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BAYESIAN INFERENCE FOR MODELING AND PREDICTING STUDENT EXAM PERFORMANCE

ABSTRACT:

In this study, I used an open-source dataset of 1000 students' records, where I examined their race, lunch type, and test preparation to check how these factors influence a student's math score. I chose Bayesian linear regression for this evaluation to quantify uncertainty and provide full posterior distributions for model parameters. The analysis reveals that students from race groups (D&E) who received only standard lunch and with test preparation show better results in their math scores. This shows that using Bayesian modeling in education data provides better insights into social and behavioral factors to shape academic outcomes.

INTRODUCTION:

The mathematics score of a student is a critical indicator for their success and a strong predictor of their future educational outcomes. Finding the factors that influence this math score can help students, educators, researchers, and teachers to design effective interventions to improve student learning.

This project analyzes how a student’s demographic, lunch type and completion of test preparation results on their performance in math exams. While traditional statistical models like ordinary least squares regression (OLS) can help us in this study but it often falls short in quantifying uncertainty and incorporating prior knowledge. To use a better model without these limitations I used Bayesian linear regression which is a probabilistic modeling approach that has full posterior distribution for parameters and allows a deeper insight. This model is not only capable of estimating the size of each variable but also helps in expressing confidence in estimating credible intervals.

The data set that used has 1000 high school student’s data which includes their demographics, academic and lunch type. By comparing a baseline frequentist regression with a Bayesian model, I highlighted the advantages of Bayesian inference in making decisions under uncertainty and quantifying uncertainty in terms of interpretation.

DATASET:

As explained in the introduction this data set contains 1000 high school students data that includes students demography like race and ethnicity, the lunch type they intake and their test preparation towards courses, along with exam scores in math, reading, and writing. This is an open source dataset that I obtained from Kaggle named as Student Performance In Exams. Below are the variables in the data set which I analyzed.

This dataset has two types of variables like every dataset a dependent and independent.

Dependent variables are Math, reading and writing scores.

Independent variables are Gender, Race/ethnicity, parental level of education, lunch and test preparation course.

A data.frame: 6 × 8

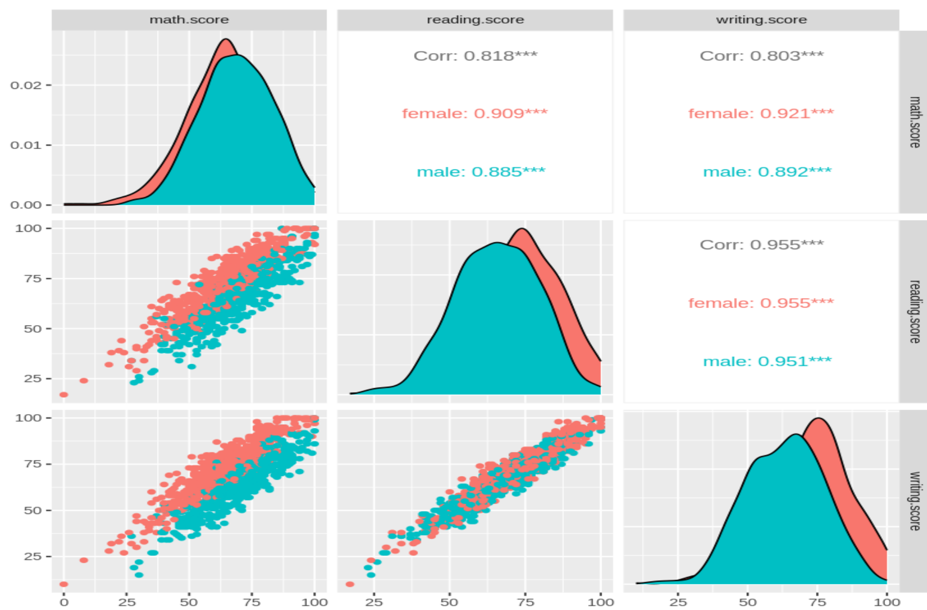
	gender	race.ethnicity	parental.level.of.education	lunch	test.preparation.course	math.score	reading.score	writing.score
	<chr>	<chr>	<chr>	<chr>	<chr>	<int>	<int>	<int>
1	female	group B	bachelor's degree	standard	none	72	72	74
2	female	group C	some college	standard	completed	69	90	88
3	female	group B	master's degree	standard	none	90	95	93
4	male	group A	associate's degree	free/reduced	none	47	57	44
5	male	group C	some college	standard	none	76	78	75
6	female	group B	associate's degree	standard	none	71	83	78

