Optimal Wiener filter design

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2023

1 Introduction

This report discusses various signal processing tasks, including signal generation, noise modeling, and Wiener filtering, along with the calculation of mean square error (MSE).

2 Signal Generation and Noise Modeling

2.1 Signal Generation

We start by generating a signal $y[n]$ using the following equation:

$$y[n] = A\cos(w_0 n + \phi)$$

where:

$$\begin{aligned} A &= 0.5 \\ w_0 &= 2\pi \times 0.05 \\ \phi & \text{ is a random phase uniformly distributed in } [0, 2\pi] \end{aligned}$$

The generated signal $y[n]$ represents the original signal.

2.2 Noise Modeling

We introduce Gaussian noise to the original signal, resulting in a noisy signal $x[n]$. The noisy signal is given by:

$$x[n] = y[n] + v[n]$$

where $v[n]$ is Gaussian noise with a mean of 0 and a standard deviation of $\sigma_v = 0.5$.

3 Signal Plots

3.1 Original Signal

The plot below shows the original signal $y[n]$.

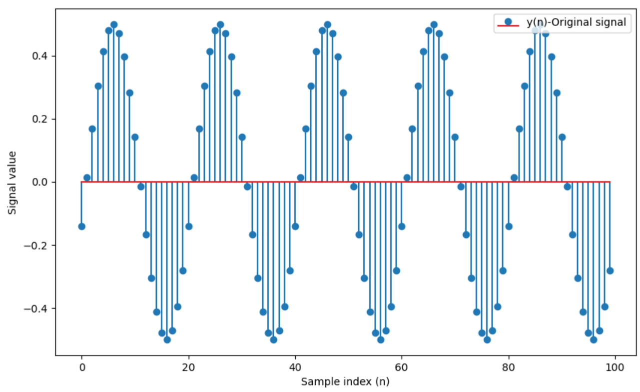

Figure 1: Original Signal $y[n]$

3.2 Noise Signal

The plot below shows the noisy signal $x[n]$, which is the original signal corrupted by Gaussian noise.

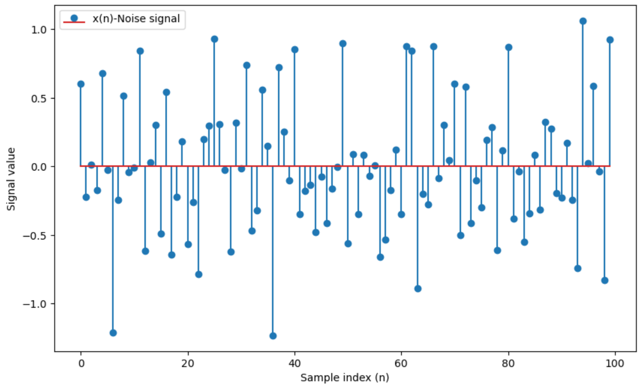

Figure 2: Noisy Signal $x[n]$

4 Wiener Filtering

To estimate the original signal $y[n]$ from the noisy signal $x[n]$, we employ the Wiener filter. The Wiener filter coefficients w are calculated using the following equations:

$$R_{hh}[k,l] = \frac{A^2}{2}\cos(w_0(k-l)) + \begin{cases} \sigma_v & \text{if } k = l \\ 0 & \text{if } k \neq l \end{cases}$$

The cross-correlation vector between the observed and original signals is given by:

$$R_{hy}[k] = \frac{A}{2}\cos(w_0 k)$$

The Wiener filter coefficients w can be calculated using the equation:

$$w = R_{hh}^{-1} R_{hy}$$

The Wiener filter coefficients for different filter lengths M are computed and displayed.

5 Signal Reconstruction and MSE Calculation

5.1 Signal Reconstruction

The noisy signal $x[n]$ is filtered using the Wiener filter coefficients to estimate the original signal $y[n]$. The filtered signal $y_h[n]$ is computed as follows:

$$y_h[n] = \sum_{i=0}^{N+M-1} \sum_{j=0}^{M-1} x[n-j]w[i]$$

6 Mean Square Error Calculation

The mean square error (MSE) is calculated to assess the performance of the Wiener filter for different filter lengths M. The MSE is defined as:

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y[i] - \hat{y}[i])^2$$

Where $\hat{y}[i]$ is the output of the Wiener filter for each sample.

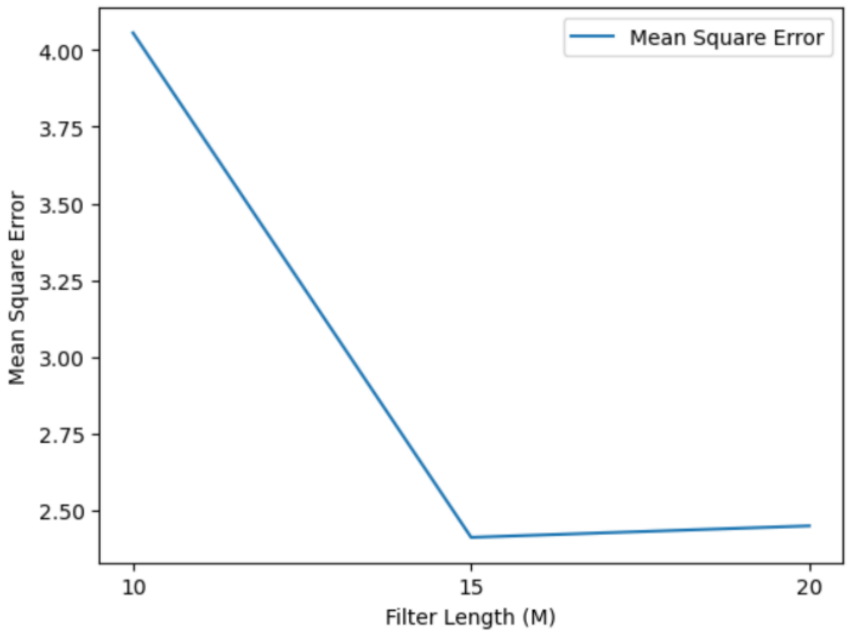

Figure 3: Mean Square Error for Different Filter Lengths

7 Conclusion

In this report, we have generated a desired signal, added noise to it, estimated Wiener filter coefficients, and evaluated the filter's performance using mean square error. The MSE results show the effectiveness of the Wiener filter in reducing noise from the signal.