



# Microcontroller Techniques

Winter Term 2016/2017

Prof. Dr. L. M. Reindl

Laboratory for Electrical Instrumentation  
Department of Microsystems Engineering – IMTEK  
University of Freiburg

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UNI  
FREIBURG

# Your instructor



## Prof. Dr. Leonhard Reindl

- 1985 Dipl.-Phys. at TU München
- 1985 cooperate research and technology (Siemens AG)
- 1997 PhD at TU Wien
- 1999 lecturer at TU Clausthal
- since 2003: professor and head of the Laboratory for Electrical Instrumentation at IMTEK, University of Freiburg



# Contact Data

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There are no set call times! Please make an appointment with the secretary (0761-203-7220) or write an e-mail.

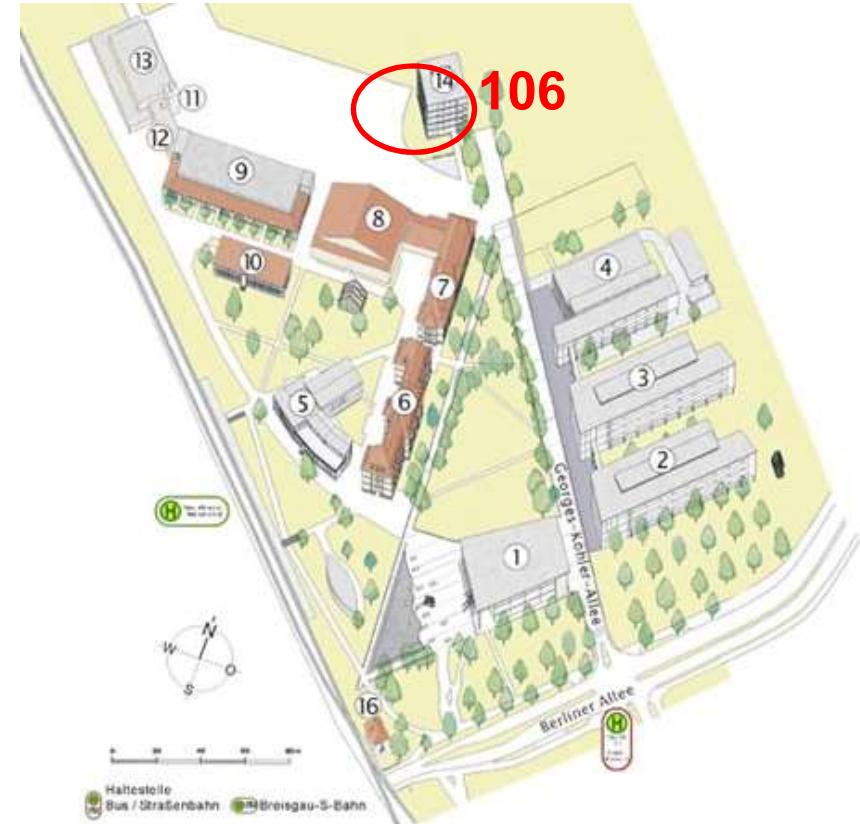
## Sebastian Stöcklin

contact person for lab course & exam

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office space: building 106, room 04-004



# ORGANISATIONAL

# Learning content and scope of the lecture



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The lecture and the practical course are just a teaser content of the field of microcontroller technology! It is not possible to treat these contents in full depth in one semester. Therefore, it is strongly recommended to rework the given topics with a textbook to obtain a deeper understanding.

# Overview of the course

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## ■ Lecture

- contents: theoretical basis & material collection about microcomputer theory
- approx. 10 lectures (participation and rework recommended)
- exam tutorial session(date & room: will be announced)
- final exam - date & room: will be announced

## ■ Practical exercises

- contents: hands-on experience with microcontrollers (MSP430) & C programming
- exercises will be performed at home, hardware available from the library of the technical faculty
- 8 out of 9 exercise sheets must be passed with more than 50% of the points
- introduction course on 26<sup>th</sup> of October (see e-mail)

# Objectives of the course

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- Basic understanding of modern data processing systems
  - digital logic
  - binary arithmetics
- Structure of microcontrollers (hardware)
  - microprocessor
  - peripherals
- Programming of microcontrollers controller (software)
  - (assembler language)
  - C
- Evaluation and selection of microprocessors
- Practical skills in systems programming in the laboratory course

# Exam

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## Permitted aids

- Non-programmable calculator
- C command summary for programming task (provided by us)

## The nature and extent of the exam

- approx. 7 to 10 tasks, 120 minutes of time.
- both theoretical tasks (based on the knowledge imparted in the lecture) and practical tasks (based on the exercises)

## What's relevant?

- Basically, all contents of the lecture and the practical exercise are relevant!

## Forums, training portal, etc.

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- ILIAS: [ilias.uni-freiburg.de](http://ilias.uni-freiburg.de)  
Course „Microcontroller Techniques“
  - Password „Lab20162017“
- Slides of the lecture, materials for the practical course, submission of exercise sheets, etc.
- Registration until 31<sup>th</sup> of October, 2016

## Additional literature

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- John Davies, „MSP430 Microcontroller Basics“, Springer, 2008.
- Matthias Sturm, „Mikrocontrollertechnik: Am Beispiel der MSP430-Familie“, Hanser, 2011.
- Marian Walter, Stefan Tappertzhofen, „Das MSP430 Mikrocontroller Buch“, Elektor-Verlag Aachen, 2011.
- Th. Flik., „Mikroprozessortechnik (RISC, CISC, Systemaufbau, Assembler und C)“, Springer, 1998.
- Beierlein Th., Hagenbruch O.: „Taschenbuch Mikroprozessortechnik“, Fachbuchverlag Leipzig, 2004.

## Introduction

# OVERVIEW OF THE LECTURE

# Structure of the lecture



- 
- Introduction
  - Chapter 0: basic knowledge
    - number systems, digital logic, ...
  - Chapter 1: calculation and control systems
    - basic logic operations in CMOS, registers, memory, concepts, ...
  - Chapter 2: Basic peripherals
    - GPIOs, ADCs, DACs, ...

# Structure of the lecture



- 
- Chapter 3: Advanced Peripherals
    - watchdog, programming interfaces, ...
  - Chapter 4: Programming
    - assembler language, C, compilers, ...
  - Chapter 5: Communication concepts
    - OSI model, topologies, UART, I2C, ...
  - Chapter 6: Outlook (among others: safe systems)

**Introduction**

# **MOTIVATION**

## Why do we need microcomputers and -controllers?

These days, it is hardly conceivable to have equipment and processes without computer assistance. Microsystems technology provides sensors and actuators which are most useful with the help of computers, being integrated into the system to control and regulate a process.

# Motivation

**Forecasts characterized by terms such as**

- Post-PC era
- Ubiquitous computing
- Pervasive computing
- Ambient intelligence
- Industry 4.0



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# Embedded Systems



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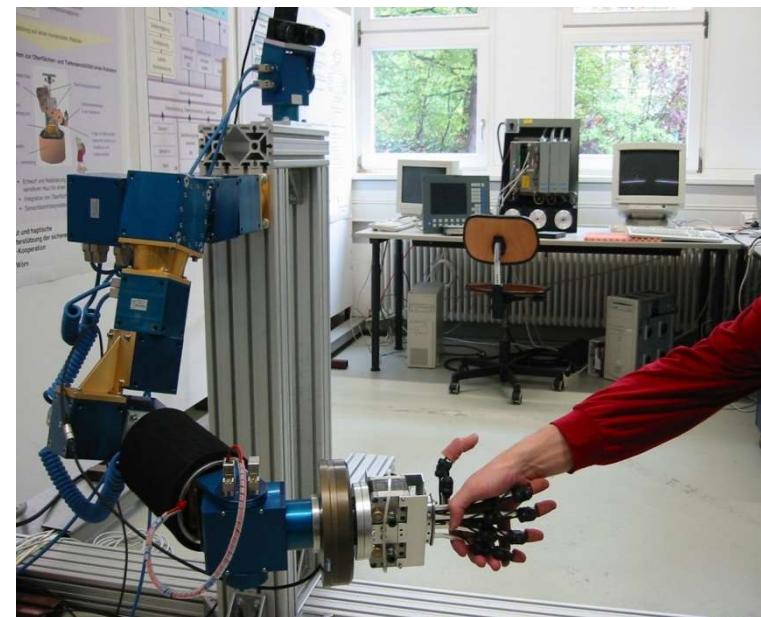
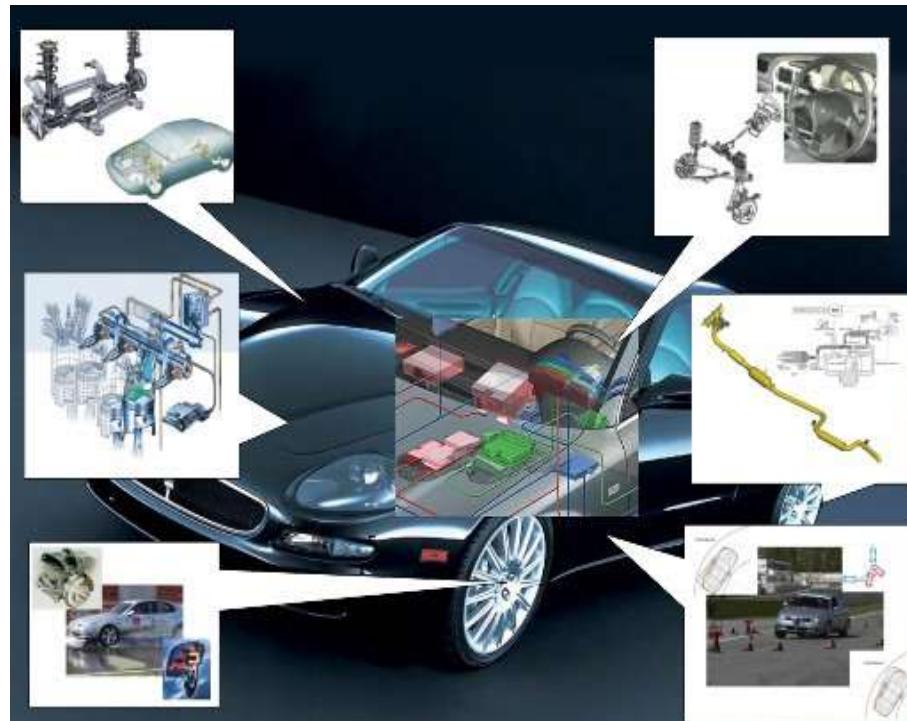
... are defined as information processing systems that

- are directly integrated into a surrounding product
- are controlling and monitoring this environment
- are adapted on the task, among other things in terms of
  - performance
  - power consumption
  - costs

# Examples of embedded systems



# Examples of embedded systems



# Examples of embedded systems



# Examples of embedded systems

## Curiosity

- 256 MB DRAM
- 2 GB flash memory (to recognize errors and do trouble shooting)
- 256 KB EEPROM
- RAD750 CPU
  - Up to 200 MHz



# Introduction

# HISTORY

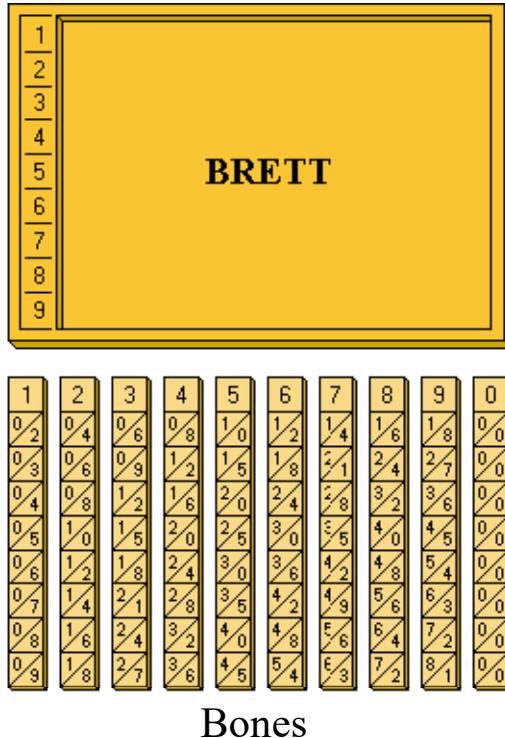
# Basics



Adam Riese's "Auf den Linien", which is equivalent to the Abacus

# Basics - Napier's bones (1617)

$$\begin{aligned}
 7 \times 1 &= 7 \\
 7 \times 2 &= 14 \\
 7 \times 3 &= 21 \\
 7 \times 4 &= 28 \\
 7 \times 5 &= 35 \\
 7 \times 6 &= 42 \\
 7 \times 7 &= 49 \\
 7 \times 8 &= 56 \\
 7 \times 9 &= 63
 \end{aligned}$$