Below is the structure of the data set which we can extract by using the command below in R.

```
str(data)

'data.frame':   1000 obs. of  8 variables:
 $ gender                     : chr  "female" "female" "female" "male" ...
 $ race.ethnicity             : chr  "group B" "group C" "group B" "group A" ...
 $ parental.level.of.education: chr  "bachelor's degree" "some college" "master's degree" "associate's degree" ...
 $ lunch                      : chr  "standard" "standard" "standard" "free/reduced" ...
 $ test.preparation.course    : chr  "none" "completed" "none" "none" ...
 $ math.score                 : int  72 69 90 47 76 71 88 40 64 38 ...
 $ reading.score              : int  72 90 95 57 78 83 95 43 64 60 ...
 $ writing.score              : int  74 88 93 44 75 78 92 39 67 50 ...
```

Scatter plot:



DATA PREPROCESSING:

Before modeling, the data was cleaned and preprocessed in R:

- Missing values: Checked using Col Sums(is.na(data)); no missing values were found.
- Duplicates: Checked and removed using duplicated () function to ensure data integrity.
- Variable types: All categorical variables were explicitly converted to factors using as.factor() to allow proper encoding by modeling functions.
- Descriptive statistics and exploratory plots: Summary statistics and value counts were reviewed to understand distributions and detect anomalies.

MODELING APPROACH:

To find the impact of a student demographic and behavioral features on math performance, I applied two regression models.

1. **Frequentist Linear Regression:** which is a traditional OLS regression model, by using the lm () function in R I was able to apply to my data set, it provided a reference point for interpretation and performance comparison with the Bayesian model.
2. **Bayesian Linear Regression:** which is a linear model that can be constructed using packages like “brms”, which helps with stan to perform posterior sampling througj No u Turn sampling (NUTS) and it is a variant of Hamiltonian Monte Carlo (HMC).

- **Model Formula**

  Math.score ~ gender + race.ethnicity + parental.education + lunch + test.preparation.course
- **Priors**:
  - Regression coefficients: Normal (0, 10) weakly informative priors
  - Residual standard deviation (sigma): Default priors used by brms
- **Sampling settings**:
  - 4 chains
  - 2,000 iterations per chain (with 500 warm-up)
  - Seed = 123 for reproducibility
- **Diagnostics**:

  Convergence was assessed through Rhat (target: 1.00), and effective sample size (ESS) metrics were used to ensure sampling reliability.

BAYESIAN PRIORS AND SAMPLING:

Priors

In Bayesian models we require prior distribution of parameters before observing the data. In this study, weakly informative priors were selected to allow the data for regularization.

- **Regression Coefficients (b)**:
  - A Normal (0, 10) prior was applied to all coefficients.
  - Before seeing the data, I assume most effects lie within ±30 points with high probability, which is appropriate for a score range of 0–100.
- **Intercept**:
  - For Intercept to take a wide range of values I used the default flat prior from brms.
- **Residual Standard Deviation (sigma)**:
  - The default prior was used, which is typically a half Student-T distribution in brms, allowing flexibility while avoiding extreme variance estimates.

These choices reflect the goal of applying minimal but meaningful regularization, especially in the absence of strong domain-specific priors.

Sampling Method

The posterior estimation in this model is taken from the Markov Chain Monte Carlo sampling through no u turn sampler, which is an adaptive form of Hamiltonian Monte Carlo. Where this no u tern sampler is efficient for high dimensional model, and it is a default from brms library.

