

# Understanding Generative Adversarial Networks

Balaji Lakshminarayanan



*Joint work with:*

*Shakir Mohamed, Mihaela Rosca, Ivo Danihelka, David Warde-Farley,  
Liam Fedus, Ian Goodfellow, Andrew Dai & others*

# Problem statement

Learn a **generative model**:

$$\mathbf{x} = \mathcal{G}_{\theta}(\mathbf{z}'); \quad \mathbf{z}' \sim q(\mathbf{z})$$

**Goal**: given samples  $x_1 \dots x_n$  from true distribution  $p^*(x)$ , find  $\theta$

**$p_{\theta}(x)$  is not available** -> can't maximize density directly

However, we can **sample from  $p_{\theta}(x)$  efficiently**

# High level overview of GANs

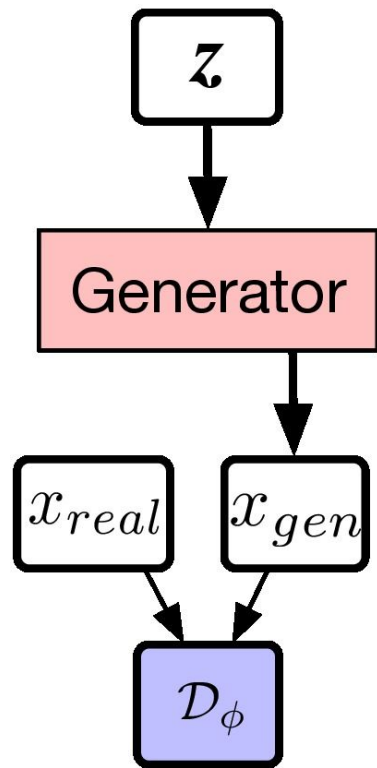
Goodfellow et al. 2014

**Discriminator:** Train a classifier to distinguish between the two distributions *using samples*

**Generator:** Train to generate samples that fool the discriminator

**Minimax game** alternates between training discriminator and generator

- Nash equilibrium corresponds to minima of Jensen Shannon divergence
- Need a bunch of tricks to stabilize training in practice



# GANs: Hope or Hype?



**Ferenc Huszar**  
@fhuszar

practicin' the alphabet with my son:

A is for AffGAN

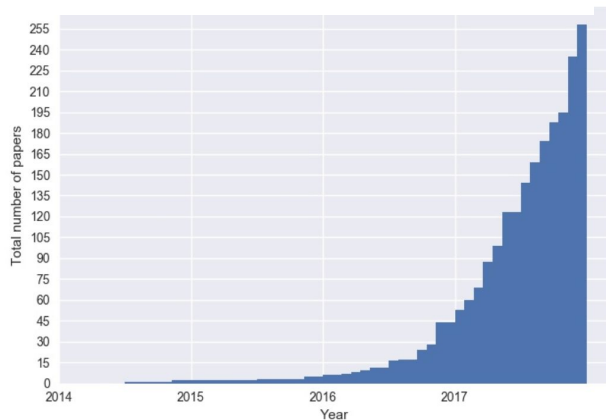
B is for B-GAN

C is for Conditional GAN

D is for DCGAN

E is for EBGAN

F is for f-GAN



<https://github.com/hindupuravinash/the-gan-zoo>



<https://github.com/junyanz/CycleGAN/blob/master/imgs/horse2zebra.gif>



[https://github.com/tkarras/progressive\\_growing\\_of\\_gans](https://github.com/tkarras/progressive_growing_of_gans)

# How do GANs relate to other ideas in probabilistic machine learning?

**Learning in implicit generative models**

*Shakir Mohamed\* and Balaji Lakshminarayanan\**

# Implicit Models

Stochastic procedure that generates data



$$\mathbf{x} = \mathcal{G}(\mathbf{z}'), \quad \mathbf{z}' \sim q(\mathbf{z})$$

$$q(\mathbf{x}) = \frac{\partial}{\partial x_1} \cdots \frac{\partial}{\partial x_d} \int_{\{\mathcal{G}(\mathbf{z}) \leq \mathbf{x}\}} q(\mathbf{z}) d^m \mathbf{z}$$

Examples: stochastic simulators of complex physical systems (climate, ecology, high-energy physics etc)

# Prescribed Models

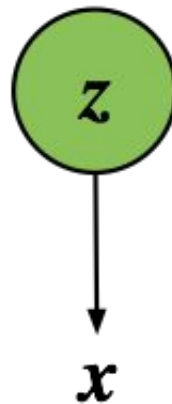
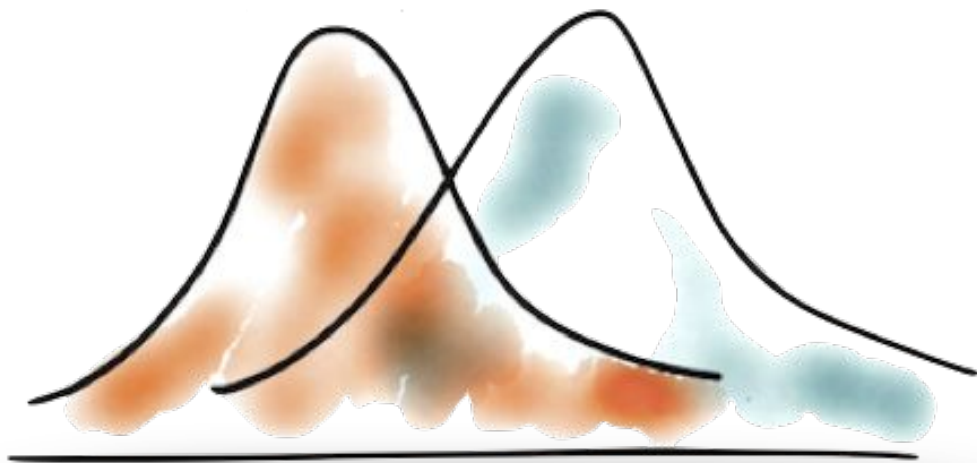
Provide knowledge of the probability of observations & specify a *conditional* log-likelihood function.



$$\mathbf{x}' \sim p(\mathbf{x} | \mathcal{G}(\mathbf{z}')), \quad \mathbf{z}' \sim q(\mathbf{z})$$

$$q(\mathbf{x}) = \int p(\mathbf{x} | \mathcal{G}(\mathbf{z})) q(\mathbf{z})$$

# Learning by Comparison



We compare the estimated distribution to the true distribution **using samples.**

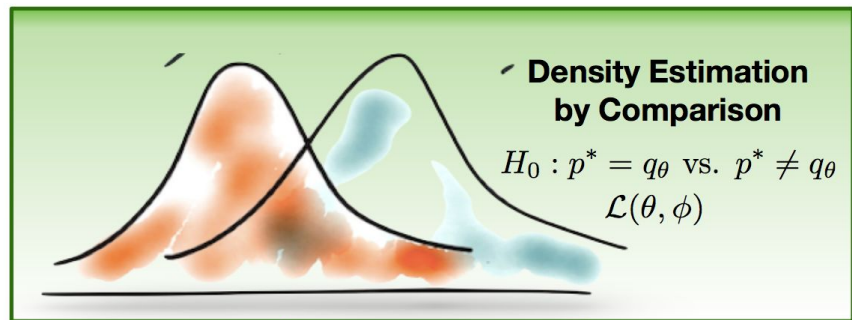
# Learning by Comparison

## Comparison

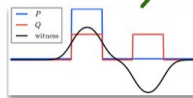
Use a hypothesis **test or comparison** to build an auxiliary model to indicate how data simulated from the model differs from observed data.

## Learning generator

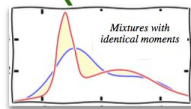
**Adjust model parameters** to better match the data distribution using the comparison.



**Density Difference**  
 $r_\phi = p^* - q_\theta$

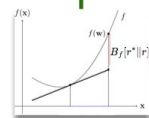


Max Mean Discrepancy

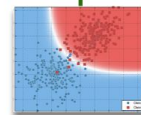


Moment Matching

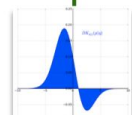
**Density Ratio**  
 $r_\phi = \frac{p^*}{q_\theta}$



Bregman Divergence



Class Probability Estimation



f-Divergence

$f(u) = u \log u - (u + 1) \log(u + 1)$



# High-level idea

Define a joint loss function  $L(\varphi, \theta)$  and alternate between:

**Comparison loss** (“discriminator”):  $\arg \min_{\varphi} L(\varphi, \theta)$

**Generative loss**:  $\arg \min_{\theta} -L(\varphi, \theta)$

Global optimum is  $q_{\theta} = p^*$  and

- Density ratio  $r_{\varphi} = 1$  or
- Density difference  $r_{\varphi} = 0$

# How do we compare distributions?

# Density Ratios and Classification

**Density  
Ratio**

$$\frac{p^*(\mathbf{x})}{q(\mathbf{x})}$$

**Bayes'  
Rule**

$$p(\mathbf{x}|y) = \frac{p(y|\mathbf{x})p(\mathbf{x})}{p(y)}$$

**Combine data**

**Assign labels**

	Real Data	Simulated Data
$\{\mathbf{x}_1, \dots, \mathbf{x}_N\} =$	$\{\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_{\hat{n}}\}$	$\{\tilde{\mathbf{x}}_1, \dots, \tilde{\mathbf{x}}_{\tilde{n}}\}$
$\{y_1, \dots, y_N\} =$	$\{+1, \dots, +1\}$	$\{-1, \dots, -1\}$

# Density Ratios and Classification

**Density ratio**

$$\frac{p^*(\mathbf{x})}{q(\mathbf{x})} = \frac{p(\mathbf{x}|y = 1)}{p(\mathbf{x}|y = -1)}$$