1	4	6	7	8	5	3	9	9
2	0	8	1	2	1	4	1	6
3	1	2	1	8	2	1	4	5
4	1	6	2	4	2	8	3	2
5	2	0	3	0	5	4	0	2
6	2	4	3	6	4	2	9	0
7	2	8	4	2	9	5	6	3
8	3	2	4	5	6	4	0	2
9	3	6	5	4	6	3	7	1

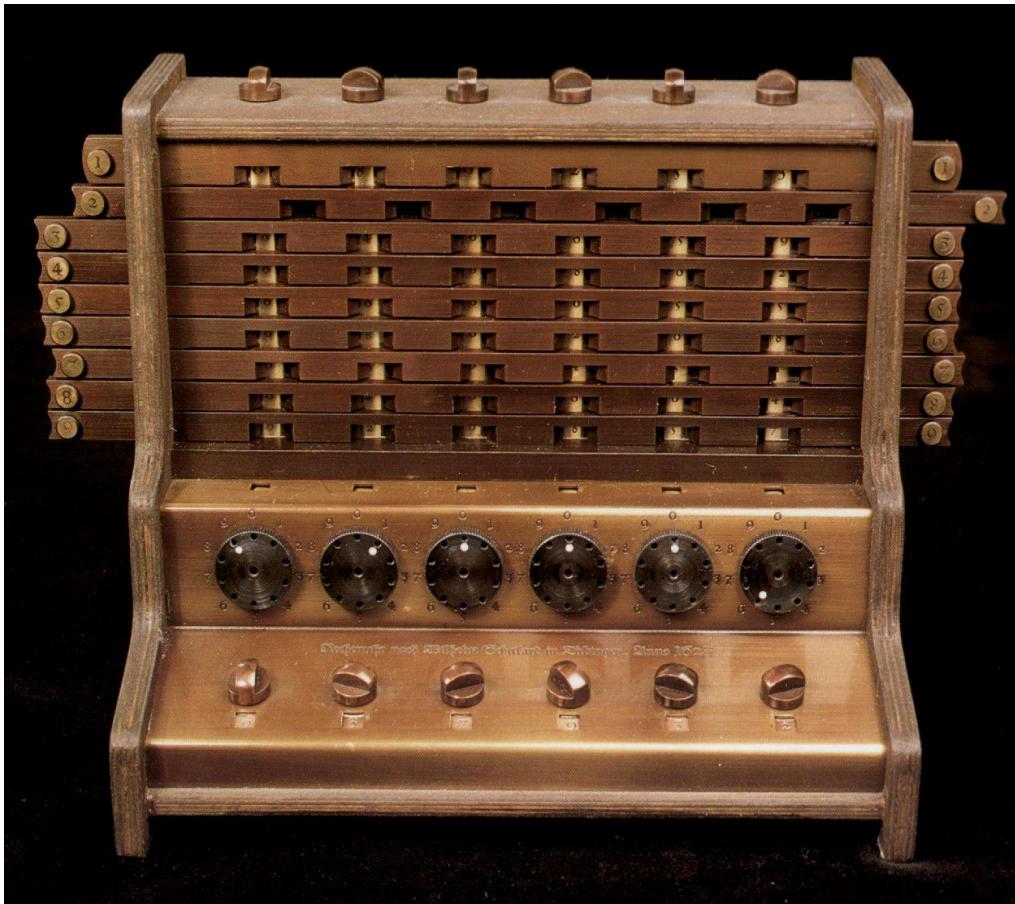
3    2    7    4    9    7    7    9    3

Computation:  $7 \times 46785399$

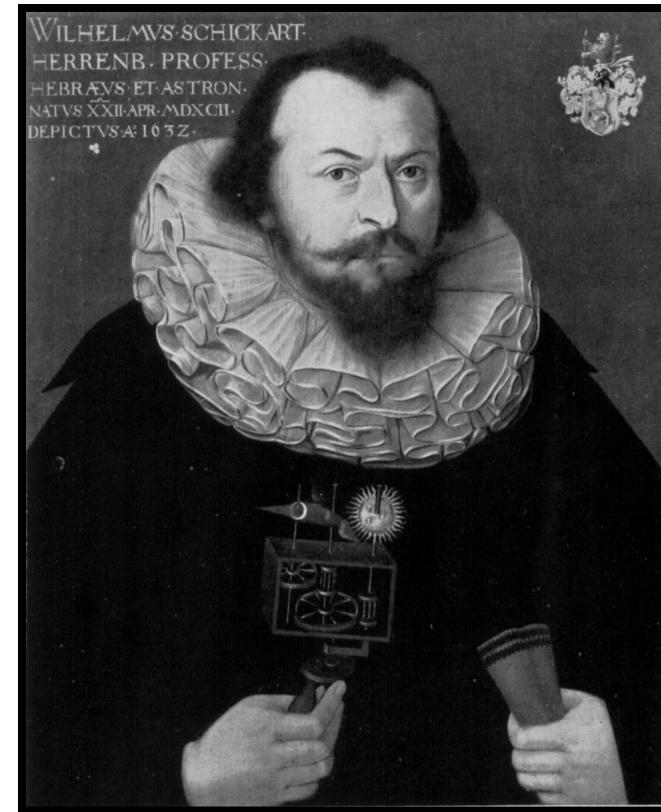
1	4	6	7	8	5	3	9	9
2	0	8	1	2	1	4	1	6
3	1	2	1	8	2	1	4	5
4	1	6	2	4	2	8	3	2
5	2	0	3	0	5	4	0	2
6	2	4	3	6	4	2	9	0
7	2	8	4	2	9	5	6	3
8	3	2	4	5	6	4	0	2
9	3	6	5	4	6	3	7	1

$$\begin{array}{r}
 46785399 \\
 \times 96431 \\
 \hline
 46785399 \\
 140356197 \\
 187141596 \\
 280712394 \\
 +421068591 \\
 \hline
 4511562810969
 \end{array}$$

# Basics - calculating clock

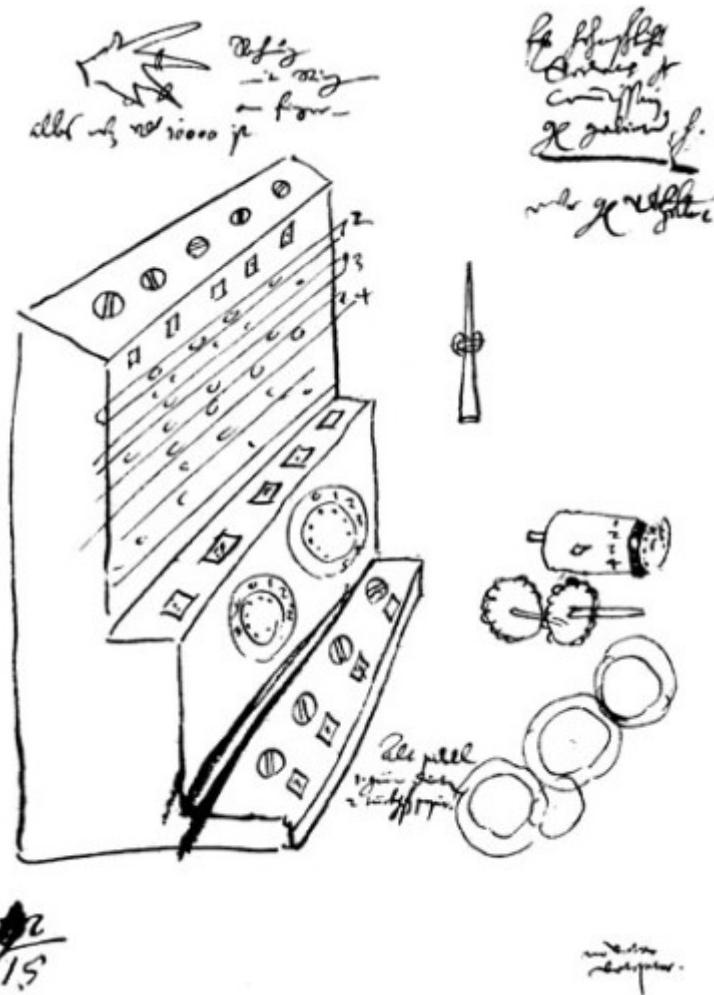


Replica of "calculating clock" of Schickard



Wilhelm Schickard 1592 - 1635

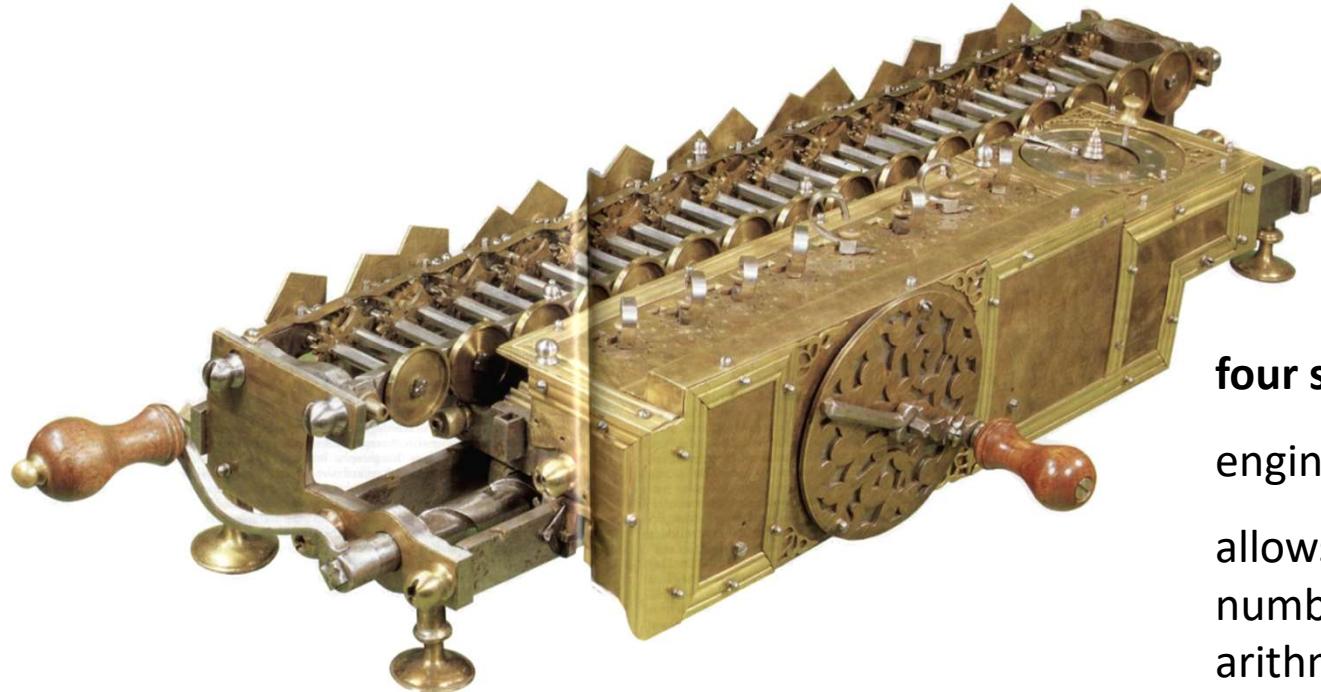
## Basics - calculating clock



# Original drawing of "calculating clock" of Schickard

- Allows summation or subtraction of numbers with up to six digits
  - "Buffer overflow" indicated by ringing of a bell
  - Napier's Bones for complex calculations

