National College of Ireland

BSc (Honours) in Computing – Full-time – Year 1 – BSHC 1
BSc (Honours) in Computing – Part-time – Year 1 – BSHCE 1
BSc (Honours) in Business Information Studies – Full-time – Year 1 – BSHBIS 1
BSc (Honours) in Business Information Studies – Part-time – Year 1 – BSHBISE 1
BSc (Honours) in Technology Management – Full-time – Year 1 – BSHTM 1
Higher Certificate in Computing – Full-time – Year 1 – HCC 1
Higher Certificate in Computing – Part-time – Year 1 – HCCE 1
Certificate in Computing – Part-time – Year 1 – CIC 1

Semester One Examinations – 2015/16

Friday 8th January 2016
2.00pm – 4.00pm

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Introduction to Mathematics for Computing

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Full marks will be awarded for complete answers to **FIVE** questions, i.e., **answer all questions**.

Duration of exam: 2 hrs
Attachments: NCI Formulae Booklet

1. **Sets**

    a) Let $A = \{a, b, c\}$, $B = \{e, f\}$ and $C = \{a, c, f\}$.

        (i)   Write down the elements of the set $A \cup C$.
        (ii)   Write down the elements of the set $(B \cap C) \cup C$.
        (iii)   Write down the elements of the set $C \, \Delta \, B$.

        [3 x 1 mark]

        (iv)   Write down the elements of $(A \times B) \cap (C \times B)$.

        [2 marks]

    b) Let $A = \{2x \mid x \in \mathbb{Z} \text{ and } -2 < x < 4\}$ and
        $B = \{ x \mid x \in \mathbb{N} \text{ and } 4 \leq x < 8 \}$.

        (i)   Write the set A using roster notation.
        (ii)   Write the set B using roster notation.
        (iii)   Is $B = A$? Explain your answer.
        (iv)   Is $B \subseteq A$? Explain your answer.
        (v)   Is $B \cap A = \varnothing$? Explain your answer.

        [5 x 1 mark]

    c) In a survey of 100 college students, the following data were obtained:

        75 were members of the Mathematics Society
        52 were members of the Computing Society
        55 were members of the Food & Drink Society
        48 were members of both the Mathematics Society and the Computing Society
        42 were members of both the Mathematics Society and the Food & Drink Society
        42 were members of both the Computing Society and the Food & Drink Society
        10 were not members of either the Mathematics Society or the Computing Society or
           the Food & Drink Society.

        (i)   Represent the above situation on a Venn Diagram.

        [5 marks]

        (ii)   How many of the students surveyed were members of the Mathematics Society and
                the Computing Society and the Food & Drink Society?

        [5 marks]

2. Relations and Functions

a) Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be two functions defined by
$f(x) = x^2 - 1$ and
$g(x) = x - 1$ where $x \in \mathbb{R}$.

   (i)   Show that $f(1) + f(-1) = g(1)$.
   (ii) Is f a surjective function? Explain your answer.
   (iii) Calculate $g \circ f(x)$.
   (iv) Calculate $f(g(x))$.

[4 x 2 marks]

b) Let $A = \{1, 2, 3\}$. Let R and S be relations on A defined by
$R = \{(x, y) \mid x, y \in A \text{ and } x < y\}$ and
$S = \{(x, y) \mid x, y \in A \text{ and } x > y\}$

   (i)   Compute the composite relation $S \circ R$.

[5 mark]

c) Let $A = \{a, b, c, d\}$ and let R be a relation on A with

$R = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c), (c, b), (d, d)\}$.

   (i)   Draw a digraph representation of the relation R.

[2 marks]

   (ii) Determine whether R is a symmetric relation.

[2 marks]

   (iii) Determine whether R is a transitive relation.

[2 marks]

   (iv) Determine whether R is a reflexive relation.

[1 mark]

3. Logic

a) Construct the truth tables for the following logical expressions:

(i) $(p \lor \overline{p}) \land (q \land \overline{q})$

[3 marks]

(ii) $(p \land \overline{p}) \lor (q \land \overline{q})$

[3 marks]

Decide whether or not the expressions in (i) and (ii) are equivalent, giving reasons for your answer.

[2 marks]

b) Consider the following quantified predicates:

$P(n):$ $\quad\forall\, n \in \mathbb{Z}, 2n \in \mathbb{Z}$

$Q(m,n):$ $\quad\exists\, m, n \in \mathbb{Z}, mn = 0$

(i) Explain why each of the quantified predicates $P(n)$ and $Q(m,n)$ is true.

[4 marks]

(ii) Write the negation of $P(n)$ using logical notation.

[4 marks]

(iii) Write the negation of $Q(m,n)$ using logical notation.

[4 marks]

4. **Probability and Statistics**
   a) Suppose 4 cards are to be drawn randomly (without replacement) from a standard pack of 52 playing cards. What is the probability that the 4 aces are drawn?

   [4 marks]

   b) The heights of all students at a college was recorded. It was found that 95% of the students are between 1.2 metres and 1.8 metres tall. Assuming that the recorded data is normally distributed, calculate
      (i) The mean height of the students
      (ii) The standard deviation of the distribution

   [2 x 3 marks]

   c) Over the course of a 20 day period, a lecturer records the number of cups of coffee that he consumes per day. The results are presented in the following frequency distribution:

Cups of Coffee Consumed	Number of Days
0	1
1	1
2	3
3	5
4	8
5	2

   Calculate

   (i) The standard deviation of the distribution.

   [10 marks]

5. **Counting**
   a) How many 9 digit telephone numbers can be formed if each number starts with **087** and no digit appears more than once?

   [3 marks]

   b) How many different ways are there to arrange the letters from the word **MATHEMATICS** such that the vowels must always come together?

   [7 marks]

   c) How many three digit numbers greater than 100 can be formed with the digits 3, 4, 5, 0, 6, 7 if repetition of digits is not allowed?

   [5 marks]

   d) A box contains 8 balloons out of which 3 are red and 5 are blue.
   In how many ways can 6 balloons be randomly chosen from the box so that there are at the most 2 red balloons?

   [5 marks]