**Bayes' substitution**

$$= \frac{p(y = +1|\mathbf{x})p(\mathbf{x})}{p(y = +1)} \bigg/ \frac{p(y = -1|\mathbf{x})p(\mathbf{x})}{p(y = -1)}$$

**Class probability**

$$\frac{p^*(\mathbf{x})}{q(\mathbf{x})} = \frac{p(y = 1|\mathbf{x})}{p(y = -1|\mathbf{x})}$$

**Computing a density ratio is equivalent to class probability estimation.**

# Class Probability Estimation

$$\mathcal{D}(\mathbf{x}; \phi) = p(\mathbf{y} = +1|\mathbf{x}) = \frac{r}{r+1}$$

Loss	Objective Function ( $\mathcal{D} := \mathcal{D}(\mathbf{x}; \phi)$ )
Bernoulli loss	$\pi \mathbb{E}_{p^*(\mathbf{x})}[-\log \mathcal{D}] + (1 - \pi) \mathbb{E}_{q_\theta(\mathbf{x})}[-\log(1 - \mathcal{D})]$
Brier score	$\pi \mathbb{E}_{p^*(\mathbf{x})}[(1 - \mathcal{D})^2] + (1 - \pi) \mathbb{E}_{q_\theta(\mathbf{x})}[\mathcal{D}^2]$
Exponential loss	$\pi \mathbb{E}_{p^*(\mathbf{x})} \left[ \left( \frac{1-\mathcal{D}}{\mathcal{D}} \right)^{\frac{1}{2}} \right] + (1 - \pi) \mathbb{E}_{q_\theta(\mathbf{x})} \left[ \left( \frac{\mathcal{D}}{1-\mathcal{D}} \right)^{\frac{1}{2}} \right]$
Misclassification	$\pi \mathbb{E}_{p^*(\mathbf{x})}[\mathbb{I}[\mathcal{D} \leq 0.5]] + (1 - \pi) \mathbb{E}_{q_\theta(\mathbf{x})}[\mathbb{I}[\mathcal{D} > 0.5]]$
Hinge loss	$\pi \mathbb{E}_{p^*(\mathbf{x})} \left[ \max \left( 0, 1 - \log \frac{\mathcal{D}}{1-\mathcal{D}} \right) \right] + (1 - \pi) \mathbb{E}_{q_\theta(\mathbf{x})} \left[ \max \left( 0, 1 + \log \frac{\mathcal{D}}{1-\mathcal{D}} \right) \right]$
Spherical	$\pi \mathbb{E}_{p^*(\mathbf{x})}[-\alpha \mathcal{D}] + (1 - \pi) \mathbb{E}_{q_\theta(\mathbf{x})}[-\alpha(1 - \mathcal{D})]; \quad \alpha = (1 - 2\mathcal{D} + 2\mathcal{D}^2)^{-1/2}$

Table 1. Proper scoring rules that can be minimised in class probability-based learning of implicit generative models.

Other loss functions for training classifier, e.g. Brier score leads to LS-GAN

Related: Unsupervised as Supervised Learning, Classifier ABC

# Divergence minimization (f-GAN)

$$\begin{aligned} D_f [p^*(\mathbf{x}) \| q_\theta(\mathbf{x})] &= \int q_\theta(\mathbf{x}) f \left( \frac{p^*(\mathbf{x})}{q_\theta(\mathbf{x})} \right) d\mathbf{x} \\ &= \mathbb{E}_{q_\theta(\mathbf{x})} [f(r(\mathbf{x}))] \\ &\geq \sup_t \mathbb{E}_{p^*(\mathbf{x})} [t(\mathbf{x})] - \mathbb{E}_{q_\theta(\mathbf{x})} [f^\dagger(t(\mathbf{x}))] \end{aligned}$$

Minimize a lower bound on f-divergence between  $p^*$  and  $q_\theta$

Choices of  $f$  recover  $\text{KL}(p^* \| q)$  (maximum likelihood),  $\text{KL}(q \| p^*)$  and  $\text{JS}(p^* \| q)$

Can use different f-divergences for learning ratio vs learning generator

# Density ratio estimation

$$\begin{aligned} B_f(r^*(\mathbf{x}) \| r_\phi(\mathbf{x})) &= \int \left( f(r^*(\mathbf{x})) - f(r_\phi(\mathbf{x})) - f'(r_\phi(\mathbf{x})) [r^*(\mathbf{x}) - r_\phi(\mathbf{x})] \right) q_\theta(\mathbf{x}) d\mathbf{x} \\ &= \mathbb{E}_{q_\theta(\mathbf{x})} [r_\phi(\mathbf{x}) f'(r_\phi(\mathbf{x})) - f(r_\phi(\mathbf{x}))] - \mathbb{E}_{p^*} [f'(r_\phi(\mathbf{x}))] + D_f[p^*(\mathbf{x}) \| q_\theta(\mathbf{x})] \\ &= \mathcal{L}_B(r_\phi(\mathbf{x})) + D_f[p^*(\mathbf{x}) \| q_\theta(\mathbf{x})] \end{aligned}$$

Optimize a Bregman divergence between  $r^*$  and  $r_\phi$

Special cases include least squares importance fitting (LSIF)

Ratio loss ends up being identical to that of f-divergence

# Moment-matching

$$\begin{aligned}\mathcal{L}(\phi, \theta) &= (\mathbb{E}_{p^*(\mathbf{x})}[s(\mathbf{x})] - \mathbb{E}_{q_\theta(\mathbf{x})}[s(\mathbf{x})])^2 \\ &= (\mathbb{E}_{p^*(\mathbf{x})}[s(\mathbf{x})] - \mathbb{E}_{q(\mathbf{z})}[s(\mathcal{G}(\mathbf{z}; \theta))])^2\end{aligned}$$

Used by

- Generative moment matching networks
- Training generative neural networks via Maximum Mean Discrepancy optimization

Connects to optimal transport literature (e.g. Wasserstein GAN)



# Summary of the approaches

## Class probability estimation

- Build a classifier to distinguish real from fake samples.
- Original GAN solution.

## Density ratio matching

- Directly minimise the expected error between the true ratio and an estimate of it.

## Divergence Minimisation

- Minimise a generalised divergence between the true density  $p^*$  and the product  $r(x)q(x)$ .
- $f$ -GAN approach.

## Moment matching

- Match the moments between  $p^*$  and  $r(x)q(x)$
- MMD, optimal transport, etc.

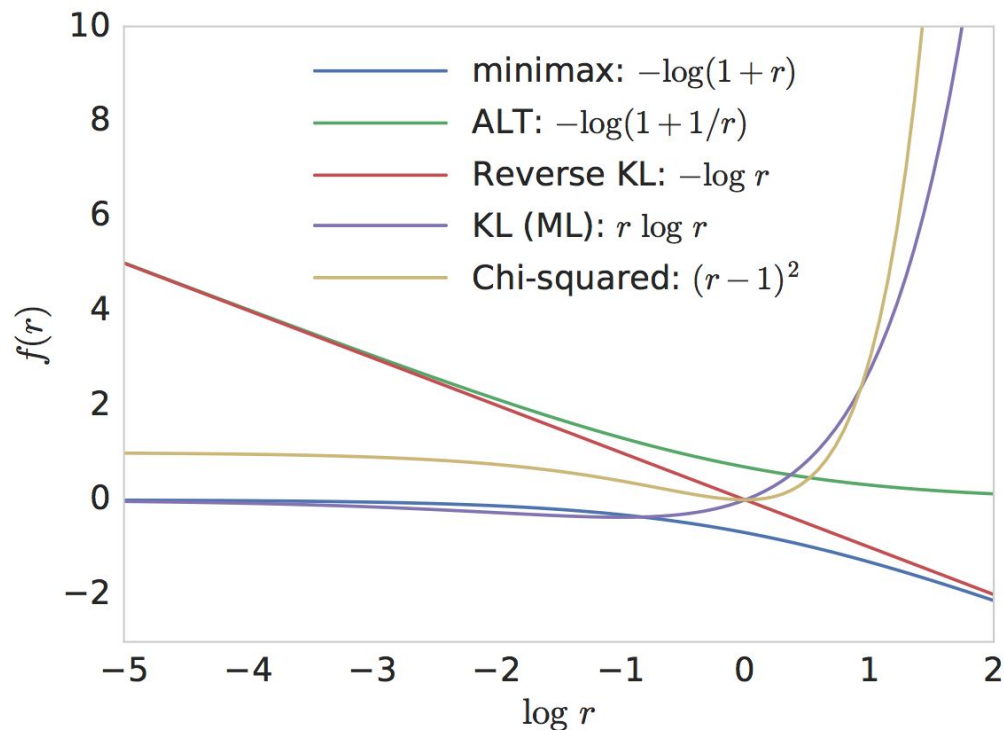
# How do we learn generator?

In GANs, the generator is **differentiable**

- Generator loss is of the following form e.g. f-divergence  $D_f = E_q [f(r)]$
- Can apply **re-parametrization trick**

$$\begin{aligned} J &= E_{q_{\theta}(\mathbf{x})}[\ell(\mathbf{x})] = E_{q(\mathbf{z})}[\ell(\mathcal{G}_{\theta}(\mathbf{z}))] \\ \nabla_{\theta} J &= \nabla_{\theta} E_{q(\mathbf{z})}[\ell(\mathcal{G}_{\theta}(\mathbf{z}))] = E_{q(\mathbf{z})}[\nabla_{\theta} \ell(\mathcal{G}_{\theta}(\mathbf{z}))] \end{aligned}$$

# Choice of f-divergence



Density ratio estimation literature has investigated choices of  $f$

However, that's only half of the puzzle. We need non-zero gradients for  $D_f = E_q[f(r)]$  to learn generator

- $r \ll 1$  early on in training
- **Non-saturating alternative loss**

We also need additional constraints on the discriminator

Figure 2. Objective functions for different choices of  $f$ .