# Basics - Four species Abacus



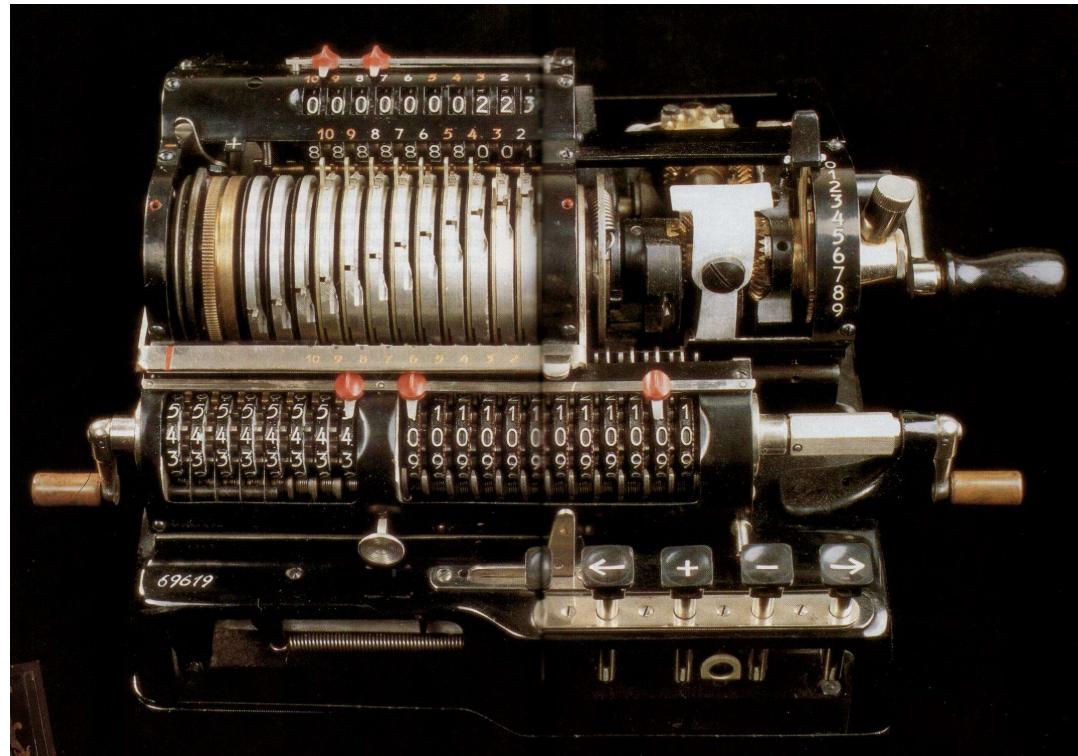
## **four species Abacus**

engineered by Leibniz in 1673

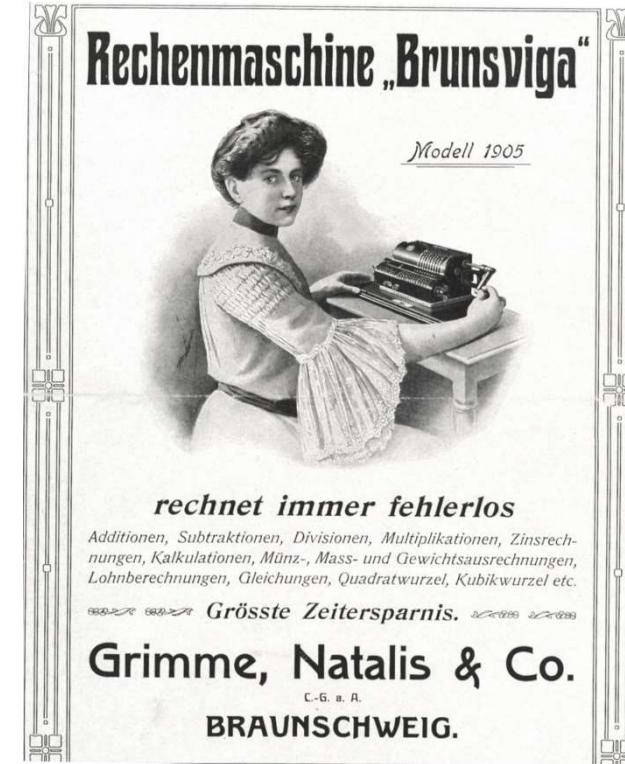
allows the processing of 8-digit numbers using the four basic arithmetic operations addition, subtraction, multiplication and division

The machine failed during demonstrations in London because of manufacturing tolerances.

# Basics - Brunsviga



Brunsviga Matador, 1905



# Basics - Konrad Zuse



**Konrad Zuse**

\* 22. June 1910 (Berlin)

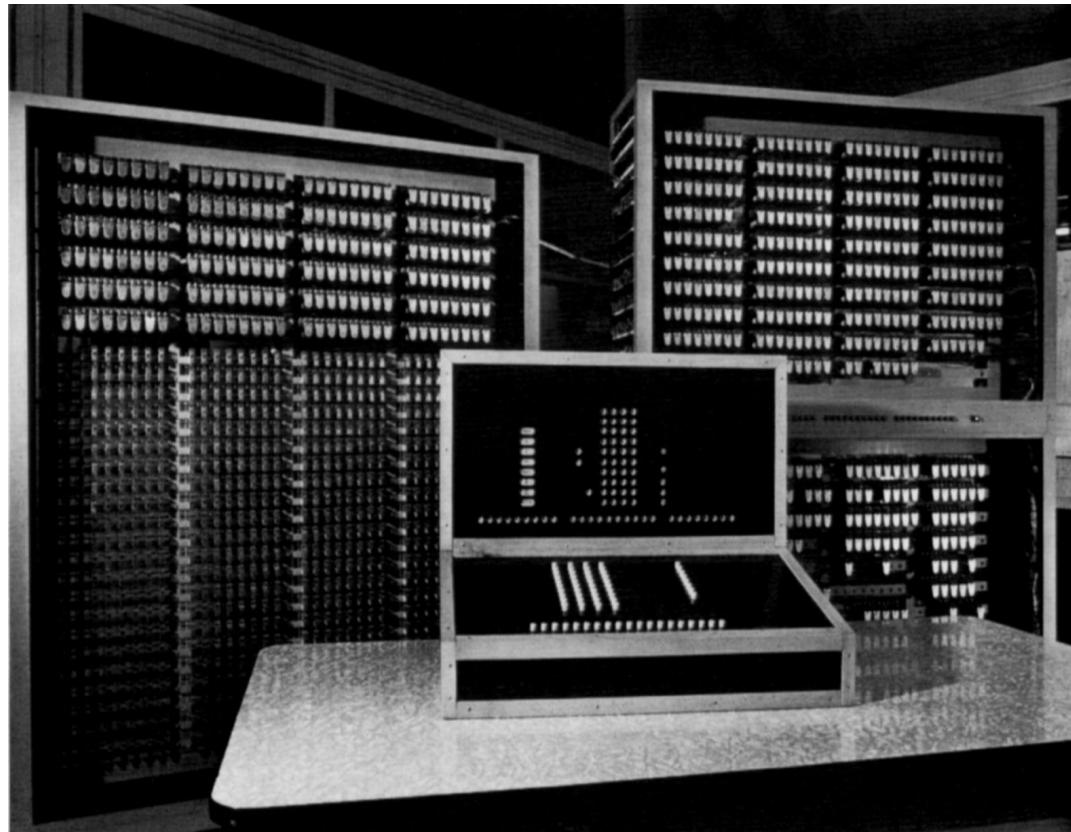
† 18. December 1995

civil engineer

inventor of the (modern)  
computer

Zuse became aware of the  
works of Charles Babbage, Alan  
Turing or Howard Aiken not  
before the end of the Second  
World War.

# Basics - Zuse Z3



## Zuse Z3, Year 1940

first operable digital  
computer with  
programmability

Calculator: 600 relays  
Memory: 1600 relays  
calculates in dual system  
floating point arithmetic  
square root calculation lasts  
about 4 seconds

# Basics - Zuse Z4

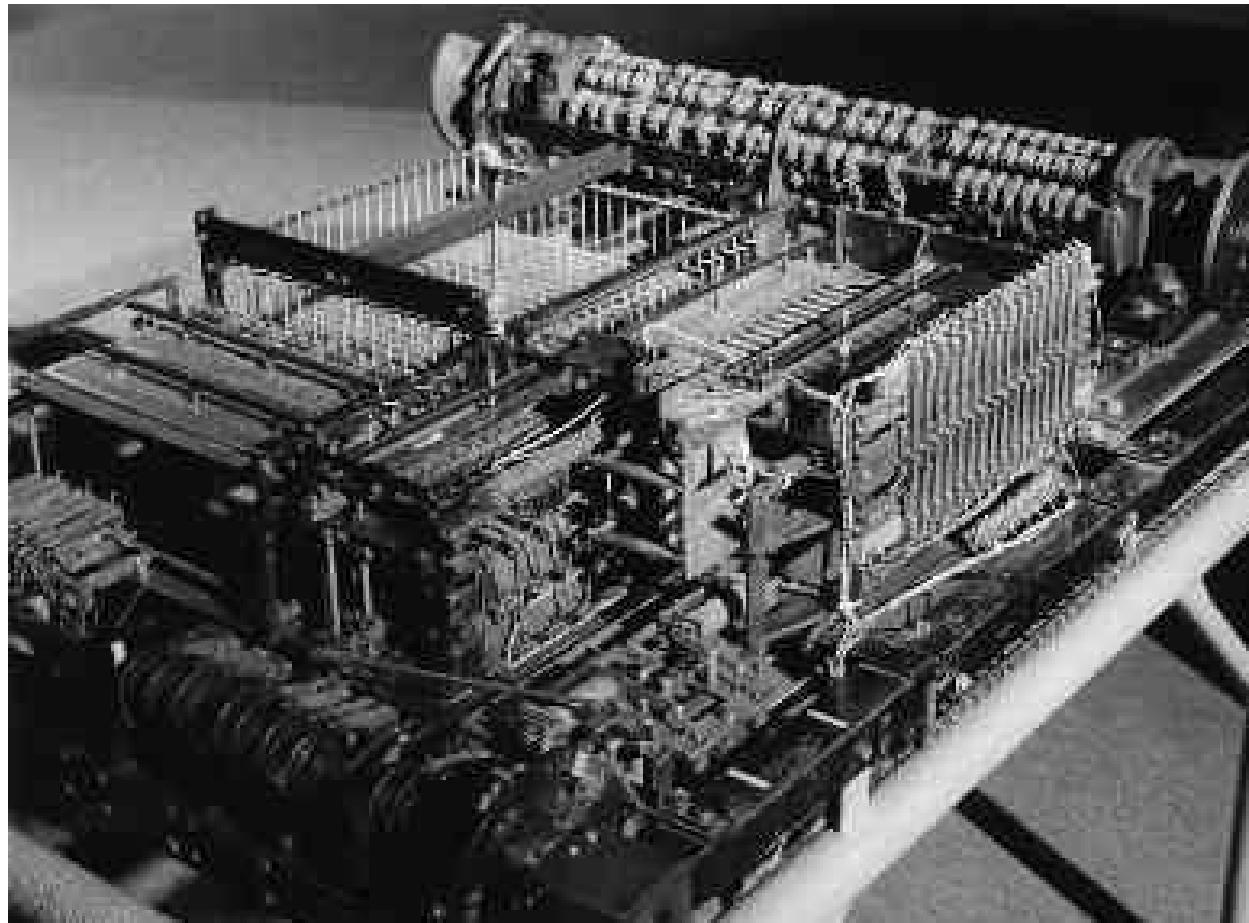


**Zuse Z4, Year 1945**

weight: 2,5 t  
2200 relays total

4 kW of electric power

# Basics – Zuse Z4



mechanical memory Z4  
for 64 floating point words with 32 bit each



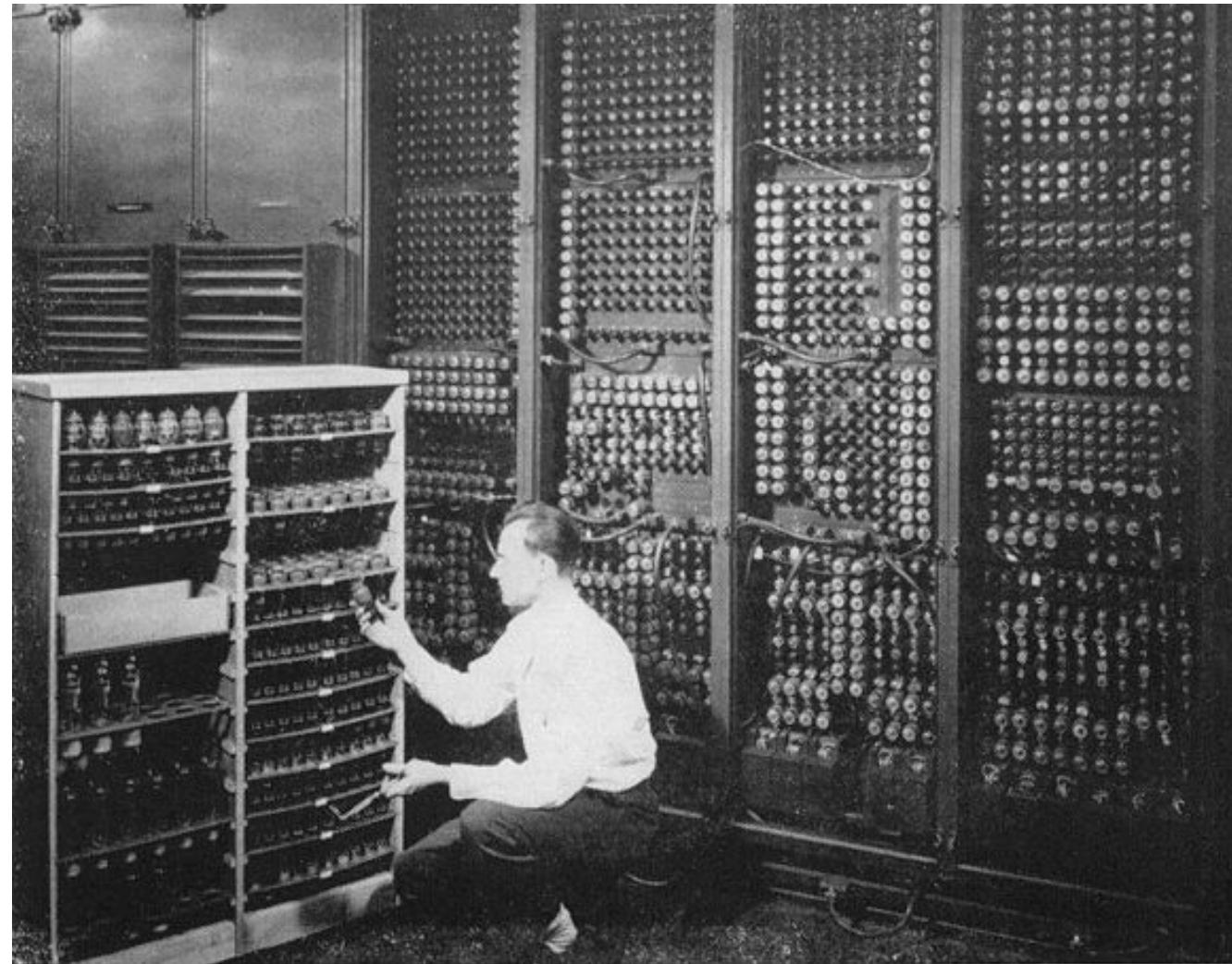
Z22, 1957, tube computer. addition 0,6 ms, multiplication 10 ms, still operable  
(55 copies were sold; Image Museum of Technology Berlin)