- Chains: 4
- Multiple chains are used to make sure that the sampler explores the posterior distribution thoroughly from different starting points, which improves the reliability of convergence diagnostics.
- Iterations per chain: 2,000
- Warm-up (burn-in): 500
- Total post-warmup draws: 6,000
- Seed: 123 (for reproducibility)

Model convergence was confirmed by checking:

- Rhat ≈ 1.00 for all parameters
- High effective sample sizes (Bulk_ESS and Tail_ESS)
- Trace plots and posterior predictive checks, which showed stable sampling behavior and good model fit.

MODEL SPECIFICATION AND COMPARISON:

In this study I have used two linear modeling approaches, one is a traditional frequentist model and another Bayesian probabilistic model to find out the influencing factors on student mathematics score. Below I delve deeply into the structure, assumptions, and outputs of each model.

1. Frequentist Linear Regression

The frequentist model was implemented using the lm() function in R. It estimates coefficients by minimizing the sum of squared residuals, also assuming the residuals are normally distributed and independent.

Model Structure:

$$\textbf{Math Score}_i = \beta_o + \sum_{j=1}^{k} \beta_j X_{ij} + €_i$$

- β_o : Intercept
- β_j: Coefficients for predictor variables (gender, race, etc.)
- $€_i \sim N(0,\sigma^2)$: Residual errors

Assumptions:

- Linearity between predictors and outcome
- Homoscedasticity (constant variance of residuals)
- Normality of residuals
- No multicollinearity
- Independence of observations

Limitations:

- Produces only point estimates and standard errors
- Uncertainty is expressed via confidence intervals, which are less intuitive than Bayesian credible intervals
- Cannot incorporate prior beliefs or uncertainty in model parameters beyond the data

2. Bayesian Linear Regression

The Bayesian model, implemented via the brms package, builds on the same structural foundation but treats all unknown quantities as random variables with probability distributions.

Model Structure:

Same linear form:

$$\textbf{Math Score}_i \sim N(\mu_i, \sigma)$$
$$\mu_i = \beta_o + \sum_{j=1}^{k} \beta_j X_{ij} + €_i$$

But now:

- Each $\beta_j \sim N(0,10)$
- σ has a default half-Student-t prior

What Makes It Different?

- Priors represent beliefs before seeing data
- Posteriors combine priors + likelihood (data evidence)
- The result is a full distribution for each parameter (not just a point estimate)

Inference:

- Estimation done through **MCMC** using NUTS

- The No-U-Turn Sampler (NUTS) is an flexible variant of Hamiltonian Monte Carlo (HMC) for sampling from the posterior. It is the brms package default because it is very efficient, especially in high-dimensional parameter spaces. NUTS automatically adjusts the path length of the sampling trajectory to avoid unnecessary computations (i.e., making "U-turns") and to more efficiently explore the posterior landscape without manual tuning. This makes it more powerful and reliable than traditional MCMC methods.
- Outputs include:
    - Posterior mean
    - Standard deviation
    - 95% credible intervals (range where the true value lies with 95% probability)
- Enables posterior predictive checks and Bayesian R^2

MODEL SELECTION:

To determine the best model for the purpose of this study, the frequentist model and Bayesian model were compared. While the frequentist model itself was an excellent starting point, the Bayesian model was eventually used due to numerous reasons:

•It provided full posterior distributions of the parameters, thus enabling more revealing interpretation through credible intervals.

•There was more conformity with observed distributions of data following posterior predictive checks.

•Bayesian R^2 indicated the same model fit as frequentist R^2 but with more accurate uncertainty quantification.

•The model is open to future development in employing prior knowledge and model fitting to more complex structures.

Therefore, the Bayesian model was selected as the final model due to its enhanced interpretability, diagnostic ability, and decision capability under uncertainty.

Although this analysis largely employed diagnostic statistics and interpretability to choose the model, other formal Bayesian model comparison statistics such as WAIC (Widely Applicable Information Criterion) or LOO-CV (Leave-One-Out Cross-Validation) could also assist in model selection. These statistics quantify predictive accuracy while penalizing model complexity and are particularly helpful when one needs to compare several Bayesian models.