# Summary: Learning in Implicit Generative Models

Unifying view\* of GANs that connects to literature on

- Density ratio estimation
  - ... but they don't focus on learning generator
- Approximate Bayesian computation (ABC) and likelihood-free inference
  - Low dimensional, better understanding of theory
  - Bayesian inference over parameters
  - Simulators are usually not differentiable (can we approximate them?)

Motivates new loss functions: [can decouple generator loss from discriminator loss](#)

GAN-like ideas can be used in other places where density ratio appears

# Comparing GANs to Maximum Likelihood training using Real-NVP

Comparison of maximum likelihood and GAN-based training of Real NVPs

*Ivo Danihelka, Balaji Lakshminarayanan, Benigno Uria, Daan Wierstra and Peter Dayan*

# Generative Models and Algorithms

## Model

**Prescribed Models**  
Directed latent variable  
models, DLGM, state  
space

## Inference

**Maximum Marginal  
Likelihood**  
Variational Inference

## Algorithm

**VAE**  
Lower bound on  
likelihood

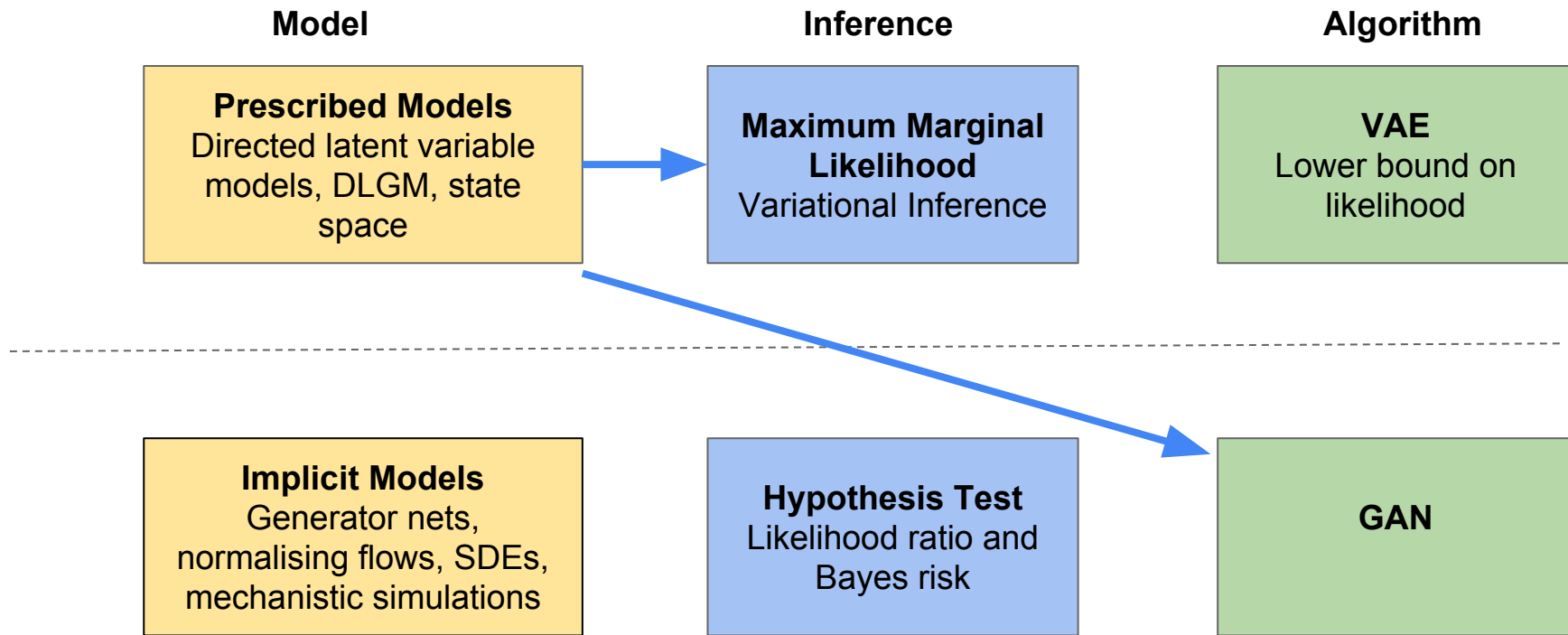
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**Implicit Models**  
Generator nets,  
normalising flows, SDEs,  
mechanistic simulations

**Hypothesis Test**  
Likelihood ratio and  
Bayes risk

**GAN**

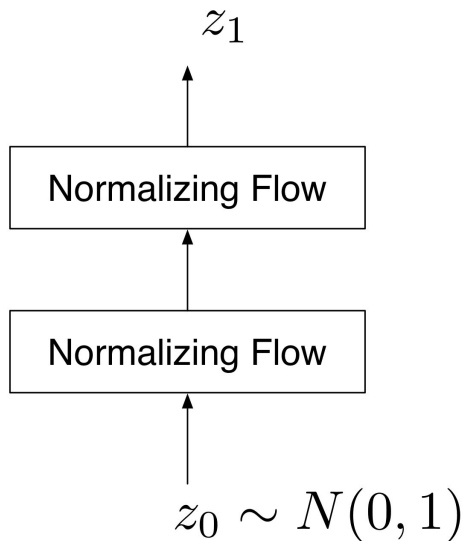
# Generative Models and Algorithms



# Comparing inference algorithms for a fixed model

Generator is Real NVP ([Dinh et al., 2016](#))

$$\log P(z_1) = \log P(z_0) - \log \left| \det \frac{dz_1}{dz_0} \right|$$



1. Train by maximum likelihood (MLE).
2. Train a generator by Wasserstein GAN.
3. Compare.

Complementary to “On the quantitative analysis of decoder-based models” by Wu et al., 2017



# Wasserstein GAN

For general distributions:

$$W_d(P_r, P_g) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x_r \sim P_r} [f(x_r)] - \mathbb{E}_{x_g \sim P_g} [f(x_g)]$$

Considering all 1-Lipschitz function  
(i.e., functions with **bounded derivatives**).

$f(x)$  is a “critic”.  
The critic should give  
high value to real samples and  
low value to generated samples.

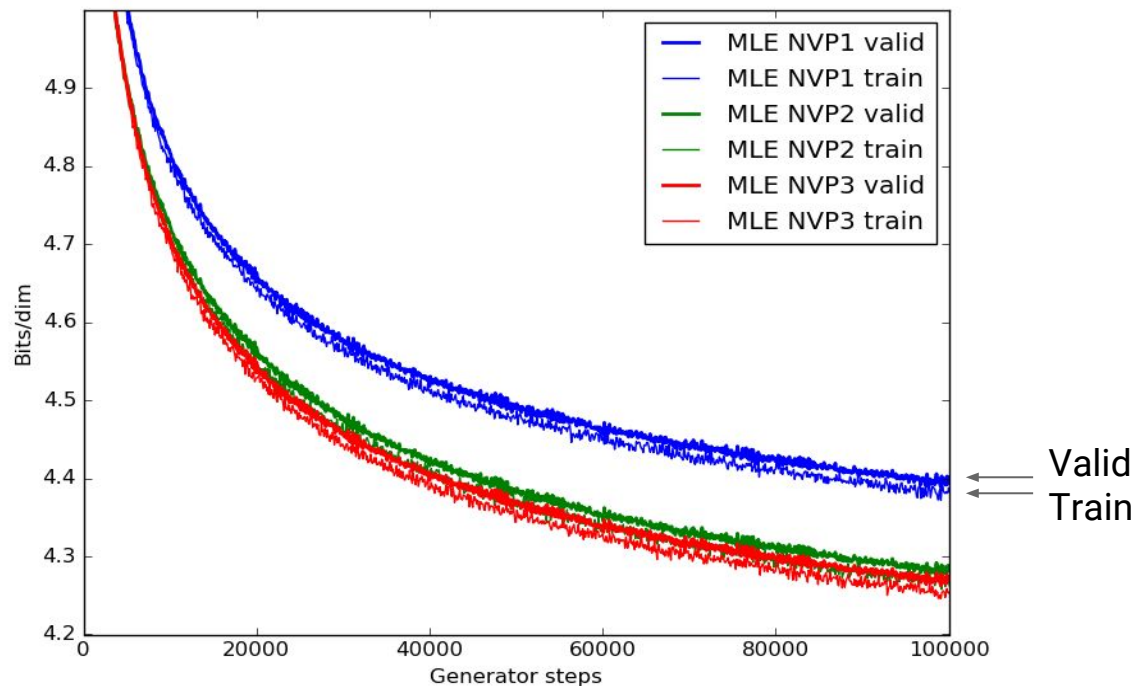
Bounded by:

- a) Weight clipping (Wasserstein GAN; “WGAN”).
- b) Gradient penalty (Improved Training; “WGAN-GP”)

