LUND UNIVERSITY

LTH FACULTY OF ENGINEERING

Home Exam in Information Theory, EITN45

May, 2019

Name: ______________________

Social id number: ______________________

Programme: ____________

Nbr of sheets: ____________

Mark the problems you solved with a cross.

1	2	3	4	5	6

Signature: ______________________

Assessment protocol

1	2	3	4	5	6	Σ	Grade

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- The exam should be solved and handed in individually.
- Start a new solution on a new sheet of paper.
- Solutions should clearly show the line of reasoning.
- Solutions are handed as pdf file at `elearning.eit.lth.se`. The solutions can either be written using computer tools (e.g. LaTeX, Word or text) or hand-written and scanned. Or a combination.
- Include the cover sheet (first page in this file) when handing in the solutions.
- Scripts and program code, such as MATLAB/Octave, python, Java or C/C++, used for derivations should be handed in as appendix to the solution. However, the solutions should be fully understandable without the code. Normal command line calculations, e.g. entropy as done in the hand in problem, are not required to hand in.
- If we are in doubt of any of the above, we need to be able to contact you for clarifications. Please, include a valid mail address on the cover sheet.
- There are six problems with 10 points each. The limits for grades are planned as:
  3: 30-40
  4: 41-50
  5: 51-60

Good luck!

Problem 1

(a) Compress the following text using LZ78.

Text: `text test of text in test text`

(b) Convert the index part of the codeword to binary form and introduce one (decodable) bit-error in the index of the fourth codeword. Decode the text.

(5+5=10p)

Problem 2

An urn contains $n-1$ white and 1 black balls. A person draws balls from the urn, one by one, until a black ball is drawn. Let X denote the number of white balls drawn before the first black, and derive its entropy when:

(a) Balls are not replaced after drawn from the urn.

(b) Balls are immediately replaced to the urn.

(5+5=10p)

Problem 3

A time discrete cat lives in an apartment, as shown in the plan in Figure 3.1. In each time instant the cat either walks to an adjacent room or stays in the same room. It chooses with equal probability among the options it has, with two exceptions:

- The cat likes to sleep, so if it is in the bedroom (A) it stays with probability $1/2$.
- Someone forgot to do the dishes, and the cat will not do it. So if it is in the kitchen (E) it will not stay.

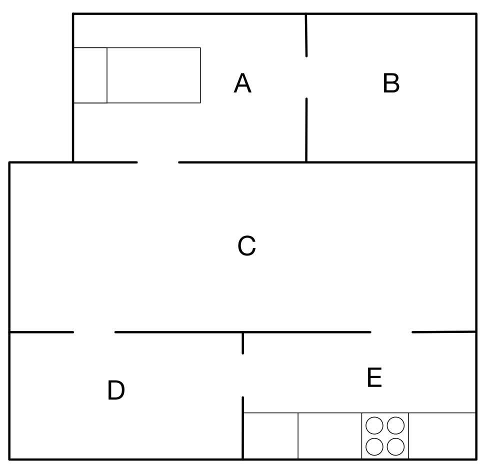

Figure 3.1: A plan of the apartment.

(a) When the owner of the apartment comes home in the evening, what is the probability that the cat is in the bedroom (A).

(b) By an advanced camera system the apartment is monitored and the cat movements recorded. What is the minimum bits required per time instant to save the cat movements?

(5+5=10p)

Problem 4

Let $\boldsymbol{p} = (p_1, p_2, \ldots, p_n)$ be a probability distribution, i.e. $p_i \geqslant 0$ and $\sum_i p_i = 1$. Show that for any m, in $1 \leqslant m \leqslant n$,

$$H(p_1, p_2, \ldots, p_n) \leqslant H(p_1, p_2, \ldots, p_m, q_m) + q_m \log(n - m)$$

where

$$q_m = 1 - \sum_{i=1}^{m} p_i$$

When is there equality in the expression?

