A Tax Based Model of Informality

Muhammad Bashir *

October, 2023

PRELIMINARY AND INCOMPLETE - PLEASE DO NOT CITE

Abstract

In low and middle-income countries, the informal sector constitutes a significant portion of GDP and employment, posing challenges for taxation and social protection. Tax evasion tends to be major driver of informality even in developed countries Pappadà & Rogoff (2023). This paper develops a theoretical model to understand how firms choose between formal and informal operations in the presence of imperfect enforcement. The model, inspired by Roy's classic self-selection theory, examines the relationship between firm productivity and profit in the context of taxation. It suggests that firms with higher productivity are more likely to operate in the formal sector if there is a wider dispersion in productivity among formal firms. This dispersion is influenced by government policies, such as tax rates, evasion and compliance costs, as well as support for formal businesses and availability of educated labor. The paper lays the groundwork for an in-depth study of how tax policy affects informality.

Keywords: Informality, Tax Evasion, Firm Registration, Roy Model, Self-selection
JEL Classification: H00, H10, H21, H26

*Contact: Muhammad Bashir mb5061@columbia.edu. I thank University of Manchester for my MSc funding and Dr. Mazhar Waseem for supervising this thesis.

I Introduction

Informality, as a concept, eludes a singular definition. The term, often mired in nuances, has been subjected to multiple interpretations in academic literature. A prevalent definition, rooted in legalistic terms, posits that a firm is informal if it operates without conforming to the legal prerequisites of registration and regulation (Perry et al., 2007). From a labor perspective, workers within the informal sector are those who are employed devoid of any formal or legally binding contract.

Recent empirical and theoretical examinations focus predominantly on the extensive and intensive facets of labor informality. These studies investigate a firm's rationale behind employing a portion of its workforce informally, i.e., without any official contract. In developed economies, complete evasionoperating without any registrationis a daunting endeavor. As a result, many firms opt for partial compliance: they register and declare a fraction of their economic and labor activities to the authorities while clandestinely maintaining informal operations and workforce. Ulyssea (2020) posits that while labor informality can be more definitively characterized, firm informality presents a more intricate scenario. Firms might strategically decide to formally register certain components of their production, leaving others unregistered. This decision often hinges on the equilibrium between the anticipated punitive actions for non-compliance and the potential savings from evading wage-related income tax. Additionally, the propensity for firms to under-declare their tax bases, encompassing sales, revenue, and profit, is well-documented in works by Best *et al.* (2015), Best *et al.* (2021), Kleven *et al.* (2011), and Carrillo *et al.* (2017).

Informality manifests in varying degrees and forms across nations. Examples include unregistered businesses, entities registered for a specific activity but clandestinely diversifying their operations, and deliberate underreporting or falsification of input invoices to tax authorities. Taking Pakistan as a case in point, street vendors epitomize informalitythey operate without registration, lack a fixed business address, and consequently evade taxation. Other informal sectors in Pakistan span the retail sector, domestic labor, agricultural activities, and self-employment. Despite agriculture's significant contribution to the Pakistani economy, the sector largely remains informal, escaping taxation and regulation. Such informal practices, albeit to different extents, are pervasive globally. Alarmingly, in developing nations, the informality even encompasses manufacturing and business enterprises. These are entities that, ideally, should be registered to leverage the efficiencies of resource utilization. Their reliance on imported inputs, which are subjected to withholding taxes, necessitates cutting-edge technologies and governmental support in enforcing sizable contracts and fostering business relationships. Their potential contributions to government projects further underscore the importance of their formal registration. The continued existence of sizable firms in the informal sector signals a glaring policy inadequacy.

Given this backdrop, for a developing nation like Pakistan, unraveling the determinants and underlying causes of informality becomes imperative, especially in the pursuit of expanding its tax base. The recurrent discourse among scholars and policymakers emphasizes the urgency of enlarging the tax base. This can be achieved by assimilating more firms and individuals into the tax net, rather than burdening the compliant populace with increased taxationa sentiment echoed particularly for the salaried demographic in Pakistan that is already under a heavy tax yoke. Furthermore, rampant informality engenders a slew of challenges ranging from ensuring equitable taxation, to framing effective fiscal policies, safeguarding debt sustainability, and ensuring the accuracy of national account metrics Pappadà & Rogoff (2023). In Pakistan, the manufacturing sector, with a 12.33% GDP share, contributes 34.5% to direct tax revenue, whereas the wholesale and retail sector, despite having an 18.33% GDP share, contributes just 2.9%. This disparity suggests horizontal tax unfairness and diverts capital from manufacturing to other sectors Javed *et al.* (2023).

Building on the aforementioned discussions, this paper delineates informality in the context of firms. A firm is categorized as informal if it conducts business activities that legally necessitate tax liability declarations to the tax authority, yet the firm consciously evades this obligation, choosing instead to function in the shadow economy, beyond the government's purview. This decision is essentially a cost-benefit analysis, where firms weigh the potential profits in both formal and informal spheres. This evaluation encompasses considerations such as evasion and compliance costs, productivity variations across sectors, labor availability, and applicable tax rates.

This deliberation by firms, in essence, is a selection dilemma. Presented with two distinct operational regimes, firms must judiciously select an appropriate mode, contingent on the prevailing economic climate. A pivotal element influencing this choice is the productivity distribution across the two regimes. Other influential determinants include compliance and evasion costs, tax rates, and the output elasticity relative to labor. Drawing inspiration from the Roy model as delineated in Borjas (1987), this research simulates the likelihood of a firm opting for formal operations as these determinants evolve.

II Related Literature

In this section I discuss some important papers that attempted to model and estimate informality. Since informality is primarily a phenomenon in the developing world as western advanced economies are highly formalized with extensive state capacity and wide tax base, therefore there is limited research on informality and there is no major work in economics of informality and its impact on tax capacity of a state. However, it is a major issue for the developing countries and understanding its causes and impacts is important. For instance, Loayza & Rigolini (2006) develop a theoretical model of the informality from a labor market perspective where the size of informal employment is determined by the relative costs and benefits of informality and the distribution of workers' skills. They use an error-correction framework to empirically examine informality's long and short-run relationships in a macroeconomic setting. Loayza & Rigolini (2006) find that in the long run, informality is larger in countries that have lower GDP per capita and impose more costs to formal firms, in the form of more rigid business regulations, less valuable police and judicial services, and weaker monitoring of informality. I use these empirical findings as basis of my model in next section. However, this paper assumes that only more productive workers join the formal sector and there is a threshold for such productivity while the selection model I present does not assume this and rather attempts to demonstrates conditions under which more productive firms join the formal sector. Moreover, this paper focused on modelling informality from labor market perspective and looks at empirical results from a general country level macroeconomic model. On the other hand, I crucially model under what conditions more productive firms will join the formal sector and when it is optimal for less productive firms to join the formal sector. Another major difference is that this paper looks at labor market informality while I consider firm level informality. Many studies show that workers who are working in informal sectors are not working there by choice but rather they do not have many options and are generally more vulnerable, weak and poor (Bonnet *et al.* (2019)). That is why modelling informality at firm level is more meaningful because firm is more likely to make this decision rather than the worker.