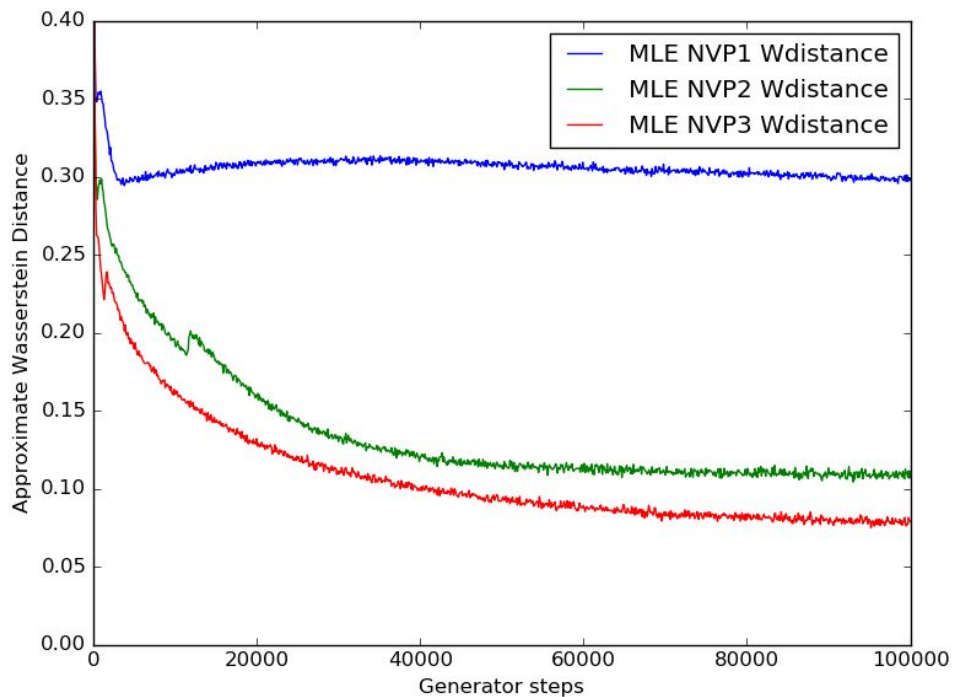
**Idea:** use an independent Wasserstein critic to evaluate generators

# Bits/dim for NVP

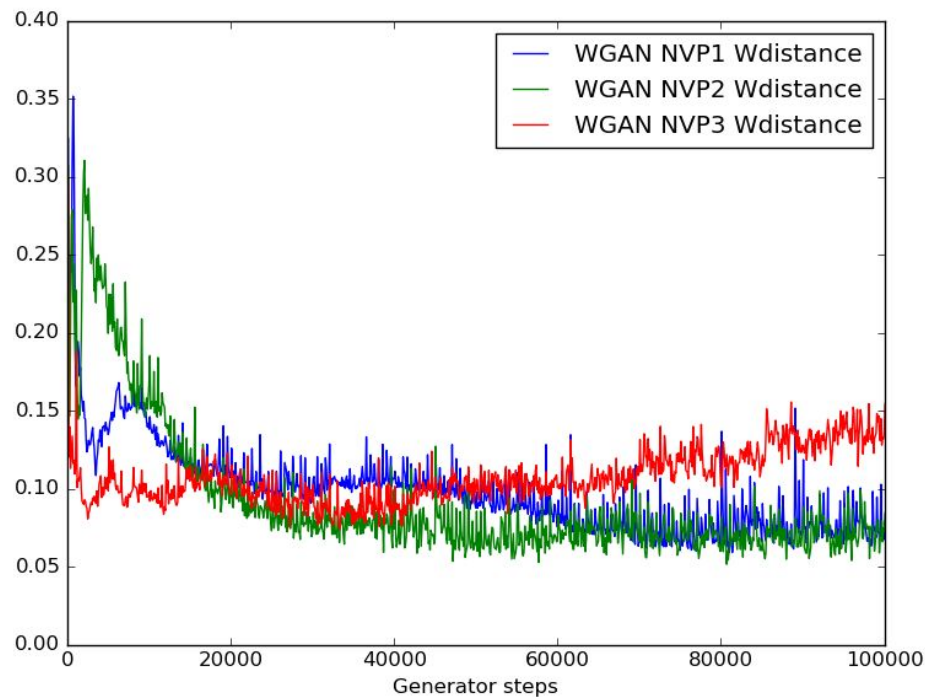
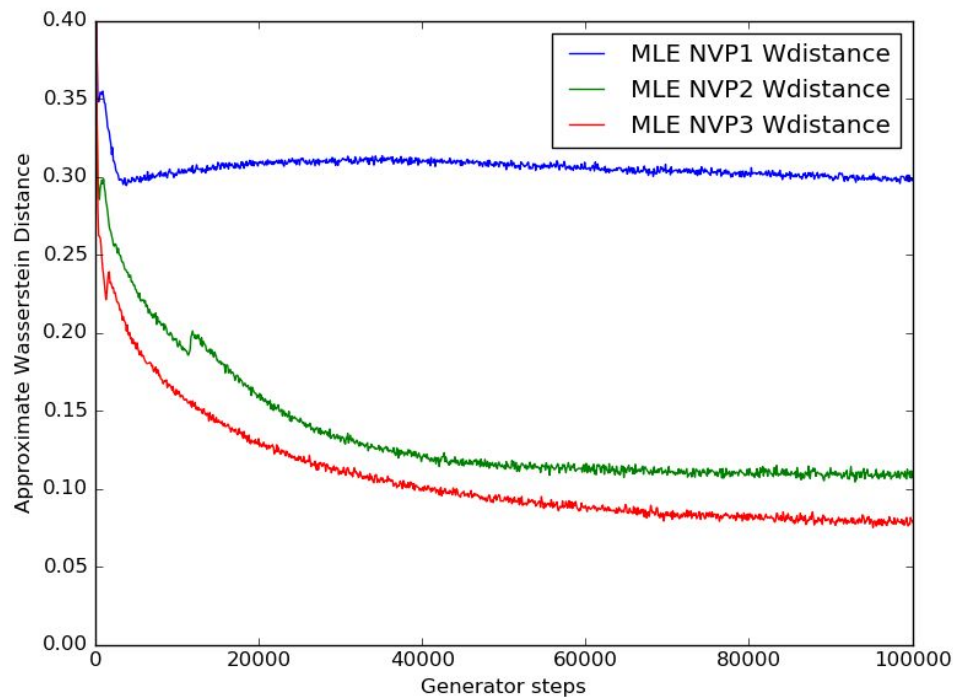
Dataset:  
CelebA 32x32.



# Wasserstein Distance for NVPs



# Wasserstein Distance Minimized by WGAN



# MLE vs. WGAN Training

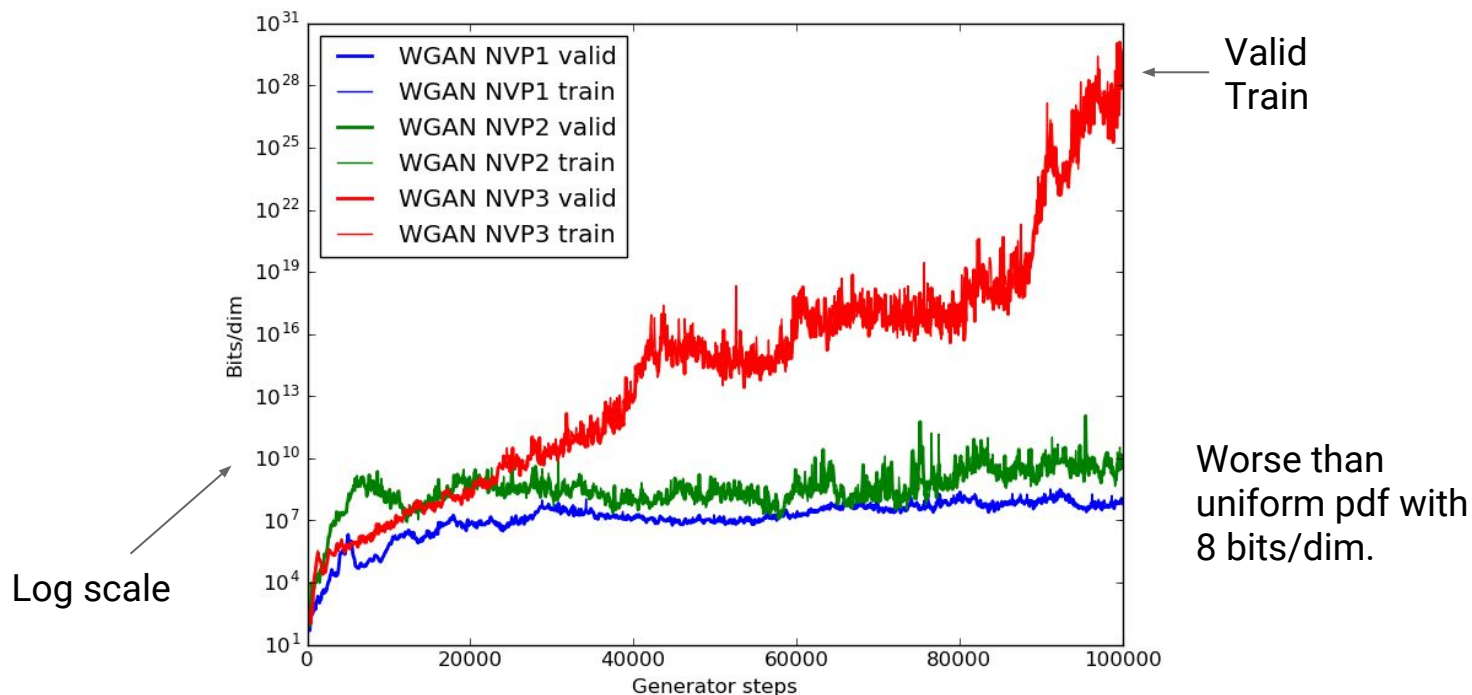




# MLE vs. WGAN Training (shallower generator)

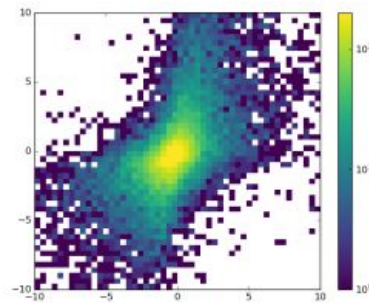
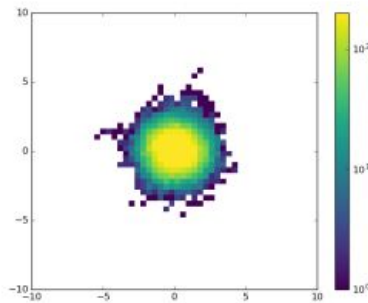