RESULTS:

Frequentist Linear Regression Results:
The frequentist model, which is traditional regression model OLS, showed the following key effects:

- Gender: Male students scored higher than females by 5 points.
- Race/Ethnicity: Students from Group D (+5.34) and Group E (+10.13) scored significantly higher than those in Group A, while Groups B and C were not significantly different.
- Parental Education: Students whose parents had only a high school or some high school education scored 4 to 5 points lower than those with an associate degree.
- Lunch Type: students who received a standard lunch got an 11-point increase in math score.
- Test Preparation: Students who did not complete a preparation course scored about 5.5 points lower than those who did.
- R-squared: 0.25, indicating that the model explains 25% of the variation in math scores.

```
Call:
lm(formula = math.score ~ gender + race.ethnicity + parental.level.of.education +
    lunch + test.preparation.course, data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-50.357  -8.744   0.166   9.001  30.655

Coefficients:
                                            Estimate Std. Error t value
(Intercept)                                  57.6305     1.8721  30.784
gendermale                                    4.9953     0.8390   5.954
race.ethnicitygroup B                         2.0408     1.6998   1.201
race.ethnicitygroup C                         2.4700     1.5918   1.552
race.ethnicitygroup D                         5.3410     1.6241   3.289
race.ethnicitygroup E                        10.1347     1.8015   5.626
parental.level.of.educationbachelor's degree   1.9661     1.5020   1.309
parental.level.of.educationhigh school       -4.8027     1.2971  -3.703
parental.level.of.educationmaster's degree     2.8884     1.9382   1.490
parental.level.of.educationsome college      -0.5827     1.2470  -0.467
parental.level.of.educationsome high school  -4.2487     1.3331  -3.187
lunchstandard                                10.8768     0.8727  12.463
test.preparation.coursenone                  -5.4947     0.8756  -6.275
                                            Pr(>|t|)
(Intercept)                                  < 2e-16 ***
gendermale                                  3.63e-09 ***
race.ethnicitygroup B                       0.230181
race.ethnicitygroup C                       0.121060
race.ethnicitygroup D                       0.001042 **
race.ethnicitygroup E                       2.41e-08 ***
parental.level.of.educationbachelor's degree 0.190831
parental.level.of.educationhigh school      0.000225 ***
parental.level.of.educationmaster's degree  0.136490
parental.level.of.educationsome college     0.640431
parental.level.of.educationsome high school 0.001482 **
lunchstandard                                < 2e-16 ***
test.preparation.coursenone                 5.22e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.17 on 987 degrees of freedom
Multiple R-squared:  0.2548,    Adjusted R-squared:  0.2457
F-statistic: 28.12 on 12 and 987 DF,  p-value: < 2.2e-16
```

Bayesian Linear Regression Results:

The results from the Bayesian model were consistent in direction and magnitude with the frequentist model but it gave a better insight through full posterior distributions and credible intervals.