A very deep and thorough analysis of informality takes place in Waseem (2018) where Waseem utilizes a natural experiment in tax policy to measure informality. He notes that there is already a relatively large informal sector in Pakistani economy and an increase in tax rate might push firms back into informality. He notes that partnerships were substantially taxed at lower rates than incorporations and that is why there was decline in the growth of incorporations, i.e, people tended to operate business as a partnership rather than an incorporation. In order to balance the tax rates, government increased the tax rate for partnerships and brought it close to incorporations. Importantly, the tax rate was increased more for partnerships than sole pro-prietorships which is another form of business. As a consequence, this created a natural identifying variation to be estimated using Differences in Differences approach. Waseem (2018) finds that treated firms (firms that now faced an increased tax rate than pre-reform tax rate) report significantly lower earnings, migrate into informality, and switch business form in response to the increase in tax rate. Importantly, Waseem (2019) notes that revenue loss was substantial i.e, three years after the reform, government was collecting even less revenue than what it would have collected without tax increase. One of the most important contribution of this paper is that existence of a thriving informal sector means it is difficult for governments to increase tax reforms as firm can move into shadow economy leaving governments without any tax revenue.

Keen & Mintz (2004) study the tax behaviours of entrepreneurs near the tax threshold. As in most value added tax designs, firms are not required to report or pay taxes until revenue reaches a certain threshold determined by the law or tax authority. They first consider the question of optimal tax design and then solve for the problem of an entrepreneur who is thinking of joining the formal sector. If the firm grows at natural rate and reports its revenue even if that crosses the exemption threshold, it will have to register with the tax authority and pay taxes. On the other hand, a firm may stop growing and continue to produce at the threshold or produce beyond threshold and misreport its revenue or growth as being below the threshold. However, in this context, both real responses and evasion responses are possible i.e, a firm may not grow beyond the threshold or continue to grow but misreport the revenue. In the next section when I develop a model of informality from a selection perspective, I borrow ideas from Keen & Mintz (2004) for specification of profit functions and proportionate taxable input used. However, they focus on behavior of entrepreneurs around the threshold and how their growth rate is impacted while I consider informality as a selection problem and discuss when positive selection takes place i.e, more productive firms from the informal sector join the formal sector.

Berdiev *et al.* (2018) build a macroeconomic model of informality where they study how economic freedom and less regulation impacts shadow economy. Using a panel of over 100 countries, they find that more economic freedom and less regulation leads to a decrease in shadow economy in a country. One important factor to increase economic freedom would be to decrease compliance costs for the registered firms. However, Berdiev *et al.* (2018) note that their main focus is to explain cross-country variations in magnitude of the shadow economy rather a model of a specific country. That is why, they only measure macroeconomic variables. Moreover, measurement of size of freedom and shadow economy is also very subjective and it makes the analysis complex when looking at the country level.

III A Tax Based Roy Model of Informality

In this section, I present and solve a basic model of informality by bringing in some ideas from labor economics literature. The main contribution is giving micro-foundations to this idea and then interpretation of the results in spirit of the Roy's model. I consider the firm problem of registering with tax authority as a selection problem of the firm where firm, based on its characteristics and economic environment, is self-selecting into either regime i.e, formality or informality. The main question analyzed through Roy's Model is under what conditions more productive firms would join formal sector and vice versa. In the next subsection, I discuss some basic background on informality before presenting the Roy's Model.

III.A Roy Model

Roy's Model is the earliest attempt in economics to model self selection or self sorting by individuals between two available choices. It was first proposed by A.D Roy in his classic paper Roy (1951). Roy studies choices of individuals sorting into two professions namely fishing and hunting. Roy assumes that there is no uncertainty about the markets, amount of available fish and rabbits, however catching fish requires more skill and effort. Based on these assumptions, he studies sorting of individuals into the two professions. Then, he asks the basic question of self selection as to whether those more suited to catch fish actually go for fishing and those most suited to hunt choose rabbit hunting as their profession. Roy argued, al though in non-mathematical terms, that the resultant selection would depend on the distribution of both skills in populations, the technology at their disposal and the correlation between their skills.

He also highlighted that the difference between wages or returns to different professions is not just random or determined by the supply and demand of labor but it also depends on the kind of people choosing the professions.

Borjas in Borjas (1987) first developed a mathematical version of Roy model in relation to migration. Borgas used same ideas and conclusions as Roy did but was able to clearly model dispersion and correlation between decision variable in two states of the world. His motivation was to answer famous question of heterogeneity of earnings among immigrants and natives in the United States. Simple statistics showed that immigrants tended to earn more as compared to natives which required an explanation. Borgas' idea was that there is a possibility that only most skilled and most motivated individuals migrate and therefore they are systematically different from natives in terms of their labor market profile. Borjas assumed that wages of an individual in his country and country of destination depends on his wage in the two countries. He assumed that log wages are decomposed into fixed average term and a random term that determines where a specific potential migrant lies in income distribution. For individual i in country j, his wage is given by

$$log(w_{ij}) = \mu_j + \epsilon_{ij}$$

where μ_j gives average log wages in country j and ϵ_{ij} is random variable that determines if the potential migrant comes from upper tail of income distribution or lower tail. Borjas (1987) assumes a fixed proportional cost of migration in proportion to wage in home country. Borjas (1987) concluded that both negative and positive selection is possible and it is not necessary that only most educated and competent would migrate. Yes, most educated and competent would migrate if the correlation between skills distribution in both states is very high i.e, your skills must be valued in the country of destination. This requires a greater dispersion in skills at the country of destination compared to home country.

Following discussion in Autor (2003) and Williams (2015), Roy's idea was that the world we observe may not be as random as we assume it to be. His first application was in the field of labor economics where he argued that people are not randomly sorted into different professions. Therefore, the outcomes of people in a specific profession such as medicine or fishery do not represent mere supply and demand outcome. But rather, it is a product of self selection or self sorting by the individuals. This was argued in Chiswick (2018) that migration benefits only to those more skilled and more motivated, however Borjas (1987) argued that it depended on the distribution of skills in home country and the country of destination.

Building on these ideas and in particular the discussion in Autor (2003) and Williams (2015), I develop a Roy Model version for informality. In the basic model, I consider an individual firm deciding whether to enter formal sector by registering itself for taxation or remain under ground by carrying out business activities in informal sector. As argued before, the choice is not apparent as both options bring some benefits and costs. Being formal, the firm has to go through the hassle of tax filing and incurs a resource cost while also paying money in terms of tax. In case of value added tax or sales tax in case of Pakistan, firm either has to pass this tax on to consumer and hence reduce demand for its product or incur this cost itself. In any case, this is a cost on firm and it wants to avoid it. Importantly, here I define formality in a very narrow sense where a firm is formal if it is registered with the tax authority and files its tax returns at the end of the tax year. A firm may well be registered with the authority and yet do informal business or underground activities. I am not considering that option here as this would add another layer of complexity in informality. The benefits of formal sector include protection from government in terms of security, legal support in case of disputes, being able to bid on government auctions and tenders. In case of informality, firm is able to avoid taxes and resource cost but there is a new cost of evasion that the firm might be fined or banned when caught and it would not be able to participate in formal legal activities in order to implement its contracts. It would also not be able to claim credit on tax it paid on inputs as it requires registration with the government.

III.B Setup of Model

For the basic model, let us define $j \in \{0, 1\}$ as business status of firm i where 0 represents being informal and 1 being formal. At any time, a formal firm may decide to move into informality and vice versa. In the production process, the firm uses a fixed tax deductable input with proportion λ i.e if firm produces output y, then tax deductable input used is λy. The non-tax deductable input is labor and production function is mainly determined by the labor input L as the tax-deductable input is used in fixed proportion λ. The firm's choice variable in any regime is L. Therefore, the production function of the firm j in state i is given by,

$$y_{ij} = f(n_{ij}L)$$

where y_{ij} is output of firm i when in business form j. For simplicity, I assume that production function takes form of $f(n_{ij}) = n_{ij}L^{\alpha}$ where $0 < \alpha < 1$ to make production function concave. The n_{ij} represents productivity of firm i in business form j and L is labor employed. We assume competitive markets for both labor with wage w and for deductable input P_I which is used in fixed proportion λ. As discussed before, two regimes offer two different productivities n_0, n_1 based on the environment that the government can supply to the firm and firms individual characteristics.

