Digital Signal Processing (CSE-4203)

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ASM

Ref. Book:

Digital Signal Processing - principles, algorithms and application (3rd edition)

- J G Proakis, D G Monolakis

Ref. Book for experiment:

Slicer_Digital_Signal_Processing_using_MATLAB_3rd_Edition

Textbook for shortcut Understanding:

Understanding Digital Signal Processing.

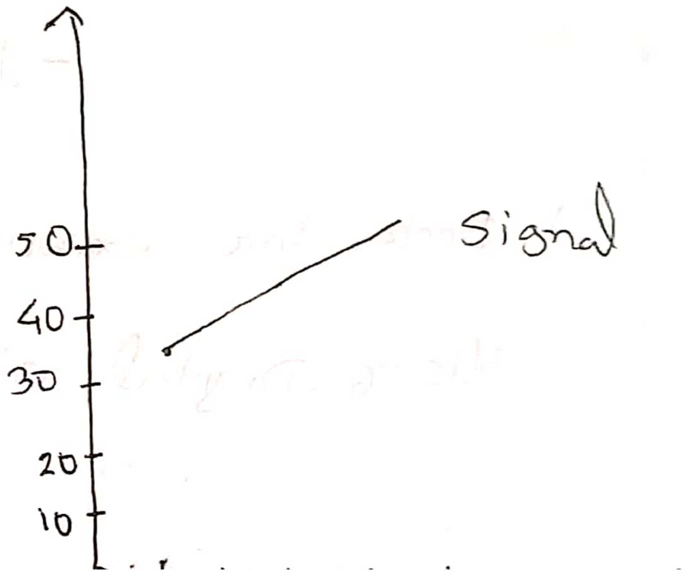
- Richard G. Lyons.

Signal

Sensor

<u>Information:</u>

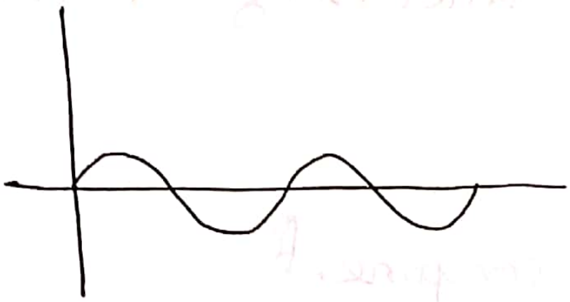
Day	marks
1	35
2	38
3	39
4	41
5	45



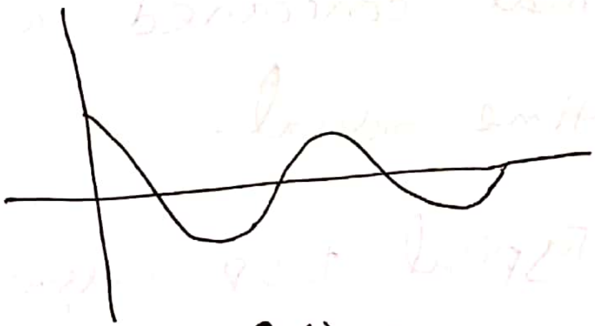
(i) Signal

(ii) System

Signal: Signal is a physical quantity that convey related information.

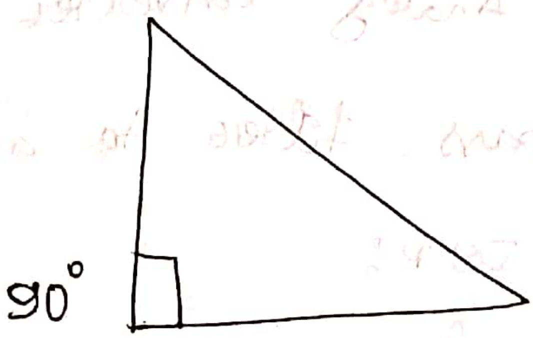


Sine



Cosine.

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$



—x—

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DSP is a processor of signal.

converting a continuously changing wave form into discrete changing waveform.

DSP is a field of numerical mathematics that concerned with processing of discrete time signal.

Typical DSP system component:

(i) Input low pass filter to avoid aliasing

(ii) Analog to Digital converter (ADC)

(iii) Digital signal processor

(iv) Digital to Analog converter (DAC)

(v) output low pass filter to avoid imaging.

Application of DSP:

(1) Biological signal.

(a) Brain signal (EEG signal)

(b) Cardiac signal (ECG)

(c) Medical image (X-RAY, PET, MRI)

Goal: Detect abnormal activity of human.
(heart attack)

Tools: Filtering, Fourier Transform.

(i) Identifying a person

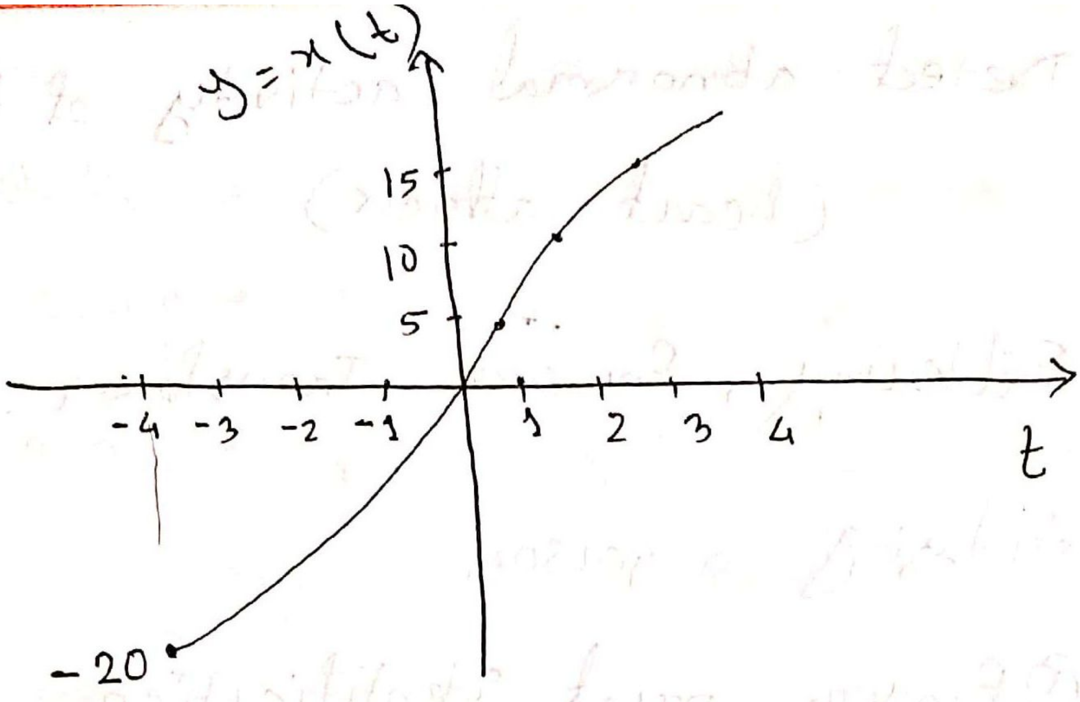
- (a) Finger print identification
- (b) Retina identification
- (c) Voice & face recognition

(ii) Sonar & Radar specification

(iii) Signal processing in communication engineering

(iv) Control system

Signal: A flow of information.



$x(t) = 5t$

if $t = 0$

$x(t) = 0$

if $t = 3$

$x(t) = 15$

if $t = -4$

$x(t) = -20$

Harmonic Oscillation

Hook's Theorem ($F = -kx$)

$$x(t) = A_i \cos(\omega_i t + \theta)$$

$$\sum_{i=1}^{n}$$

—x—

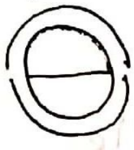
Harmonic Oscillation:

$$x(t) = \sum_{i=1}^{N} A_i(t) \sin(2\pi F_i(t)\, t + \theta_i(t))$$

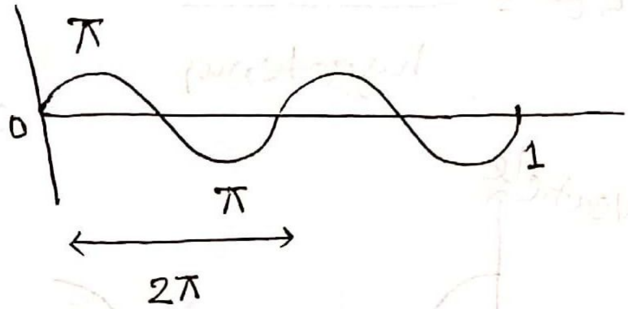
Ω $\qquad$ $\pi = 180°$

ω $\qquad$ $\pi = 3.1416$

π



π $\qquad$ 2π



1 Radian / 1 cycle

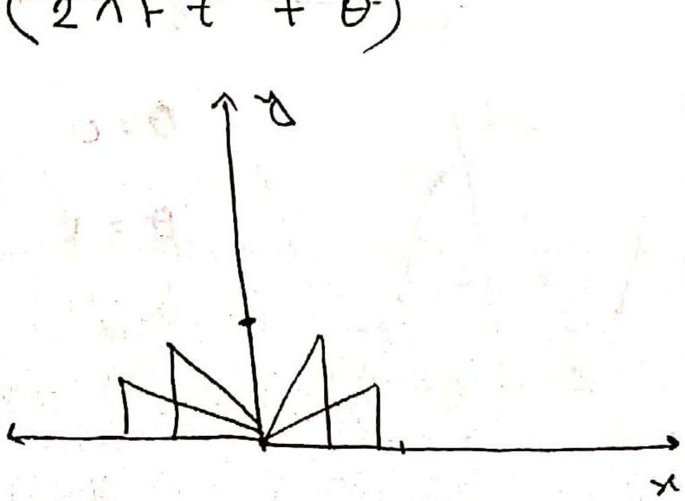
$T = 1s$

Frequency, $f = 2$ Hz

$$x_a(t) = A \cos(\Omega t + \theta)$$

$$= A \cos(2\pi F t + \theta)$$

$\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenus}}$



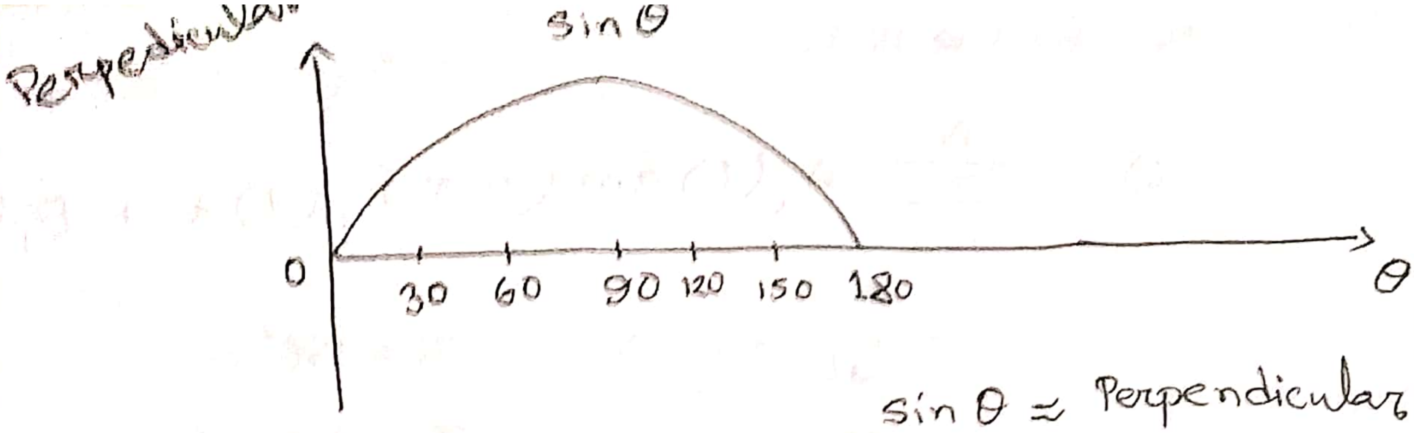
$\theta = 0$ Per. $= 0$

$\theta = 30$ increases

$\theta = 60$ increases

$\theta = 90$ per. $= 1$

Hyp. $= 1$



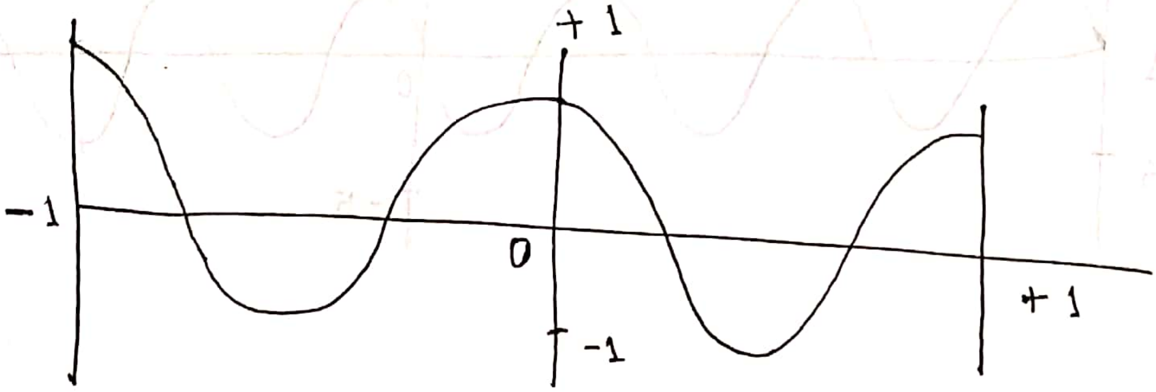
$\sin\theta =$ Perpendicular

$$\cos\theta = \frac{\text{Verticle}}{\text{hypotenus}}$$

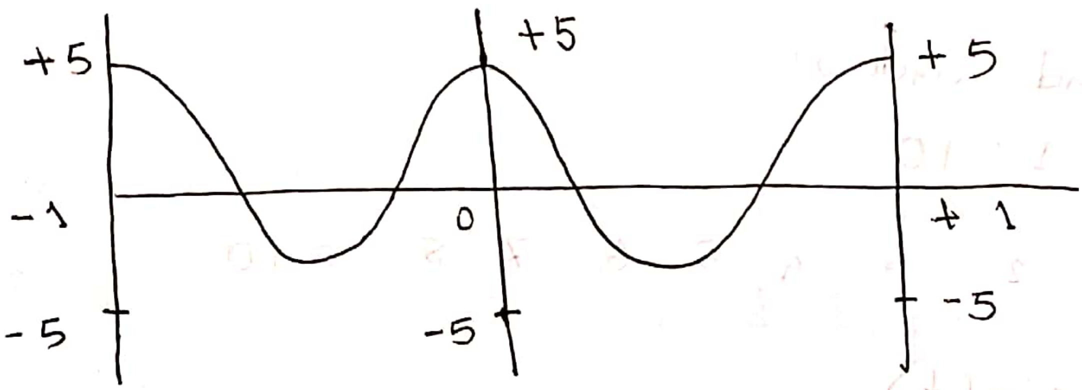
$\cos\theta =$ Verticle

$x_{(a)}(t) = A \cos(2\pi F t + \phi)$ $-\infty < t < \infty$

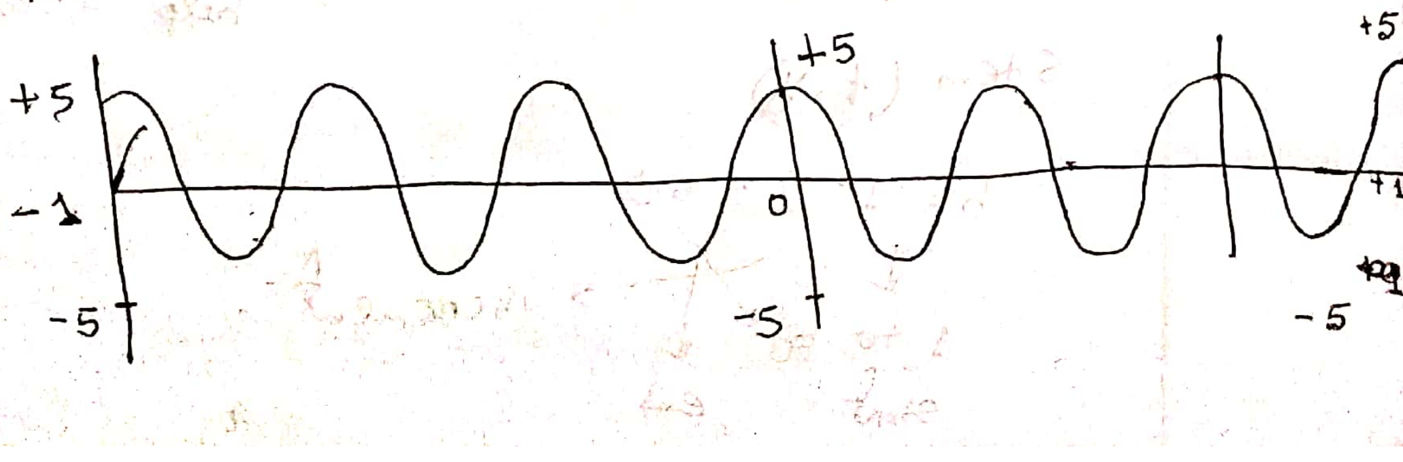
$x_a(t) = 1 \cos(2\pi 1 t + \phi)$; $A = 1$; $F = 1$.