Key Posterior Estimates and 95% Credible Intervals

```
 Family: gaussian
  Links: mu = identity; sigma = identity
Formula: math.score ~ gender + race.ethnicity + parental.level.of.education + lunch + test.preparation.course
   Data: data (Number of observations: 1000)
  Draws: 4 chains, each with iter = 2000; warmup = 500; thin = 1;
         total post-warmup draws = 6000

Regression Coefficients:
                                           Estimate Est.Error l-95% CI u-95% CI
Intercept                                     57.98      1.84    54.41    61.55
gendermale                                     4.95      0.84     3.35     6.57
race.ethnicitygroupB                           1.69      1.62    -1.57     4.82
race.ethnicitygroupC                           2.12      1.54    -0.97     5.10
race.ethnicitygroupD                           4.96      1.56     1.84     7.89
race.ethnicitygroupE                           9.69      1.72     6.29    13.05
parental.level.of.educationbachelorsdegree     1.97      1.46    -0.91     4.88
parental.level.of.educationhighschool         -4.73      1.28    -7.26    -2.18
parental.level.of.educationmastersdegree       2.85      1.89    -0.82     6.55
parental.level.of.educationsomecollege        -0.52      1.24    -2.97     1.95
parental.level.of.educationsomehighschool     -4.17      1.30    -6.79    -1.66
lunchstandard                                 10.81      0.87     9.08    12.49
test.preparation.coursenone                   -5.46      0.89    -7.19    -3.73
                                           Rhat Bulk_ESS Tail_ESS
Intercept                                  1.00     3350     3979
gendermale                                 1.00     8464     4952
race.ethnicitygroupB                       1.00     3524     4581
race.ethnicitygroupC                       1.00     3357     3980
race.ethnicitygroupD                       1.00     3424     3984
race.ethnicitygroupE                       1.00     3709     4256
parental.level.of.educationbachelorsdegree 1.00     4757     4889
parental.level.of.educationhighschool      1.00     4484     4929
parental.level.of.educationmastersdegree   1.00     6055     4397
parental.level.of.educationsomecollege     1.00     4484     4682
parental.level.of.educationsomehighschool  1.00     4688     4954
lunchstandard                              1.00     6415     4736
test.preparation.coursenone                1.00     7462     4466

Further Distributional Parameters:
      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sigma    13.17      0.29    12.61    13.77 1.00     7857     4260

Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
and Tail_ESS are effective sample size measures, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
```

Predictor	Estimate	95% CI	Interpretation
Gender-male	4.95	(3.35, 6.57)	Males score higher
race.group-D	4.96	(1.84, 7.89)	Significant positive
race.group-E	9.69	(6.29, 13.05)	Strong effect
Lunch-standard	10.81	(9.08, 12.49)	High certainty & large effect
test prep: none	-5.46	(-7.19, -3.73)	Clear negative impact
low parental education	-4.7	Credible intervals below 0	Negative effect

Model Fit and Diagnostics:

- All Rhat values are 1.00, indicating excellent convergence.
- Effective sample sizes were large for all parameters.
- Posterior predictive checks showed close alignment between predicted and observed distributions.
- Bayesian R^2 was comparable to the frequentist R^2, confirming there is similar model fit.

Summary of Effects

- From the results I found that lunch type and test preparation are the strongest and most reliable predictors for the math performance. Where Race/ethnicity shows meaningful effects for groups D&E. Bayesian model added value in finding the probabilities by explicitly quantifying uncertainty in positive or negative effect.

DISCUSSION AND INTERPRETATION:

The results from frequentist and Bayesian linear regression models provide evidence that demographic and behavioral factors significantly influence student performance in their mathematics score. By using Bayesian inference, I was able not only to estimate the effects of these predictors but also to quantify the uncertainty and confidence around those estimates, offering deeper insights than traditional approaches.

1. Impact of Race/Ethnicity

The models show that students from Race Groups D and E consistently score higher than those in Group A, where Group E outperformed by nearly 10 points. From the specific social or cultural factors that were not captured in the dataset, these findings would have reflected group-based results in access to resources, community support, or educational quality. On the other hand, Race B and C showed effects with wide credible intervals that included zero, which suggested no impact on math scores for these groups under the current model.

Interpretation: Race/ethnicity, which are a proxy for broader social context, this may influence student outcomes such as neighborhood effects, bias in education, or differing expectations across groups.

2. Influence of Lunch Type

One of the consistent predictors of math performance was the type of lunch received. Students who received standard lunch scored about 11 points higher, with very narrow credible intervals, indicating high confidence in this area.

Interpretation: Lunch type is the most common factor for socioeconomic status. The results which I received from this factor show that students who have standard lunch type have provided better nutrition to focus on their studies to gain better scores.

3. Role of Test Preparation Courses

Completion of a test preparation course shows result with a 5.5-point increase in math scores, again with high certainty like other predictor variables. This showcase about the value of structured, focused academic preparation and even modest interventions can bring measurable improvements.