Let us drop i and consider the production function of a single firm in both states as determined by the economic environment,

$$y_0 = f(n_0L) = n_0L^{\alpha}$$

$$y_1 = f(n_1L) = n_1L^{\alpha}$$

The after-tax output price Q when market price is P is given by

$$Q = P(1 + \tau)$$

where τ is the tax rate. I assume a single fixed tax rate which can be relaxed later. However, this assumption simplifies the derivation. Moreover, I assume that firm any in case collects the tax from consumer and only choice that firm makes is whether to report it to the tax authority or not.

The cost of evasion when the firm is formally registered is defined as

$$evasion\ cost = \gamma y_0$$

where γ is cost incurred for every unit of output evaded. It may include cost involved in misreporting or using machinery to evade as well as cost incurred when you are caught by the tax authority. Replacing the value of y_1,

$$evasion\ cost = \gamma f(n_0L)$$

Later, I define $\gamma' = \frac{\gamma}{1+\tau}$. I assume constant marginal cost of evasion for every extra unit of output because it gets difficult to misreport, hide and simply bribe bureaucrats when you are a bigger firm with a huge business. That is why it makes sense to assume a proportional evasion cost. We also note that evasion cost is linear and therefore convex in output evaded y.

Similarly, I define compliance cost per unit of output as δ and therefore,

$$compliance\ cost = \delta f(n_1 L)$$

In case of compliance costs as well, I assume a proportional cost of compliance i.e, it is higher for large firms with more output than smaller firms. This again makes sense because if you produce more, you have more trading partners and a relatively complex trading network, which makes it difficult to comply with tax authorities and fill the tax forms. Moreover, tax authority officers might be more suspicious of you and impose extra costs in terms of unnecessary audits.

As in standard production theory, firm optimizes its profit in both states. I assume inelastic demand for the product and firm is able to sell entire output. This is again a simplifying assumption and generalizing it would not change qualitative analysis.

The profits are defined by

$$\pi^1 = \{P - P_I\lambda - \delta\} f(n_0 L) - wL \quad formal\ case \tag{1}$$

$$\pi^0 = (1+\tau)\{P - P_I\lambda - \gamma'\} f(n_1 L) - wL \quad informal\ case \tag{2}$$

In these specifications, I assumed that firm collects tax anyway whether it is registered or unregistered, the real choice that firm makes is whether to report that tax and its business to tax authority or simply keep it to itself. So, when firm is formal, it pays $\tau P - \tau P_I\lambda$ to government as it gets an input tax rebate of $\tau P_I\lambda$. This gives specification in equation (1). When firm does not register, it keeps entire revenue i.e, $(1+\tau)Py_0$ but it also does not get rebate on its input and therefore the firm has already paid $\tau P_I\lambda$.

III.C Optimal Profit Under Formality

Firm first optimizes its profits in both cases and then chooses the state that has more profit. Consider first optimization of π^1,

$$\max_L \pi^1$$

Since we assume production function to be $f(nL) = L^\alpha$ where $0 < \alpha < 1$ i.e., the production function is concave, we get the solution by deriving first order conditions.

$$\max_L \{P - P_I\lambda - \delta\} n_1 L^\alpha - wL$$

The first order condition is

$$\frac{d\pi^1}{dL} = n_1\alpha\{P - P_I\lambda - \delta\}L^{\alpha-1} - w = 0$$

$$\implies L^{\alpha-1} = \frac{w}{n_1\alpha(P - P_I\lambda - \delta)}$$

$$\implies L^\star = \left[\frac{w}{n_1\alpha(P - P_I\lambda - \delta)}\right]^{\frac{1}{\alpha-1}}$$

Using optimal expression for $L^\star$ from (3) in (1).

$$\pi^1 = \{P - P_I\lambda - \delta\} n_1 L^\alpha - wL$$

$$\pi^1 = \{P - P_I\lambda - \delta\} n_1 \left[\frac{w}{n_1\alpha(P - P_I\lambda - \delta)}\right]^{\frac{\alpha}{\alpha-1}} - w\left[\frac{w}{n_1\alpha(P - P_I\lambda - \delta)}\right]^{\frac{1}{\alpha-1}}$$

$$\pi^1 = \{P - P_I\lambda - \delta\} n_1 \left[\frac{w}{n_1\alpha(P - P_I\lambda - \delta')}\right]^{\frac{\alpha}{\alpha-1}} - w\left[\frac{w}{n_1\alpha(P - P_I\lambda - \delta)}\right]^{\frac{1}{\alpha-1}}$$

$$\pi^1 = (P - P_I\lambda - \delta)\, n_1 \frac{w^{\frac{\alpha}{\alpha-1}}}{(n_1\alpha(P - P_I\lambda - \delta))^{\frac{\alpha}{\alpha-1}}} - \frac{w^{\frac{\alpha}{\alpha-1}}}{(n_1\alpha(P - P_I\lambda - \delta))^{\frac{1}{\alpha-1}}}$$

$$\pi^1 = n_1^{-\frac{1}{\alpha-1}} \left(\frac{P - P_I\lambda - \delta}{\alpha(P - P_I\lambda - \delta)^{\frac{\alpha}{\alpha-1}}}\right) w^{\frac{\alpha}{\alpha-1}}$$

Taking log of optimal profit for the firm,

$$\ln(\pi^1) = -\frac{1}{\alpha-1}\ln(n_1) + \ln\left(\frac{P - P_I\lambda - \delta}{[\alpha(P - P_I\lambda - \delta)]^{\frac{\alpha}{\alpha-1}}} \cdot w^{\frac{\alpha}{\alpha-1}}\right)$$

Let us define the constant market determined term,

$$A = \ln\left(\frac{P - P_I\lambda - \delta}{[\alpha(P - P_I\lambda - \delta)]^{\frac{\alpha}{\alpha-1}}} \cdot w^{\frac{\alpha}{\alpha-1}}\right)$$

$$A = \ln\left(\frac{(P - P_I\lambda - \delta)^{\frac{\alpha}{\alpha-1}}}{\alpha^{\frac{1}{1-\alpha}}} \cdot w^{\frac{\alpha}{\alpha-1}}\right)$$

Although the firm exactly knows its productivity based on its technology and environment, it is a random variable for the econometrician. Therefore,

$$\ln(\pi^1) = A - \frac{1}{\alpha-1}\ln(n_1)$$

III.D Optimal Profit Under Informality

When firm is registered, firm's profit is given by

$$\pi^0 = (1+\tau)\{P - P_I\lambda - \gamma'\}f(n_0 L) - wL$$

Using definition of production function

$$\pi^0 = (1+\tau)\{P - P_I\lambda - \gamma'\}n_0 L^\alpha - wL$$

First order condition for optimization is given by

$$\frac{d\pi^0}{dL} = \alpha n_0(1+\tau)\{P - P_I\lambda - \gamma'\}L^{\alpha-1} - w = 0$$