$x_a(t) = 5 \cos(2\pi 1 t + \phi)$



$x_a(t) = 5 \cos(2\pi 3 t + \phi)$



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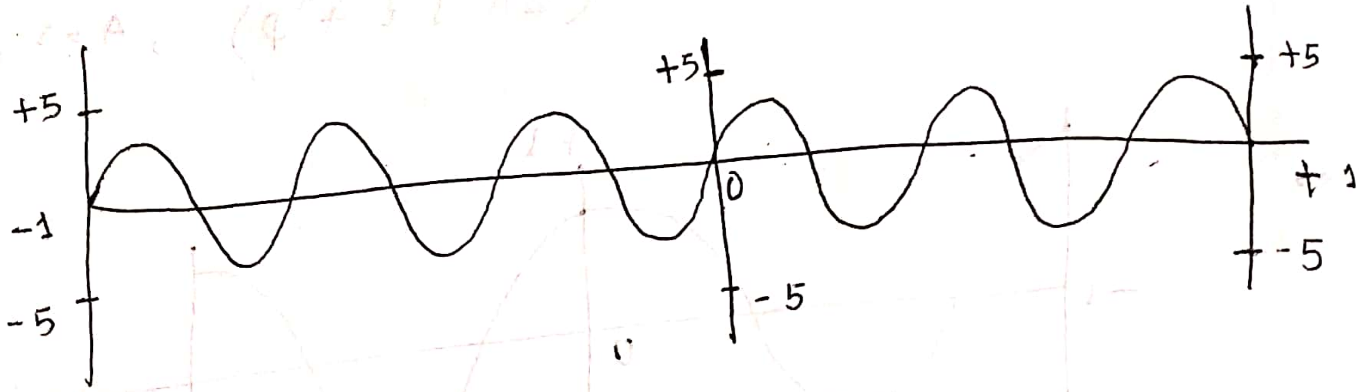
$x_a(t) = 5\cos(2\pi 3t + 90)$



—— x ——

"LAB"

17.01.19

Command window:

t = 1 : 10

1 2 3 4 5 6 7 8 9 10

y = sin(t)

Plot (t, y)

Stem (t, y)

help stem

b = 1 : 2 : 50

1 to 50 — increment

start — end

Row vector (3, :)

Col. vector (:, 3)

——— x ———

20.01.19

Continuous time Sinosoidal signal:

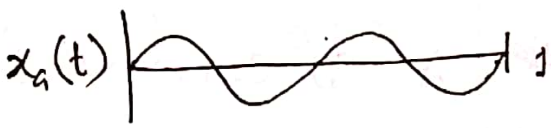
$$x_a(t) = A \cos(\Omega t + \theta)$$

A = amplitude

$\Omega = 2\pi F$

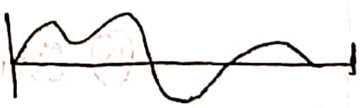
Properties of continuous time sionosoidal signal:

① For every fixed value of frequency F, $x_a(t)$ is periodic

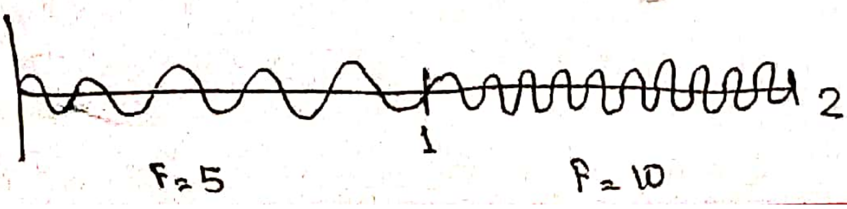


periodic

⑪ every distinct value of f, $x_a(t)$ will be distinct



non-periodic



(iii) Oscillation rate will increase, if the frequency F increases.

Discrete time Sinosoidal signal:

$$x(n) = A\cos(\omega n + \theta)$$

(i) Continuous signal (Analog)
(ii) Discrete Time signal (Analog)
(iii) Digital signal

$x_a(t) = A\cos(\Omega t + \theta)$; continuos

$= A\cos(2\pi F t + \theta)$

$\downarrow$ ADC

$x(n) = A\cos(2\pi F n + \theta)$; Discrete.

A - D Converter :

(i) Sampling
(ii) Quantization
(iii) Coding

Sampling:

when $t = 1$ s

if interval = 0.01

No. of Sample = 100

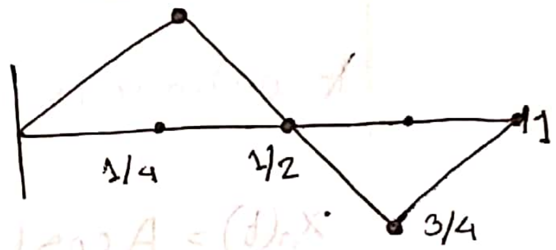
if interval = 0.001

No. of Sample = 1000

4 points to sample

$n = (1, 2, 3, 4)\,T$

Discrete time Analog Signal

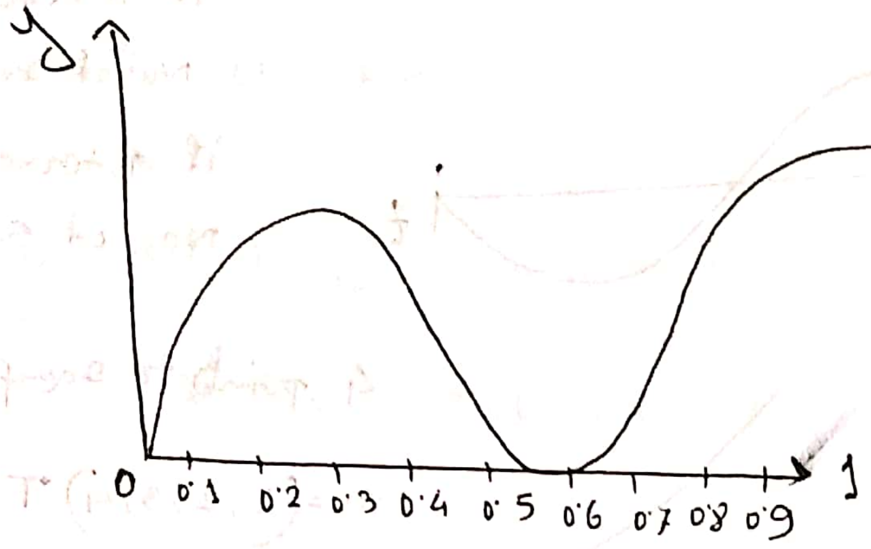


Data loss

22.01.19

```
t = 0: 0.1 : 1 / 0 : 0.01 : 1 / 0 : 0.001 : 1.

% x = 0.9 .^ t

x = sin (2*pi * t) (

% stem (t, x)

plot (t, x)

axis ( [0, 1, -1, 1] )

grid on
```

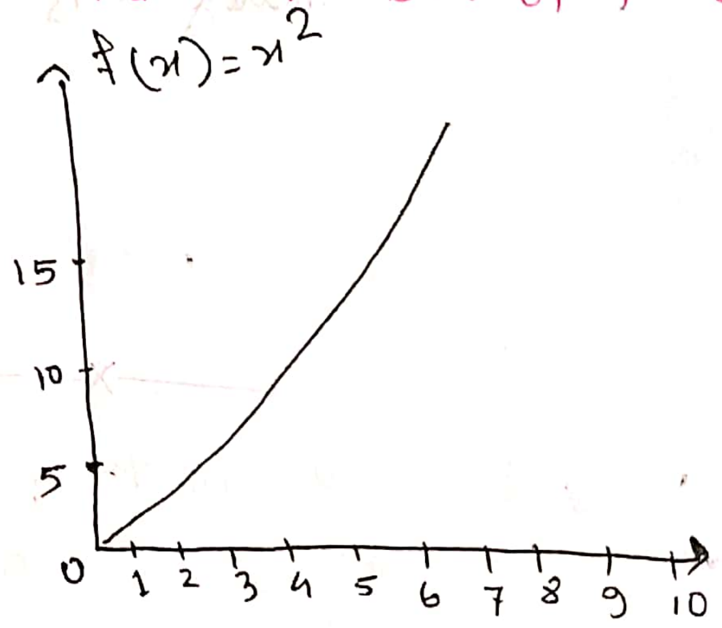
$x(t) = A\cos(2\pi f t + \theta)$



* Continuous Signal → infinite number of points.

$x_a(t) = A\cos(2\pi f t + \theta)$

$x_a(n) = x_a(nT)\ [t = nT]$



$t = nT$

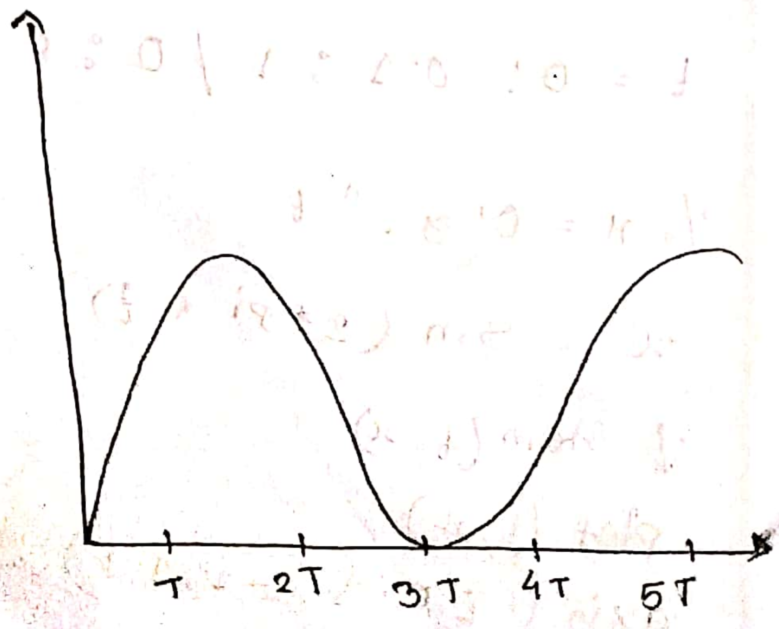
$= 5T$

$t = 0.1 = 10$ points

$t = 0.01 = 100$ "

$t = 0.001 = 1000$ "

$nT = t$



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$nT = 1000$

$5T = 1000$

$$T = \frac{1000}{5} = 200$$

$\therefore x_a(n) = A \cos(2\pi f n + \theta)$

$x_a(t) = A \cos(\Omega t + \theta)$

$x_a(t) = A \cos(2\pi f t + \theta)$

$x_a(nT) = A \cos(2\pi f nT + \theta)$

$x_a(nT) = A \cos(2\pi f Tn + \theta)$ ——— (i)

Discrete time Sinosoidal signal,

$x(n) = A \cos(\omega n + \theta)$

$x(n) = A \cos(2\pi f n + \theta)$ ——— (ii)

from (i) & (ii)

$f = FT$

$$\Rightarrow f = F \frac{1}{F_s}$$

$$\Rightarrow f = \frac{F}{F_s}$$

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Sampling rate / sampling Theorem:

→ If the frequency rate is higher, then it will need more memory.

→ If the frequency rate is smaller, then the data will loss.

Nyquist Theorem:

Sampling rate must be at least twice as high as the highest frequency you want to represent.

$$F_s >= 2F_{max} \text{ [Nyquist rate]}$$

$F_s = 2$

$T = 2 \times 1$

$F_s = 2.$

—x—

"LAB"

24.01.19

Scaler $x = 10$

Vector $x = [1, 2, 3, 4]$; row vector

Vector → Row Vector, Column vector

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$; Column vector..

Save b → To use the variable b along with all the variable in workspace for further use.

load b → To reuse variable b, & all the variables in that workspace.

dir

Save
load
sqrt

$x = 100$

$y = sqrt(x)$

$y = 10$

who → shows the used variable

whos → Variables name, with size, Bytes, class & attributes.

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format long → 16 digits after point

format bank → 2 " "

format short → 4 " "

format long e

format short e

format rat → ratio (0.5 → 1/2)

a = zeros(3) → creates a 3×3 matrix, with each value zero.

b = ones(3) → creates a 3×3 matrix, with each value one.

d = magic(3) → creates a 3×3 matrix, with random values (integer)

f = rand(3) → creates a 3×3 matrix, with random values (fraction numbers).

a = [2, 3, 4, 5, 6]

b = a(1, 4) → finds out the value of row 1 & column 4 from a.

b = a(1:3) → takes the value from 1 to 3 index.

b = a(1, 3:5) → Takes the value of row 1, from 3 to 5 index.

f = [1; 2; 3; 4] → creates a column vector

$$f = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

b = a(1, :) → Takes all the ~~values~~ column values of row 1.

k = a(:, 3) → Takes all the ~~~~ row values of column 3.

(:)

↑ row ↑ column

a(:, :, 1)

—x—

27.01.19

Discrete Time Signal:

$F_s = 2 \times F_{max}$ F_s = Sampling frequency

$x(n) = A \cos(2\pi F n)$ → discrete time signal.

$x_1 = 1 \times \cos(2\pi(10).t)$ $F_s = 40$

→ $A = 1$ (Analog)

Analog $= A \cos(2\pi F t)$

Discrete $= A \cos(2\pi f n)$

$f = \frac{F_{max}}{F_s}$

$= \frac{10}{40} = 0.25$

$f = \frac{f_{max}}{F_s}$

$\Rightarrow \frac{1}{2} = \frac{F_{max}}{F_s}$

$\Rightarrow F_s = 2 \times F_{max}$

∴ Discrete Time signal

$= A \cos(2\pi(0.25)n)$.

Quantization:

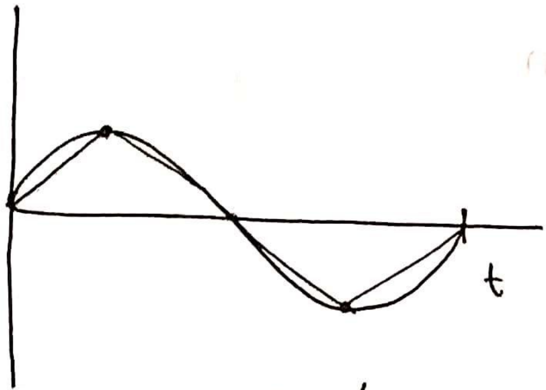
$t = nT$

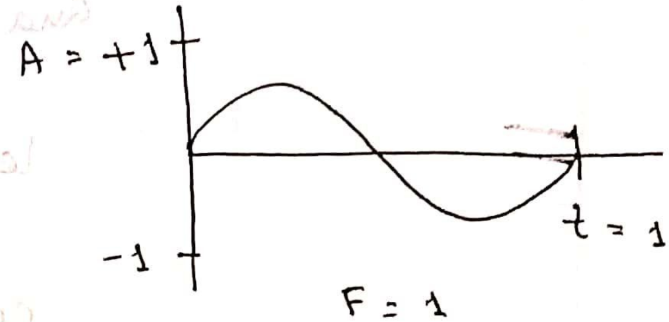
$$x_a(t) = A \cos(2\pi F t + 90)$$

$A = 1$

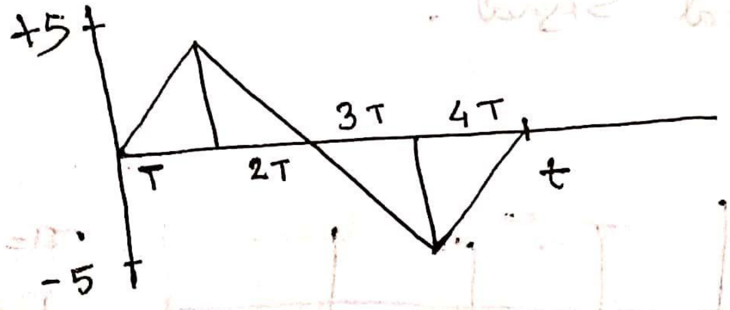
$F = 1$

$t = 1$





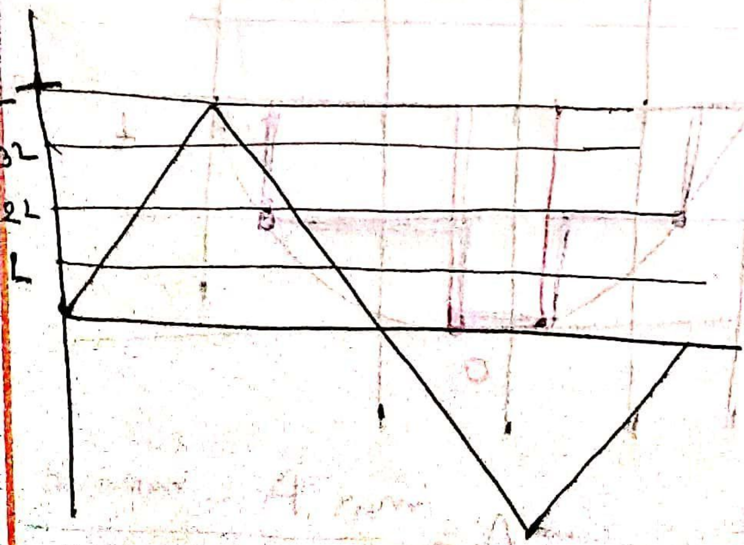
$n = 4$



Sampling rate

* we get Discrete time Signal (DTS) by sampling the Analog signal. (x-axis)

y-axis → Quantization.