# Bits/dim for NVPs Trained by WGAN



# Summary

- Wasserstein distance can compare models.
- Wasserstein distance can be approximated by training a critic.
- Training by WGAN leads to nicer samples but significantly worse log-probabilities.
- Latent codes from WGAN training are non-Gaussian



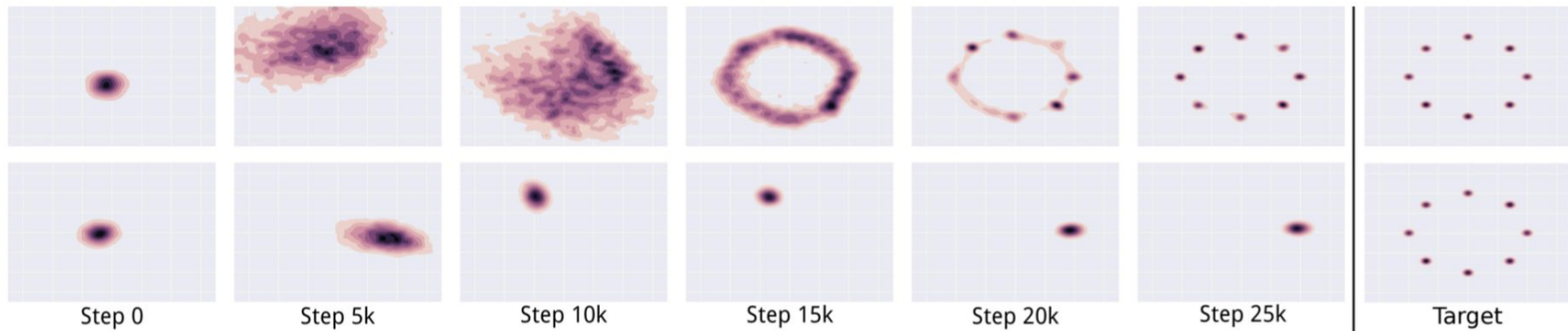


# How do we combine VAEs and GANs to get the best of both worlds?

**Variational approaches for auto-encoding generative adversarial networks**

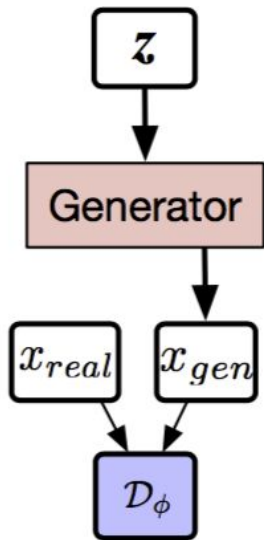
*Mihaela Rosca\*, Balaji Lakshminarayanan\*, David Warde-Farley and Shakir Mohamed*

# Motivating problem: Mode collapse

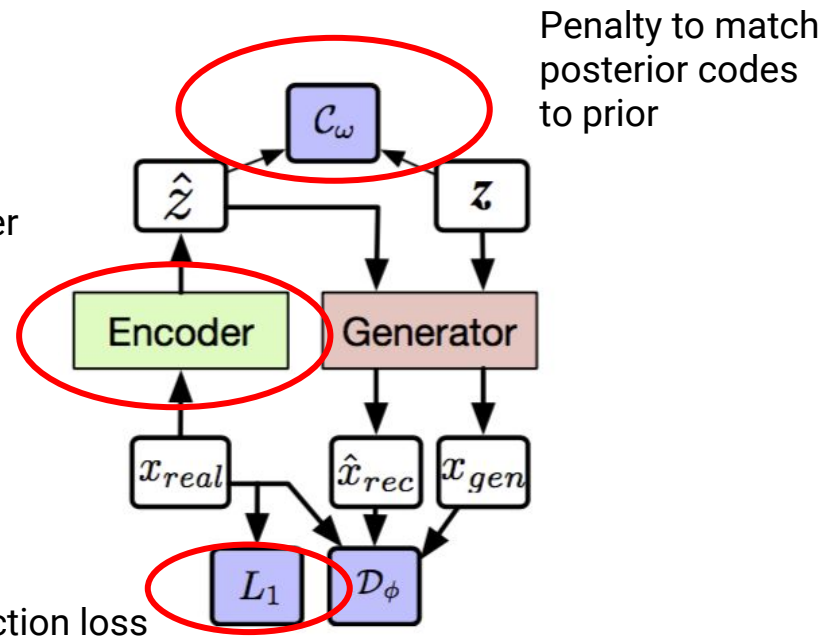


- MoG toy example from “Unrolled GAN” paper
- VAEs have other problems, but do not suffer from mode-collapse
  - Can we add auto-encoder to GANs?

# Adding auto-encoder to GANs

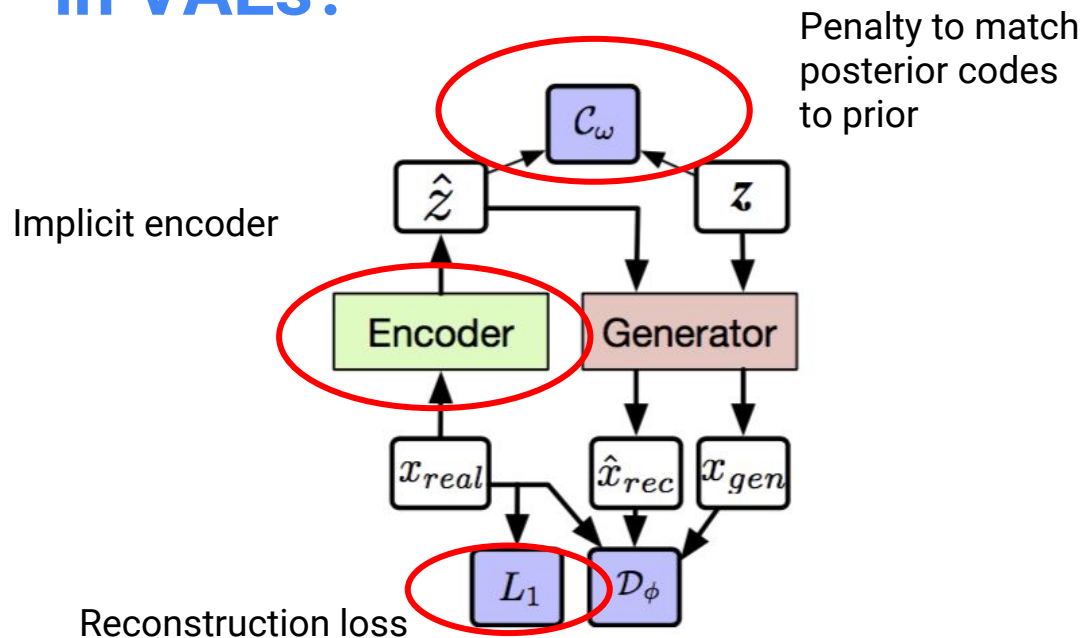


Implicit encoder



Reconstruction loss

# How does it relate to Evidence Lower Bound (ELBO) in VAEs?

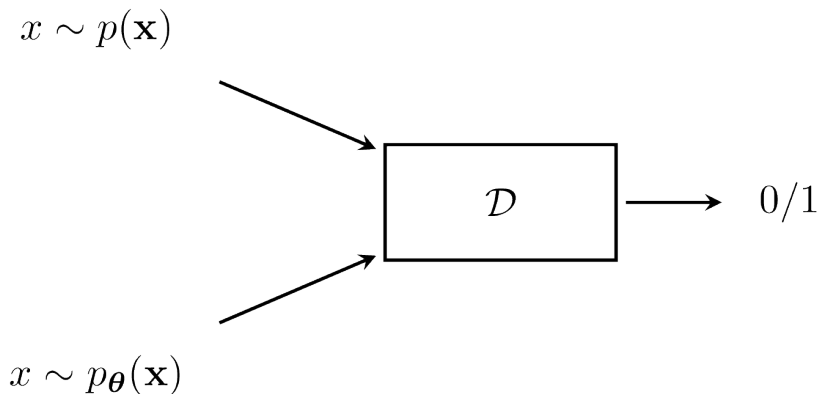


$$\log p_{\theta}(\mathbf{x}) = \log \int p_{\theta}(\mathbf{x}|\mathbf{z})p(\mathbf{z})d\mathbf{z} \geq \mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})] - \text{KL}[q_{\eta}(\mathbf{z}|\mathbf{x})||p(\mathbf{z})]$$

# Recap: Density ratio trick

Estimate the ratio of two distributions only from samples, by building a binary **classifier** to distinguish between them.

$$\frac{p(\mathbf{x})}{p_{\theta}(\mathbf{x})} = \frac{\mathcal{D}(x)}{1 - \mathcal{D}(x)}$$



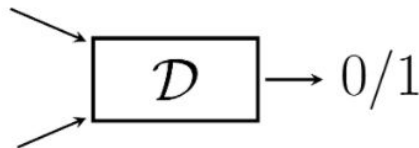
# Revisiting ELBO in Variational Auto-Encoders

$$\log p_{\theta}(\mathbf{x}) = \log \int p_{\theta}(\mathbf{x}|\mathbf{z})p(\mathbf{z})d\mathbf{z} \geq \mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})] - \text{KL}[q_{\eta}(\mathbf{z}|\mathbf{x})||p(\mathbf{z})]$$

LIKELIHOOD TERM

$$\begin{aligned}\mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})] &= \mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})}[\log(\frac{p_{\theta}(\mathbf{x}|\mathbf{z})}{p(\mathbf{x})}p(\mathbf{x}))] \\ &= \mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})}[\underbrace{\log \frac{p_{\theta}(\mathbf{x}|\mathbf{z})}{p(\mathbf{x})}}_{\text{ratio}}] + \underbrace{\mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})}[\log p(\mathbf{x})]}_{\text{constant}}\end{aligned}$$

$$\mathbf{x} \sim p^*(\mathbf{x})$$



$$\mathbf{x} \sim p_{\theta}(\mathbf{x})$$

# Revisiting ELBO in Variational Auto-Encoders

$$\log p_{\theta}(\mathbf{x}) = \log \int p_{\theta}(\mathbf{x}|\mathbf{z})p(\mathbf{z})d\mathbf{z} \geq \mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})] - \text{KL}[q_{\eta}(\mathbf{z}|\mathbf{x})||p(\mathbf{z})]$$