Solving for optimal L,

$$L^{\alpha-1} = \frac{w}{\alpha n_0(1+\tau)\{P - P_I\lambda - \gamma'\}}$$

$$\implies L = \left[\frac{w}{\alpha n_0(1+\tau)\{P - P_I\lambda - \gamma'\}}\right]^{\frac{1}{\alpha-1}}$$

Using this optimal L in profit function,

$$\pi^0 = (1+\tau)\{P - P_I\lambda - \gamma'\}n_0 \left[\frac{w}{\alpha n_0(1+\tau)\{P - P_I\lambda\}}\right]^{\frac{\alpha}{\alpha-1}}$$

Taking log of this expression we get,

$$\ln(\pi^0) = -\frac{1}{\alpha-1}\ln(n_0) + \ln\left((1+\tau)\{P - P_I\lambda + \gamma'\}\left[\frac{w}{\alpha(1+\tau)\{P - P_I\lambda - \gamma'\}}\right]^{\frac{\alpha}{\alpha-1}}\right)$$

Let us define as before B.

$$B = \ln\left((1+\tau)\{P - P_I\lambda - \gamma'\}\left[\frac{w}{\alpha(1+\tau)\{P - P_I\lambda - \gamma'\}}\right]^{\frac{\alpha}{\alpha-1}}\right)$$

$$B = \ln\left(\frac{[(1+\tau)(P - P_I\lambda - \gamma')]^{\frac{1}{1-\alpha}}}{\alpha^{\frac{\alpha}{\alpha-1}}}\right)w^{\frac{\alpha}{\alpha-1}}$$

Therefore,

$$\ln(\pi^0) = B - \frac{1}{\alpha-1}\ln(n_0)$$

We are able to derive optimal form of profits in both regimes. As we assumed production function is concave and $0 < \alpha < 1$, therefore $\alpha - 1$ is negative and $-(\alpha - 1)$ is positive. As a result, we can multiply expressions for both profits with $-(\alpha-1)$ without changing the sign. After multiplying, I redefine A, B so that expected value of log productivity is 0. I define these modified constants as A', B' and centered modified productivity variables as N_0, N_1. Therefore, optimal profit takes the form

$$\ln(\pi^0) = B' + N_0$$

$$\ln(\pi^1) = A' + N_1$$

Here, N_0 and N_1 are modified productivity in both regimes so that expected value is 0 for both regimes. Here, modified constants are defined as

$$A' = (1-\alpha)A + \eta_1$$

$$B' = (1-\alpha)B + \eta_0$$

where $\eta_1 = E[ln(n_1)]$ and $\eta_0 = E[ln(n_0)]$. And therefore, re-centered productivity is

$$N_0 = \ln(n_0) - \eta_0$$

$$N_1 = \ln(n_1) - \eta_1$$

Moreover, B' is the average profit of the firm if she decides to operate in the informal sector. It can be approximated by average profit of all firms in the informal sector. It, in a sense, captures observable factors such as labor productivity α, evasion costs through γ, wage rate w and share of tax deductable input . Similarly, A' is average profit of firm in the formal state. On the other hand, $_j$ describes how firm's profits are distributed across this modified mean value.

III.E Productivity

Although a firm observes its productivity n_{ij} exactly, it would be a random variable for the econometrican looking at its decision. It is important to argue here that there is a distribution of firms in both regimes. There is a distribution of productivity of firms in the formal sector and there is a distribution of productivity of firms in the informal sector. However, this is only true when an econometrician looks at the entire universe of firms. For an individual firm, it exactly knows its productivity and maximizes according to it. The productivity of a firm may well be determined by many factors in the regime under consideration. For application of Roy's Model, it would be important how productivity is correlated across the two regimes i.e., the correlation between $\ln(n_0)$ and $\ln(n_1)$. Moreover, another important determinant in Borjas (1987) version of Roy model is dispersion of productivity across the two regimes i.e. variance of $\ln(n_0)$ and $\ln(n_1)$.

III.F Roy Model: Analysis

As discussed before, both N_0 and N_1 are random variables in the data containing entire universe of firms. Additionally, we defined these so that the mean of both variables is 0. Therefore,let us assume that these are jointly normally distributed.

This joint distribution is given by

$$\begin{pmatrix} N_0 \\ N_1 \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{0,1} \\ \sigma_{0,1} & \sigma_1^2 \end{bmatrix} \right)$$

where σ_0^2 and $\sigma_{0,1}$ are variance of N_0 and N_1 respectively while $\sigma_{0,1}$ is covariance between productivity of a firm in the two different regimes. The productivity in formal and informal state are correlated because there are idiosyncratic firm specific factors that determine its productivity as well as exogenous factors determined by government and environment that differ across the two regimes. Similarly, we define $\rho_{0,1}$ as correlation between σ_0, σ_1.

$$\rho_{0,1} = \frac{\sigma_{0,1}}{\sigma_0 \sigma_1}$$

III.G Roy Model: Selection

Now we need to calculate probability of a firm joining the formal sector. A firm decides to register with tax authority if $\pi^1 > \pi^1$. Following Borjas (1987), define indicator function I,

$$I = \ln(\pi^1) - \ln(\pi^0)$$

So, firm joins formal sector is $I > 0$. Therefore,

$$I = A' - B' + N_1 - N_0$$

$$\implies I = A' - B' + N_1 - N_0$$

Define v as a new random variable,

$$v = N_1 - N_0$$

Then, using properties of N_0 and N_1,

$$v \sim N(0, \sigma_v^2)$$

where variance of v,σ_v^2 is given by

$$\sigma_v^2 = \sigma_0^2 + \sigma_1^2 - 2\sigma_{0,1}$$

Now I is a function of a random variable. Every individual firm is assumed to exactly know its indicator function I, while an econometrican looks at the distribution and considers it a random variable. The probability of a specific firm moving from the informal sector to the formal sector is calculated as

$$P = Pr[I > 0] = Pr[A' - B' + N_1 - N_0 > 0]$$

$$P = Pr[N_1 - N_0 > B' - A']$$

Replacing $N_1 - N_0$ with v and dividing with the standard deviation of v, we get

$$P = Pr\left[\frac{v}{\sigma_v} > \frac{B' - A'}{\sigma_v}\right]$$

Define $z = \frac{B'-A'}{\sigma_v^2}$. Since we assumed v to be normally distributed, then the above probability is given by

$$P = Pr[\frac{v}{\sigma_v} > z] = 1 - Pr[\frac{v}{\sigma_v} \leq z]$$

(a) $$\implies P = 1 - \Phi(z)$$

where Φ is cumulative distribution function of standard normal distribution as $\frac{v}{\sigma_v^2}$ is a standardized random variable. The equation (a) gives expression for probability of a firm joining the formal sector. When z goes up, the probability of firm joining the formal sector goes down.

III.H Comparative Statics of Probability

The expression for z is

(b) $$z = \frac{B' - A'}{\sigma_v}$$

where

$$A' = (1 - \alpha) \ln \left(\frac{(P - P_I\lambda - \delta)^{\frac{\alpha}{\alpha-1}}}{\alpha^{\frac{1}{\alpha-1}}} \right) w^{\frac{\alpha}{\alpha-1}} + \eta_1$$

III.H.1 Effect of Compliance Cost: δ

When δ increases, compliance with tax rules and tax returns becomes more expensive for the firm. Consequently, A' decreases, leading to an increase in z. This results in an increase in $\Phi(z)$ and a decrease in $1 - \Phi(z)$. In this context, $\Phi(z)$ and $1 - \Phi(z)$ represent probabilities; therefore, the likelihood of a firm joining the formal sector decreases, making it less likely to enter the formal sector.