$2^1 = 2$ $\quad$ $2^3 = 8$

$2^2 = 4$ $\quad$ $2^4 = 16$ → memory bit to save

Sampling

↓

Discrete (Analog) [DTS]

↓

Quantization

↓

level

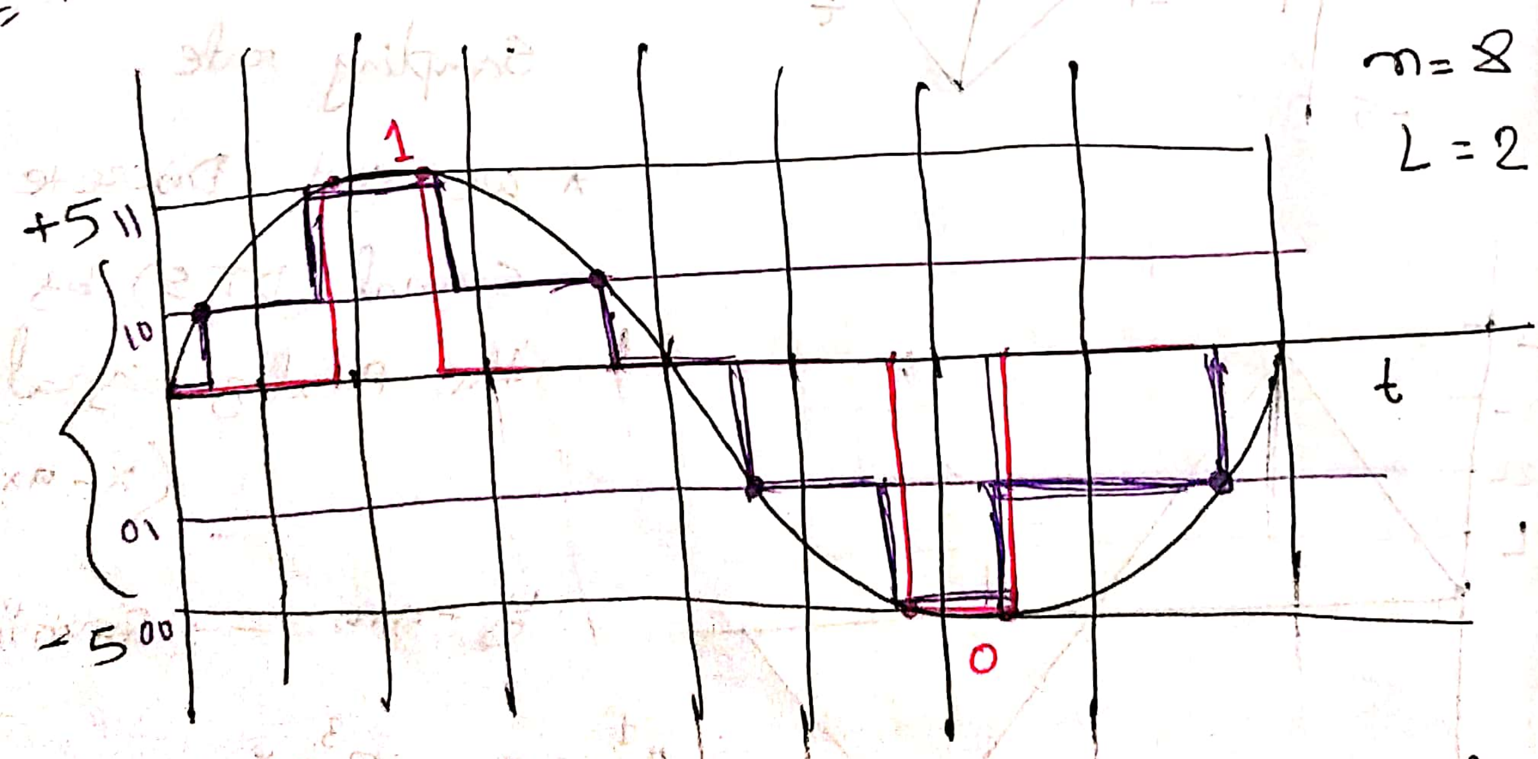
↓

coding

↓

Digital signal.

=4



* more level recovers more information.

$$\Delta = \frac{\max A - \min A}{L-1}$$

$$= \frac{5-(-5)}{\ldots} = 10$$

Analog

↓

Sampling

↓

Discrete (Analog) [DTS]

↓

Quantization

↓

level

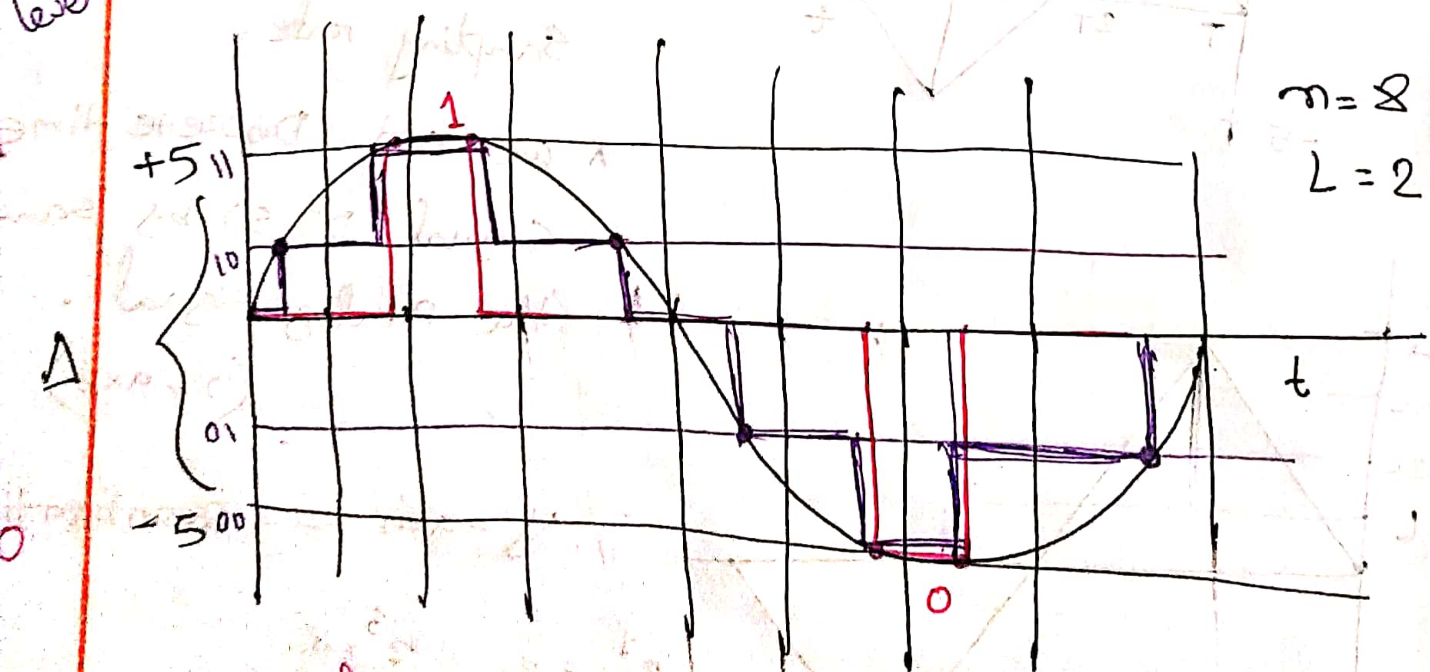
↓

coding

↓

Digital signal.

level = 4



* more level recovers more information.

$$\Delta = \frac{\max A - \min A}{L-1}$$

$$= \frac{5-(-5)}{2-1} = 10$$

Analog

↓

Sampling

↓

Discrete (Analog) [DTS]

↓

Quantization

↓

level

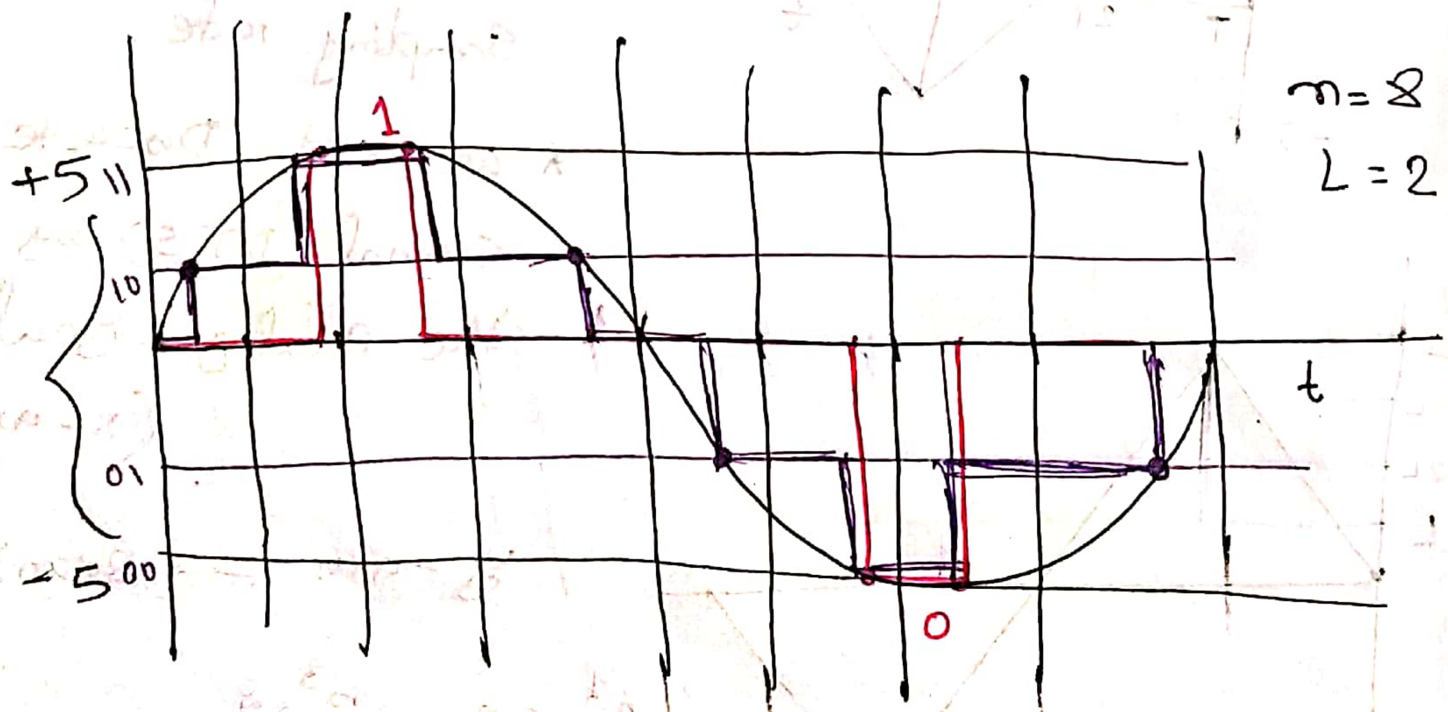
↓

coding

↓

Digital signal.

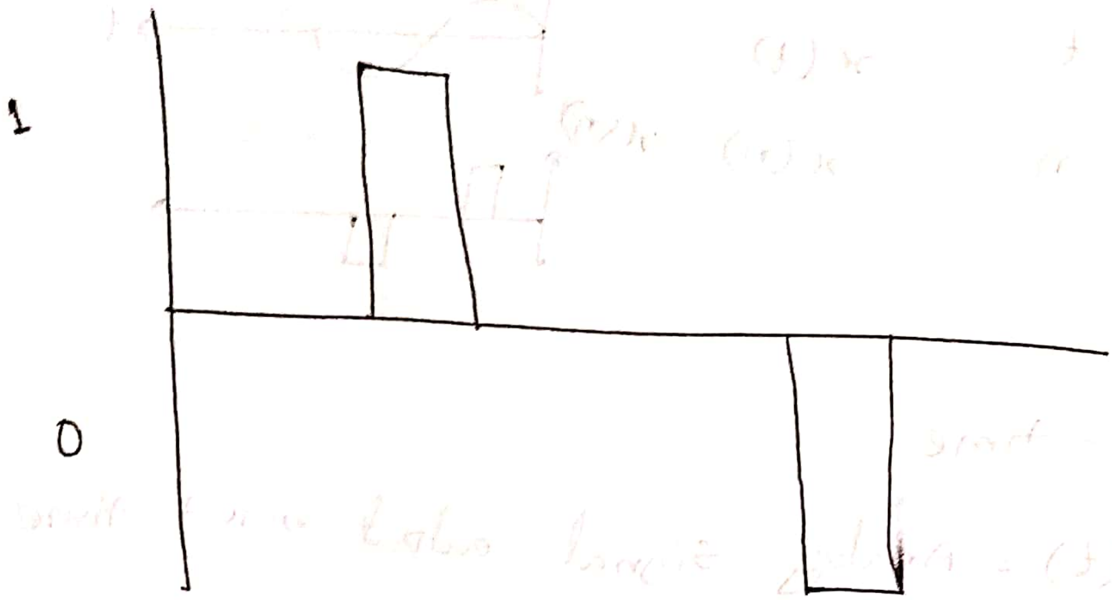
=4



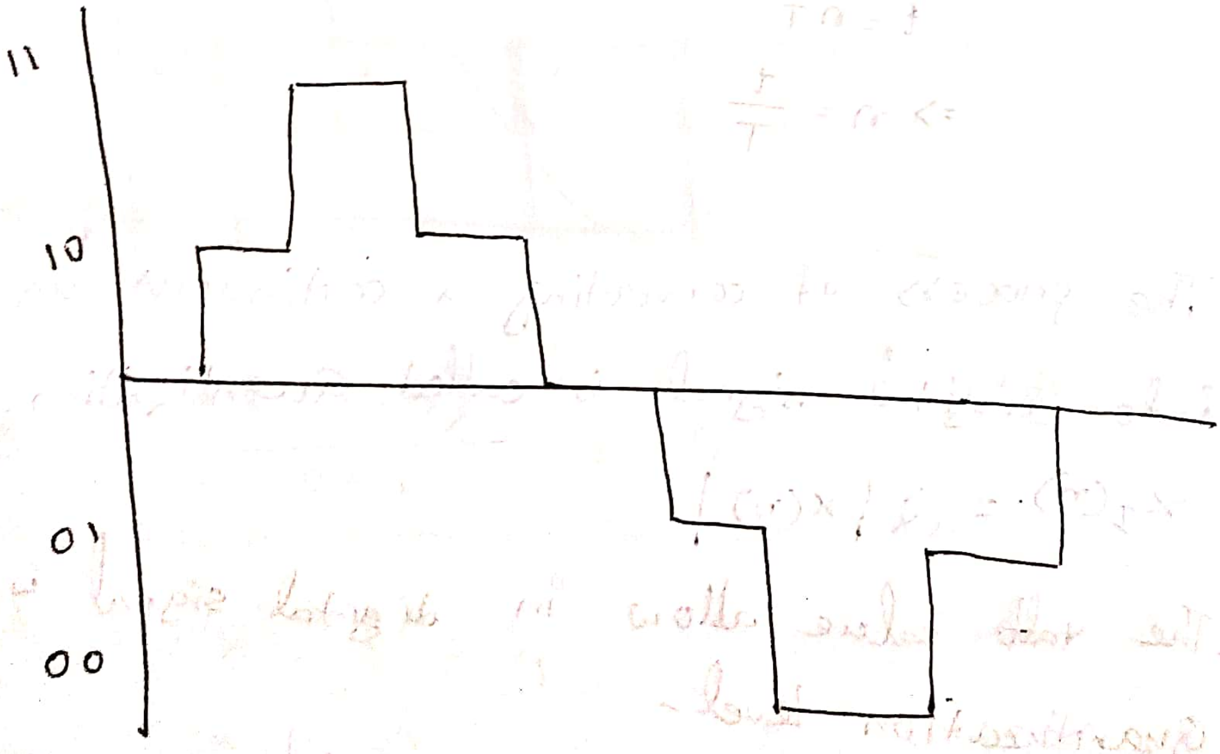
* more level recovers more information.

$$\Delta = \frac{\max A - \min A}{L-1}$$

$$= \frac{5-(-5)}{2-1} = 10$$



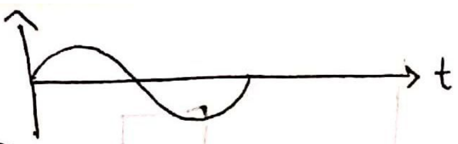
when L = 2



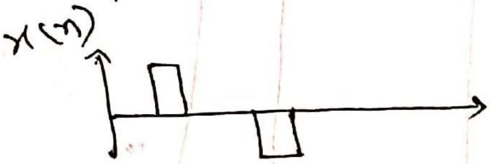
when L = 4

——x——

t $x(t)$



n $x(n)$



t = time

$x(t)$ = Analog signal output w.r.t. time

n = No. of sample

$x(n)$ = Discrete time signal.

$$t = nT$$

$$\Rightarrow n = \frac{t}{T}$$

(i) * The process of converting a continuous signal into digital signal is called Quantization.

$$x_q(n) = Q|x(n)|$$

(ii) ⇒ The ~~valt~~ value allow in digital signal is called Quantization level.

