numerical Analysis
Homework 5

Due Time: 22:00 , Tuesday, 04/04, 2023.

Instructor: Min-Hui Lo

- **Regulation**
  1. **NO PLAGIARISM and NO LATE ASSIGNMENTS.**
- **Submission**
  1. Please write down your answers (including discussions and figures) in the same order as the problem sheet in the word/pdf file.
  2. You should upload zip file,including code and pdf (or word) file via NTU COOL.
  3. zip file name: ”*hw{hw number}_g{group id}.zip” (e.g. hw01_g01.zip)*

---

1. **Iterative method**

$$\begin{cases} 9\text{x}_1 - 2x_2 + 3x_3 + 2x_4 = 54.5 \\ 2\text{x}_1 + 8x_2 - 2x_3 + 3x_4 = -14 \\ \text{-}3\text{x}_1 + 2x_2 + 11x_3 - 4x_4 = 12.5 \\ \text{-}2\text{x}_1 + 3x_2 + 2x_3 + 10x_4 = -21 \end{cases}$$

1. Solve the system until the relative error (ϵ) falls below 5%.

$$\left| \frac{x_i^{k+1} - x_i^k}{x_i^k} \right| < \epsilon, i = 1, 2, ..., n \tag{1}$$

   (a) Use Gauss-Seidel iterative method

   (b) Use Jacobi iterative method

2. Plot the evolutions for each solution (X_1, X_2, X_3, X_4) for (a) and (b) (with each iteration step).
3. Discuss which one (a) or (b) is more efficient? Why?

2. **Applied Linear Equations to solve diurnal cycle of soil temperature**

When the surface is heated, the heat will diffuse through the soil. The diffusion can be represented by the equation:

$$\frac{\partial T}{\partial t} = \frac{\kappa}{C_v}\frac{\partial^2 T}{\partial z^2}$$

We can rewrite the system of equation by finite difference methods and boundary conditions in the form of:

$$\begin{bmatrix} 1+2M & -M & 0 & \dots & \dots & \dots & 0 \\ -M & 1+2M & -M & 0 & \dots & \dots & \vdots \\ 0 & -M & 1+2M & -M & \ddots & \dots & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 & \vdots \\ \vdots & \dots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \dots & \dots & 0 & -M & 1+2M & -M \\ 0 & \dots & \dots & \dots & 0 & -M & 1+M \end{bmatrix} \begin{bmatrix} T_1^{n+1} \\ T_2^{n+1} \\ \vdots \\ \vdots \\ \vdots \\ T_{N-1}^{n+1} \\ T_N^{n+1} \end{bmatrix} = \begin{bmatrix} T_1^n + MT_s^{n+1} \\ T_2^n \\ \vdots \\ \vdots \\ \vdots \\ T_{N-1}^n \\ T_N^n \end{bmatrix}$$

where T_s^{n+1} denote the surface temperature when $t = n+1$, T_N^{n+1} denote the N^{th}-layer soil temperature when $t = n+1$ and $M = \frac{\kappa}{C_v}$. Assume $\frac{\kappa}{C_v} \approx 5\times 10^{-7}\ m^2/s$

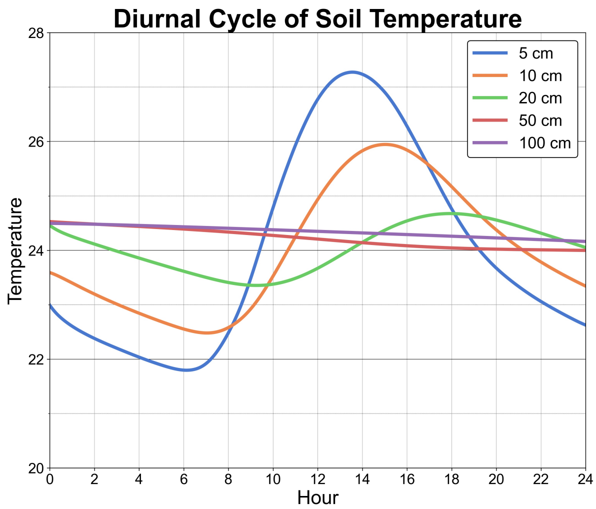
1. Given T_s and $T_1^0, T_2^0, ..., T_{N-1}^0, T_N^0$, which can be obtained from *Tsoil.nc* (or *T_ini.txt* and *T_s.txt*). Solve the tridiagonal system of equations with $\Delta t = 60\ s$ and $\Delta z = 0.05\ m$ using

   (a) Gauss-elimination method

   (b) Jacobi method

   (c) Gauss-Seidel method

   and compare their differences. The result will look similar to:



Variables :

T_ini: the initial soil temperature at $t = 0$ (variant in z)

T_s: the surface temperature used for boundary conditions (variant in time)

File :

T_soil.nc contains T_ini and T_s. You can open the file by xarray package or netCDF4 package. If you do not know how to open nc file, you can use txt file (*T_ini.txt* and *T_s.txt*) instead.

Hint :
The tridiagonal system of equations can be abbreviated to $AT^{n+1} = B$. If you want to access the soil temperature at $t = n + 1$, you will need the soil temperature at $t = n$. You need to solve Ax=B totally 1439 times to get the diurnal cycle of soil temperature.
Note that A is invariant in timestep, while B is variant in timestep.

2. Complete the following table.

	Gauss-elimination method	Jacobi method	Gauss-Seidel method
Numerical or Analytical method			
Whether initial conditions (I.C.s) are needed?			
Computational efficiency (please rank)			

Hint :
To rank the computational efficiency, you can use datetime or time package to get the process time. For example:

```
### Method 1
from time import process_time
start = process_time()
'''
Write your code here
'''
end   = process_time()
print(end-start)

### Method 2
import datetime
start = datetime.datetime.now()
'''
Write your code here
'''
end   = datetime.datetime.now()
print(end-start)
```