III.H.2 Effect of Evasion Cost: γ

The expression for B' is given by

$$B' = (1-\alpha)\ln\left(\frac{[(1+\tau)(P - P_I\lambda - \gamma')]^{\frac{1}{1-\alpha}}}{\alpha^{\frac{\alpha}{\alpha-1}}}\right) + \eta_0$$

As γ is defined to be per unit evasion cost, when γ goes up, γ' also goes up and as a result B' also goes up and z goes down. Therefore, $1 - \Phi(z)$ goes up because $\Phi(z)$ is now smaller and firm is more likely to join formal sector. When it costs more to evade a unit of output, firms are more likely to operate in formal sector.

III.H.3 Effect of Tax Rate: τ

Similarly, when tax rate goes up, B' also goes up which results in higher value of z and lower probability of joining the formal sector. This again one would expect from standard economic theory.

III.H.4 Effect of Covariance between Productivity: σ_v^2

Since σ_v^2 is given by

$$\sigma_v^2 = \sigma_0^2 + \sigma_1^2 - 2\sigma_{0,1}$$

when productivity in both regimes is highly correlated, σ_v^2 goes down and this increases z. As a result, probability of joining formal sector goes down as firm can get same productivity while remaining in informal sector and keeping tax revenue with itself.

III.I Selection Revisited

I derived probability of joining the formal sector for a typical firm. Now, I attempt to calculate expected profits for those who are able to join formal sector in both regimes. I also calculate the counter factual profit. This will help me disentangle the crucial idea of selection of firms in both sectors. First I reproduce expressions for profits.

$$\ln(\pi^0) = B' + N_0$$

$$\ln(\pi^1) = A' + N_1$$

I calculate below the expected profits of a firm in both regimes in case it is a formal firm. This derivation closely follows Williams (2015).

$$\text{(3)} \quad E[\ln(\pi^0)|I > 0] = E[B' + N_0|\frac{v}{\sigma_v} > z] = B' + E[N_0|\frac{v}{\sigma_v} > z] = B' + \sigma_0 E\left[\frac{N_0}{\sigma_0}|\frac{v}{\sigma_v} > z\right]$$

Note that,

$$Var(N_0) = Var\left(\ln(n_0) - \eta_0\right) = Var(\ln(n_0)) = \sigma_0^2$$

Similarly,

$$Var(N_1) = Var\left(\ln(n_1) - \eta_1\right) = Var(\ln(n_1)) = \sigma_1^2$$

and as a result,

$$N_0 \sim N(0, \sigma_0^2)$$

$$N_1 \sim N(0, \sigma_1^2)$$

I focus the term $E\left[\frac{N_0}{\sigma_0}|\frac{v}{\sigma_v} > z\right]$. Since $v = N_1 - N_0$ and N_0, N_1 are jointly normal, their difference v is also normal. Hence,

$$\begin{pmatrix} N_0 \\ N_1 - N_0 \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{0,1} - \sigma_0^2 \\ \sigma_{0,1} - \sigma_0^2 & \sigma_0^2 + \sigma_1^2 - 2\sigma_{0,1} \end{bmatrix}\right)$$

Using property 2 from Appendix A,

$$E\left[\frac{N_0}{\sigma_0}\middle|v\right] = \rho_{0,v}\left(\frac{\sigma_0}{\sigma_v}\right)v$$

where

$$\rho_{0,v} = \frac{\sigma_{0,v}}{\sigma_0\sigma_v}$$

Using Law of Iterated Expectations,

$$E\left[\frac{N_0}{\sigma_0}\middle|\frac{v}{\sigma_v}>z\right]=E\left(E\left(\frac{N_0}{\sigma_0}\middle|\frac{v}{\sigma_v}\right)\middle|\frac{v}{\sigma_v}>z\right) \tag{4}$$

Define $s=\frac{v}{\sigma_v}$. So, s is standard normal i.e, N(0,1), using property A.3.

$$E\left[\frac{N_0}{\sigma_0}\middle|s\right]=\frac{1}{\sigma_0}E[N_0|s]=\frac{1}{\sigma_0}\frac{\sigma_{0,s}}{\sigma_s^2}s=\frac{1}{\sigma_0}\frac{\frac{cov(N_0,v)}{\sigma_v}}{1}\frac{v}{\sigma_v}=\frac{\sigma_{0,v}}{\sigma_0\sigma_v}\frac{v}{\sigma_v}=\rho_{0,v}\frac{v}{\sigma_v} \tag{5}$$

Using this expression in (4),

$$E[\ln(\pi^0)|I>0)]=B'+\sigma_0E\left(\rho_{0,v}\frac{v}{\sigma_v}\middle|\frac{v}{\sigma_v}>z\right)=B'+\sigma_0\rho_{0,v}E\left(\frac{v}{\sigma_v}\middle|\frac{v}{\sigma_v}>z\right)$$

Using A.4,

$$E[\ln(\pi^0)|I>0)]=B'+\sigma_0\rho_{0,v}\left(\frac{\phi(z)}{1-\Phi(z)}\right) \tag{c}$$

Using similar arguments, we can show

$$E[\ln(\pi^1)|I>0)]=A'+\sigma_1\rho_{1,v}\left(\frac{\phi(z)}{1-\Phi(z)}\right) \tag{d}$$

Simplifying (c)

$$E[\ln(\pi^0)|I>0)]=B'+\sigma_0\frac{\sigma_{0,v}}{\sigma_0\sigma_v}\left(\frac{\phi(z)}{1-\Phi(z)}\right)=B'+\frac{\sigma_{0,1}-\sigma_0^2}{\sigma_v}\left(\frac{\phi(z)}{1-\Phi(z)}\right)$$

$$=B'+\frac{\sigma_0\sigma_1}{\sigma_v}\left(\frac{\sigma_{0,1}}{\sigma_0\sigma_1}-\frac{\sigma_0}{\sigma_1}\right)=B'+\frac{\sigma_0\sigma_1}{\sigma_v}\left(\rho_{o,1}-\frac{\sigma_0}{\sigma_1}\right) \tag{e}$$

Similarly simplifying (d),

$$E[\ln(\pi^1)|I>0)]=A'+\sigma_1\frac{\sigma_{1,v}}{\sigma_1\sigma_v}\left(\frac{\phi(z)}{1-\Phi(z)}\right)=A'+\frac{\sigma_{0,1}-\sigma_1^2}{\sigma_v}\left(\frac{\phi(z)}{1-\Phi(z)}\right)$$

$$=A'+\frac{\sigma_1\sigma_1}{\sigma_v}\left(\frac{\sigma_{0,1}}{\sigma_1\sigma_1}-\frac{\sigma_1}{\sigma_1}\right)\left(\frac{\phi(z)}{1-\Phi(z)}\right)=A'+\frac{\sigma_0\sigma_1}{\sigma_v}\left(\frac{\sigma_0}{\sigma_1}-\rho_{o,1}\right)\left(\frac{\phi(z)}{1-\Phi(z)}\right) \tag{f}$$

We decompose expected profit into an average value and a term that depends on how log productivities are correlated in the two states. Let us define Q_0, Q_1 as below

$$Q_0 = \frac{\sigma_0 \sigma_1}{\sigma_v} \left(\rho_{o,1} - \frac{\sigma_0}{\sigma_1} \right) \left(\frac{\phi(z)}{1 - \Phi(z)} \right)$$

$$Q_1 = \frac{\sigma_0 \sigma_1}{\sigma_v} \left(\frac{\sigma_1}{\sigma_0} - \rho_{o,1} \right) \left(\frac{\phi(z)}{1 - \Phi(z)} \right)$$

Essentially these are the two average values of productivities for those in formal sector in both states i.e,

$$Q_0 = E[N_0|I > 0] = \frac{\sigma_0 \sigma_1}{\sigma_v} \left(\rho_{o,1} - \frac{\sigma_0}{\sigma_1} \right) \left(\frac{\phi(z)}{1 - \Phi(z)} \right)$$

and

$$Q_1 = E[N_1|I > 0] = \frac{\sigma_0 \sigma_1}{\sigma_v} \left(\frac{\sigma_1}{\sigma_0} - \rho_{o,1} \right) \left(\frac{\phi(z)}{1 - \Phi(z)} \right)$$

III.J Results: Cases of Selection

Having derived expected profits of formal firms in both states of the world, we can analyze what kind of selection is taking place and in which scenario it would be optimal for firms to move to formal sector and what scenario it is optimal to remain informal.