## Basics – ENIAC

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- Electronic Numerical Integrator And Calculator
- In the early 40s (1944), the ENIAC was developed under the direction of J. Presper Eckert and John Mauchly at the University of Pennsylvania
- 17.468 vacuum tubes
- 30 tons
- 200 kW
- Performance equivalent to four species calculator (+, -, ·, :)
- clock speed of 100 kHz
- multiplication requires 3 ms
- Purpose of the development was the computer aided design of ballistics (e.g. calculating the trajectories of projectiles) for the U.S. Army in WW2.

# Basics - ENIAC



Replacing a bad tube meant checking among ENIAC's 19,000 possibilities.

Microcontroller Techniques – Prof. Dr. L. M. Reindl

# Basics – Mark I



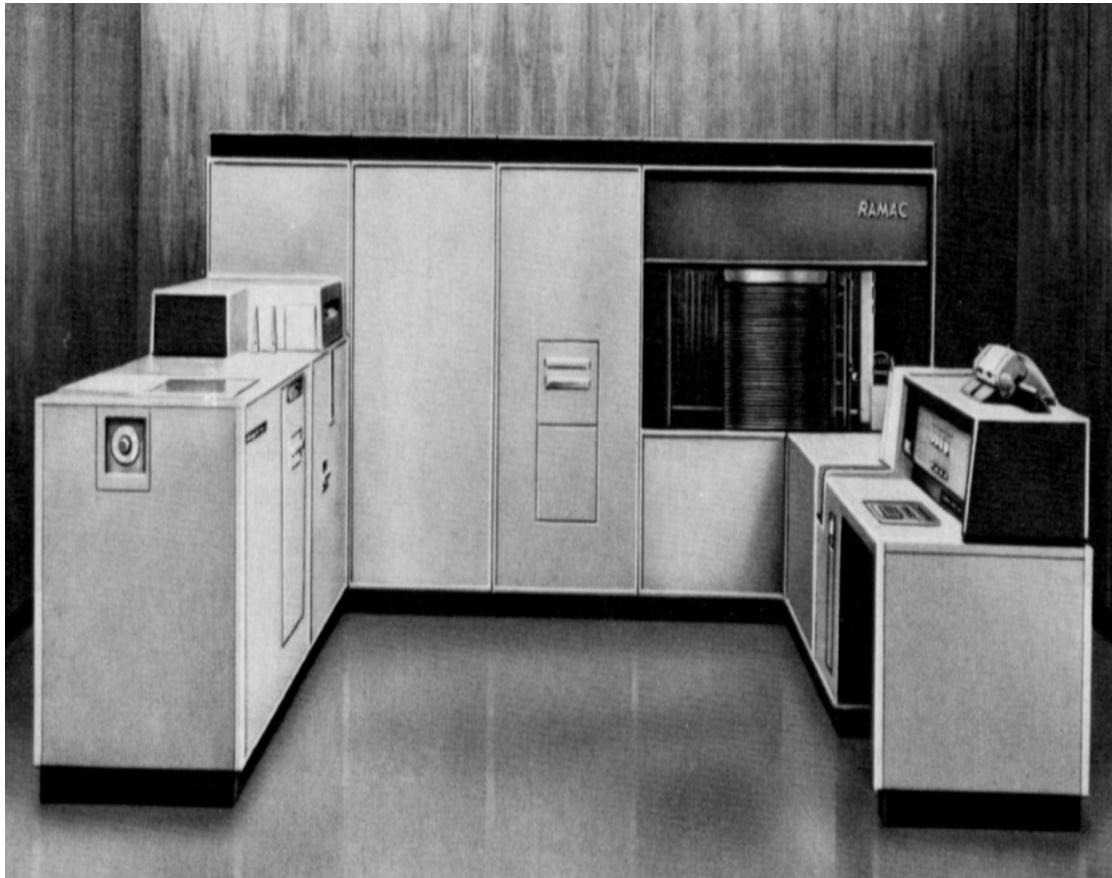
Harvard Mark I, the first stored-program computing machine of the United States, 1944, 23-digit decimal numbers, 16 m long, 2.5 m high

# Basics – Mark IV



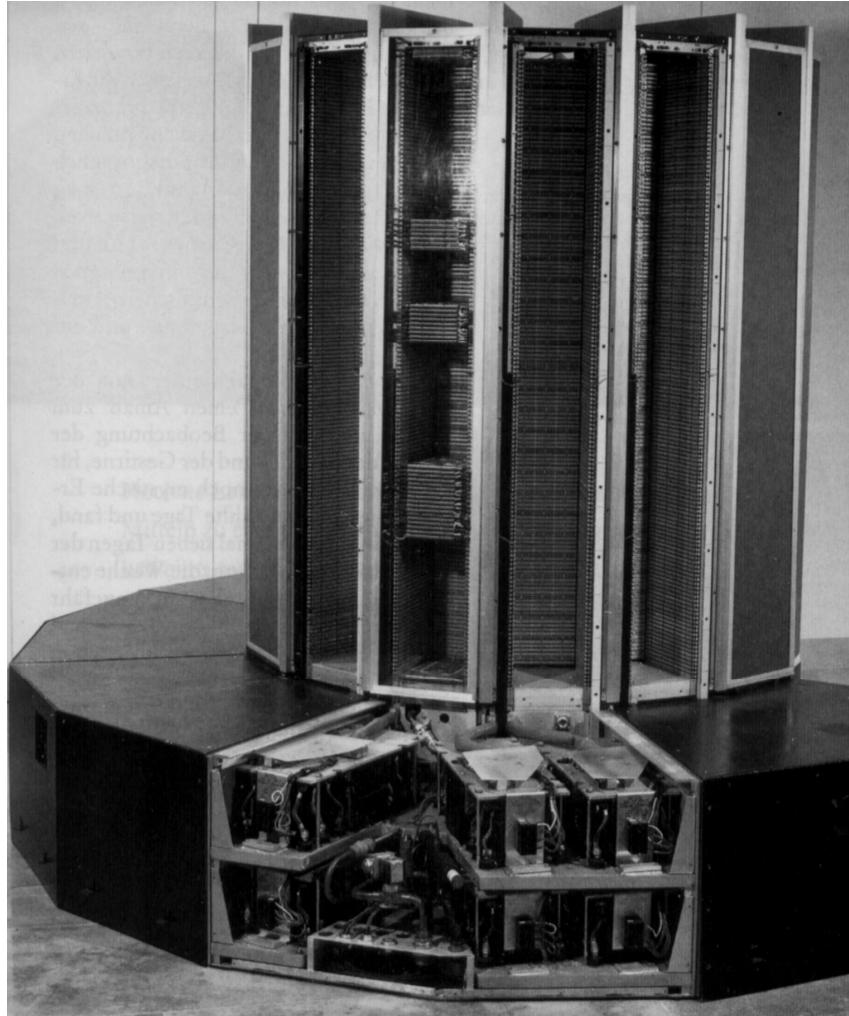
Harvard Mark IV, built by Aiken in 1952;  
decimal computer with magnetic drum, 4000 cells, clock 16 kHz, 4000 tubes

# Basics – IBM 305



IBM 305 "RAMAC", in  
use until 1963,  
33 additions per  
second, 3500 tubes,  
1250 relays,  
ferrite core memory  
100 places

# Basics Cray

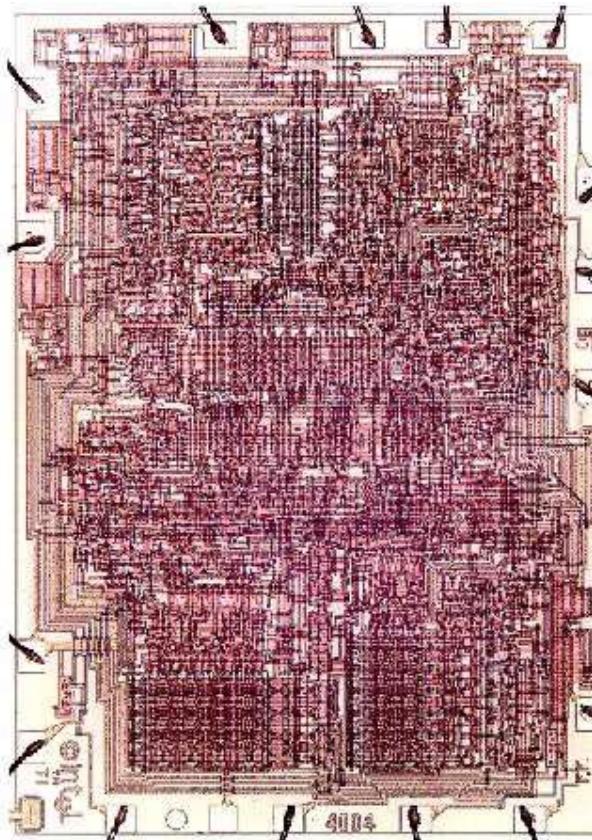


Cray-Supercomputer

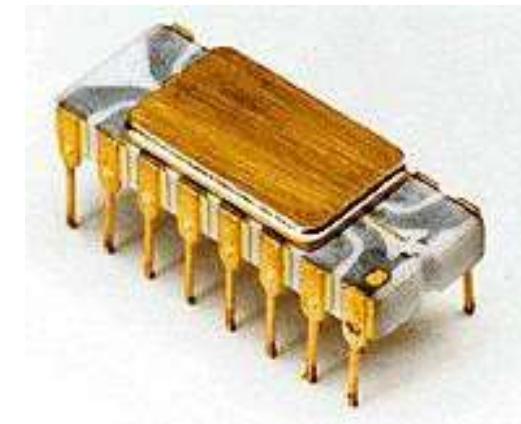
first Version was launched in 1976

# Intel 4004 (1971)

- 2300 Transistors
- word length of 4 bit
- up to 740 kHz



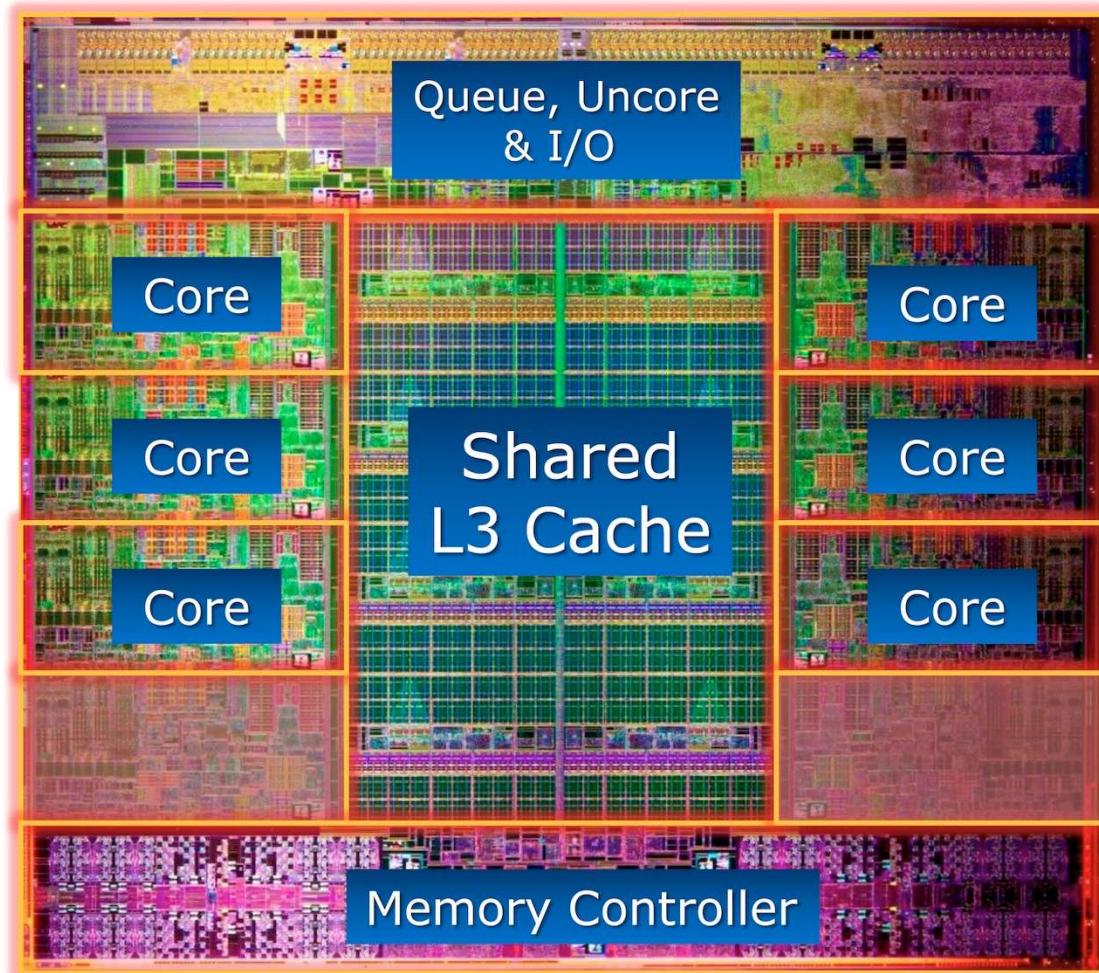
Intel Microprocessor Hall of Fame



[hcs.ucla.edu/cluster1.htm](http://hcs.ucla.edu/cluster1.htm)