Interpretation: preparation is the most common factor that can tell most of the results that we are expected in this study. This factor is main in bringing better insights and interventions in preparation programs for underperforming schools or institutions.

4. Parental Education and Gender

However, this factor is not the focus for this analysis, while gender and parental education level also emerged as meaningful predictors. Male students scored slightly higher, and for students whose parents had lower educational background showed score lower.

Interpretation: These results will help in broader research on gender dynamics for STEM education to bring better academic outcomes.

Bayesian Modeling Advantages

The use of Bayesian inference added considerable value by providing credible intervals that directly show the probability of parameter values, by allowing uncertainty in transparent communication and supporting decisions from probabilistic reasoning, not just by the binary significance tests

This approach is especially beneficial in educational policy, where understanding about an intervention's effect can guide resource allocation, equity measures, and strategic planning.

CONCLUSION:

In this study I applied both frequentist and Bayesian linear regression techniques to analyze the factors that effecting student performance in their mathematics score. Using a dataset of 1,000 students, I analyzed the impacts of demographic attributes and behavioral factors on math scores.

Both modeling approaches identified lunch type, test preparation completion, and some race/ethnicity groups (D and E) as significant predictors for math scores. Among these, socioeconomic status, proxied by lunch type, showed effect in underscoring the critical role of economic resources in academic outcomes of students.

The Bayesian framework provided advantages by quantifying uncertainty through full posterior distributions and credible intervals, instead of depending on p-values and point estimations. This probabilistic perspective provided a richer understanding of each predictor's impact.

In conclusion, the results highlighted that there is a need to address socioeconomics to support academic preparation to improve educational outcomes. Future research could expand this analysis by incorporating additional behavioral, environmental, and institutional factors, and by exploring causal relationships for better outcomes.

RECOMMENDATIONS FOR FUTURE WORK:

While this project effectively demonstrated the advantage of Bayesian modeling for student mathematics performance prediction, parts of it can be improved or extended:

1. 1.Extension to Other Scores: The next models could include reading and writing scores as additional response variables, possibly in a multivariate setting, to mirror subject interactions.
2. 2.Hierarchical Modeling: The addition of multilevel models (e.g., students in schools or areas) would allow finer analysis and the inclusion of group-level factors.

3. 3.Extra Predictors: Factors such as school spending, class size, attendance, or family income might improve model explanatory power and capture more socioeconomic effects.
4. 4.Informative Priors: In cases where expert opinion or historical data are available, informative priors could increase prediction accuracy and validity, particularly when there are small data sets.
5. 5.Metrics for Model Comparison: Employing information criteria such as WAIC or LOO-CV would enable more formal model comparison across competing Bayesian models.
6. 6.Interactive Visualizations: Adding interactive dashboards or Shiny apps to the output would enable findings to become more easily communicable to policy-makers and teachers.
7. By achieving these developments, future work can identify more clearly the determining factors of student performance and inform educational policy more effectively.

Limitations of the Study

- While this study highlights the determinants of student mathematics performance applying Bayesian linear regression, some limitations should be highlighted:
- Limited Predictors
  There are few demographic and behavior variables included in the dataset. Some important variables like school quality, teaching performance, parental income, or psychological variables were not measured, which would exert a strong influence on performance.
- Assumption of Linearity
  Linear models have linear relationships between predictors and outcomes. Non-linear interactions or effects among variables might be not fully modeled.
- No Causal Inference
  The research is observational in nature and does not enforce causality. The observed relations (e.g., test scores and lunch type) could not be presumed as causal effects without another experimental or quasi-experimental design.
- Priors Were Weakly Informative
  Even though weak priors regularize estimates, they do not automatically utilize available expert data, or prior research to the best degree. More informative priors would be useful in the attempt to improve model performance.
- No Out-of-Sample Validation
  The model's prediction performance was not tested and validated on an independent test set or cross-validation, which precludes generalizability testing.
- Lack of Model Comparison Using Information Criteria
  WAIC or LOO-CV were not used to compare models in this explicit way, though they are standard utilities in Bayesian model checking.