1. **Positive Selection:** $Q_0 > 0$ **and** $Q_1 > 0$. In this case, formal firm's average productivity is greater than 0 whether it operates in formal state or informal state. In this case, productive firms from informal sector are joining formal sector and this is a case of positive selection. This positive selection takes place, if and only if $\rho_{0,1} > \frac{\sigma_0}{\sigma_1}$ and $\rho_{0,1} < \frac{\sigma_1}{\sigma_0}$ which is only true if $\rho_{0,1} > \frac{\sigma_0}{\sigma_1}$. This requires a relative high correlation between log productivity in formal and informal state or more dispersion in productivity in formal sector than informal sector. This would be a scenario determined by government policy i.e, for given informal sector distributions of firm productivity, how does formal environment look?

2. **Negative Selection:** $Q_0 < 0$ **and** $Q_1 < 0$. In this case, formal firm's average productivity is negative in both states of the world. This again requires high correlation between productivities in both states and relatively less dispersion in distribution of productivities in formal state.

3. **Switch Selection:** $Q_0 < 0$ **and** $Q_1 > 0$. In this case, unproductive firms join the formal sector and end up being above average in productivities. This might mean that firms operate in sectors that rely on formal legal procedures a lot and cannot do well while being informal. This requires that $\rho_{0,1} < \min(\frac{\sigma_1}{\sigma_0}, \frac{\sigma_0}{\sigma_1})$. This requires that there is a very low correlation between productivities in two states of the world.

The main conclusion of this version of Roy's Model is that selection depends on distribution of productivity in formal sector and informal sector. If the productivity is more dispersed in formal sector, the model predicts positive selection i.e., more productive firms would prefer joining formal sectors while unproductive firms would stay in informal sector. This seems to be a desirable outcome. On the other hand, if being more productive is not rewarded in the formal sector, there would be negative selection i.e., less productive firms would join formal sector and it would not help the government much in its long-term goal of more growth and tax revenue. This would be the case where there is a firm owned by someone with power and it only bids on government projects. This firm would not compete in the normal market since it can free ride in the government sector without much competition. As more more productive firms are not rewarded and there is little variation in profits, less productive firms will join this sector. This is called negative selection.

Government can help more productive firms with better services such as an efficient and fair judicial system for conflict resolution and enforcement of contracts, standardizing the procedures for the business as well as offering better public services and efficiently solving adverse selection and moral hazard problems.

III.K Simulation

In this section, I visualize the basic relationships for ease of interpretation. One can note that most of these results can be seen directly through comparative statics. Simulation helps see how probability of joining formal sector varies with respect to different parameters. I use the following values for baseline simulation.

$$\delta = 1.2,\ \gamma = 1,\ \alpha = 0.6,\ \eta_1 = 0,\ \eta_0 = 0,\ \tau = .16,\ w = 1$$

In order to generate normal productivity random variables in two states of the world, I use following mean and variance-covariance matrices.

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ \Sigma = \begin{bmatrix} 8 & 1 \\ 1 & 5 \end{bmatrix}$$

There is no particular reason to choose specific values for the parameters. I only make sure that Σ is positive definite and parameters lead to a well-defined z.

III.K.1 Analysis: Labor Productivity α

Since we assumed that labor is not tax deductable, when α goes up i.e. labor is more productive, firm chooses to operate in informal sector and probability of joining the formal sector goes down. This shows a classic case of substitution. This is illustrated in Fig. I. We also note that α can be interpreted as elasticity of output with respect to labor. Let us represent this elasticity of output with respect to labor as e.Then,

$$e = \frac{d\log(y)}{d\log(L)}$$

Using definition of the production function,

$$\log(y_i) = \log(n_i L^\alpha)$$

$$\log(y_i) = \log(n_i) + \alpha \log(L)$$

Therefore,

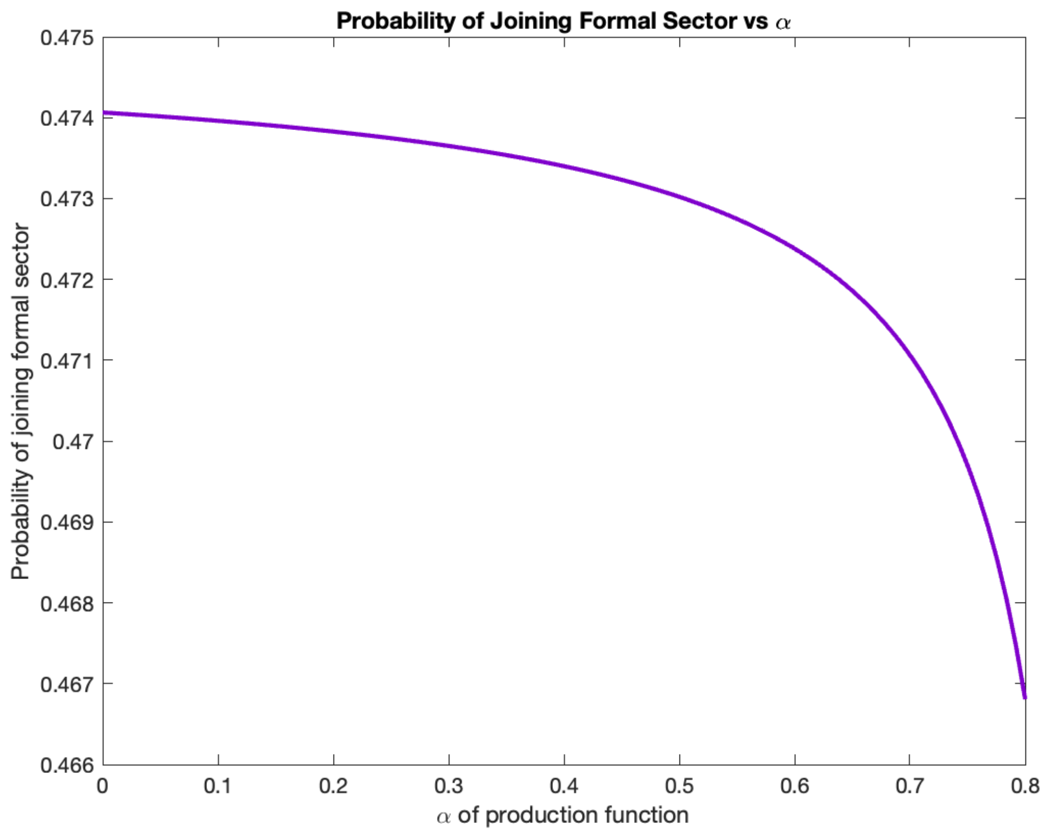
$$e = \alpha$$

Therefore, the Fig.II shows that as elasticity of output with respect to non-tax deductable input goes up, the probability of joining the formal sector goes down. It makes sense because now there is more freedom for the firm to move between different inputs and as a result optimize the profit.