# Intel Core i7-3960X (2012) – High-End-PCs

- 2270 million transistors
- 32 nm process
- 3300 to 3900 MHz
- 6 cores / 12 threads
- 435 mm<sup>2</sup>
- 2011 pins
- 130 watts



# Technology development



Year	Name	Size (cu. ft.)	Power (watts)	Performance (adds/sec)	Memory (KB)	Price	Price- performance vs. UNIVAC	Adjusted price (2003 \$)	Adjusted price- performance vs. UNIVAC
1951	UNIVAC I	1,000	125,000	2,000	48	\$1,000,000	1	\$6,107,600	1
1964	IBM S/360 model 50	60	10,000	500,000	64	\$1,000,000	263	\$4,792,300	318
1965	PDP-8	8	500	330,000	4	\$16,000	10,855	\$75,390	13,135
1976	Cray-1	58	60,000	166,000,000	32,000	\$4,000,000	21,842	\$10,756,800	51,604
1981	IBM PC	1	150	240,000	256	\$3,000	42,105	\$5,461	154,673
1991	HP 9000/ model 750	2	500	50,000,000	16,384	\$7,400	3,556,188	\$9,401	16,122,356
1996	Intel PPro PC (200 MHz)	2	500	400,000,000	16,384	\$4,400	47,846,890	\$4,945	239,078,908
2003	Intel Pentium 4 PC (3.0 GHz)	2	500	6,000,000,000	262,144	\$1,600	1,875,000,000	\$1,600	11,452,000,000

# Number of transistors



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Processor	Year of introduction	Transistors
4004	1971	2,250
8008	1972	2,500
8080	1974	5,000
8086	1978	29,000
286	1982	120,000
386	1985	275,000
486 DX	1989	1,180,000
Pentium	1993	3,100,000
Pentium II	1997	7,500,000
Pentium III	1999	24,000,000
Pentium 4	2000	42,000,000

[www.intel.com/research/silicon/mooreslaw.htm](http://www.intel.com/research/silicon/mooreslaw.htm)

# Moore's Law



Moore's Law graph, 1965

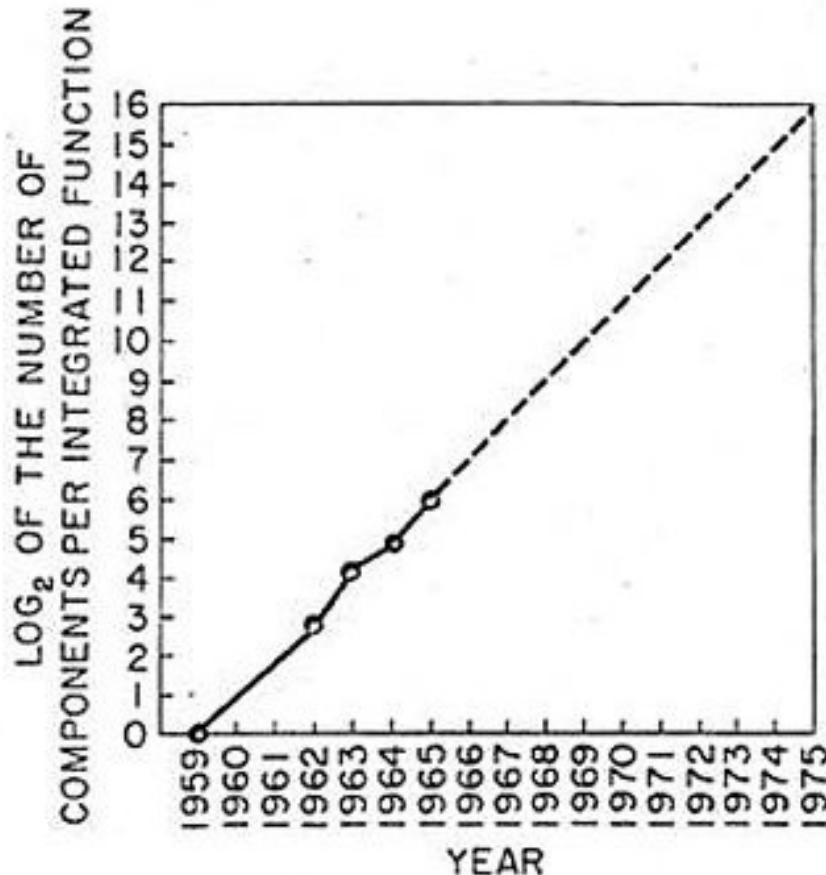


Fig. 2 Number of components per integrated function for minimum cost per component extrapolated vs time.

Gordon E. Moore

\* 03.01.1929, San Francisco

1954 Ph.D. in Chemistry & Physics  
Univ. California, Berkeley

1968 founding member Intel Corp.  
(Executive Vice President)

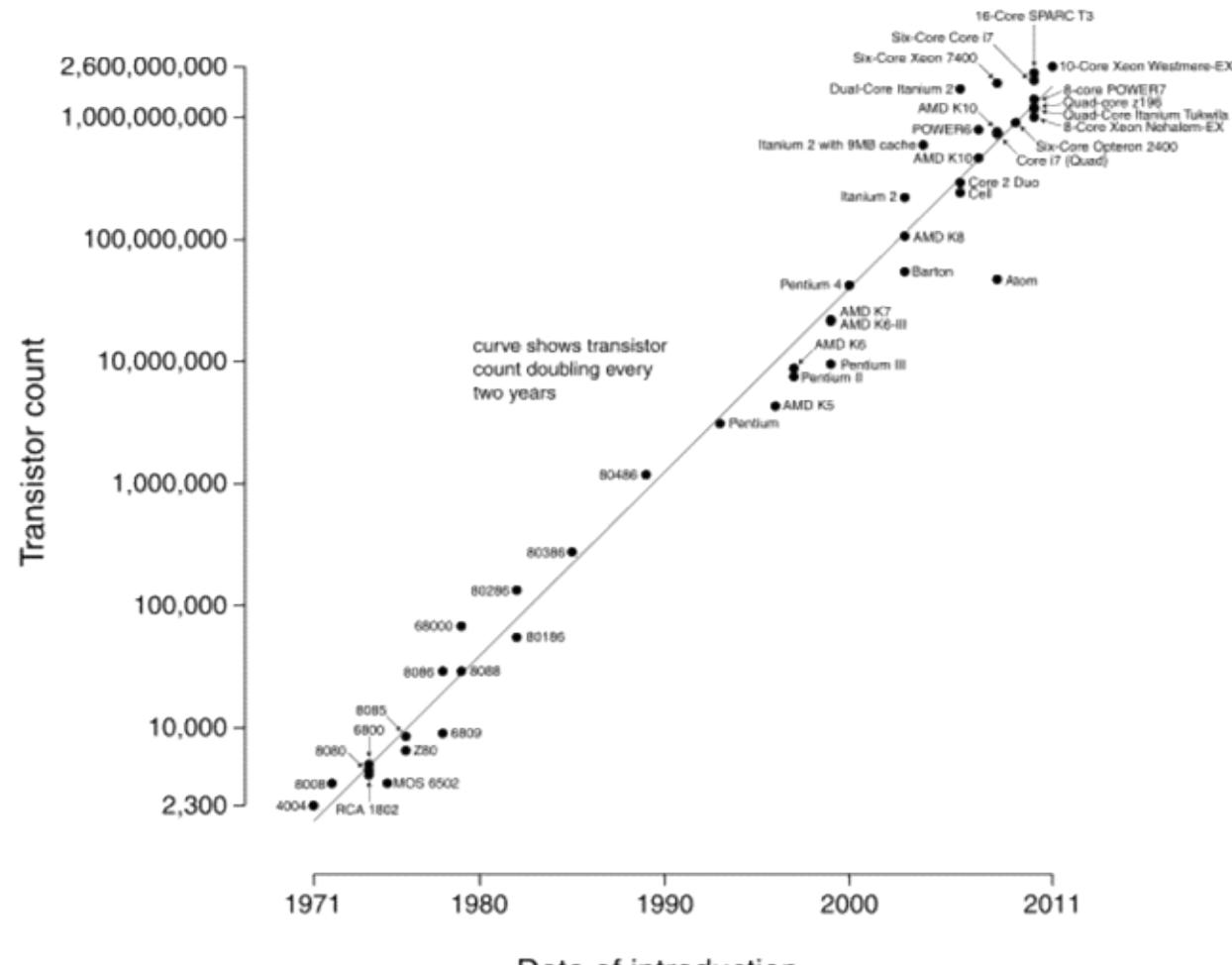
1987 retirement

1997 Chairman Emeritus

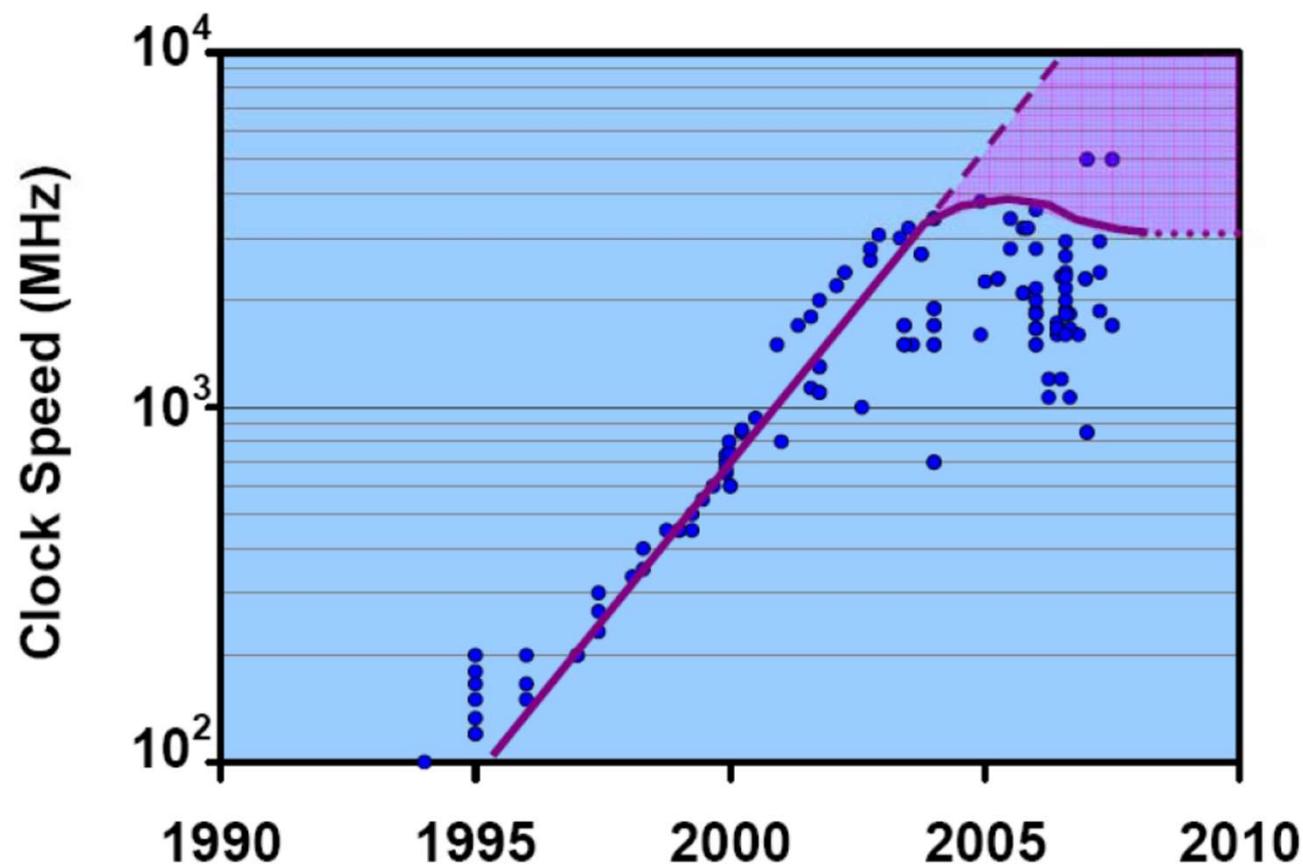


# Moore's Law

Microprocessor Transistor Counts 1971-2011 & Moore's Law



# Clock speed



# Power consumption

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When maintaining the computer architecture, it applies:

- computing power  $\sim$  clock rate

In CMOS technology, it holds:

- power  $\sim$  load capacity  $\cdot$  supply voltage<sup>2</sup>  $\cdot$  clock rate
  - clock rate should be increased in new generations
  - voltage is already small (5 V  $\rightarrow$  1.5 V in the last years)
  - load capacity (number of transistors) is constant or increasing
  - Power  $\uparrow\uparrow$
  - Problem: heat dissipation

# Change in strategy

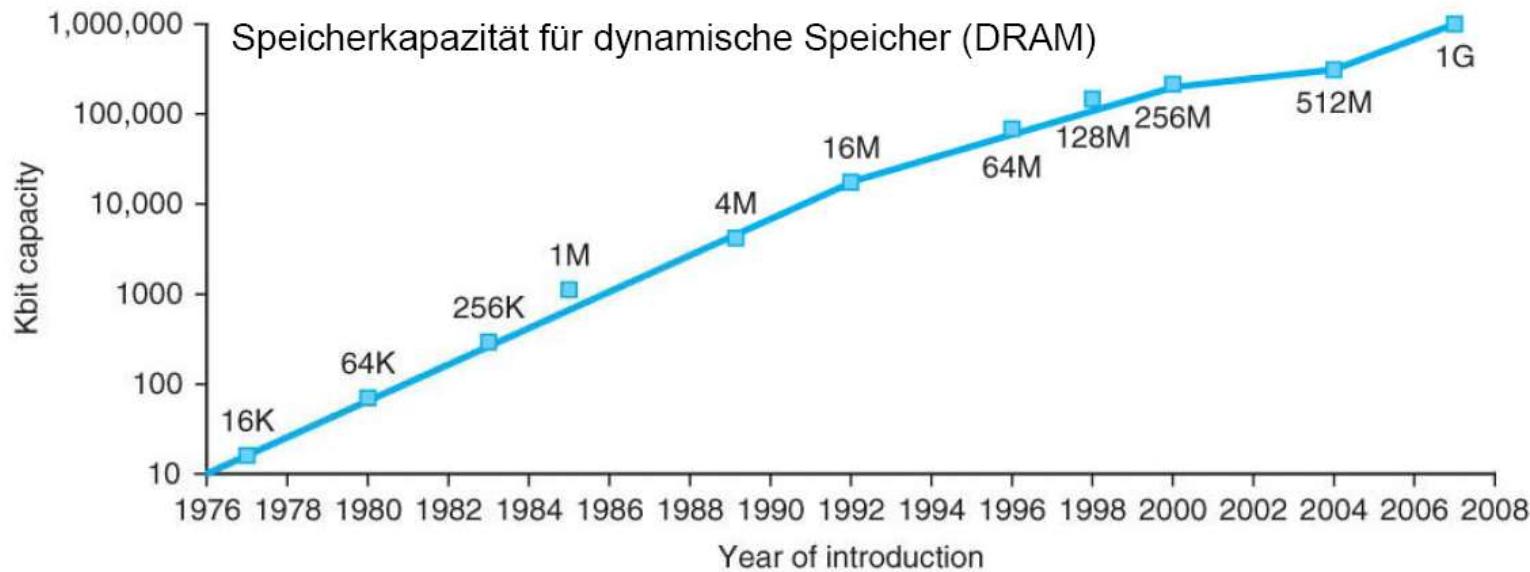
## Change of the design strategy

- Lower clock speeds, but several processor cores
- According to Moore's Law, the required footprint of an IC is supposed to shrinking by half every 12 - 24 months
  - Aim: doubling the number of cores every 2 years

Product	AMD Opteron X4 (Barcelona)	Intel Nehalem	IBM Power 6	Sun Ultra SPARC T2 (Niagara 2)
Cores per chip	4	4	2	8
Clock rate	2.5 GHz	~ 2.5 GHz ?	4.7 GHz	1.4 GHz
Microprocessor power	120 W	~ 100 W ?	~ 100 W ?	94 W

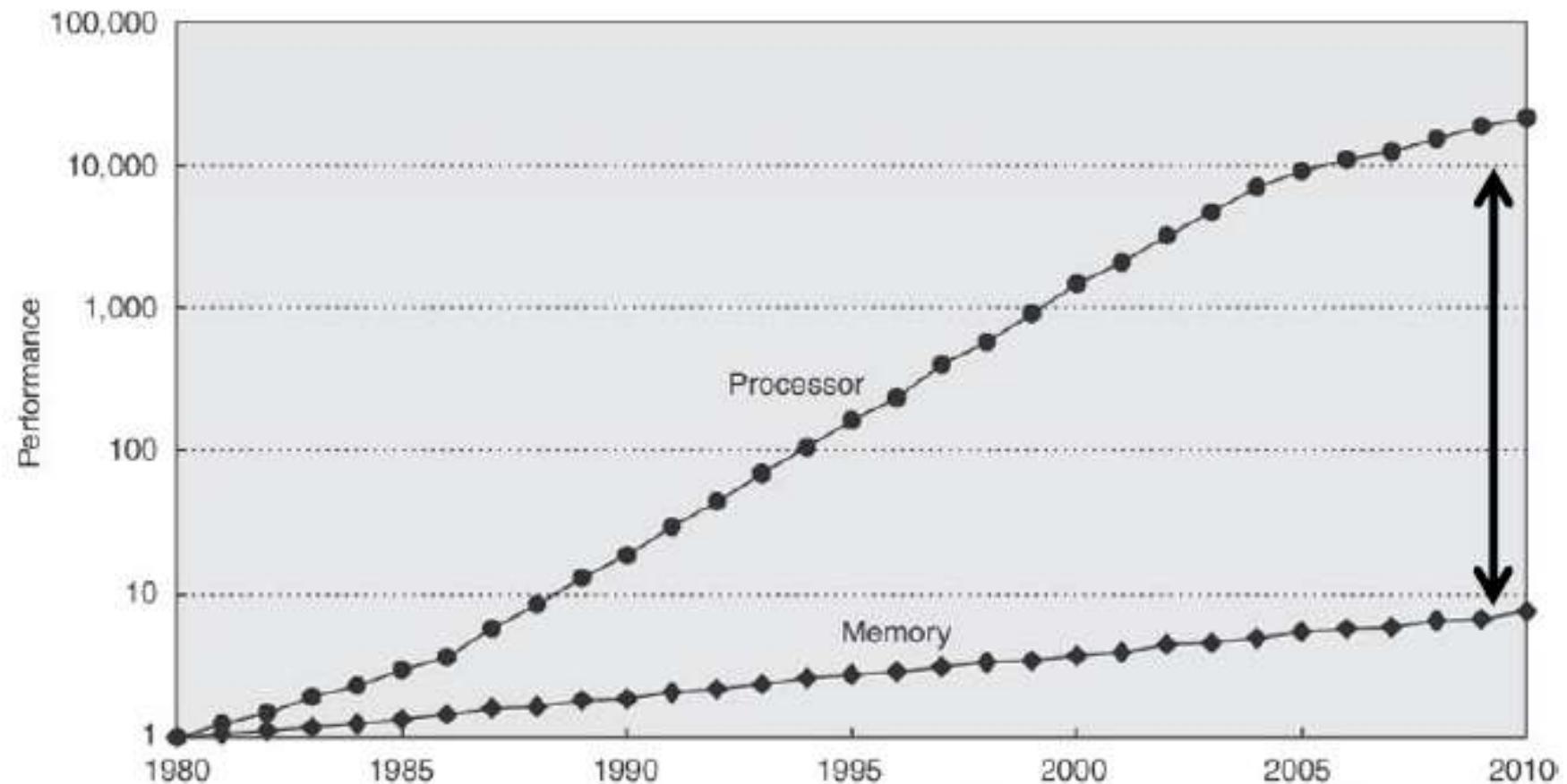
- Problems: Parallel programming, load distribution, communication between the cores, synchronization, ...

# Storage capacity

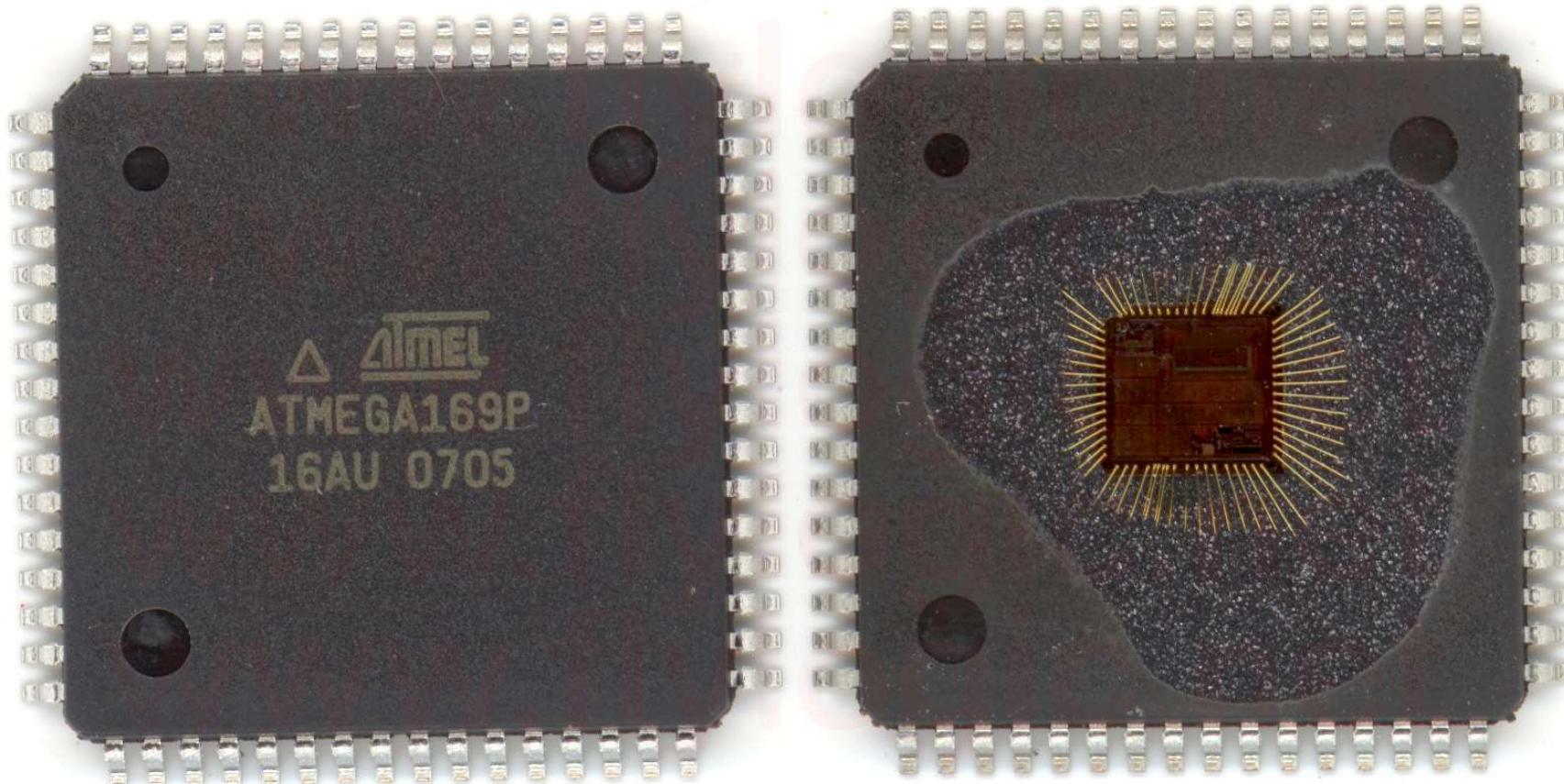


- Storage density increased by 60 % per year; only 30 % of increase in the last years
- access times decreases by approximately 30 % in 5 years