APPINDEX:

Data set

```
data = read.csv("/content/StudentsPerformance.csv")
head(data)
```

A data.frame: 6 × 8

	gender	race.ethnicity	parental.level.of.education	lunch	test.preparation.course	math.score	reading.score	writing.score
	<chr>	<chr>	<chr>	<chr>	<chr>	<int>	<int>	<int>
1	female	group B	bachelor's degree	standard	none	72	72	74
2	female	group C	some college	standard	completed	69	90	88
3	female	group B	master's degree	standard	none	90	95	93
4	male	group A	associate's degree	free/reduced	none	47	57	44
5	male	group C	some college	standard	none	76	78	75
6	female	group B	associate's degree	standard	none	71	83	78

```
str(data)
```

```
'data.frame':   1000 obs. of  8 variables:
 $ gender                     : chr  "female" "female" "female" "male" ...
 $ race.ethnicity             : chr  "group B" "group C" "group B" "group A" ...
 $ parental.level.of.education: chr  "bachelor's degree" "some college" "master's degree" "associate's degree" ...
 $ lunch                      : chr  "standard" "standard" "standard" "free/reduced" ...
 $ test.preparation.course    : chr  "none" "completed" "none" "none" ...
 $ math.score                 : int  72 69 90 47 76 71 88 40 64 38 ...
 $ reading.score              : int  72 90 95 57 78 83 95 43 64 60 ...
 $ writing.score              : int  74 88 93 44 75 78 92 39 67 50 ...
```

Summary

```
summary(data)
```

```
    gender          race.ethnicity     parental.level.of.education
 Length:1000        Length:1000        Length:1000
 Class :character   Class :character   Class :character
 Mode  :character   Mode  :character   Mode  :character



    lunch           test.preparation.course   math.score     reading.score
 Length:1000        Length:1000             Min.   :  0.00   Min.   : 17.00
 Class :character   Class :character        1st Qu.: 57.00   1st Qu.: 59.00
 Mode  :character   Mode  :character        Median : 66.00   Median : 70.00
                                            Mean   : 66.09   Mean   : 69.17
                                            3rd Qu.: 77.00   3rd Qu.: 79.00
                                            Max.   :100.00   Max.   :100.00
 writing.score
 Min.   : 10.00
 1st Qu.: 57.75
 Median : 69.00
 Mean   : 68.05
 3rd Qu.: 79.00
 Max.   :100.00
```

Converting character categorical into numerical categorical

```
data$'gender' = as.factor(data$'gender')
data$'race.ethnicity' = as.factor(data$'race.ethnicity')
data$'parental.level.of.education' <- as.factor(data$'parental.level.of.education')
data$'lunch' <- as.factor(data$'lunch')
data$'test.preparation.course' <- as.factor(data$'test.preparation.course')
```