(iii) Distance Δ between two Quantization level.

(iv) Encode each sample value in order to store b-bit memory location.

if $b = 2$ bit

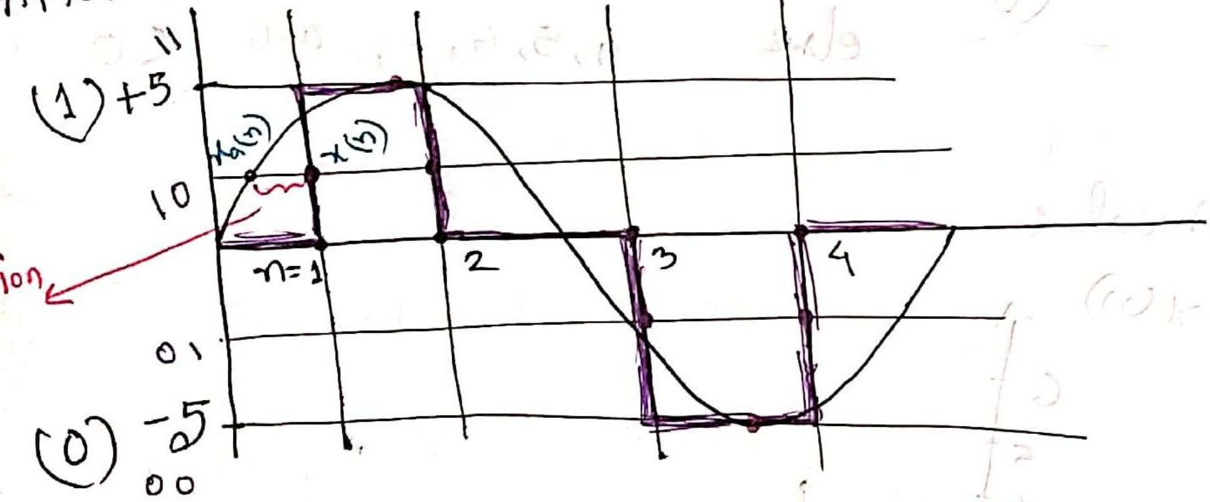
$$\boxed{\text{quantization level, } L = 2^b = 2^2 = 4}$$

if $b = 3$ bit,

$$L = 2^3 = 8$$

$$\Delta = \frac{max - min}{L - 1}$$

After Quantization → Digital signal.



$$\therefore \Delta = \frac{+5-(-5)}{2-1} = 10$$

$x(n) = [1, 0]$

$x(n) = [10, 11, 10, 01, 00, 01]$

if $b = 2$, $L = 4$

$$\Delta = \frac{5-(-5)}{4-1} = \frac{10}{3} = 3.33$$

* Quantization error:

$$e_q(n) = x_q(n) - x(n)$$

* limit of quantization error,

$$-\frac{\Delta}{2} \le e_q(n) \le \frac{\Delta}{2}$$

—x—

Signal Representation:

(i) Graphical representation

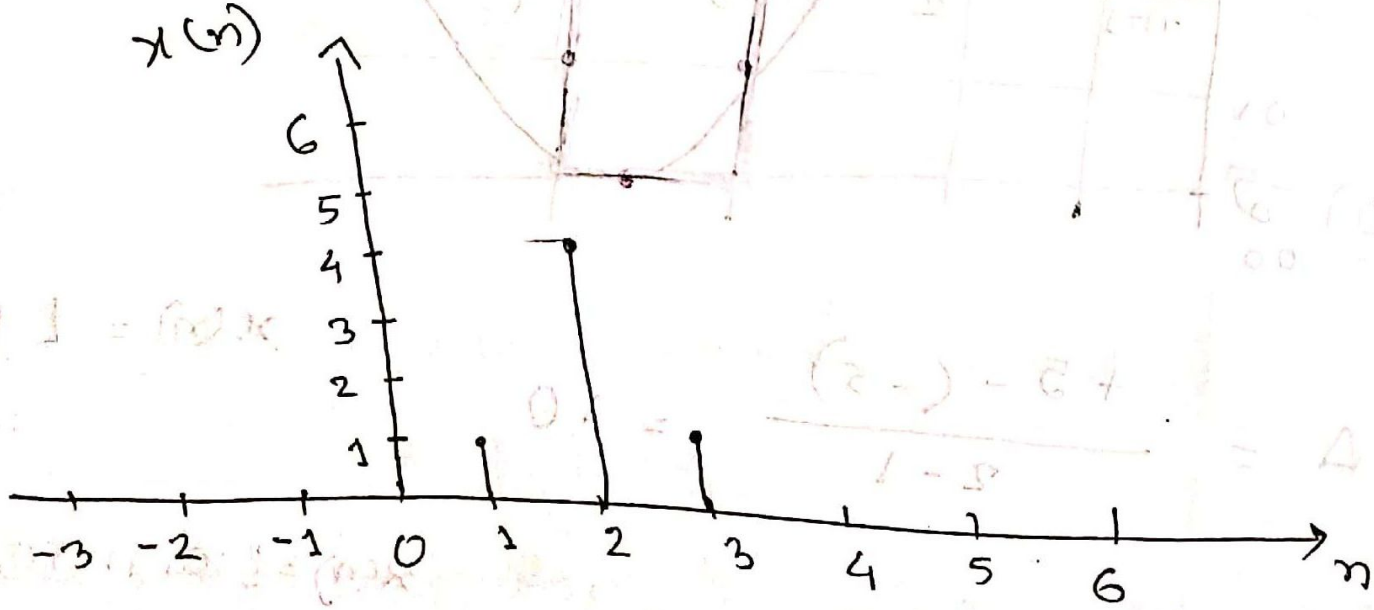
(ii) functional "

(iii) Tabular "

(iv) sequential "

$$x(n) = \begin{cases} 1 & \text{when } n = 1, 3 \\ 4 & \text{" } n = 2 \\ 0 & \text{else } 4, 5, 6, 7, \text{ or } < 0 \end{cases}$$

Graphical:



Tabular:

n	-3	-2	-1	0	1	2	3	4	5	6	7
$x(n)$	0	0	0	0	1	4	1	0	0	0	0

Sequential:

$$x(n) = \{ \ldots\ldots 0\ 0\ \underset{\uparrow}{0}\ 1\ 4\ 1\ 0\ 0\ 0\ 0 \}$$

Elementary of Discrete time signal:

(i) Unit sample → computers starting

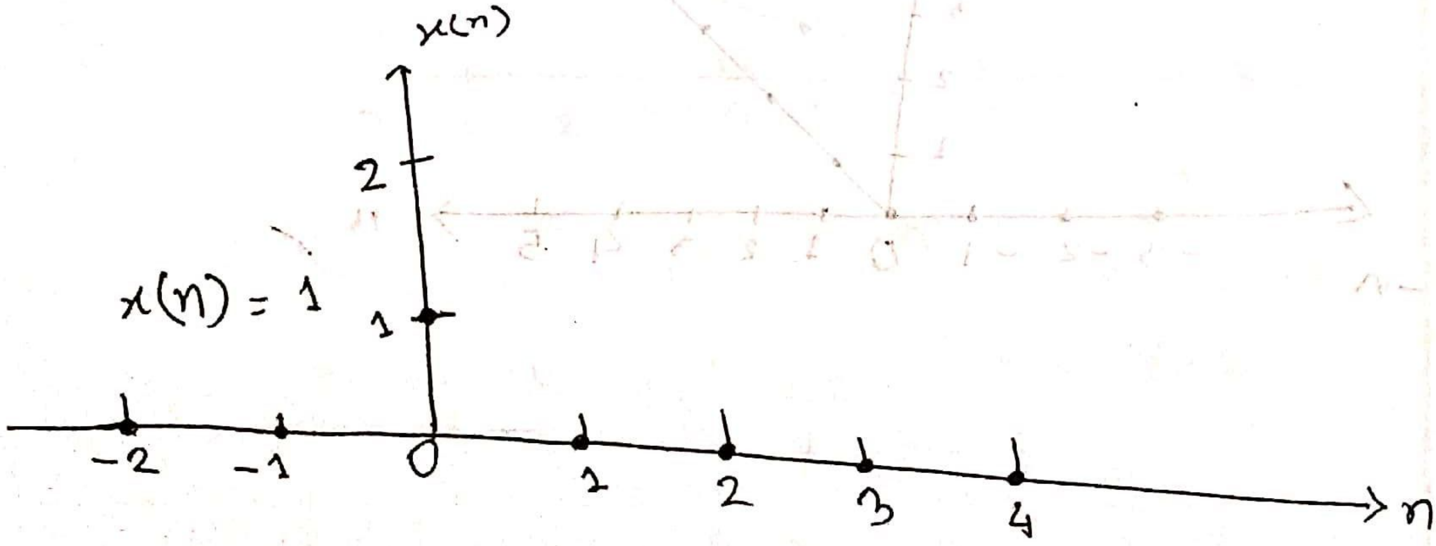
(ii) Unit step

(iii) Unit ramp

(iv) Exponential signal

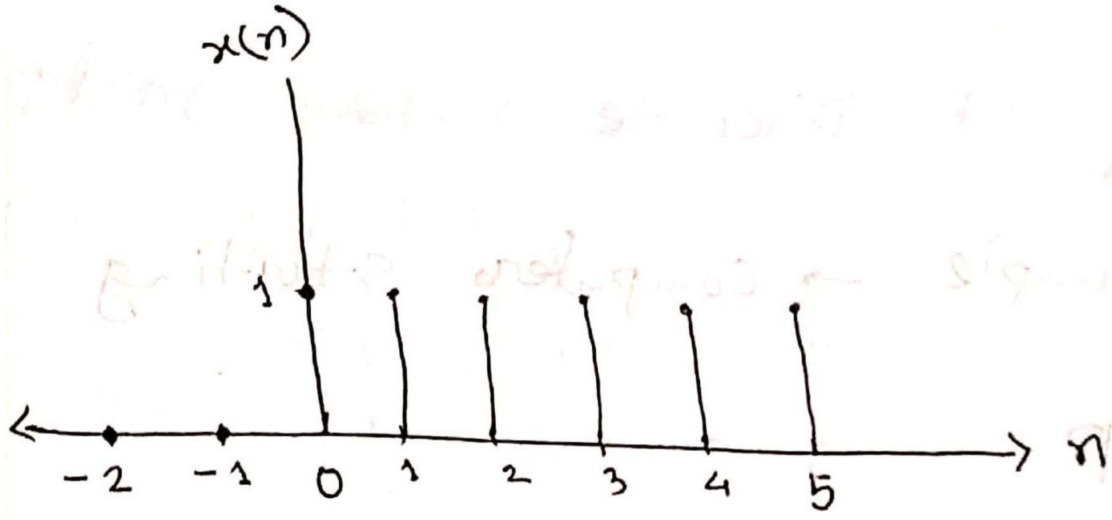
Unit sample / Impulse signal:

$$x(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



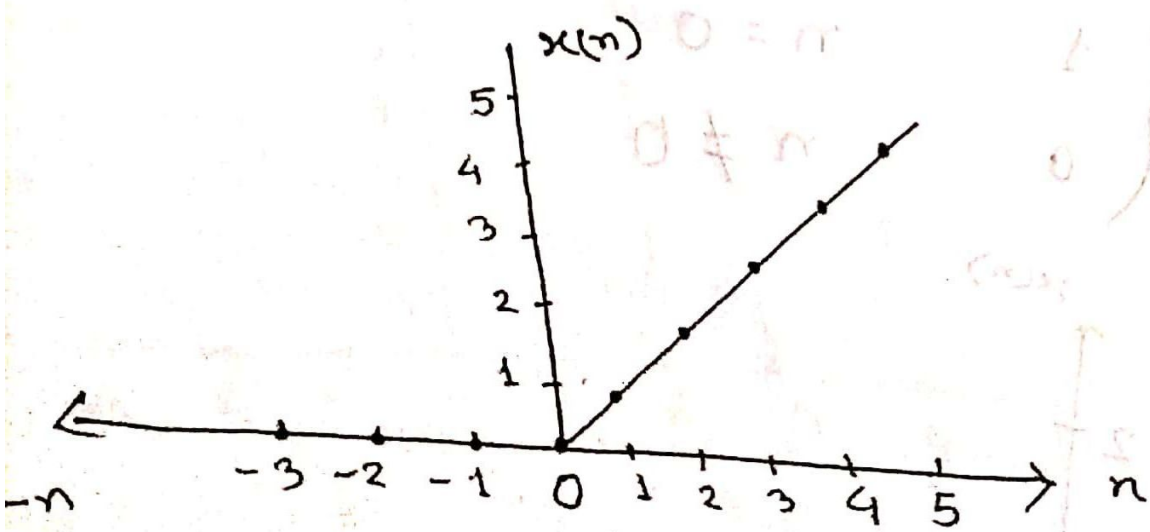
Unit Step:

$$x(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



Unit ramp:

$$x(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



) Exponential Signal:

$x(n) = a^n$ for all n

$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \ldots$

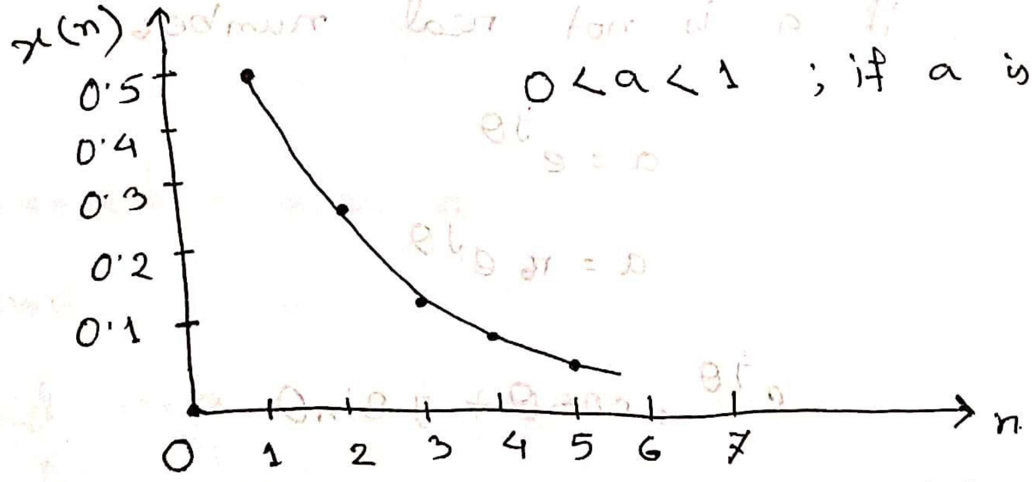
$0 < a < 1$

$x(n) = 0.5^1 = 0.5$

$x(n) = 0.5^2 = 0.25$

$= 0.5^3 = 0.125$

$= 0.5^4 = 0.0625$



$0 < a < 1$; if a is real.

