Convergence analysis using steepest descent algorithm

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1 Introduction

This report presents an analysis of the Steepest Descent Algorithm applied to an optimization problem. The algorithm aims to find the optimal filter coefficients in a signal processing problem. We will describe the problem, the algorithm, and visualize its performance.

2 Problem Description

We consider an optimization problem where we want to find the filter coefficients $\mathbf{w} = [w_1, w_2]$ that minimize a cost function $J(\mathbf{w})$. The optimization problem can be formulated as follows:

$$\min_{\mathbf{w}} J(\mathbf{w}) = \sigma_d^2 - \mathbf{p}^T\mathbf{w} + \mathbf{w}^T\mathbf{R}\mathbf{w} \tag{1}$$

where

$$\begin{aligned}
\mathbf{w} &= [w_1, w_2] \text{ is the vector of filter coefficients,}\\
\sigma_d^2 &\text{ is the variance of the desired signal,}\\
\mathbf{p} &= [0, 0.2939] \text{ is the cross-correlation vector,}\\
\mathbf{R} &= \begin{bmatrix} 1 & 0.4045 \\ 0.4045 & 1 \end{bmatrix} \text{ is the autocorrelation matrix.}
\end{aligned}$$

We want to minimize $J(\mathbf{w})$ by adjusting the values of w_1 and w_2.

3 Algorithm Description

The Steepest Descent Algorithm is used to find the optimal filter coefficients $\mathbf{w}$. The algorithm iteratively updates the coefficients according to the following rule:

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu(\mathbf{R}\mathbf{w}_k - \mathbf{p}) \tag{2}$$

where

$$\begin{aligned}
\mathbf{w}_k &\text{ is the filter coefficients at iteration } k,\\
\mu &\text{ is the step size parameter.}
\end{aligned}$$

The algorithm continues for a specified number of iterations until convergence.

4 Results

The algorithm's performance is visualized in Figure 1. The figure shows the error performance surface and the convergence path of the Steepest Descent Algorithm.

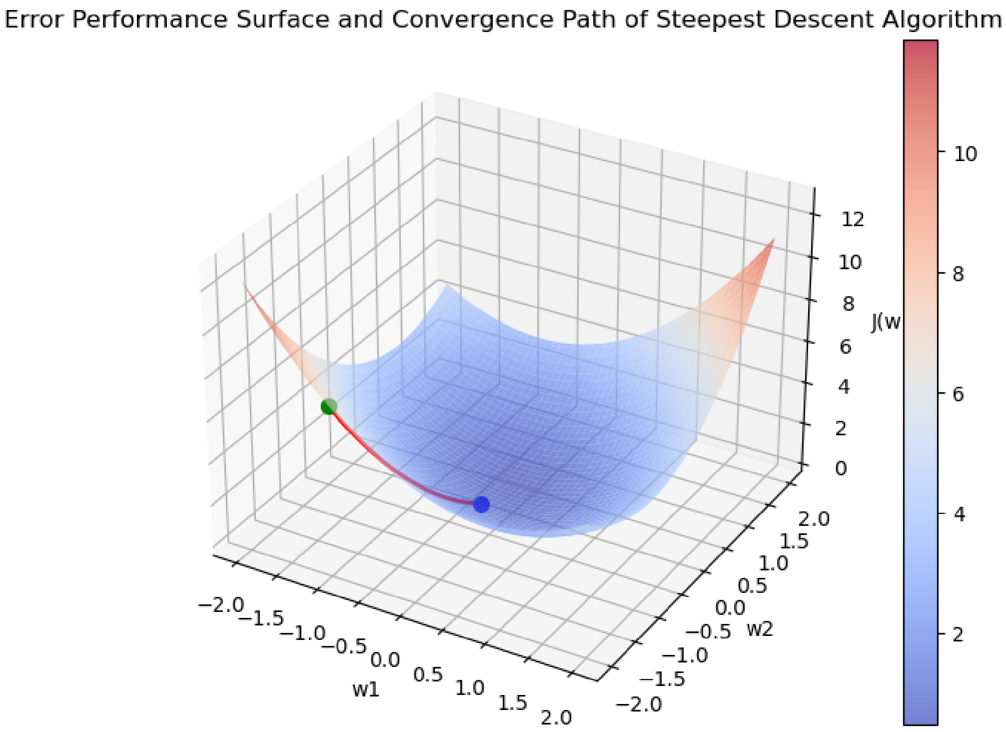

Figure 1: Error Performance Surface and Convergence Path of Steepest Descent Algorithm

5 Conclusion

The Steepest Descent Algorithm successfully finds the optimal filter coefficients $\mathbf{w}$ that minimize the cost function $J(\mathbf{w})$. This optimization technique can be applied to various signal-processing problems.