THE KL TERM

$$-\text{KL}[q_{\eta}(\mathbf{z}|\mathbf{x})||p(\mathbf{z})] = \mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})} \left[ \log \frac{p(\mathbf{z})}{q_{\eta}(\mathbf{z}|\mathbf{x})} \right]$$

$$\mathbf{z} \sim p(\mathbf{z})$$



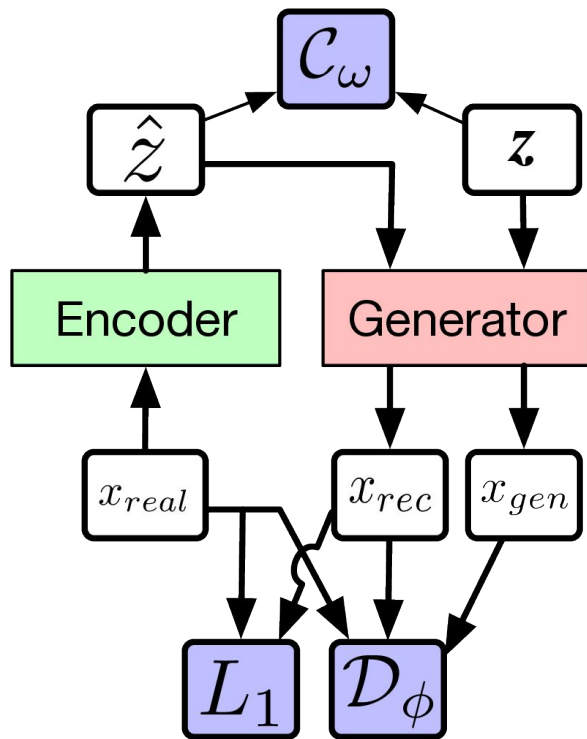
$\rightarrow 0/1$

$$\mathbf{z} \sim q_{\eta}(\mathbf{z}|\mathbf{x})$$

Encoder can be implicit!

More flexible distributions

# Putting it all together



$$-\text{KL}[q_\eta(\mathbf{z}|\mathbf{x})||p(\mathbf{z})] \approx \mathbb{E}_{q_\eta(\mathbf{z}|\mathbf{x})} \left[ \log \frac{\mathcal{C}_\omega(\mathbf{z})}{1 - \mathcal{C}_\omega(\mathbf{z})} \right]$$

$$\mathbb{E}_{q_\eta(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] \approx \left[ \log \frac{\mathcal{D}_\phi(\mathcal{G}_\theta(\mathbf{z}))}{1 - \mathcal{D}_\phi(\mathcal{G}_\theta(\mathbf{z}))} \right]$$

$$\mathbb{E}_{q_\eta(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] \approx \mathbb{E}_{q_\eta(\mathbf{z}|\mathbf{x})} [-\lambda ||\mathbf{x} - \mathcal{G}_\theta(\mathbf{z})||_1]$$



## Combining VAEs and GANs

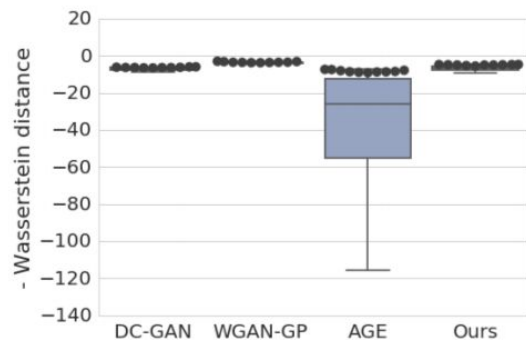
$$\log p_{\theta}(\mathbf{x}) = \log \int p_{\theta}(\mathbf{x}|\mathbf{z})p(\mathbf{z})d\mathbf{z} \geq \mathbb{E}_{q_{\eta}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})] - \text{KL}[q_{\eta}(\mathbf{z}|\mathbf{x})||p(\mathbf{z})]$$

- Likelihood: Reconstruction vs “synthetic likelihood” term
- KL: Analytical vs “code discriminator”
- Can recover various hybrids of VAEs and GANs

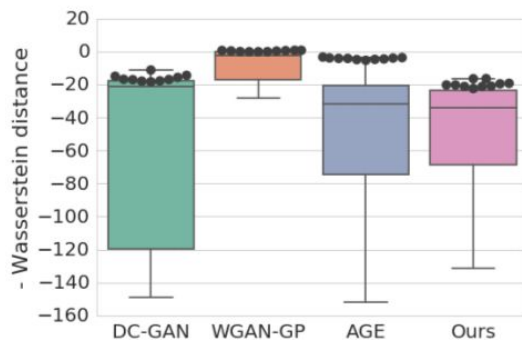
Algorithm	Likelihood		Prior		
	Observer	Ratio estimator ("synthetic")	KL (analytic)	KL (approximate)	Ratio estimator
VAE	✓		✓		
DCGAN		✓			
VAE-GAN	✓	*	✓		
Adversarial-VB	✓				✓
AGE	✓			✓	
$\alpha$ -GAN (ours)	✓	✓			✓

Table 1: Comparison of different approaches for training generative latent variable models.