# Processor-memory gap

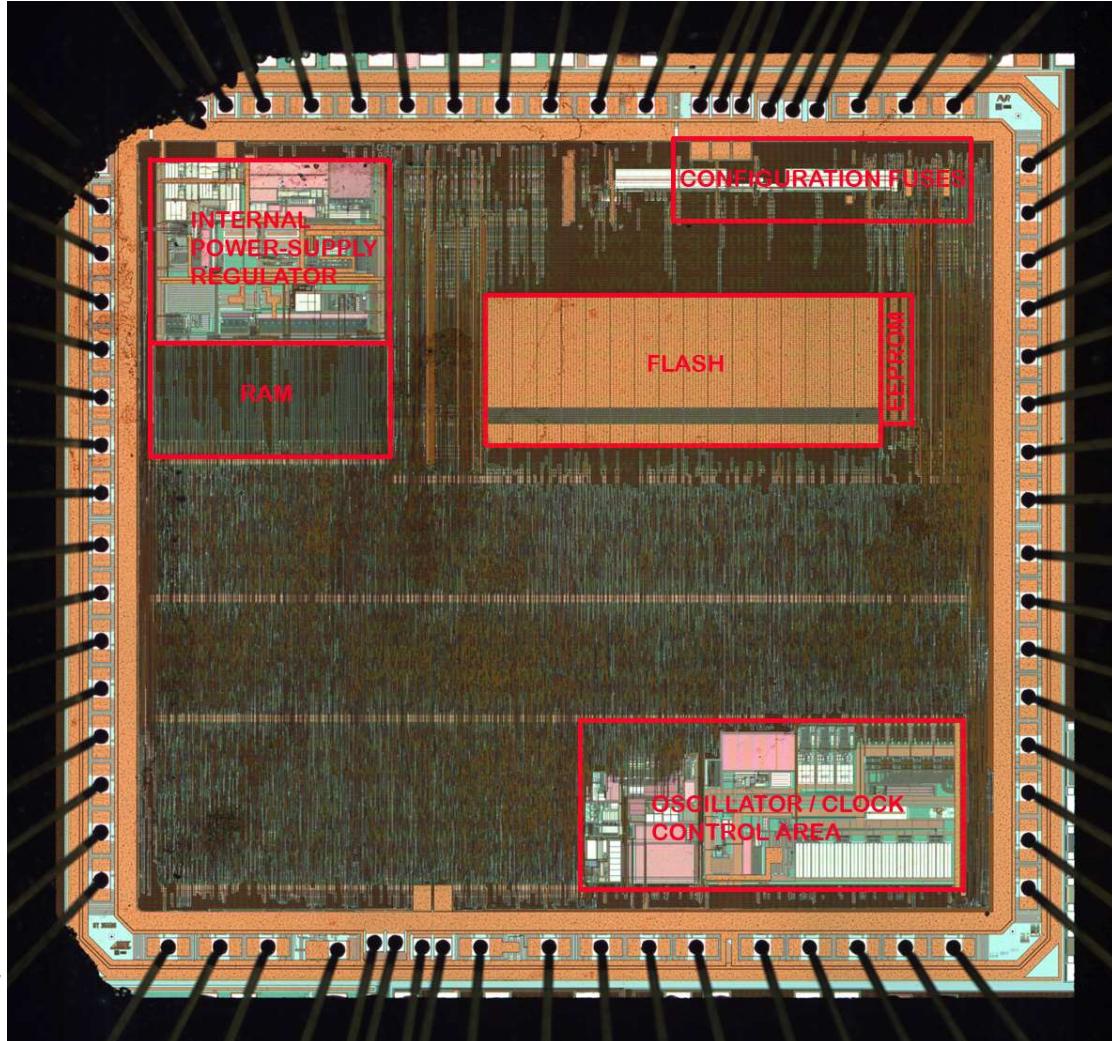


# Here it is: The microcontroller!



<http://blog.ioactive.com/2007/11/atmega169p-quick-peek.html>

# Here it is: The microcontroller!



[http://blog.ioactive.com/  
2007/11/atmega169p-  
quick-peek.html](http://blog.ioactive.com/2007/11/atmega169p-quick-peek.html)

# State of the lecture



- 
- ✓ Introduction
  - Chapter 0: basic knowledge
    - Number systems, digital logic, ...
  - Chapter 1: calculation and control system
    - Basic operations in CMOS, registers, memory, concepts, ...
  - Chapter 2: Basic peripherals
    - GPIO, ADC, ...

...



# 0. Basic knowledge

Digital signals, character and number representations, digital logic

## 0. Basic knowledge

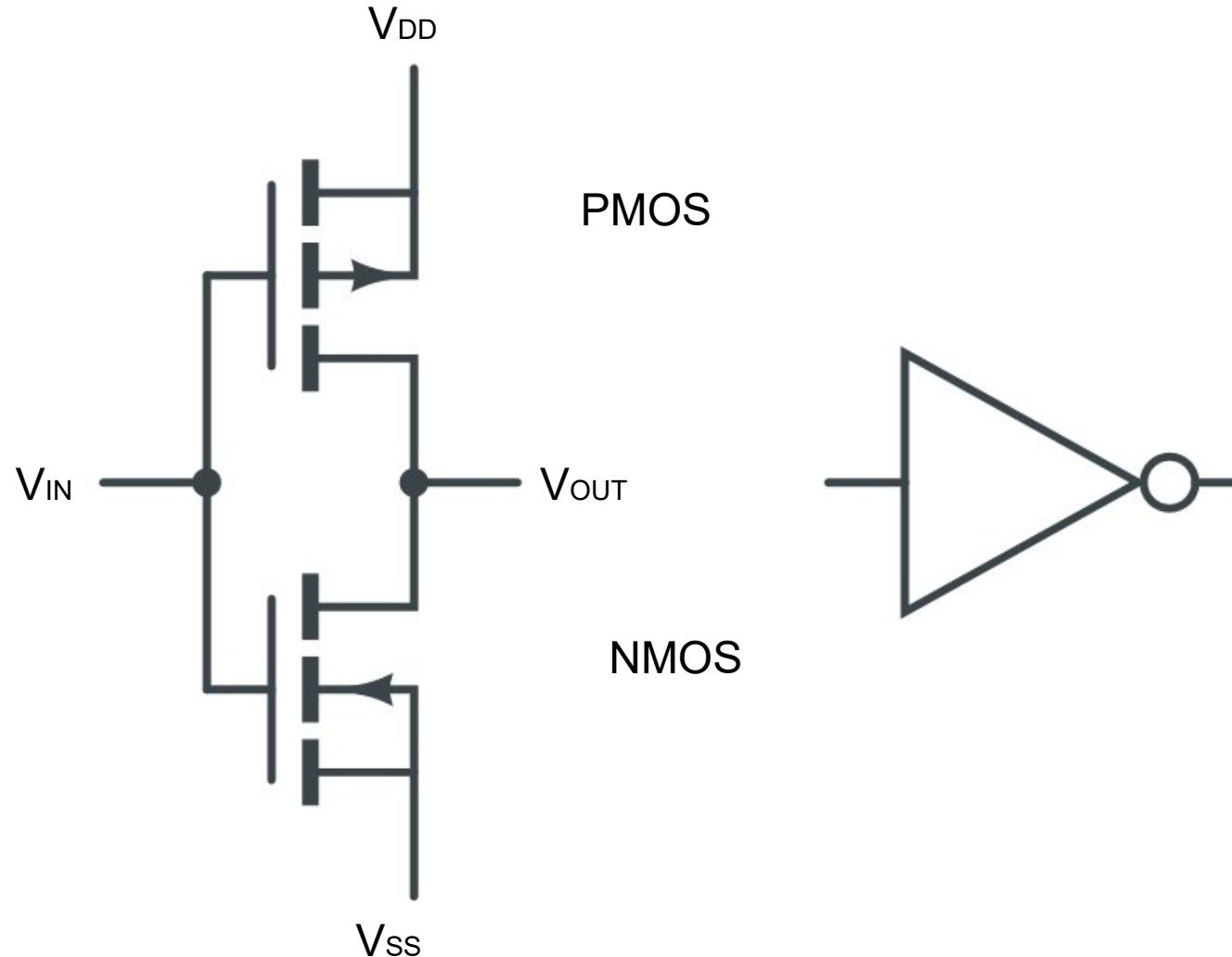
# INTRODUCTION - DIGITAL SIGNALS

# High, Low, Z, X

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- High signal
  - Connection to the (supply) voltage
  - 24 V, 12 V, 5 V, 3.3 V, 1.8 V, ... (generally referred as  $V_{DD}$  for CMOS)
  - Logic 1
- Low signal
  - Connection to ground
  - $V_{SS}$  (for CMOS usually GND)
  - Logic 0
- Z
  - High impedance
  - „floating wire“
  - Neither 1 nor 0
- X
  - any signal

# CMOS Inverter im Detail



## 0. Basic knowledge

# CHARACTER- & NUMBER ENCODING

# It's all about the numbers

- Computers can't process any text
- Everything which is processed must be represented as a number
- Letters are translated to numbers  
(see, for example, ASCII  
(American Standard Code for  
Information Interchange))

000	NUL	033	!	066	B	099	c	132	ä	165	ñ	198	å	231	þ
001	Start Of Header	034	"	067	C	100	d	133	â	166	˜	199	À	232	߱
002	Start Of Text	035	#	068	D	101	e	134	ã	167	°	200	ܲ	233	ܻ
003	End Of Text	036	\$	069	E	102	f	135	ç	168	݂	201	ܰ	234	ܻ
004	End Of Transmission	037	%	070	F	103	g	136	è	169	ܶ	202	ܲ	235	ܻ
005	Enquiry	038	&	071	G	104	h	137	é	170	݂	203	ܰ	236	ܻ
006	Acknowledge	039	?	072	H	105	i	138	è	171	݂	204	ܰ	237	ܻ
007	Bell	040	(	073	I	106	j	139	í	172	݂	205	-	238	-
008	Backspace	041	)	074	J	107	k	140	݂	173	݂	206	ܰ	239	ܰ
009	Horizontal Tab	042	*	075	K	108	l	141	݂	174	݂	207	݂	240	݂
010	Line Feed	043	+	076	L	109	m	142	ܰ	175	݂	208	ܰ	241	ܰ
011	Vertical Tab	044	,	077	M	110	n	143	ܰ	176	݂	209	ܰ	242	݂
012	Form Feed	045	-	078	N	111	o	144	ܰ	177	݂	210	ܰ	243	݂
013	Carriage Return	046	.	079	O	112	p	145	ܰ	178	݂	211	ܰ	244	ܰ
014	Shift Out	047	/	080	P	113	q	146	ܰ	179	ܰ	212	ܰ	245	ܰ
015	Shift In	048	0	081	Q	114	r	147	ܰ	180	ܰ	213	ܰ	246	ܰ
016	Delete	049	1	082	R	115	s	148	ܰ	181	ܰ	214	ܰ	247	ܰ
017	-- frei --	050	2	083	S	116	t	149	ܰ	182	ܰ	215	ܰ	248	ܰ
018	-- frei --	051	3	084	T	117	u	150	ܰ	183	ܰ	216	ܰ	249	ܰ
019	-- frei --	052	4	085	U	118	v	151	ܰ	184	ܰ	217	ܰ	250	ܰ
020	-- frei --	053	5	086	V	119	w	152	ܰ	185	ܰ	218	ܰ	251	ܰ
021	Negative Acknowledge	054	6	087	W	120	x	153	ܰ	186	ܰ	219	ܰ	252	ܰ
022	Synchronous Idle	055	7	088	X	121	y	154	ܰ	187	ܰ	220	ܰ	253	ܰ
023	End Of Transmission Block	056	8	089	Y	122	z	155	ܰ	188	ܰ	221	ܰ	254	ܰ
024	Cancel	057	9	090	Z	123	{	156	ܰ	189	ܰ	222	ܰ	255	ܰ
025	End Of Medium	058	:	091	\	124		157	ܰ	190	ܰ	223	ܰ	256	ܰ
026	Substitute	059	:	092	\	125	}	158	ܰ	191	ܰ	224	ܰ	257	ܰ
027	Escape	060	<	093	)	126	~	159	ܰ	192	ܰ	225	ܰ	258	ܰ
028	File Separator	061	=	094	^	127	o	160	ܰ	193	ܰ	226	ܰ	259	ܰ
029	Group Separator	062	>	095	-	128	ܰ	161	ܰ	194	ܰ	227	ܰ	260	ܰ
030	Record Separator	063	?	096	`	129	ܰ	162	ܰ	195	ܰ	228	ܰ	261	ܰ
031	Unit Separator	064	@	097	a	130	ܰ	163	ܰ	196	ܰ	229	ܰ	262	ܰ
032		065	A	098	b	131	ܰ	164	ܰ	197	ܰ	230	ܰ	263	ܰ

Bildquelle: [www.chip.de](http://www.chip.de)

# Examples of text encoding

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- Hällü Wörld  
(ANSI)
- H,,ll W"rld  
(OEM 852)
- HĂăllĂĹ' WĂ rld  
(Windows 1250)
  
- 01001000 11100100 01101100 01101100 11111100 00100000  
01010111 11110110 01110010 01101100 01100100

# Number representations

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There are three most frequently encountered number representations in the microcomputer area:

- Binary
  - 0, 1, 10, 11, 100, ...
- Decimal
  - 0, 1, ..., 9, 10, 11, ...
- Hexadecimal
  - 0, 1, ..., 9, A, ... F, 10, 11, ...

