JPEG Compression

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JPEG (Joint Photographic Experts Group) is a widely used image compression method that reduces the storage and transmission requirements of digital images. It utilizes the Discrete Cosine Transform (DCT) and quantization to achieve compression.

1 JPEG Compression Steps

1.1 Block Division

The input image is divided into non-overlapping blocks of a fixed size, typically 8×8 pixels.

1.2 Discrete Cosine Transform (DCT)

For each block, the DCT is applied to obtain the frequency domain representation. The 2D DCT of a block B is given by:

$$DCT(u,v) = \frac{1}{4}C(u)C(v)\sum_{x=0}^{7}\sum_{y=0}^{7}B(x,y)\cos\left(\frac{(2x+1)u\pi}{16}\right)\cos\left(\frac{(2y+1)v\pi}{16}\right)$$

where u and v are frequency indices, $C(u) = C(v) = \frac{1}{\sqrt{2}}$ for $u = v = 0$, and $C(u) = C(v) = 1$ otherwise.

1.3 Quantization

The DCT coefficients are quantized using a quantization matrix Q. The quantization process is given by:

$$Q_{ij} = \frac{\text{round}(DCT_{ij}/Q_{ij})}{Q_{ij}}$$

where DCT_{ij} is the DCT coefficient and Q_{ij} is the corresponding element of the quantization matrix.The quantization matrix used in the JPEG compression method is as follows:

$$Q = \begin{bmatrix} 6 & 4 & 4 & 6 & 10 & 16 & 20 & 24 \\ 5 & 5 & 6 & 8 & 10 & 23 & 24 & 22 \\ 6 & 5 & 6 & 10 & 16 & 23 & 28 & 22 \\ 6 & 7 & 9 & 12 & 20 & 35 & 32 & 25 \\ 7 & 9 & 15 & 22 & 27 & 44 & 41 & 31 \\ 10 & 14 & 22 & 26 & 32 & 42 & 45 & 37 \\ 20 & 26 & 31 & 35 & 41 & 48 & 48 & 40 \\ 29 & 37 & 38 & 39 & 45 & 40 & 41 & 40 \end{bmatrix}$$

The quantization matrix controls the level of compression in the JPEG algorithm. Smaller values in the matrix lead to more compression, while larger values preserve more image details.

1.4 Inverse Quantization

To reconstruct the quantized DCT coefficients, the inverse quantization is performed:

$$DCT_{ij} = Q_{ij} \times Q_{ij}$$

1.5 Inverse DCT (IDCT)

The inverse DCT is applied to the quantized DCT coefficients to obtain the pixel values of the compressed block:

$$B(x,y) = \frac{1}{4}\sum_{u=0}^{7}\sum_{v=0}^{7}C(u)C(v)DCT(u,v)\cos\left(\frac{(2x+1)u\pi}{16}\right)\cos\left(\frac{(2y+1)v\pi}{16}\right)$$

2 SSIM Calculation

The Structural Similarity Index (SSIM) is a widely used metric to assess the quality of images by measuring the structural similarity and luminance differences between a reference and a distorted image.

2.1 Luminance Comparison

The luminance comparison is based on the mean and variance of the pixel intensities in the reference image (x) and the distorted image (y):

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$\sigma_x^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_x)^2 \quad \sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}(y_i - \mu_y)^2$$

$$\sigma_{xy} = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)$$

2.2 Contrast Comparison

The contrast comparison accounts for the differences in image contrast:

$$\mathrm{cov}_{xy} = \sigma_{xy}$$

$$\mathrm{contrast}(x, y) = \frac{2\sigma_x\sigma_y + C_1}{\sigma_x^2 + \sigma_y^2 + C_1}$$

where C_1 is a small constant to prevent division by zero.

2.3 Structure Comparison

The structure comparison measures the similarity in image structure:

$$\mathrm{structure}(x, y) = \frac{2\sigma_{xy} + C_2}{\sigma_x^2 + \sigma_y^2 + C_2}$$

where C_2 is another small constant.