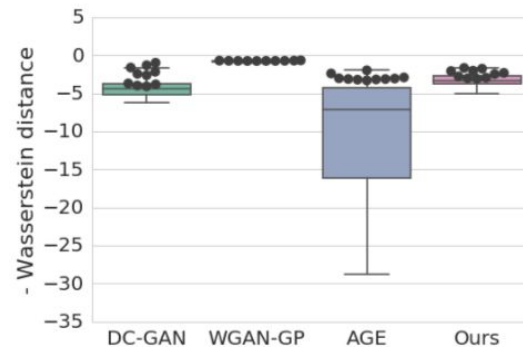
# Evaluating different variants



(a) ColorMNIST



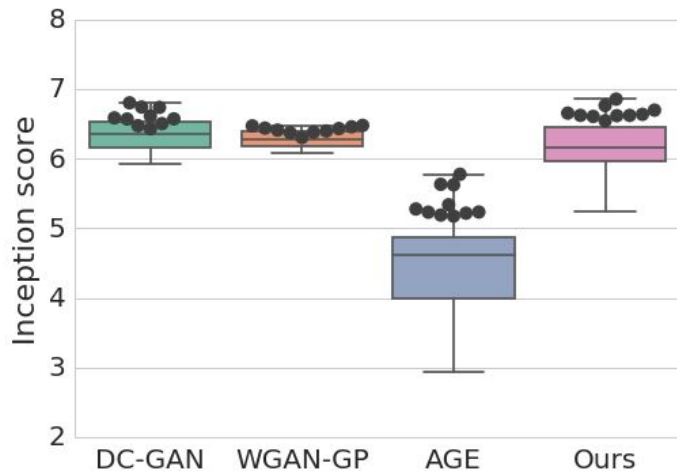
(b) CelebA



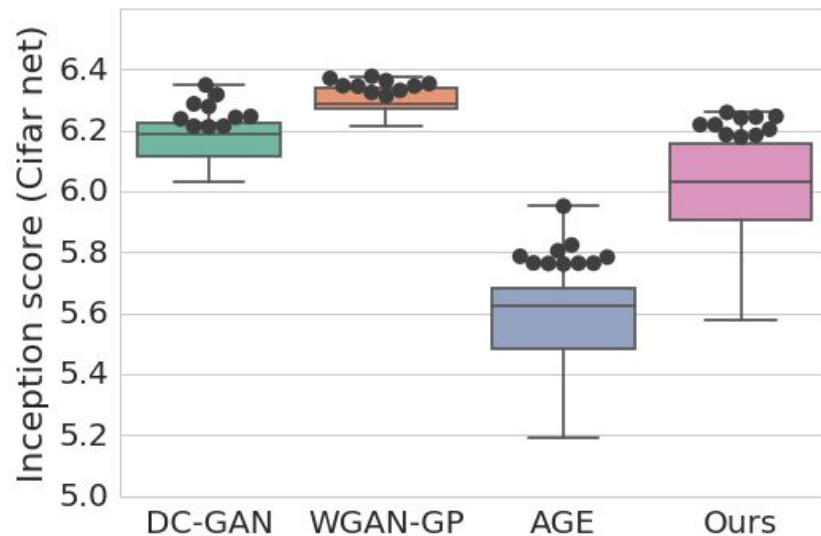
(c) CIFAR-10

Our VAE-GAN hybrid is competitive with state-of-the-art GANs

# Cifar10 - Inception score



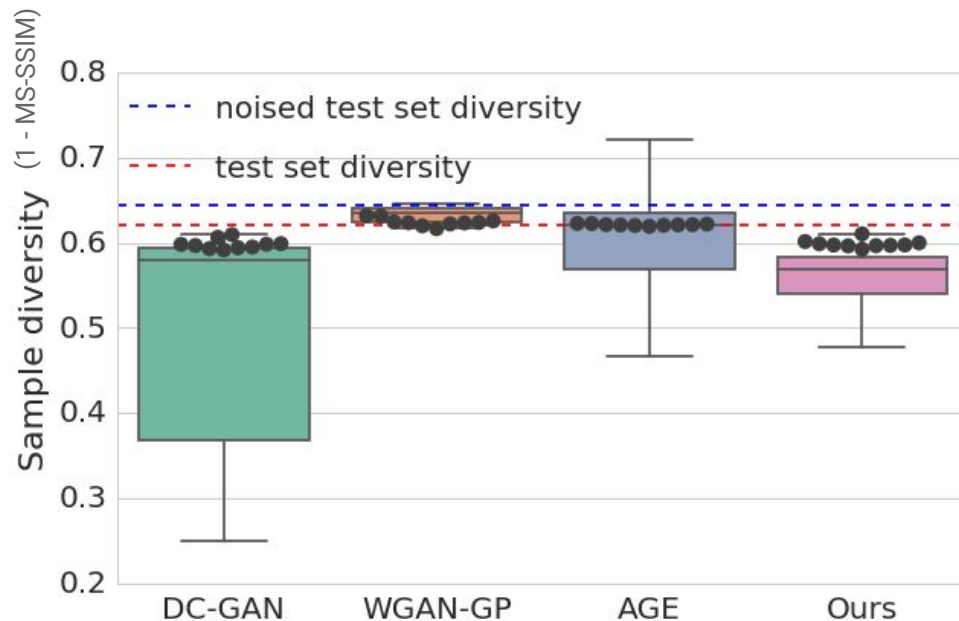
Classifier trained on Imagenet



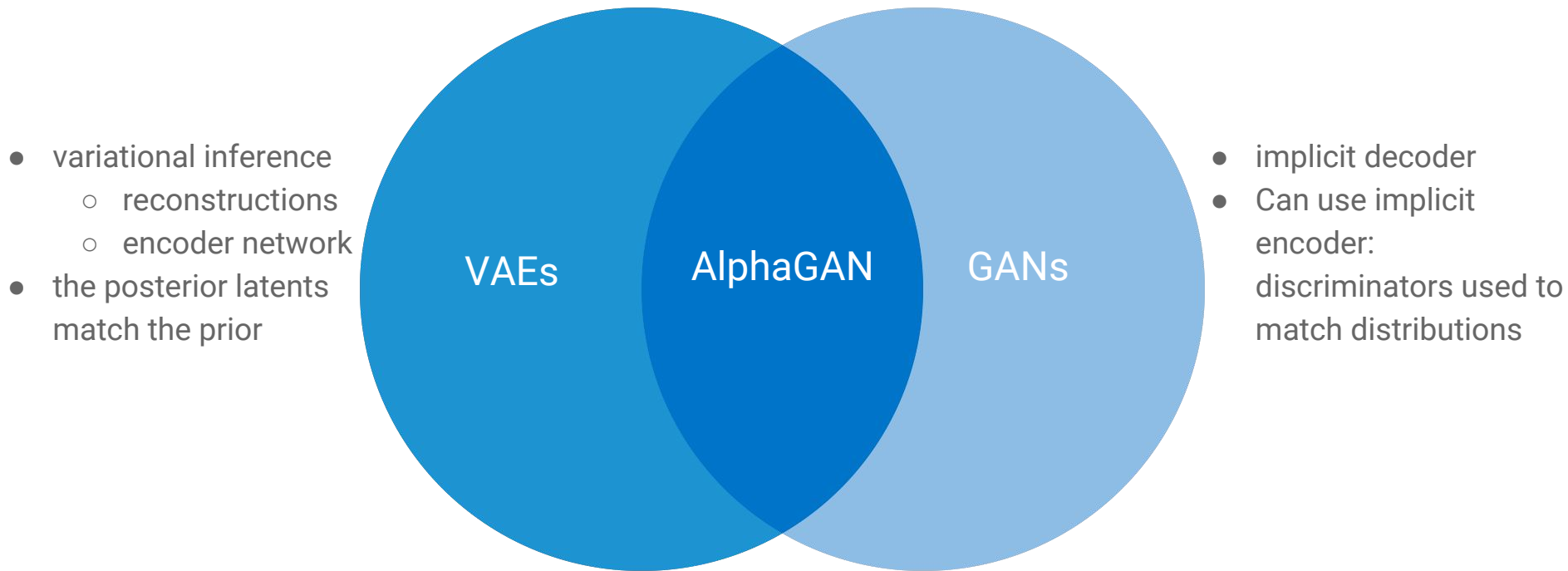
Classifier trained on Cifar10

Improved Techniques for Training GANs T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen

# CelebA - sample diversity



# Summary: VAEs and GANs



# Bridging the gap between theory & practice

**Many paths to equilibrium: GANs do not need to decrease a divergence at every step**

*William Fedus\*, Mihaela Rosca\*, Balaji Lakshminarayanan, Andrew Dai, Shakir Mohamed & Ian Goodfellow*

# Differences between GAN theory and practice

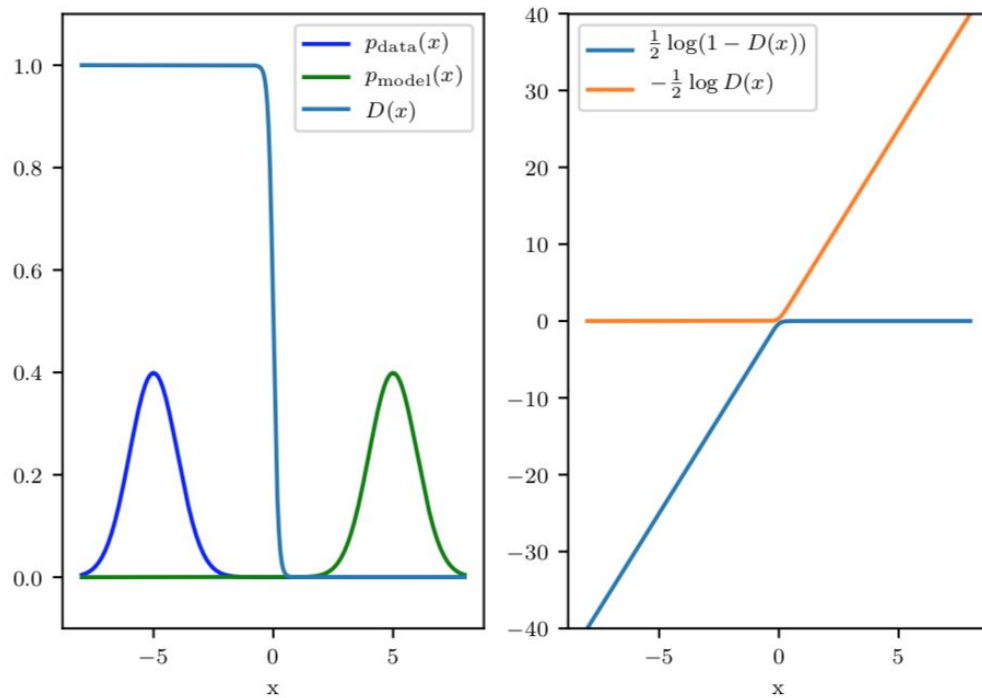
Lots of new GAN variants have been proposed (e.g. Wasserstein GAN)

- Loss functions & regularizers motivated by new theory
- Significant difference between theory and practice

How do we bridge this gap?

- Synthetic datasets where theory predicts failure
- Add new regularizers to original non-saturating GAN

# Non-Saturating GAN





# Gradient Penalties for Discriminators

$$\tilde{J}^{(D)}(D, G) = - \mathbb{E}_{x \sim p_{\text{data}}} [\log D(x)] - \mathbb{E}_{z \sim p_z} [\log(1 - D(G(z)))] + \lambda \mathbb{E}_{\hat{x} \sim p_{\hat{x}}} [(\|\nabla_{\hat{x}} D(\hat{x})\|_2 - 1)^2]$$

To formalize the above, both proposed gradient penalties of the form:

$$\mathbb{E}_{\hat{x} \sim p_{\hat{x}}} [(\|\nabla_{\hat{x}} D(\hat{x})\|_2 - 1)^2],$$

where  $p_{\hat{x}}$  is defined as the distribution defined by the sampling process:

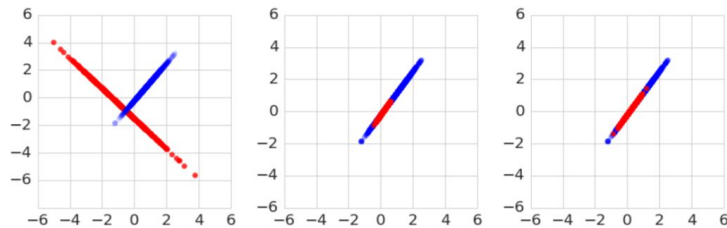
$$x \sim p_{\text{data}}; \quad x_{\text{model}} \sim p_{\text{model}}; \quad x_{\text{noise}} \sim p_{\text{noise}}$$

$$\textbf{DRAGAN} \quad \tilde{x} = x + x_{\text{noise}}$$

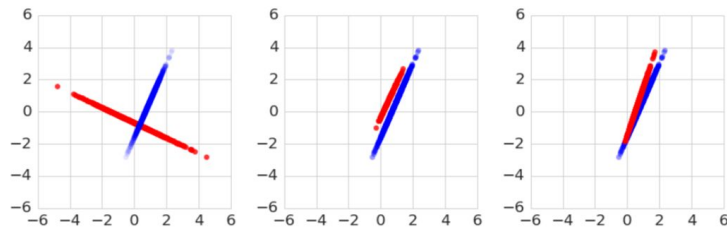
$$\textbf{WGAN-GP} \quad \tilde{x} = x_{\text{model}}$$

$$\alpha \sim U(0, 1)$$

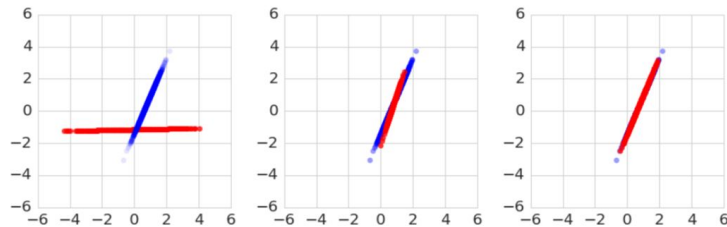
$$\hat{x} = \alpha x + (1 - \alpha)\tilde{x}.$$



(a) Non-saturating GAN training at 0, 10000 and 20000 steps.