(10p)

Problem 5

A friend of yours claims to have constructed an ingenious transmission system by use of a complicated construction of signal mapping and interleaving. The system is such that the resulting channel can be seen as the construction in Figure 5.1, where each box is a BSC with crossover probability p. Your friend claims that the system will be better than two consecutive independent uses of a BSC (with the crossover probability p). But you are sceptic. Instead you claim that this does not perform better than just using the BSC with two independent binary variables after each other. On your way home you start to doubt your statement. Who was right?

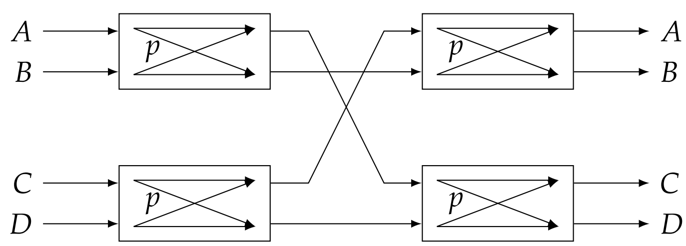

Figure 5.1: A DMC that is equivalent to your friends complicated system.

(10p)

Problem 6

Normally, when we talk about a channel with Gaussian noise the distribution of the noise is not dependent on the actual signal transmitted. However, in fiber optic communication this is typically not true. Often a modulation scheme called on-off-keying (OOK) is used on fiber links. This is a binary modulation which transmits a puls ("on") for a one and nothing ("off") for a zero.

The difference between the two cases one and zero, is that if a pulse is transmitted it can be affected by noise due to e.g. imperfections in the fiber, but if no signal is transmitted this is not the case. Actually, in the ideal case, if nothing is transmitted, nothing should be received since the fiber is a passive element. But in a real system there can be active (optical) components along the fiber path, which will introduce noise. Furthermore, there are noise added to both zeros and ones in the electrical domain (e.g. in transmitter, receiver and converters along the way).

Taking the above into account, it is reasonable to assume that the noise variance for a transmitted one is three times as large as for a transmitted zero, i.e. (see also Figure 6.1)

$$N_{0,1} = 3N_{0,0} \tag{6.1}$$

(a) Derive an expression for the mutual information $I(X;Y)$ between the transmitted variable X and the received variable Y. Notice, that you will not be able to derive all the integrals in the expression, but as much as possible should be derived in the expression.

(b) Make a plot of the mutual information $I(X;Y)$ as a function of the SNR in the intervall -5 dB $\leqslant E_s/N_0 \leqslant 20$ dB. Define E_s/N_0 as the signal to noise ratio for a transmitted one, $E_{s,1}/N_{0,1}$.

You will need to solve parts of the derivations with numerical methods, and the programs (or scripts or commands) should be appended to the solutions. But the solution in it-self must be understandable without the program listing. In derivations like this is advisable to fix either the signal level or the noise level throughout the derivations.

(c) Show how your expressions in part (a) can be used to show that for large signal to noise ratios the mutual information flattens out at 1, i.e. that $I(X;Y) \approx 1$, for large E_s/N_0.

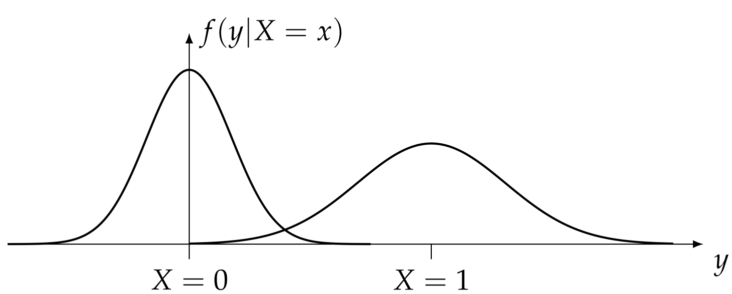

Figure 6.1: The density functions for the received value conditioned on the transmitted.

(3+4+3=10p)