III.L Compliance Cost

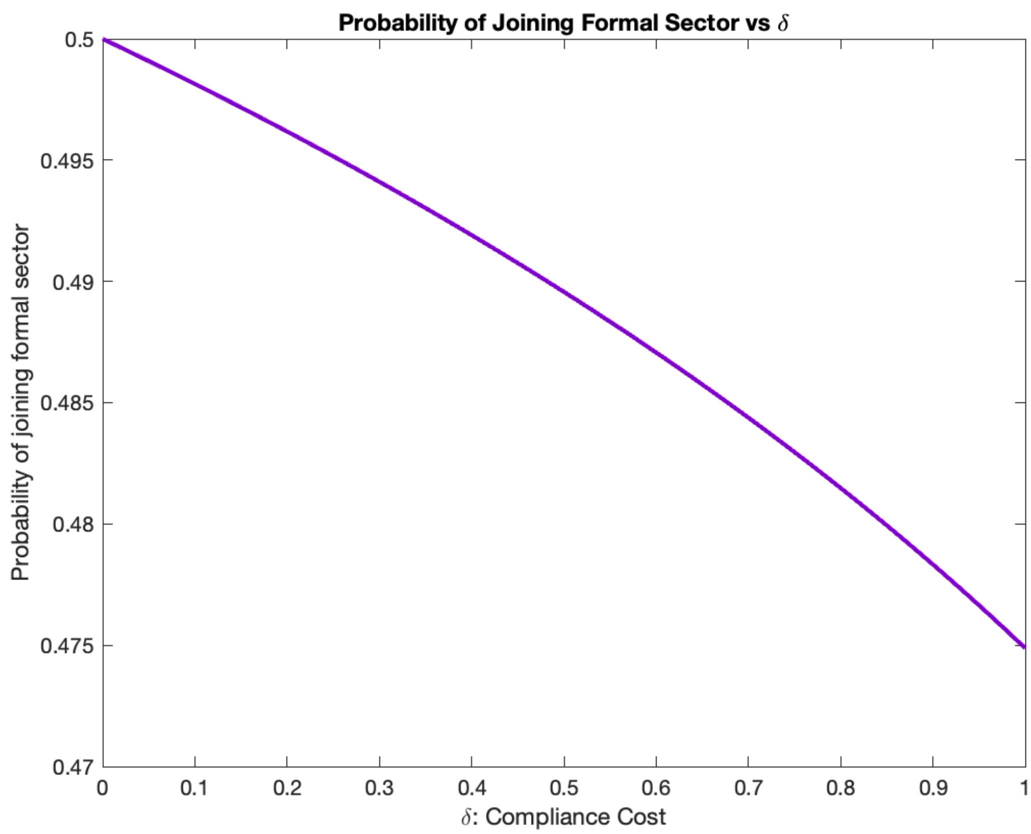
As expected, higher compliance cost reduces the probability of joining the formal sector for a typical firm. Even when there is no compliance cost, firm still does not

FIGURE I: IMPACT OF LABOR PRODUCTIVITY ON INFORMALITY



join formal sector with probability 1 because there are other factors as well such as evasion cost and tax rate that determine this decision.

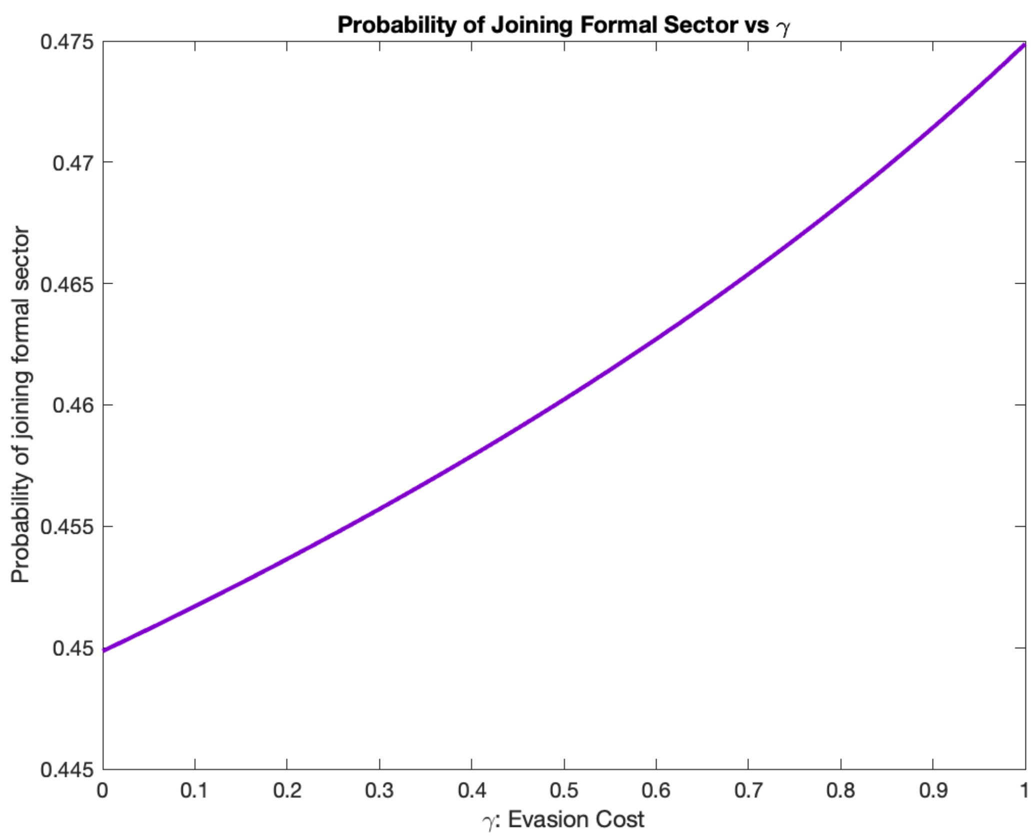
FIGURE II: IMPACT OF COMPLIANCE COST ON INFORMALITY



III.L.1 Analysis: Evasion Cost γ

As expected, an increase in evasion cost leads to increase in probability of a firm joining the formal sector. Although we note that even when evasion cost is 0, the probability of a firm joining formal sector is not 0 which we might expect given that there is no cost to evade taxes. This is because we assumed that firm might still benefit form joining the formal sector because it offers higher productivity.