2.4 SSIM Index

The SSIM index is a combination of the three comparisons and is given by:

$$\mathrm{SSIM}(x, y) = \mathrm{luminance}(x, y) \times \mathrm{contrast}(x, y) \times \mathrm{structure}(x, y)$$

2.5 Interpretation

The SSIM index ranges from -1 to 1, with 1 indicating identical images and 0 indicating no structural similarity. Negative values indicate dissimilarity.

3 Results

JPEG compression uses the DCT and quantization to reduce the size of images while maintaining a reasonable level of visual quality. The compression factor and quality trade-off can be controlled by adjusting the quantization matrix.

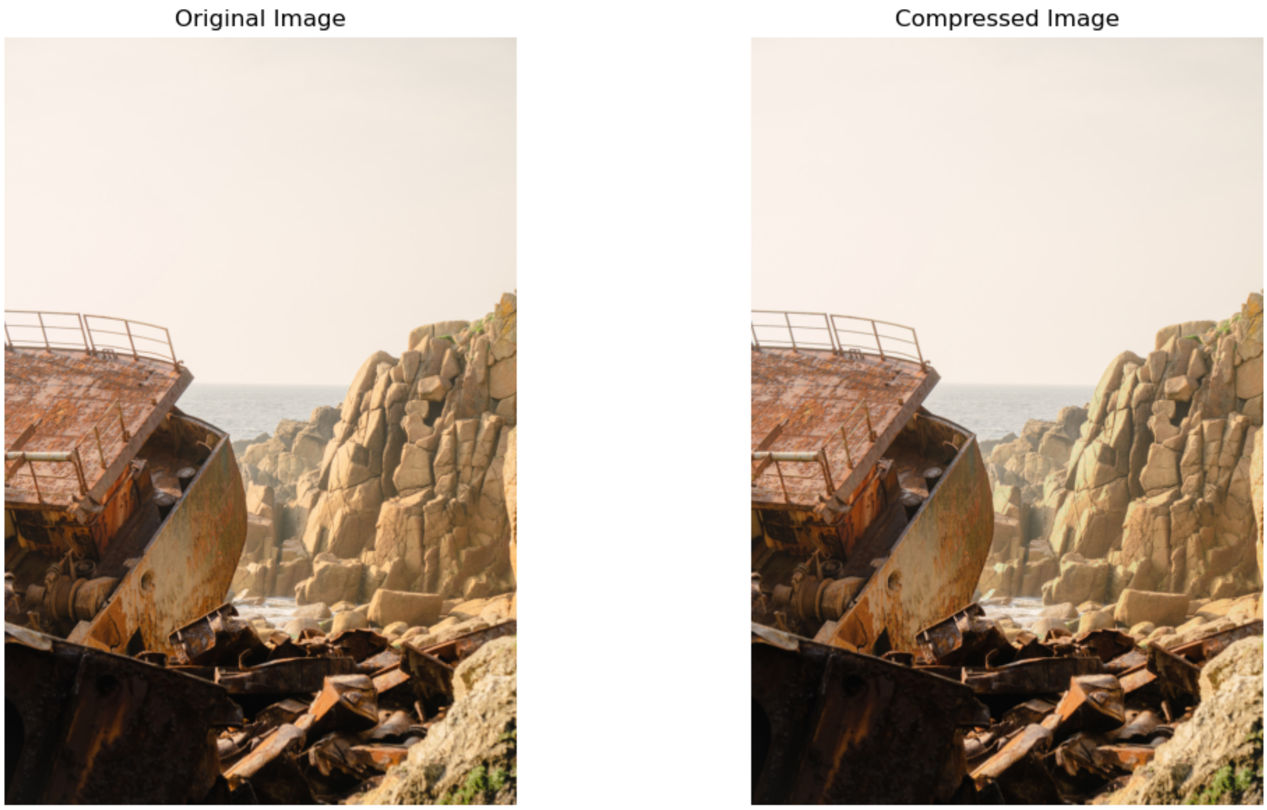

Figure 1: JPEG Compression of an image