$a > 0$

$x(n) = a^n$; $n = 1 \quad 2 \quad 3 \quad 4 \quad 5 \ldots$

$a = 2$

$= 2^1 = 2$

$= 2^2 = 4$

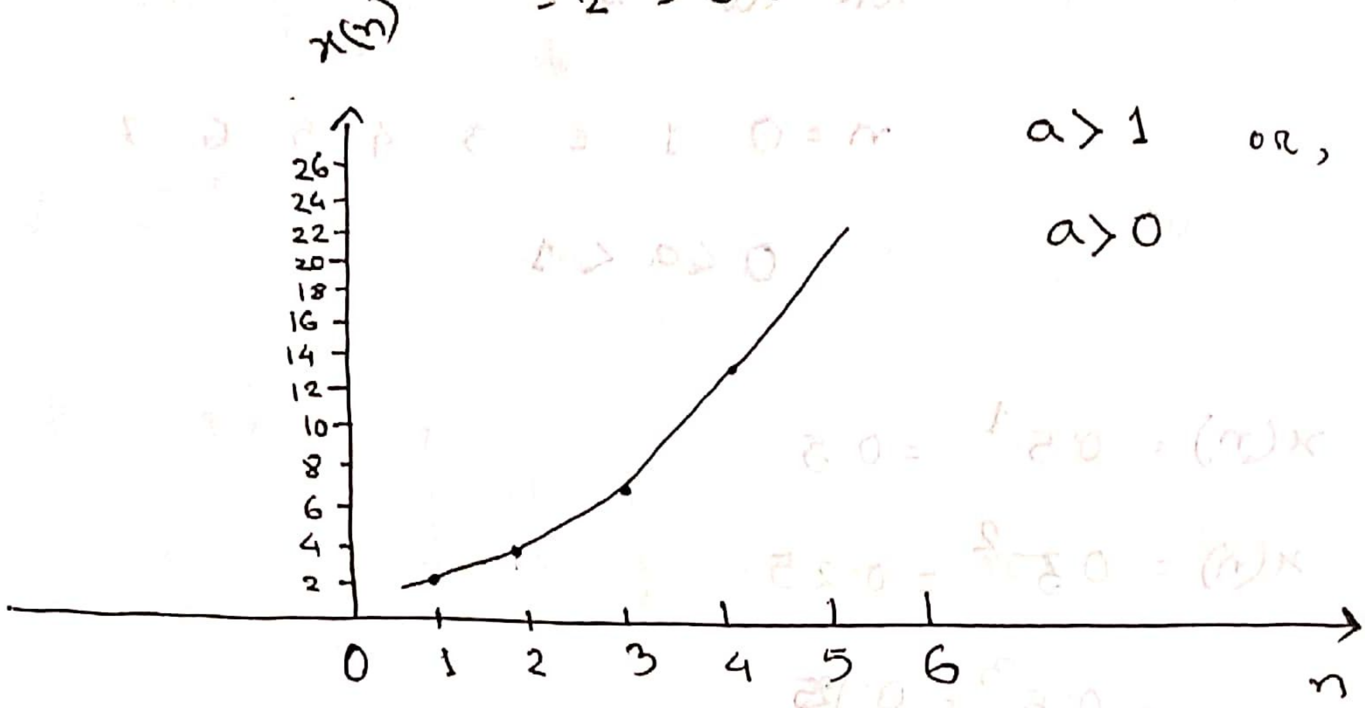
$= 2^3 = 8$

$= 2^4 = 16$

Scanned by CamScanner

$= 2^5 = 32$

$= 2^6 = 64$



$a > 1$ or, $a > 0$

$x(n) = a^n$

if a is not real number

$$a = e^{j\theta}$$

$$a = r e^{j\theta}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$a = r(\cos\theta + j\sin\theta)$$

$$x(n) = a^n$$

$$= \left(r(\cos\theta + j\sin\theta)\right)^n$$

$$= r^n(\cos\theta + j\sin\theta)^n \quad \left[r^n e^{j\theta n} = x(n)\right]$$

Scanned by CamScanner

$$a = r e^{j\theta}$$

$$a^n = r\,(e^{j\theta})^n$$

$$= r^n \cdot e^{j\theta n}$$

$$= r^n (\cos\theta\, n + j \sin\theta\, n)$$

Real of exponential, $Re = r^n \cos\theta n$

Imaginary part, $Im = r^n \sin\theta n$

———×———

"LAB"

31. 01. 19

Sinusoidal sequence

cosine "

unit step

Unit sample

unit ramp

exponential sequence,

———×———

Discrete signal classification:

1. Energy of a signal & power of a signal
2. Periodic / Aperiodic Signal
3. Symmetric / Asymmetric signal.
   (even) (odd)

$$y = \int_{t=0}^{1} x(t)$$

$$y = \sum_{n=1}^{5} x(n)$$

$-\alpha \le n \le \alpha$ (energy of a signal)

Energy of a signal:

$$y = \sum_{n=-\alpha}^{\alpha} |x(n)|^2$$

$$E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2$$

$E = 0.546$ (Energy of a signal)

$$P = \frac{1}{2N-1} \sum_{n=-N}^{N} |x(n)|^2$$

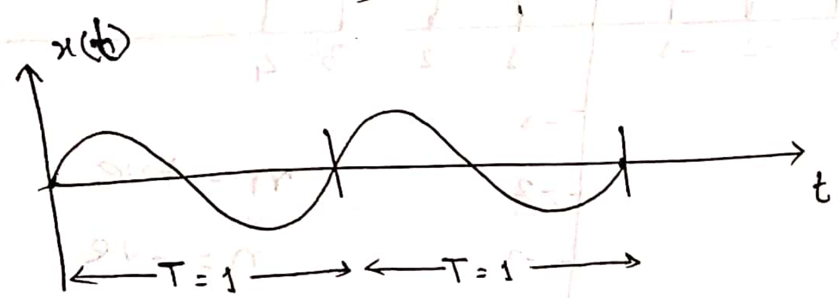
(power of a signal)

(11) Periodic signal:

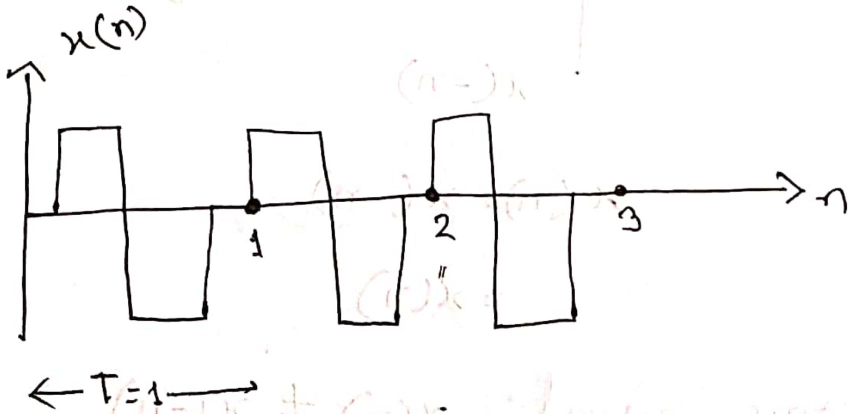
Some signal repeats themselve after a fixed time interval such signal is called periodic signal.

$$x(t) = x(t+T)$$

$$x(n) = x(n+N)$$



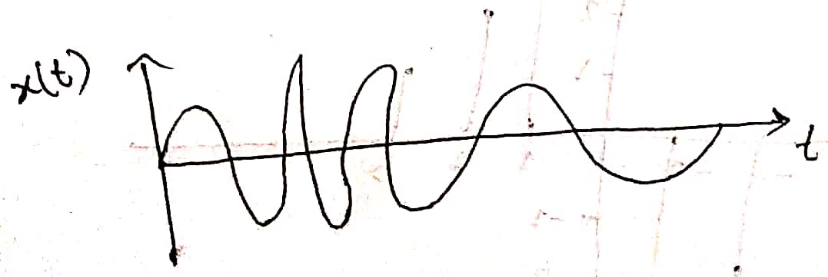
(Analog)



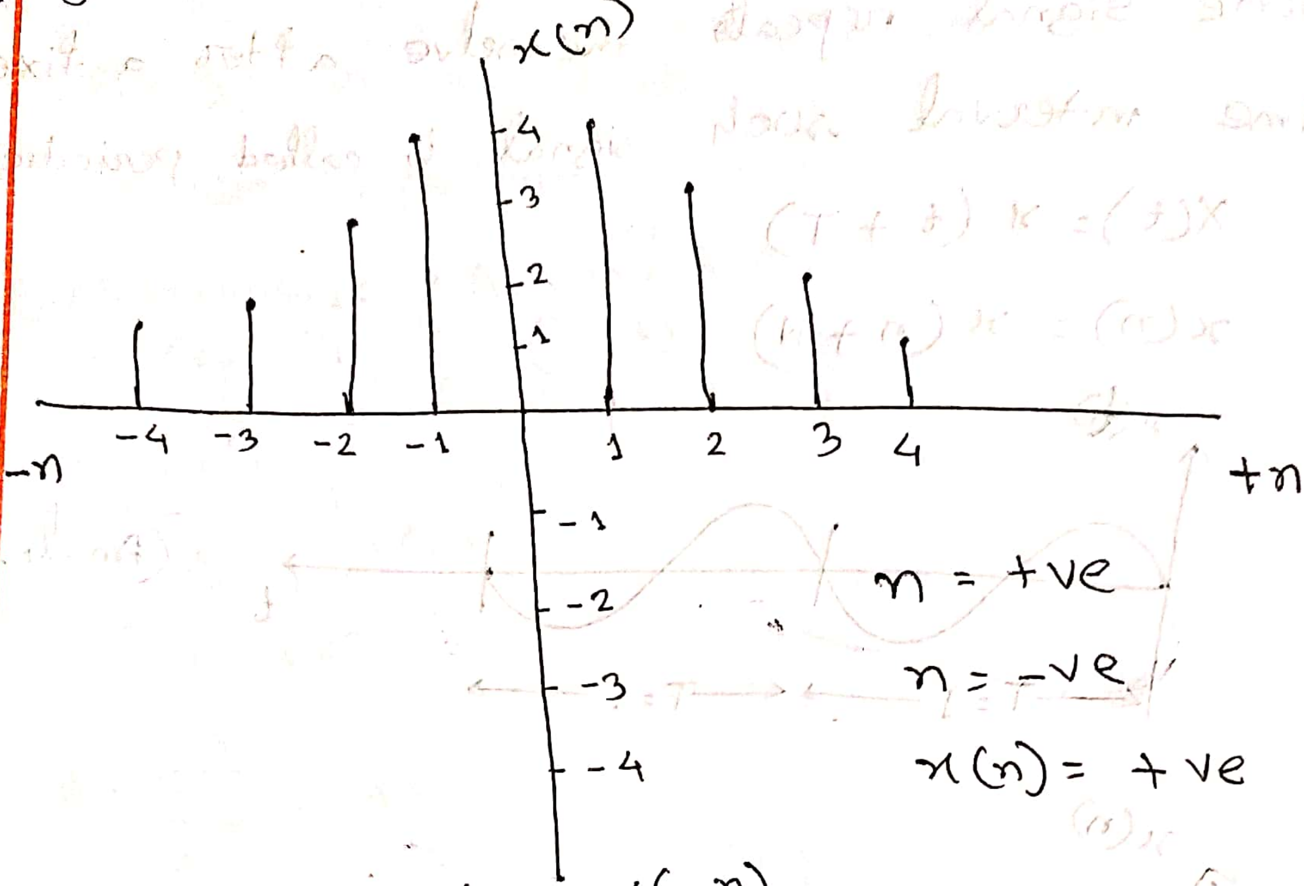
(Discrete)

Non-periodic:

$$x(t) \neq x(t+T)$$



(III) Symmetric Signal:



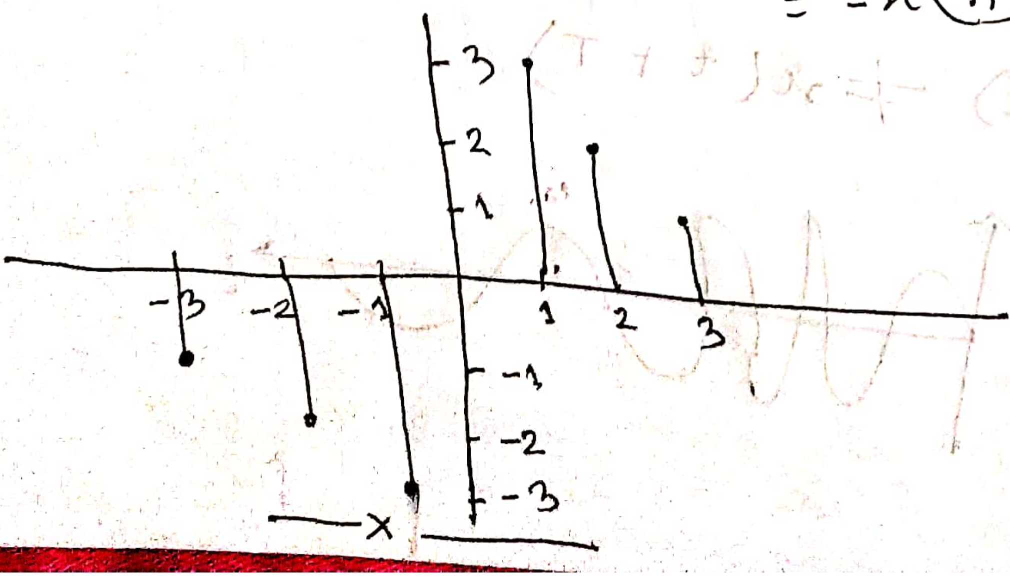
$x(-n)$

$$x(n) = x(-n)$$

$$= x(n)$$

Asymmetric signal: $x(n) \neq x(-n)$

$$= -x(n)$$



Operation of DSP:

1. Addition
2. Subtraction
3. Summation
4. Multiplication
5. Delay/Shifting.

Addition:

$x_1(n) = A\cos(2\pi f n)$ ⟶ Discrete sinosoidal signal

$x_2(n) = A\cos(2\pi f n)$

$x_3(n) = x_1(n) + x_2(n)$

$n = 0, 1, 2, 3$

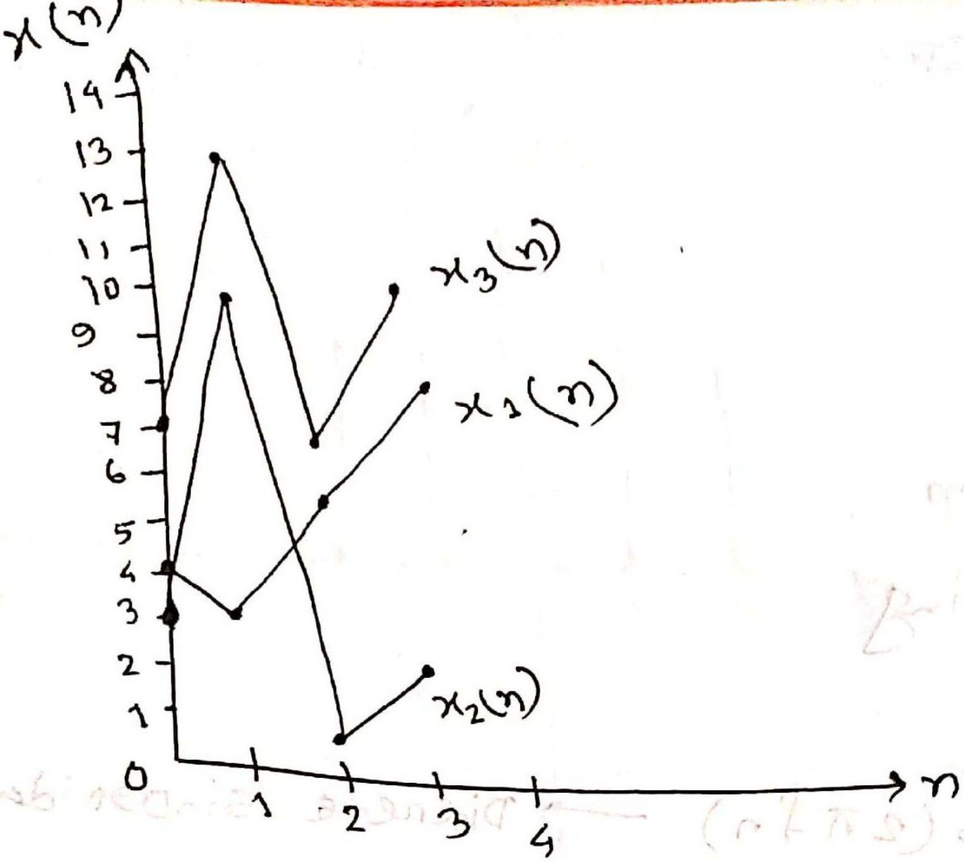
$x_1(n) = \underset{\downarrow}{4}, 3, 6, 8.$