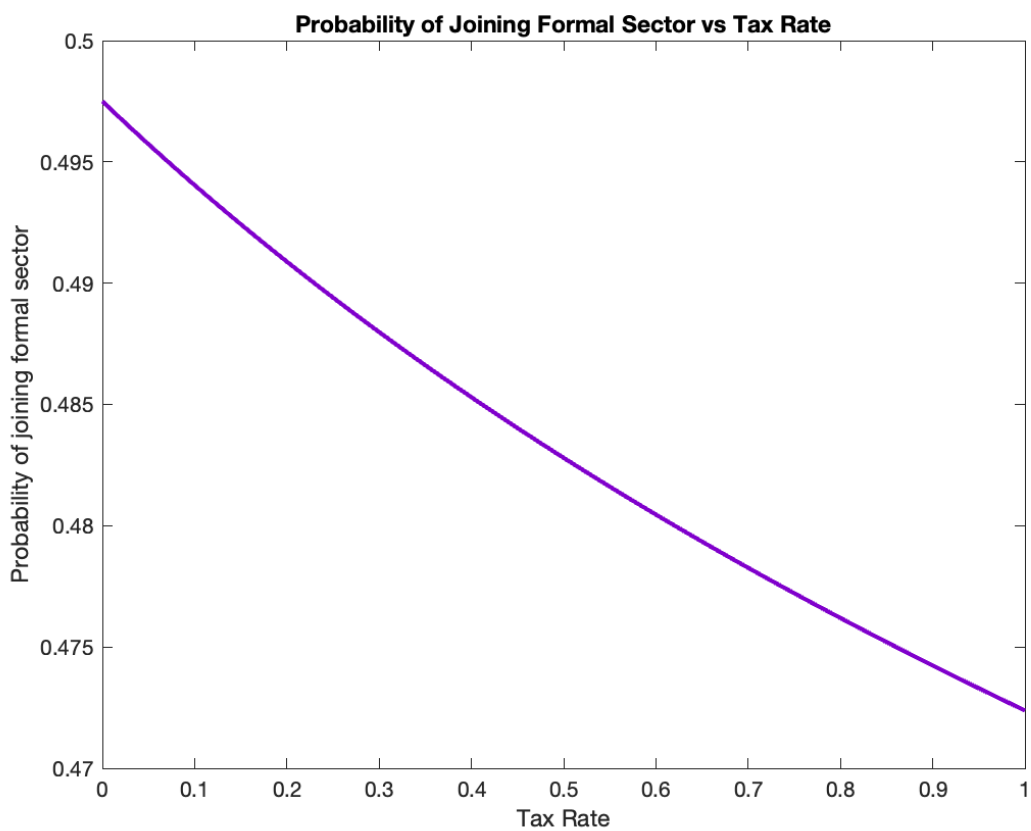
FIGURE III: IMPACT OF EVASION COST ON INFORMALITY



III.L.2 Analysis: Tax Rate τ

As standard economic theory predicts, higher tax rate decreases probability of joining the formal sector for a typical firm and this is revealed by the simulation. As discussed before, a zero tax rate does not lead a firm to join formal sector for sure but rather it only increases the probability.This is because there are other factors determining the decision except τ.

FIGURE IV: IMPACT OF TAX RATE ON INFORMALITY



IV Conclusion

Informality remains a persistent challenge in numerous developing economies, necessitating a comprehensive understanding of its underlying determinants. To dissect this multifaceted issue, the current paper employs a firm-centric model, positing that it is the firm that ultimately decides to either register with tax authorities or operate informally.

In framing the issue as a selection problem, this study elucidates how a firm self-selects into one of two distinct regimes, formal or informal, 525based on specific policy variables and prevailing economic conditions. Each regime presents a unique set of economic variables that the firm must optimize against.

In our baseline model, we eliminate environmental uncertainty, allowing firms to precisely calculate their compliance and evasion costs, as well as accurately gauge the production functions available in either regime. Our central finding is that firms with greater productivity are more inclined to join the formal sector when the dispersion in productivity is higher in this sector, a situation generally occurring when the formal economy rewards productivity. Conversely, during negative selection, less productive firms are more likely to formalize when there is lower variability in productivity in the formal sector, typically in contexts where rent-seeking is prevalent.

Some limitations of the model are that I assumed proportional evasion and compliance costs while a more general model would assume convex cost functions. The model would become less tractable if we assume general convex cost functions and that would not lead to clean analysis of selection. Another limitation is that I assumed a simple Cobb-Doughles production structure where productivity acts as a multiplier. This is an important assumption because we cannot decompose optimal profit into a constant term and log productivity without assuming this form for the production function. However, this assumption is not too restrictive because this production structure is most common in the literature.

References

AUTOR, DAVID. 2003. Lecture note: self-selection–The Roy model. *Retrieved Nov*, **14**, 2010.

BERDIEV, AZIZ N, SAUNORIS, JAMES W, & SCHNEIDER, FRIEDRICH. 2018. Give me liberty, or I will produce underground: Effects of economic freedom on the shadow economy. *Southern Economic Journal*, **85**(2), 537–562.

BEST, MICHAEL, SHAH, JAWAD, & WASEEM, MAZHAR. 2021. *Detection Without Deterrence: Long-Run Effects of Tax Audit on Firm Behavior*. Tech. rept. Tech. rep., Mimeo, University of Manchester.

BEST, MICHAEL CARLOS, BROCKMEYER, ANNE, KLEVEN, HENRIK JACOBSEN, SPINNEWIJN, JOHANNES, & WASEEM, MAZHAR. 2015. Production versus revenue efficiency with limited tax capacity: theory and evidence from Pakistan. *Journal of political Economy*, **123**(6), 1311–1355.

BONNET, FLORENCE, VANEK, JOANN, & CHEN, MARTHA. 2019. Women and men in the informal economy: A statistical brief. *International Labour Office, Geneva*, **20**.

BORJAS, GEORGE J. 1987. *Self-selection and the earnings of immigrants*. Tech. rept. National Bureau of Economic Research.

CARRILLO, PAUL, POMERANZ, DINA, & SINGHAL, MONICA. 2017. Dodging the taxman: Firm misreporting and limits to tax enforcement. *American Economic Journal: Applied Economics*, **9**(2), 144–164.

CHISWICK, BARRY R. 2018. The effect of Americanization on the earnings of foreign-born men. *Pages 111–136 of: Interdisciplinary Perspectives on the New Immigration*. Routledge.

JAVED, UMAIR, ASAD, SHER AFGHAN, & SARWAR, HAMID ATEEQ. 2023 (5). *Understanding Informality in the Wholesale, Retail, Real Estate Sectors*. Accessed: 2023-10-03.

KEEN, MICHAEL, & MINTZ, JACK. 2004. The optimal threshold for a value-added tax. *Journal of Public Economics*, **88**(3-4), 559–576.

KLEVEN, HENRIK JACOBSEN, KNUDSEN, MARTIN B, KREINER, CLAUS THUSTRUP, PEDERSEN, SØREN, & SAEZ, EMMANUEL. 2011. Unwilling or unable to cheat? Evidence from a tax audit experiment in Denmark. *Econometrica*, **79**(3), 651–692.

LOAYZA, NORMAN, & RIGOLINI, JAMELE. 2006. *Informality trends and cycles*. Vol. 4078. World Bank Publications.

PAPPADÀ, FRANCESCO, & ROGOFF, KENNETH. 2023. Rethinking the Informal Economy and the Hugo Effect. *Available at SSRN 4538487*.

ROY, ANDREW DONALD. 1951. Some thoughts on the distribution of earnings. *Oxford economic papers*, **3**(2), 135–146.

ULYSSEA, GABRIEL. 2020. Informality: Causes and consequences for development. *Annual Review of Economics*, **12**, 525–546.

WASEEM, MAZHAR. 2018. Taxes, informality and income shifting: Evidence from a recent Pakistani tax reform. *Journal of public economics*, **157**, 41–77.

WASEEM, MAZHAR. 2019. Information, asymmetric incentives, or withholding? understanding the self-enforcement of value-added tax. *Understanding the Self-Enforcement of Value-Added Tax (March 19, 2019)*.

WILLIAMS, HEIDI L. 2015. Self-selection: The Roy model.