Frequentist linear regression

```
# model fir
lm_model <- lm(math.score ~ gender + race.ethnicity + parental.level.of.education +
                  lunch + test.preparation.course, data)
# summary
summary(lm_model)
```

```
Call:
lm(formula = math.score ~ gender + race.ethnicity + parental.level.of.education +
    lunch + test.preparation.course, data = data)

Residuals:
    Min      1Q  Median      3Q     Max 
-50.357  -8.744   0.166   9.001  30.655 

Coefficients:
                                             Estimate Std. Error t value
(Intercept)                                   57.6305     1.8721  30.784
gendermale                                     4.9953     0.8390   5.954
race.ethnicitygroup B                          2.0408     1.6998   1.201
race.ethnicitygroup C                          2.4700     1.5918   1.552
race.ethnicitygroup D                          5.3410     1.6241   3.289
race.ethnicitygroup E                         10.1347     1.8015   5.626
parental.level.of.educationbachelor's degree   1.9661     1.5020   1.309
parental.level.of.educationhigh school        -4.8027     1.2971  -3.703
parental.level.of.educationmaster's degree     2.8884     1.9382   1.490
parental.level.of.educationsome college       -0.5827     1.2470  -0.467
parental.level.of.educationsome high school   -4.2487     1.3331  -3.187
lunchstandard                                 10.8768     0.8727  12.463
test.preparation.coursenone                   -5.4947     0.8756  -6.275
                                             Pr(>|t|)    
(Intercept)                                   < 2e-16 ***
gendermale                                   3.63e-09 ***
race.ethnicitygroup B                        0.230181    
race.ethnicitygroup C                        0.121060    
race.ethnicitygroup D                        0.001042 ** 
race.ethnicitygroup E                        2.41e-08 ***
parental.level.of.educationbachelor's degree 0.190831    
parental.level.of.educationhigh school       0.000225 ***
parental.level.of.educationmaster's degree   0.136490    
parental.level.of.educationsome college      0.640431    
parental.level.of.educationsome high school  0.001482 ** 
lunchstandard                                 < 2e-16 ***
test.preparation.coursenone                  5.22e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.17 on 987 degrees of freedom
Multiple R-squared:  0.2548,    Adjusted R-squared:  0.2457 
F-statistic: 28.12 on 12 and 987 DF,  p-value: < 2.2e-16
```

Bayesian Linear Regression

```r
library(brms)
bayes_model <- brm(
  formula = math.score ~ gender + race.ethnicity + parental.level.of.education +
              lunch + test.preparation.course,
  data = data,
  family = gaussian(),
  prior = set_prior("normal(0, 10)", class = "b"),
  chains = 4,
  iter = 2000,
  warmup = 500,
  cores = 4,
  seed = 123
)
summary(bayes_model)
```

```
 Family: gaussian
  Links: mu = identity; sigma = identity
Formula: math.score ~ gender + race.ethnicity + parental.level.of.education + lunch + test.preparation.course
   Data: data (Number of observations: 1000)
  Draws: 4 chains, each with iter = 2000; warmup = 500; thin = 1;
         total post-warmup draws = 6000

Regression Coefficients:
                                           Estimate Est.Error l-95% CI u-95% CI
Intercept                                     57.98      1.84    54.41    61.55
gendermale                                     4.95      0.84     3.35     6.57
race.ethnicitygroupB                           1.69      1.62    -1.57     4.82
race.ethnicitygroupC                           2.12      1.54    -0.97     5.10
race.ethnicitygroupD                           4.96      1.56     1.84     7.89
race.ethnicitygroupE                           9.69      1.72     6.29    13.05
parental.level.of.educationbachelorsdegree     1.97      1.46    -0.91     4.88
parental.level.of.educationhighschool         -4.73      1.28    -7.26    -2.18
parental.level.of.educationmastersdegree       2.85      1.89    -0.82     6.55
parental.level.of.educationsomecollege        -0.52      1.24    -2.97     1.95
parental.level.of.educationsomehighschool     -4.17      1.30    -6.79    -1.66
lunchstandard                                 10.81      0.87     9.08    12.49
test.preparation.coursenone                   -5.46      0.89    -7.19    -3.73
                                           Rhat Bulk_ESS Tail_ESS
Intercept                                  1.00     3350     3979
gendermale                                 1.00     8464     4952
race.ethnicitygroupB                       1.00     3524     4581
race.ethnicitygroupC                       1.00     3357     3980
race.ethnicitygroupD                       1.00     3424     3984
race.ethnicitygroupE                       1.00     3709     4256
parental.level.of.educationbachelorsdegree 1.00     4757     4889
parental.level.of.educationhighschool      1.00     4484     4929
parental.level.of.educationmastersdegree   1.00     6055     4397
parental.level.of.educationsomecollege     1.00     4484     4682
parental.level.of.educationsomehighschool  1.00     4688     4954
lunchstandard                              1.00     6415     4736
test.preparation.coursenone                1.00     7462     4466

Further Distributional Parameters:
      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sigma    13.17      0.29    12.61    13.77 1.00     7857     4260

Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
and Tail_ESS are effective sample size measures, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
```