(b) GAN-GP training at 0, 10000 and 20000 steps.

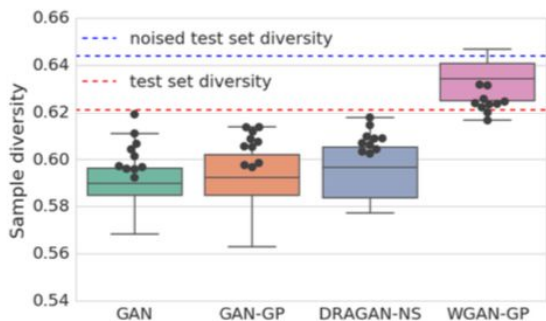


(c) DRAGAN-NS training at 0, 10000 and 20000 steps.

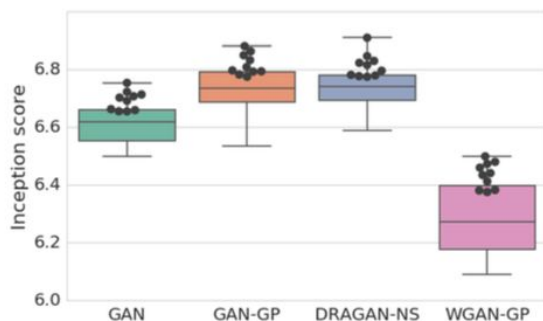
Comparisons on synthetic dataset where Jensen Shannon divergence fails

- Gradient penalties lead to better performance

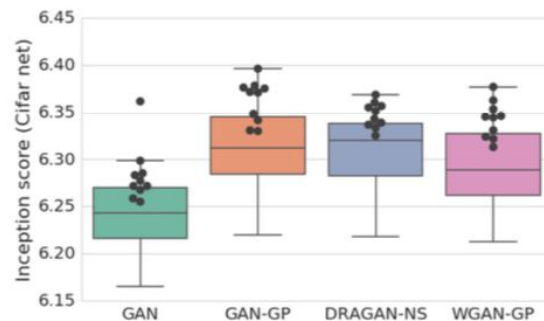
# Results on real datasets



(a) CelebA

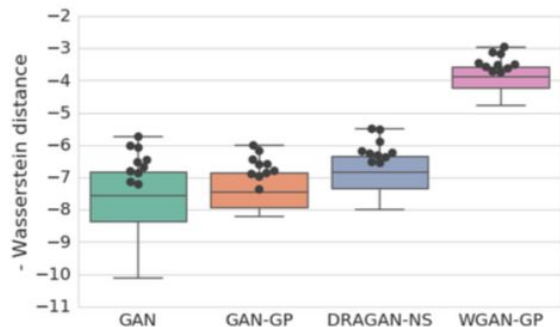


(b) Inception Score (ImageNet)

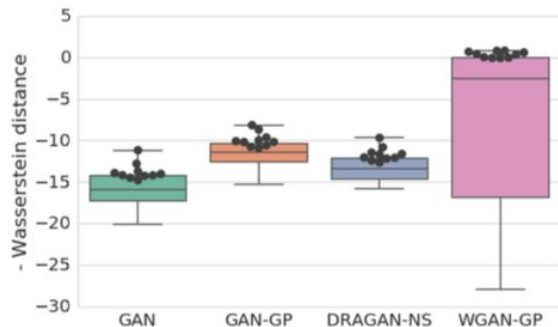


(c) Inception Score (CIFAR)

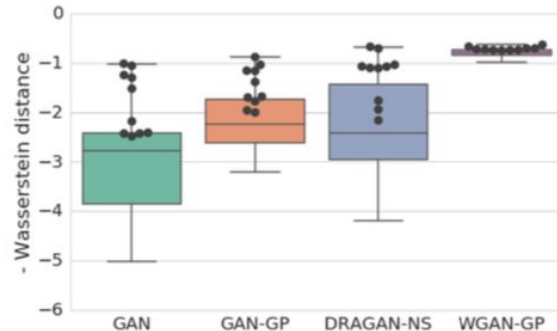
# Results on real datasets



(a) Color MNIST



(b) CelebA



(c) CIFAR-10

# Summary

Some surprising findings:

- Gradient penalties stabilize (non-Wasserstein) GANs as well
- Think not just about the ideal loss function but also the optimization

*“In theory, there is no difference between theory and practice. In practice, there is.”*

- Better ablation experiments will help bridge this gap and move us closer to the holy grail

# Other interesting research directions

# Overloading GANs and “Adversarial training”

Originally formulated as a minimax game between a discriminator and generator

Recent insights:

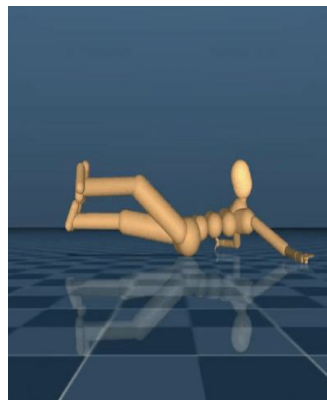
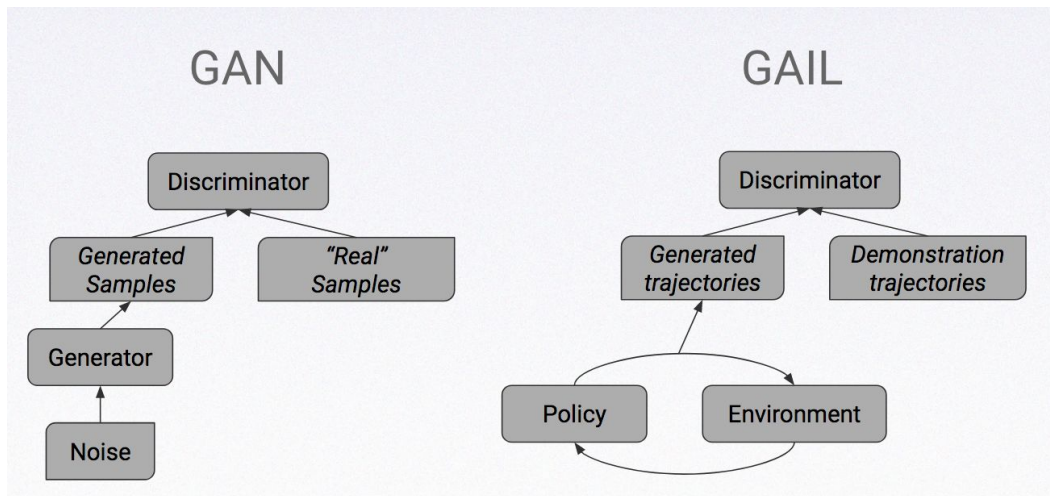
- **Density ratio trick**: discriminator estimates a density ratio. Can replace density ratios and f-divergences in message passing with discriminators.

$$r_{\phi}(\mathbf{x}) = \frac{p^*(\mathbf{x})}{p_{\theta}(\mathbf{x})} = \frac{p(\mathbf{x}|y=1)}{p(\mathbf{x}|y=0)} = \frac{p(y=1|\mathbf{x})}{p(y=0|\mathbf{x})} = \frac{\mathcal{D}_{\phi}(\mathbf{x})}{1 - \mathcal{D}_{\phi}(\mathbf{x})}$$

- **Implicit/Adversarial variational inference**: Implicit models can be used for flexible variational inference (require only samples, no need for densities)
- **Adversarial loss**: Discriminator provides a mechanism to “learn” what is realistic, this is better than using a (gaussian) likelihood to train generator.

# GANs for imitation learning

Use a separate network (discriminator) to “learn” what is realistic  
Adversarial imitation learning: RL Reward comes from a discriminator



**Learning human behaviors from motion capture by adversarial imitation**

Josh Merel, Yuval Tassa, Dhruva TB, Sriram Srinivasan, Jay Lemmon, Ziyu Wang, Greg Wayne, Nicolas Heess



# Lots of other exciting research

- Research
  - Using ideas from convergence of Nash equilibria
  - Connections to RL (actor-critic methods)
  - Control theory (e.g. numerics of GANs)
- Applications
  - Class-conditional generation,
  - Text-to-image generation
  - Image-to-image translation
  - Single image super-resolution
  - Domain adaptation

And many more ...

# Summary

## Ways to stabilize GAN training

- Combine with Auto-encoder
- Gradient penalties

## Tools developed in GAN literature are intriguing even if you don't care about GANs

- Density ratio trick is useful in other areas (e.g. message passing)
- Implicit variational approximations
- Learn a realistic loss function than use a loss of convenience
- How do we handle non-differentiable simulators?
  - Search using differentiable approximations?

# Thanks!

**Learning in implicit generative models**, Shakir Mohamed\* and Balaji Lakshminarayanan\*

**Variational approaches for auto-encoding generative adversarial networks**, Mihaela Rosca\*, Balaji Lakshminarayanan\*, David Warde-Farley and Shakir Mohamed

**Comparison of maximum likelihood and GAN-based training of Real NVPs**, Ivo Danihelka, Balaji Lakshminarayanan, Benigno Uria, Daan Wierstra and Peter Dayan

**Many paths to equilibrium: GANs do not need to decrease a divergence at every step**, William Fedus\*, Mihaela Rosca\*, Balaji Lakshminarayanan, Andrew Dai, Shakir Mohamed and Ian Goodfellow

**Slide credits:** *Mihaela Rosca, Shakir Mohamed, Ivo Danihelka, David Warde-Farley, Danilo Rezende*

Papers available on my webpage <http://www.gatsby.ucl.ac.uk/~balaji/>