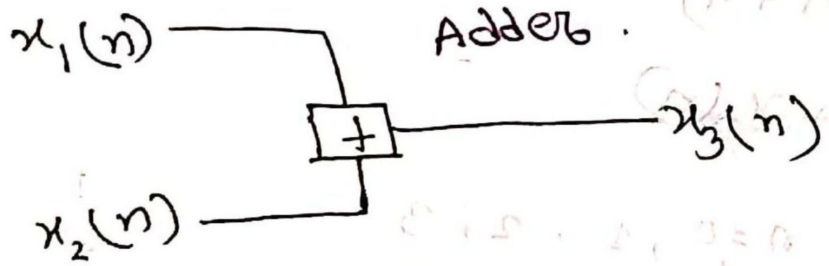
$x_2(n) = 3 \quad 10 \quad 0'5 \quad 2$

$x_3(n) = x_1(n) + x_2(n)$

$= \{ \underset{\uparrow}{7}, 13, 6'5, 10 \}$

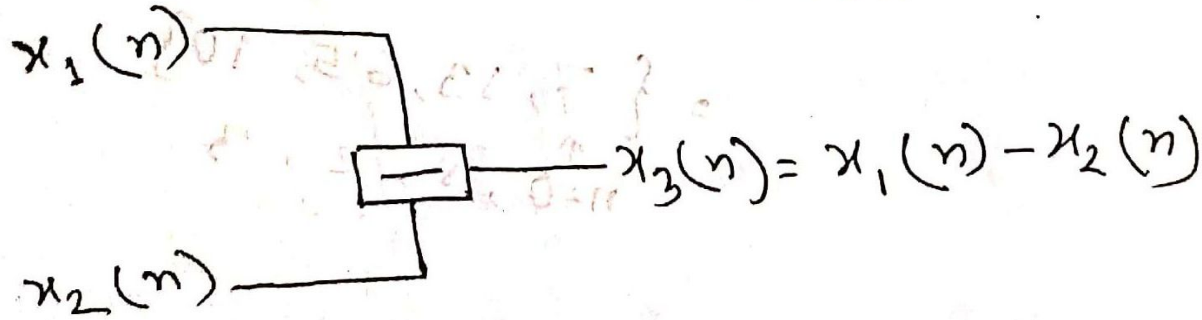
$n = 0, 1, 2, 3$





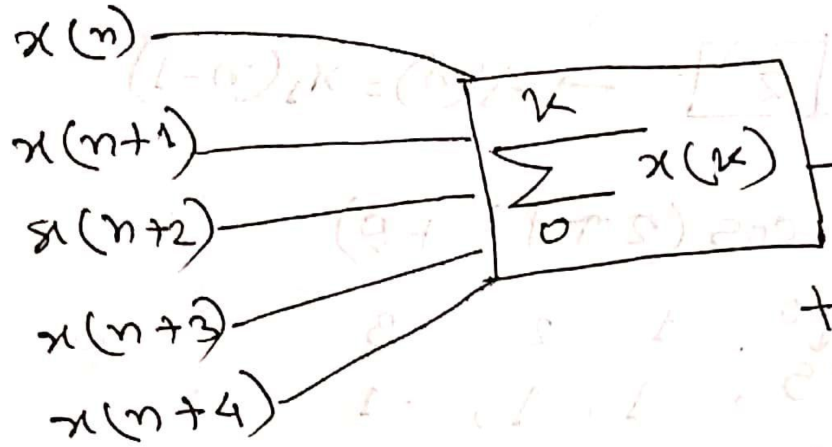
Addition operation of DSP.

Subtraction:



Subtraction operation of DSP.

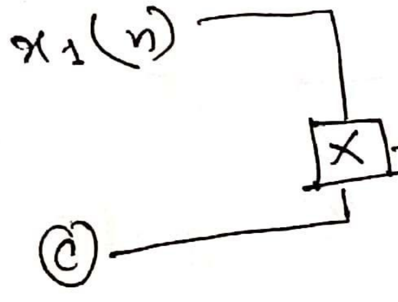
Summation:



$y(n) = x(n) + x(n+1) + x(n+2) + x(n+3) + x(n+4)$

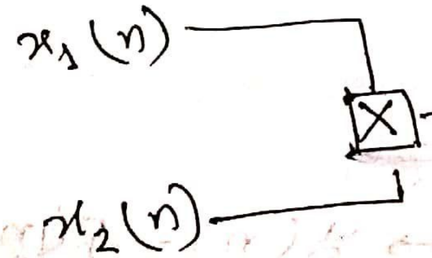
Multiplication:

- multiplied by constant c
- multiplied by sequence of signal



$y(n) = c \;.*\; x_1(n)$ → element by element multiplication

Multiplier



$y = x_1(n) * x_2(n)$

(V) Delay/ Shifting Operation:

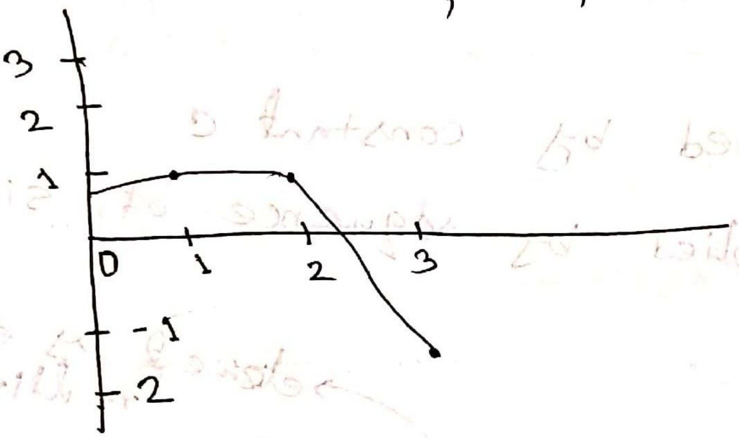
$x_1(n) \longrightarrow \boxed{z^{-1}} \longrightarrow y(n) = x_1(n-1)$

$x(n) = A \cdot \cos(2\pi f t + \theta)$

$$x(n) = \underset{\substack{n=0\\ \downarrow}}{0'5},\ \underset{1}{1},\ \underset{2}{1},\ \underset{3}{-1}$$

$\uparrow$

$n = 0'5,\ 1,\ 2,\ 3$



$y(n) = x_1(n-1) = n = 0$

$x(-1)$

$= x(0)$ $n = 1$

$x(1)$

$x(2)$

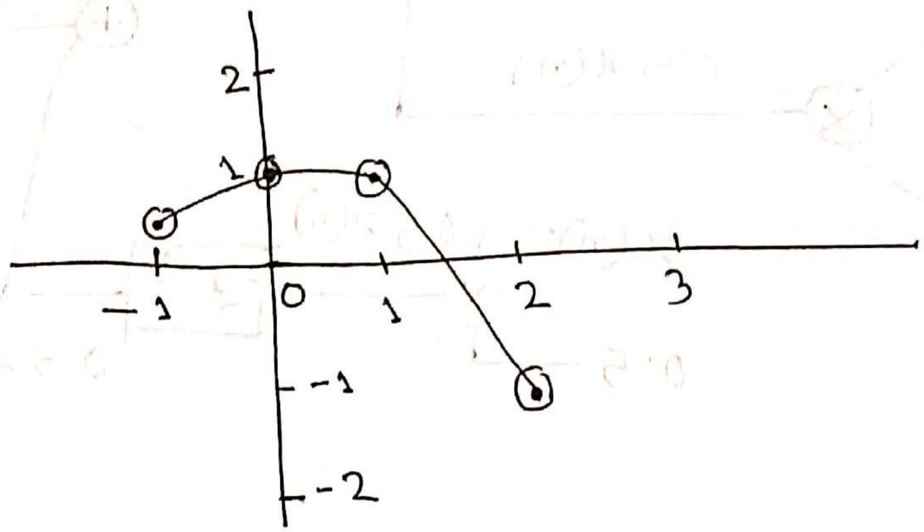
$p(n) \longrightarrow \boxed{z^{-1}} \longrightarrow y(n) = p(n-1)$ $n=0$

$p(0) = n = 0$ $p(-1)$

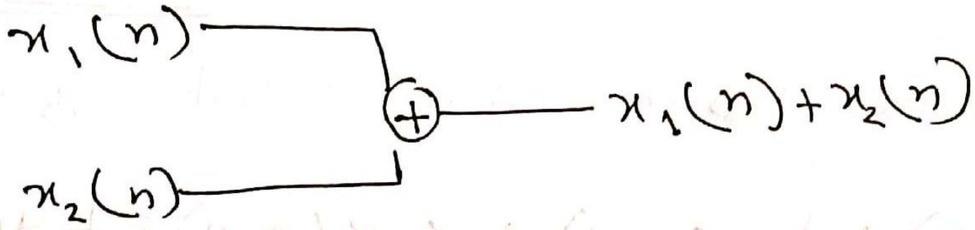
$n=0$ $n=1$ $n=2$ $n=3$

$p(n) = 0.5 \quad 1 \quad 1 \quad -1$

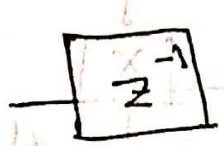
$n = -1 \quad 0 \quad 1 \quad 2$



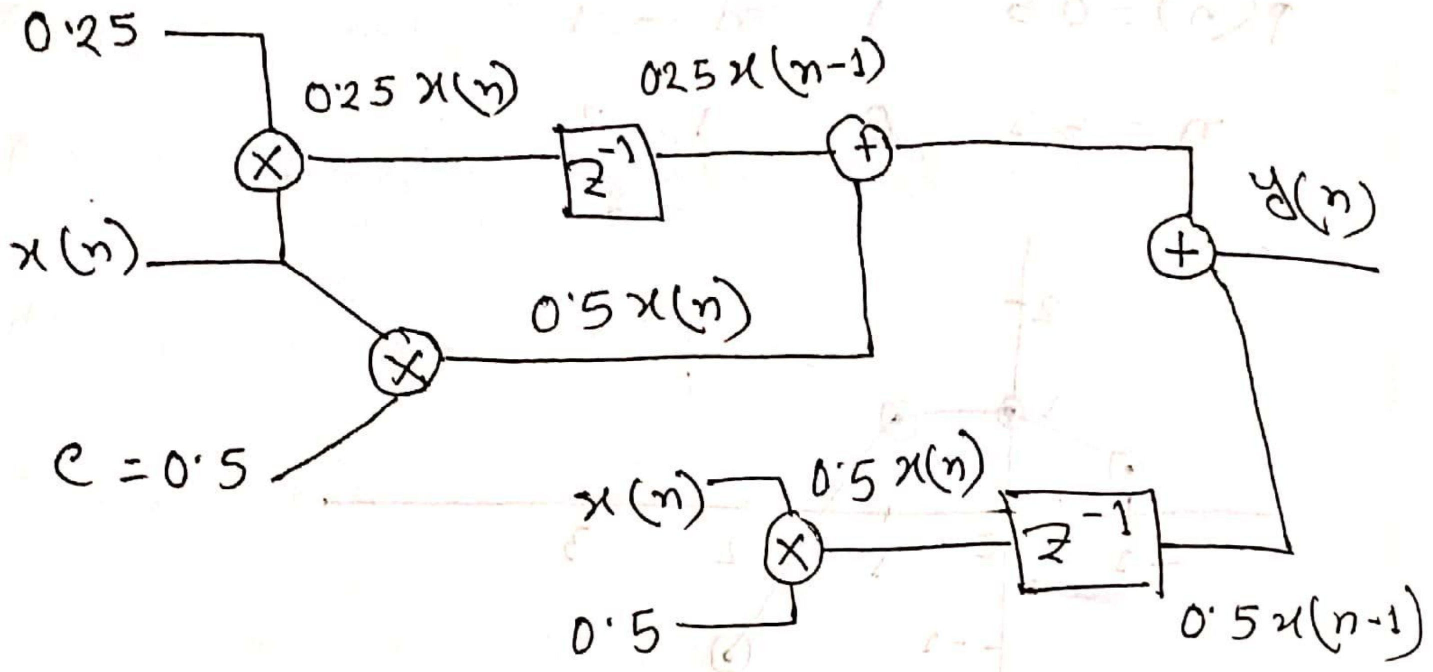
$x_1(n) + x_2(n)$



$$y(n) = \frac{1}{4} x(n-1) + \frac{1}{2} x(n) + \frac{1}{2} x(n-1)$$



Shifting operation.

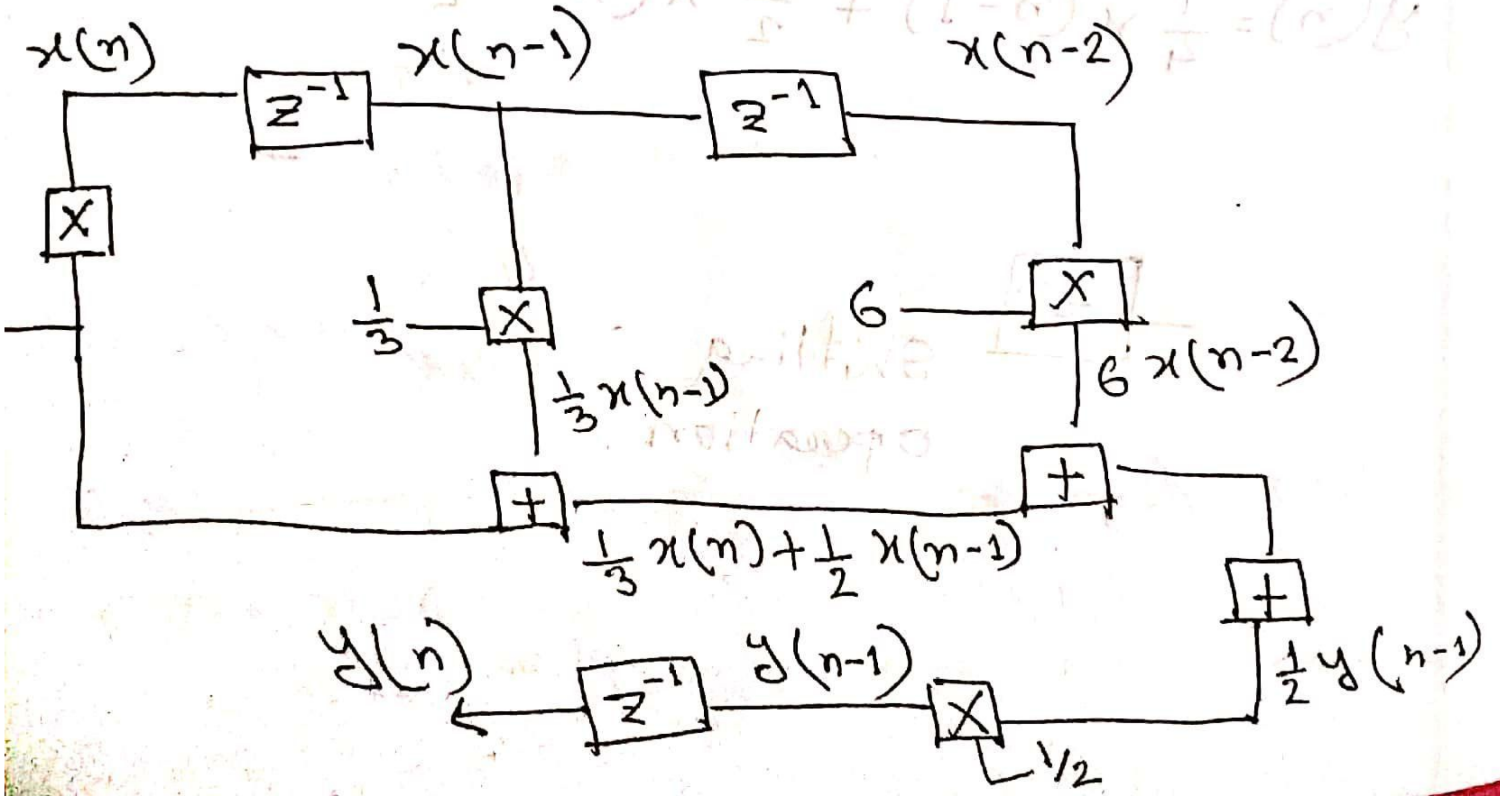


— x —

06.02.19

Sketch the system used to following equation:

$$y = \frac{1}{8}x(n) + \frac{1}{3}x(n-1) + 6\,x(n-2) + \frac{1}{2}y(n-1)$$



System:

A system is a mathematical model of a physical process that relate to input signal & output signal.

Let, x is the input signal & y is the output signal.

$y = Tx$ → Transformation

T = Adder
Subtraction
Multiplier
Shifter.