# Positional notation

## General representation of integers

- Most number encodings are based on a positional notation
- Integers are represented by:

$$A = a_{B-1}a_{B-2} \dots a_1a_0$$

with  $a_i = \{0, 1, \dots, r - 1\}$  and the position's value  $r^i$

- The value of a number is defined by

$$A = \sum_{i=0}^{B-1} a_i r^i$$

- Usual bases are  $r = 2$  (binary),  $r = 10$  (decimal) and  $r = 16$  (hexadecimal)

# Positional notation

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- 124
  - 0111 1100 (sometimes written as: 0111 1100<sub>b</sub> or 0111 1100<sub>2</sub>)
  - 0x7C [spoken „hex(decimal) seven C“; alternatively: 7Ch or 7C<sub>16</sub>)
- 255
  - 1111 1111
  - 0xFF
- 256
  - 0001 0000 0000
  - 0x100 (spoken „hexadecimal one-zero-zero“)
  - If limited to 8 bits: 0000 0000 (0x00)

# Basic arithmetic operations in binary system



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Identical to basic arithmetic operations in the decimal system:

## addition

$$\begin{array}{r} 0001 \\ + 0110 \\ \hline = 0111 \end{array}$$

## subtraction

$$\begin{array}{r} 0111 \\ - 0110 \\ \hline = 0001 \end{array}$$

## Identical with long numbers

$$\begin{array}{r} 01000000 \\ - 00111111 \\ \hline = 00000001 \end{array}$$

## multiplication

$$\begin{array}{r} 0101 \cdot 0011 \\ 0000 \\ 0000 \\ 0101 \\ 0101 \\ \hline = 0001111 \end{array}$$

## attention:

$$\begin{array}{r} 1111 \\ + 0001 \\ \hline = 10000 \end{array}$$

For a fixed word width of 4 bits  
the result is a zero!

# Negative numbers

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- $5 = 0101$
- $127 = 0111\ 111$
  
- And -127?
- computer don't know a negative sign on the bit level

→ encoding is required

# Number encoding: magnitude and sign

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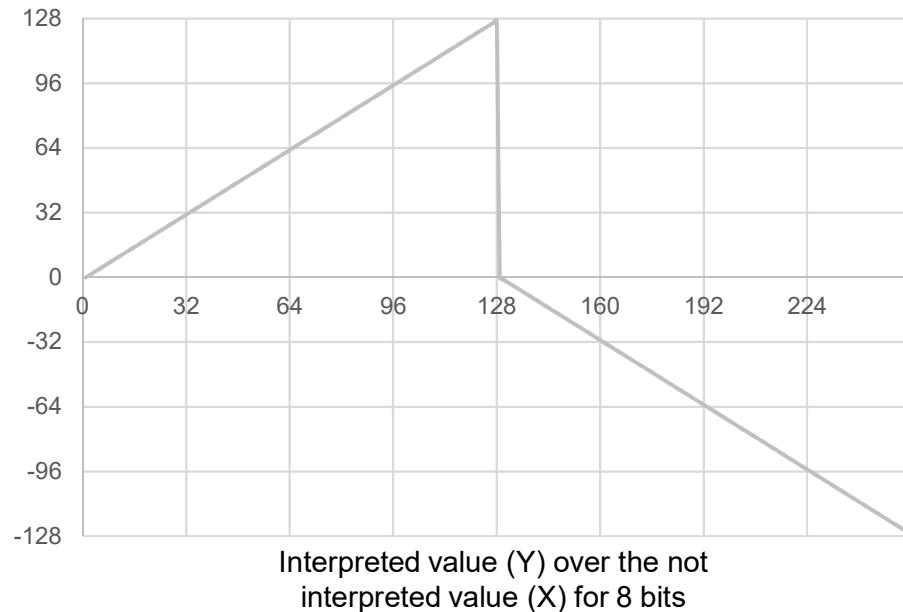
- The MSB ( $a_{B-1}$ ) determines the sign:

$$A = \begin{cases} \sum_{i=0}^{B-2} a_i r^i & \text{for } a_{B-1} = 0 \\ -\sum_{i=0}^{B-2} a_i r^i & \text{for } a_{B-1} = 1 \end{cases}$$

- The range is  $-(2^{B-1}-1) \dots 2^{B-1}-1$

# Number encoding: magnitude and sign

- Most significant bit indicates sign, the magnitude is remaining
- Example
  - 4 = 0000 0100
  - -4 = 1000 0100
  - 127 = 0111 1111
  - -127 = 1111 1111
  - $0000\ 0100 + 0000\ 0001$  ( $4 + 1$ )  
= 0000 0101 (5)
- Two issues:
  - $0000\ 0100 + 1000\ 0100 = ?$   
(1000 1000 would be -8, which is completely wrong ...)
  - $1000\ 0000 \triangleq 0000\ 0000$



# Number encoding: one's complement

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- For negative numbers, the complement is formed (by inverting).
- The MSB ( $a_{B-1}$ ) is determined to be the sign :

$$A = \begin{cases} \sum_{i=0}^{B-2} a_i r^i & \text{für } a_{B-1} = 0 \\ 2^{B-1} - 1 - \sum_{i=0}^{B-2} a_i r^i & \text{für } a_{B-1} = 1 \end{cases}$$

- The range is  $-(2^{B-1} - 1) \dots 2^{B-1} - 1$

# Number encoding: one's complement

- Most significant bit is the sign.
- If the number is negative, the magnitude is inverted
- Example

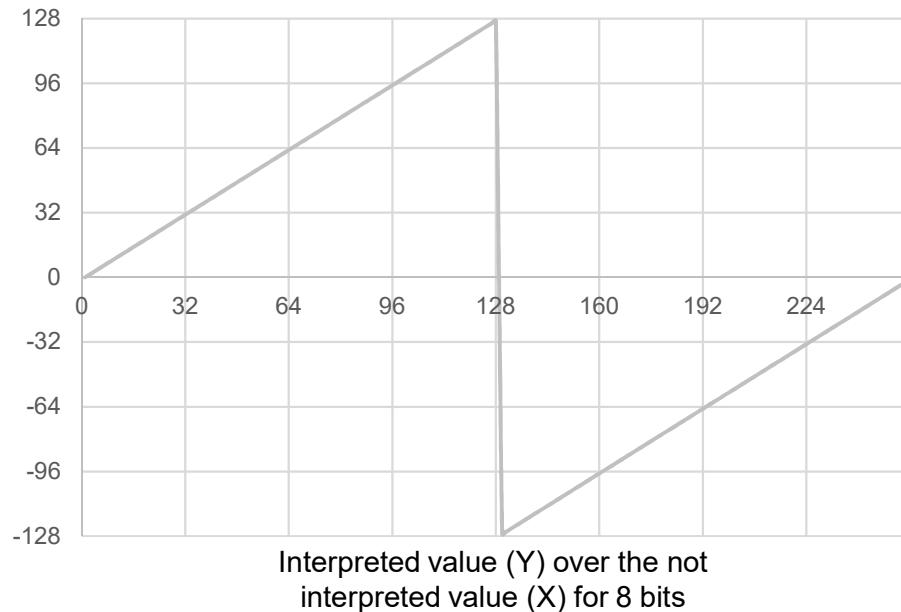
➤ 4 = 0000 0100

➤ -4 = 1111 1011

➤ 127 = 0111 1111

➤ -127 = 1000 0000

➤  $0000\ 0100 + 1111\ 1011$  ( $4 + (-4)$ )  
= 1111 1111 (0)



- Again two problems
- $0000\ 0110 + 1111\ 1010$  ( $6 + (-5)$ )=?
- $1111\ 1111 \triangleq 0000\ 0000$

# Number encoding: two's complement

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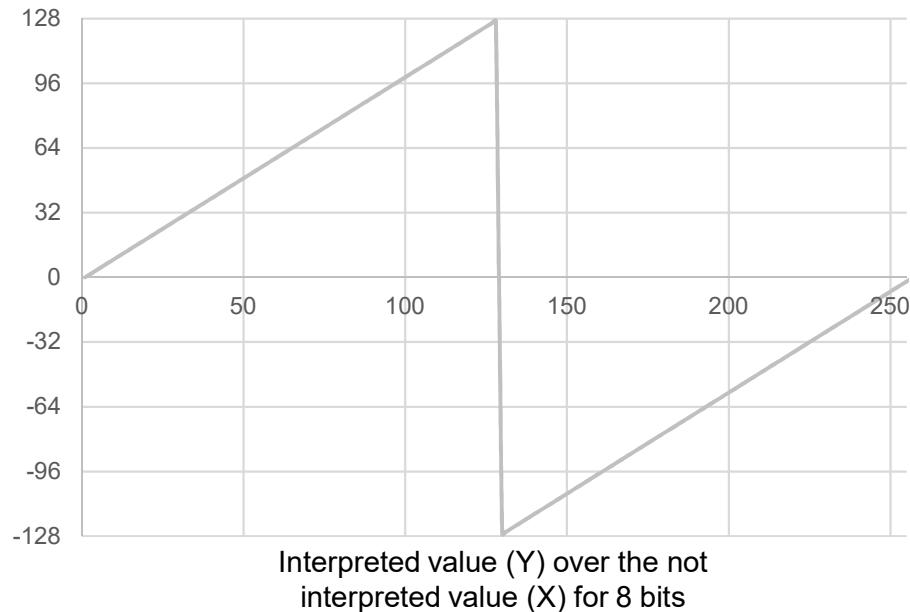
- For negative numbers, the 2's complement is formed by inverting and adding with one.
- The MSB ( $a_{B-1}$ ) still determines the sign :

$$A = \begin{cases} \sum_{i=0}^{B-2} a_i r^i & \text{für } a_{B-1} = 0 \\ 2^{B-1} - \sum_{i=0}^{B-2} a_i r^i & \text{für } a_{B-1} = 1 \end{cases}$$

- The range is  $-2^{B-1} \dots 2^{B-1} - 1$

# Number Coding: two's complement

- Like one's complement - but negative numbers are incremented by one.
- Example
  - 4 = 0000 0100
  - -4 = 1111 1100
  - 127 = 0111 1111
  - -127 = 1000 0001
  - 0 = 0000 0000
  - -128 = 1000 0000
  - $0000\ 0110 + 1111\ 1011\ (6 + (-5)) = 0000\ 0001$



# Binary arithmetic



M&S (magnitude & sign)	One's (one's complement)	Two's (two's complement)
0001 + 1000		
1010 + 0111		
0110 + 0111		
0110 - 0111		
1010 - 0111		
1111 - 1000		

# Binary arithmetic



M&S	One's	Two's
0001 + 1000	0001 $(1 + (-0))$	1001 $(1 + (-7))$
1010 + 0111		
0110 + 0111		
0110 - 0111		
1010 - 0111		
1111 - 1000		

# Binary arithmetic

M&S	One's	Two's
0001 + 1000	0001 $(1 + (-0))$	1001 $(1 + (-7))$
1010 + 0111	0101 $(-2 + 7)$	0001 $(-5 + 7)$
0110 + 0111		
0110 - 0111		
1010 - 0111		
1111 - 1000		

# Binary arithmetic

M&S	One's		Two's	
0001 + 1000	0001	(1 + (-0))	1001	(1 + (-7))
1010 + 0111	0101	(-2 + 7)	0010	(-5 + 7)
0110 + 0111	01101	(6 + 7)	01101	(6 + 7)
0110 - 0111				
1010 - 0111				
1111 - 1000				

# Binary arithmetic

M&S	One's	Two's
0001 + 1000	0001 $(1 + (-0))$	1001 $(1 + (-7))$
1010 + 0111	0101 $(-2 + 7)$	0001 $(-5 + 7)$
0110 + 0111	01101 $(6 + 7)$	01101 $(6 + 7)$
0110 - 0111	1001 $(6 - 7)$	1111 $(6 - 7)$
1010 - 0111	11001 $(-2 - 7)$	10011 $(-5 - 7)$
1111 - 1000	1111 $(-7 - 0)$	0111 $(0 - (-7))$