Discrete Time System (DTS):

A DTS is a device or an algorithm that can operate a discrete time signal

$y = TX$

$y[n] = T\{x[n]\}$

$y(t) = Tx(t)$

$x[n] \rightarrow T[\,] \rightarrow y[n]$ (output)

Scanned by CamScanner

i) $y(n) = x(n)$; idetity system

ii) $y(n) = x(n-1)$; Unid delay system

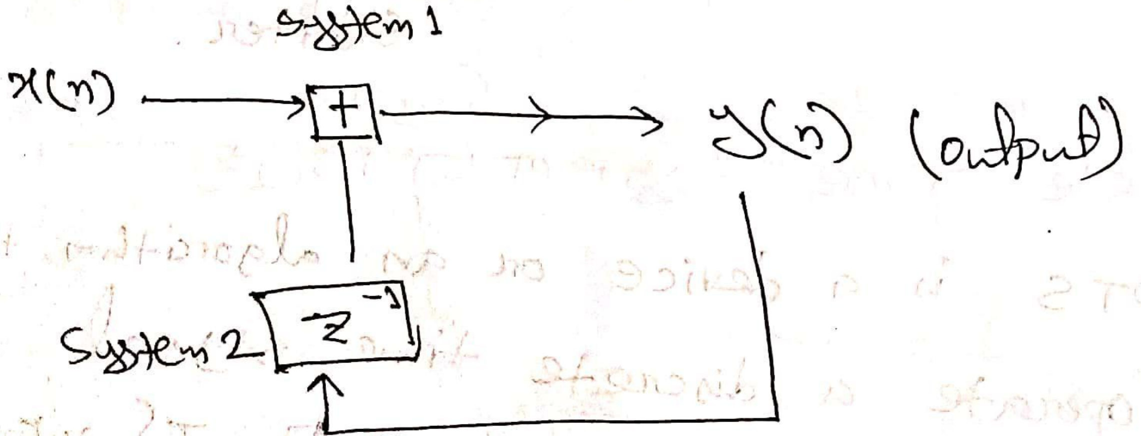
iii) $y(n) = x(n+1)$; Unid Advance system

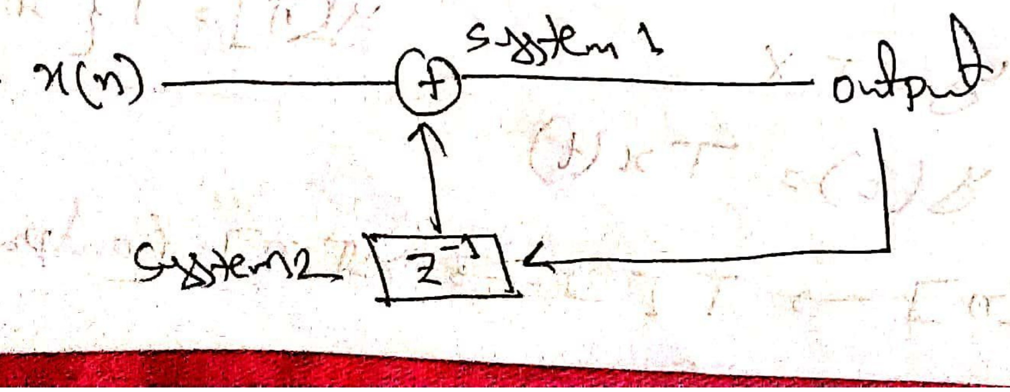
iv) $y(n) = \frac{1}{3}[x(n) + x(n-1) + y(n-1)]$; average filter.

* Feedback system:

When an output of a first system is the input of second system and output of second is the input of first system.

$$y(n) = x(n) + y(n-1)$$





$y(n) = x(n) + 3 * y(n)$

* Different types of system:

$x = 3$

$y = 3 * x$

$y = 9$

$\boxed{\begin{array}{l} y = x^4 \\ y = |x^2| \end{array}}$ → non-linear.

$y = -\frac{1}{2} x$

$y = Tx$
$= -\frac{1}{2} x$

(i) Linear system.

(ii) Non-linear system

(iii) Time invariant system

(iv) Time variant system

(v) Causal system

(vi) Non causal system

(vii) Stable / Unstable system

(viii) Linear Time invariant system.

—x—

"LAB" 07.02.19.

① Plot a signal on Matlab that have 3 frequency, 5 Amplitude during 5 seconds.

$$x(t) = A \cos(2\pi f t)$$

$A \downarrow 5$, $f \downarrow 3$, $t \downarrow 5$

t = 0 : 0.0001 : 1 ; if 1 second duration.

```
y = 5 * cos (2 * pi * 3 * t);
plot (t, y);
grid on;
xlabel ('Time');
ylabel ('Amplitude');
title ('Sinusoidal sequence');
```

fliplr

—x—

Linear System:

A system is called linear if it satisfies super-position principle.

Weighted sum of input signal be equal to the corresponding weighted sum of the output signal.

$$T[a_1 x_1 + a_2 x_2] = a_1 T[x_1] + a_2 T[x_2]$$

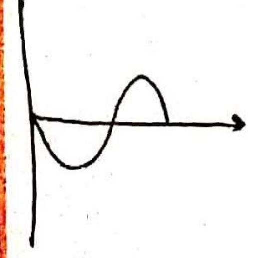
$$= a_1 y_1 + a_2 y_2$$

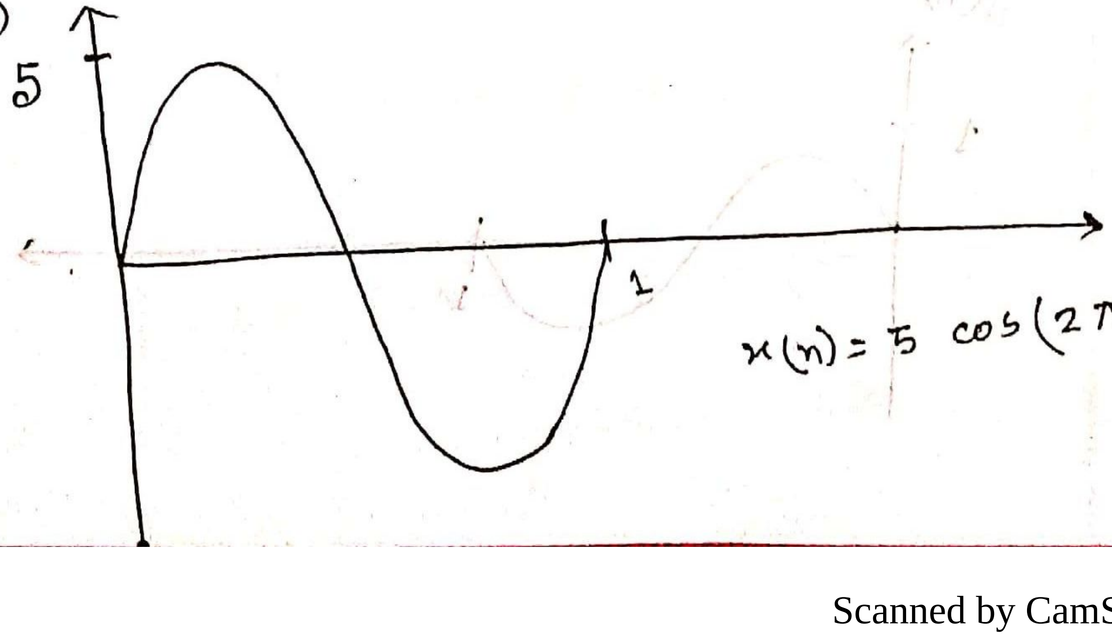
$$a_1 x_1 + a_2 x_2 = a_1 y_1 + a_2 y_2$$

$$x(n) = A \cos(2\pi f t + \theta)$$

$$= 5 \cos(2 . \pi . 1 . 1 + 90^\circ)$$

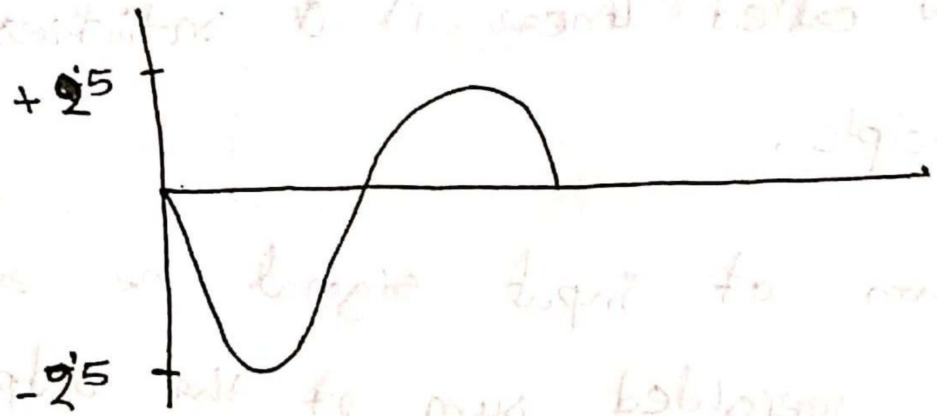
$$y(n) = -\frac{1}{2} x(n) \quad \Rightarrow T[x(n)] = y(n)$$





$$x(n) = 5 \cos(2\pi . 1 . 1 + 90^\circ)$$

$$y(n) = -\frac{1}{2} x(n)$$



Non-linear System:

If a system produces a non zero output for zero input that called non-linear system.

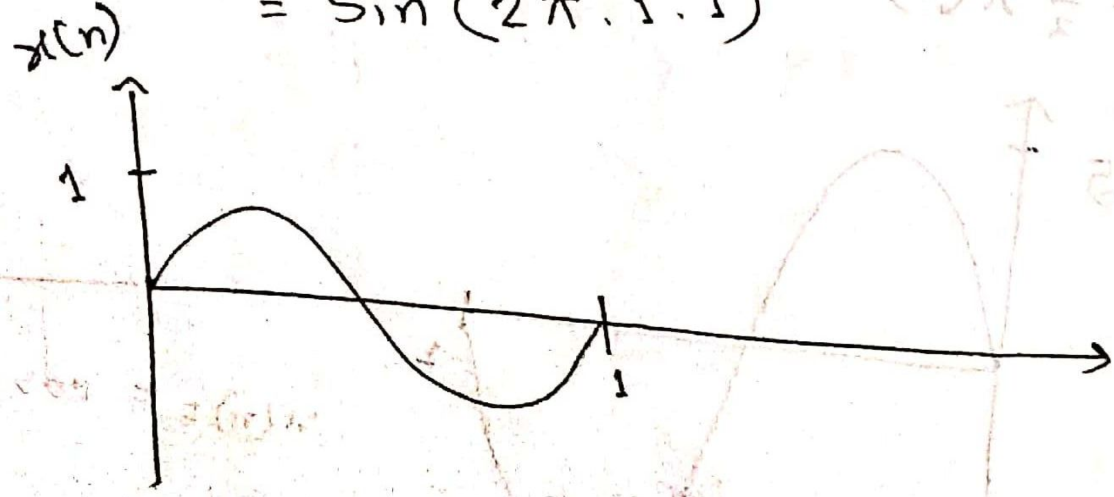
Not satisfies superposition principle.

$$y(n) = [x(n)]^2$$

$$x(n) = \sin(2\pi \cdot 1 \cdot 1 + \theta) = A \sin(2\pi f t + \theta)$$

$$= 1 \cdot \sin(2\pi \cdot 1 \cdot 1 + 0)$$

$$= \sin(2\pi \cdot 1 \cdot 1)$$



$$y(n) = [x(n)]^2$$

$$= x(n) * x(n)$$

$$= \sin(2\pi \cdot 1 \cdot 1) * \sin(2\pi \cdot 1 \cdot 1)$$

$$\left[\sin(a) * \sin(b) = \frac{\cos(a-b)}{2} - \frac{\cos(a+b)}{2} \right]$$

$$= \frac{\cos(2\pi \cdot 1 \cdot 1 - 2\pi \cdot 1 \cdot 1)}{2} - \frac{\cos(2\pi \cdot 1 \cdot 1 + 2\pi \cdot 1 \cdot 1)}{2}$$

$$= \frac{\cos(0)}{2} - \frac{\cos(2\pi \cdot 2 \cdot 1)}{2}$$

$$= \frac{1}{2} - \frac{\cos(2\pi \cdot 2 \cdot 1)}{2}$$; 0, 2 frequency cosine wave.

$x(n) = \sin(2\pi \cdot 3 \cdot t_s + \theta)$; 0, 6 frequency cosine wave.

$$y(n) = [x_1(n) + x_2(n)]^2$$

$$= \underbrace{[x_1(n)]^2}_{0,2} + 2 \cdot x_1(n) \cdot x_2(n) + \underbrace{[x_2(n)]^2}_{0,6}$$

$$\left[2 * \sin(2\pi \cdot 1 \cdot 1) * \sin(2\pi \cdot 3 \cdot 1) \right]$$

$$\left[2 * \sin(a) \sin(b) = \cos(a-b) - \cos(a+b) \right]$$

$= 2 * \sin(2\pi . 1 . 1) * \sin(2\pi . 3 . 1)$

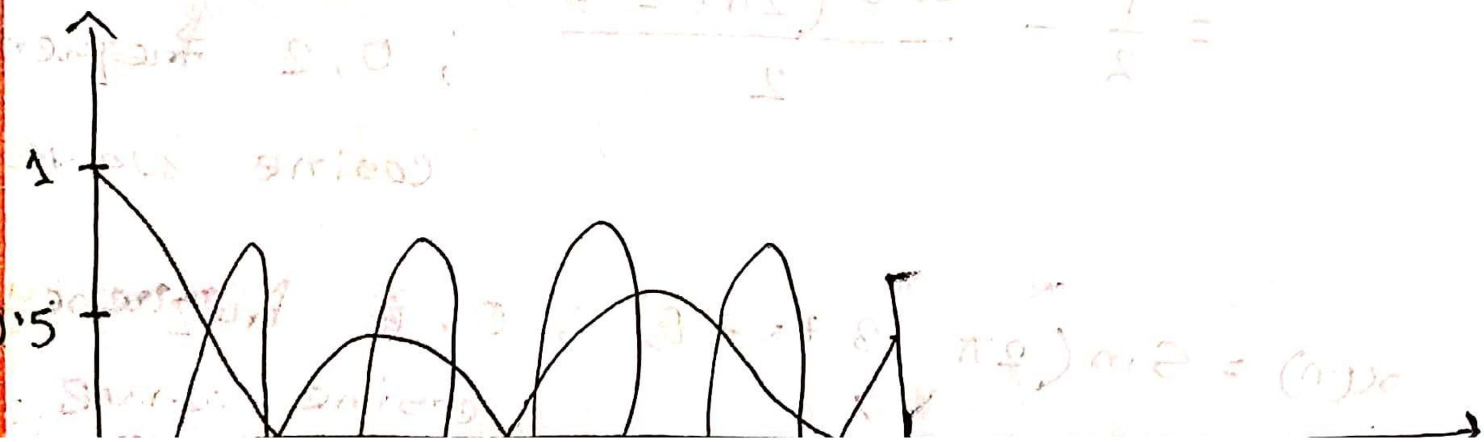
$= \cos(2\pi . 1 1 - 2\pi . 3 . 1) - \cos(2\pi . 1 . 1 + 2\pi . 3 . 1)$

$= \cos(2\pi . 2 . 1) - \cos(2\pi . 4 . 1)$

$[\cos(-x) = \cos(x)]$

$$= \frac{1}{2} - \underbrace{\frac{\cos(2\pi . 2 . 1)}{2}}_{\text{Fig 1}} + \underbrace{\cos(2\pi . 2 . 1)}_{\text{Fig 2}} - \underbrace{\cos(2\pi . 4 . 1)}_{\text{Fig3}}$$

$$+ \frac{1}{2} - \underbrace{\frac{\cos(2\pi . 6 . 1)}{2}}_{\text{Fig 4}}$$



"LAB"

14.02.19

$y(n) = x(n) + x(n-1) - 0.7\, y(n-1)$ at $-20 \le n \le 100$

$y(k+2) + 2y(k+1) + \frac{1}{2} * y(k) = x(k+1) + x(k)$

$\underline{\underline{x}} = [1, 5, 10, 15]$ → signal

x^2 x^3

quadr

$\underline{\underline{y}}$ = filter (a, b) ⊗ → Sampling

y = filt(b, a, x)

a/b = a →

b

b/a = ?

a/b = □ linea

x^1

y^1

ut signal exp

linear / non-linear linear

$\underline{a} = [1 \; 0]$

$\underline{b} = [1 \; 0.9]$

$y_2 - y_1 = \square m$

$m = n$

linear / non-linear

(x_1, y_1)

(x_2, y_2)

x_3, y_3

an

$y_3 - y_2 = my = ax + b$

Linear Time Invariant system:

$$x(n) \times \delta(n-k)$$

δ = unit impulse signal.

$$y(n) = \sum_{k=\text{lower}}^{\text{upper limit}} x(k) \times h(n-k).$$

$$y(n) = x(n) * h(n)$$

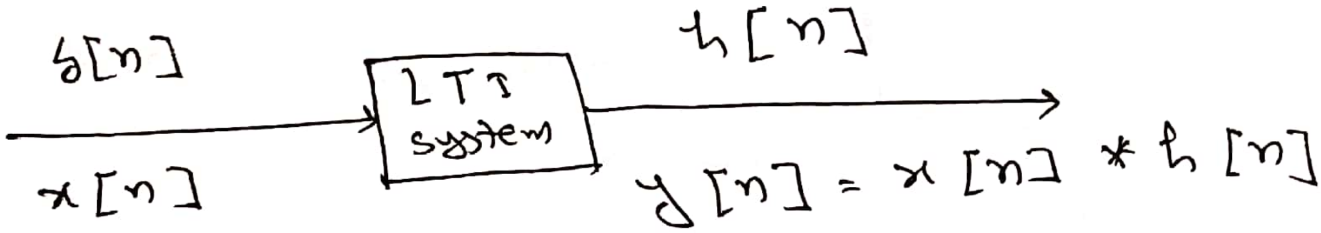


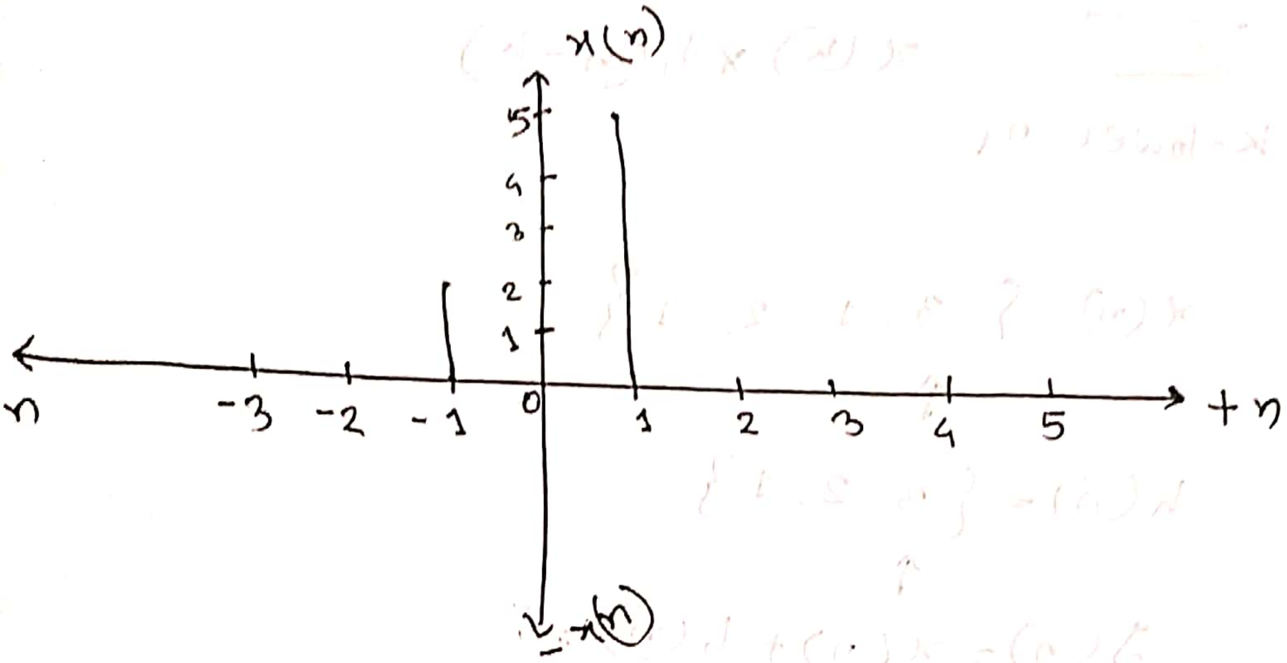
Fig: Discrete time LTI system.

Three different process of computing convolution sum:

1. Mathematical approach
2. Tabular approach
3. Graphical approach.

$$x(n) = \begin{array}{cccccc} & & n=0 & & & \\ [\,1 & 2 & 5 & 6 & 7 & 8\,] \\ -2 & -1 & 0 & 1 & 2 & 3 \\ & & \uparrow & & & \end{array}$$

$$h(n) = \begin{array}{ccc} -1 & 0 & 1 \\ \hline \end{array}$$
$$h(n) = [\,1 \quad \underset{\uparrow}{2} \quad 4\,]$$




```
for (i=0; i<=n; i++)
{ for (j=0; j<=k; j++)
   {
   }
}
```

Lower limit $= N_1 = -2$

upper limit $= N_2 = +3$

for $x(n) = \begin{array}{cccccc} -2 & -1 & 0 & 1 & 2 & 3 \\ {[}1 & 2 & \underset{\uparrow}{5} & 6 & 7 & 8{]} \end{array}$

$h(n) = N_2$

lower $P_1 = -1$

upper $P_2 = +1$

for $h(n) = \begin{array}{ccc} -1 & 0 & 1 \\ {[}1 & 2 & 4{]} \end{array}$

$y(n) = N_1 + N_2$

upper limit $= n_2$

$$\sum_{k = \text{lower } n_1}^{} x(k) \times h(n-k)$$

$x(n) = \{3, 1, 2, -1\}$
$\uparrow$

$h(n) = \{3, 2, 1\}$
$\uparrow$

$y(n) = x(n) + h(n)$

length of $y(n) = 4+3 = 7$

lower limit of $y(n) = 0+0 = 0$

upper limit of $y(n) = 3+2 = 5$

$n = 4+3 = 7 - 1 = 6$

$k = 0+0 = 0$

$k = 3+2 = 5$

$$y(n) = \sum_{k=\text{lower}}^{\text{upper limit}} x(k) \times h(n-k)$$

$$\left\{ y(n) = \sum_{k=0}^{5} x(k) * h(n-k) \right\}$$

when $n=0$, $k = 0,1,2,3,4,5$.

$$x(0) * h(0-k) + x(1) * h(1-k) + x(2) * h(2-k)$$
$$+ x(3) * h(3-k)$$

$$(0) \Rightarrow x(0) * h(0-0) + x(1) * h(0-1) + x(2) * h(0-2)$$
$$+ x(3) * h(0-3) + x(4) * h(0-4) + x(5) * h(0-5)$$

when, $n = 1$, $k = 0,1,2,3,4,5$,

$$\Rightarrow x(0) * h(1-0) + x(1) * h(1-1) + x(2) * h(1-2)$$
$$+ x(3) * h(1-3) + x(4) * h(1-4) + x(5) * h(1-5)$$

if $n=0$

$h(n-k) = h(-k)$

$$\begin{array}{ccccc} & -1 & n=0 & 1 & 2 \\ h(k) = \{ & 1, & \underset{\uparrow}{2}, & 1, & -1 \} \end{array}$$

folding operation

$$h(-k) = \{-1, 1, \underset{\uparrow}{2}, 1\}$$

1. Folding
2. Shifting
3. Multiplication
4. Summation.

—— X ——

19.02.19.

LTI :

$y(n) = x(n) * \delta(n-k)$

$y(n) = \sum_{k=-\infty}^{\infty} x(k) * h(n-k$

- → Linear
- → Time Invariant.

$y(n) = x(n) * h(n)$ → Convolution Sum.

$y(n) = T[x(n)]$; T = LTI System.

$= x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) * h(n-k)$$

$x[n] = \{2 \quad \underset{\uparrow}{0} \quad 3 \quad 1\}$

$y[n] = \{1 \quad \underset{\uparrow}{3} \quad 4\}$

$n = 4+3-1 = 6$

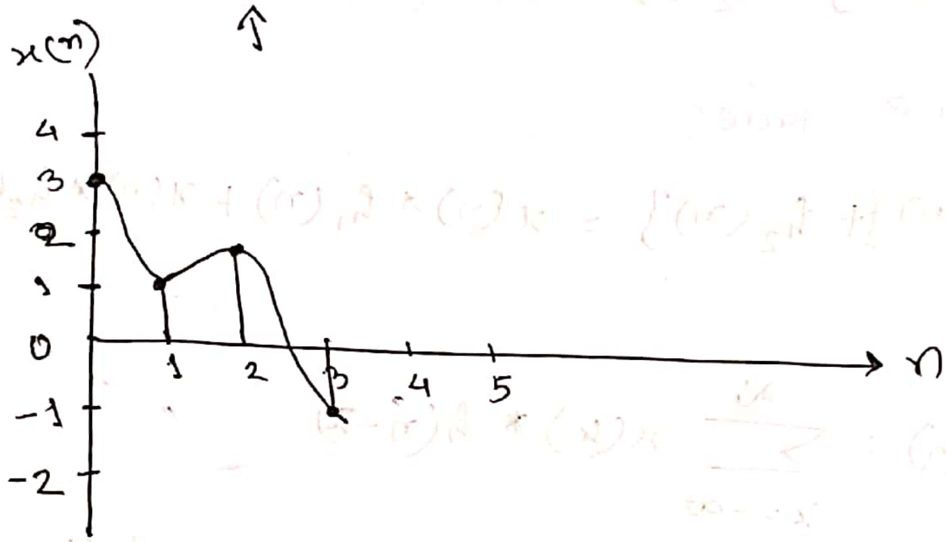
k lower limit $= -1-1 = -2$

k upper " $= 2+1 = 3$

$$y(n) = \sum_{k=-2}^{3} x(k) * h(n-k)$$

$$\begin{array}{cccccc} & n=0 & 1 & 2 & 3 & \\ x = [& 3 & 1 & 2 & -1 &] \\ & \uparrow & & & & \end{array}$$

$$\begin{array}{ccccc} & n=0 & 1 & 2 & \\ h = [& 3 & 2 & 1 &] \\ & \uparrow & & & \end{array}$$



$$y(n) = \sum_{k=0}^{5} x(k) * h(n-k)$$

When $n=0$,

$$y(0) = x(0) * h(0) + x(1) * h(0-1) + x(2) * h(-2) + x(3) * h(-3) + x(4) * h(-4) + x(5) * h(-5)$$

$$= 3*3 + 1*0 + 2*0 + 0 + 0 + 0$$

$$= 9$$

Butterworth Filter
IIR Filter
FIR Filter
Low pass "
High " "

—— X ——

Properties of the convolution sum:

1. Commutative Rule:

$$x(n) * h(n) = h(n) * x(n)$$

2. Associative Rule:

$$\{x(n) * h_1(n)\} * h_2(n) = x(n) * \{h_1(n) * h_2(n)\}$$

3. Distributive Rule:

$$\{x(n) * \{h_1(n)\} + h_2(n)\} = x(n) * h_1(n) + x(n) * h_2(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) * h(n-k)$$

$$y(n) = x(n) * h(n) \longrightarrow \text{convolution sum.}$$

$n - k = p \quad \Rightarrow n - p = k$

$k = -\infty$

$p = -\infty$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) * h(n-k)$$

$$= \sum_{p=-\infty}^{\infty} h(p) * x(n-p)$$

* Types of LTI system:

$x(0)$ $x(-1)$

(i) Causal LTI system (depends on present & delay system)

$x(0)$ $x(-1)$ $(x(0))$

(ii) Anti- " " " (" " " , " & advance ")

(iii) Stable " " (input sequence is bounded)

(iv) Non- " " "

—x—

26. 02. 19

"LAB" 26.02.19.

```
x = [3 4 5 7 2 0]
h = [3 4 2 ]

x = [3 4 5 7 2 0 0 0 0 0]
h = [3 4 2 0 0 0 0 0 0 0]
y = zeros(1,9)
for n = 1:9
    for k = 1:n
        y(n) = y(n) + x(k) * h(n-k+1)
    end
end
stem (n, y(n))
```

```
Syms z n;
ztrans (1/4 ^ n)
iztrans (1/4 ^ n)
```

27.02.19.

Z- transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cancel{x} * z^{-n}$$

Polar Form:

$$z = re^{j\theta}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] * z^{-1}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left(re^{j\theta}\right)^{-n}$$

if $e^{j\theta} = 1$

$$= \sum_{n=-\infty}^{\infty} x(n)\ r^{-n}$$

$|X(z)|$ is finite if the sequence $x(n)\ r^{-n}$ is absolutely summable.

$$= \sum_{n=-3}^{3} x(n)\ r^{-n}$$ [not summable, because array index can not be (-) minus]

$$\Rightarrow X(z) = \sum_{-3}^{-1} x(n) z^{-n} + \sum_{0}^{3} x(n) z^{-n}$$

$$= x(-3) z^{-(-3)} + x(-2) z^{-(-2)} + x(-1) z^{-(-1)}$$

$$\cancel{= x(3) z^{3} - x(2) z}$$

$$= x(-3) z^{3} + x(-2) z^{2} + x(-1) z$$

$$= \sum_{n=1}^{3} x(-n) z^{n}$$

$$x = \begin{array}{ccccc} 3 & 2 & \underset{\uparrow}{4} & 6 & 5 \\ -2 & -1 & n=0 & 1 & 2 \end{array}$$

$x_1 = [-1 \quad -2]$

$= [2 \quad 3]$

$x_2 = [4 \quad 6 \quad 5]$

One sided Z-transform:

$$\sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\Rightarrow \sum_{n=0}^{\infty} x(n) z^{-n}$$

ROC (Region Of Convergence):

Right-Sided Signal

Left - " "

Two - " "

Finite duration "

Z- Transform Properties:

Linearity:

$$X[a\, x_1(n)] \longrightarrow Z\,|\,a\, x_1(n)\,|$$

$$= a\; x_1(z)$$

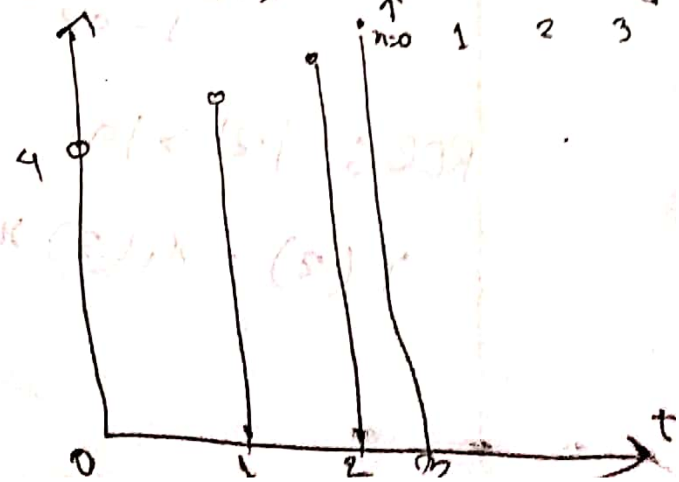
$$Z[a\, x_1(n) + k\, x_2(n)] = a\, x_1(z) + k\, x_2(z)$$

Time-Shift:

$$x(n) = X(z)$$

$$x(n-1) = z^{-1}\, x(z)$$

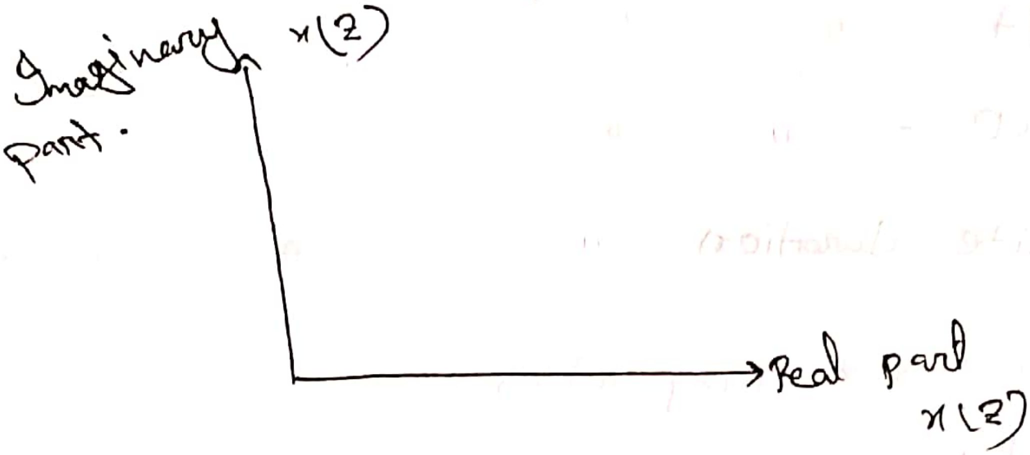
$x(n) = [4 \;\; 6 \;\; 8 \;\; 9]$



Scanned by CamScanner

$x(n) = x(z)$

$x(n-1) = z^{-1} x(z) x(z)$



Time Advance:

Time Reveral:

Convolution:

$$x_1(n) * x_2(n) \xleftrightarrow{z} x_1(z)\, x_2(z)$$

$x_1(n) = a^n u(n)$ and $x_2(n) = u(n)$

Eq $x_1(z) = \dfrac{1}{1 - az^{-1}}$ $\qquad$ $x_2(z) = \dfrac{1}{1 - z^{-1}}$

ROC: $|z| > |a|$ $\qquad$ ROC: $|z| > 1$

$$Y(z) = x_1(z)\, x_2(z) = \frac{1}{(1 - az^{-1})(1 - z^{-1})}$$