



Algorithm AS 32: The Incomplete Gamma Integral

Author(s): G. P. Bhattacharjee

Source: *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, Vol. 19, No. 3 (1970), pp. 285-287

Published by: [Blackwell Publishing](#) for the [Royal Statistical Society](#)

Stable URL: <http://www.jstor.org/stable/2346339>

Accessed: 19/09/2010 12:09

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=black>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Royal Statistical Society and Blackwell Publishing are collaborating with JSTOR to digitize, preserve and extend access to *Journal of the Royal Statistical Society. Series C (Applied Statistics)*.

<http://www.jstor.org>

Algorithm AS 32

The Incomplete Gamma Integral

By G. P. BHATTACHARJEE

Department of Mathematics, I.I.T., Kharagpur (India)

LANGUAGE

ASA Standard Fortran

DESCRIPTION AND PURPOSE

For given values of $x(\geq 0)$, $p(>0)$ and the complete gamma function $\Gamma(p)$, the function subprogram computes the incomplete gamma ratio defined by

$$I(x, p) = \frac{1}{\Gamma(p)} \int_0^x \exp(-t) t^{p-1} dt.$$

Numerical method

The subprogram uses the series expansion (c.f. Pearson, 1922) viz.

$$I(x, p) = \frac{\exp(-x) x^p}{\Gamma(p+1)} \left[1 + \sum_{r=1}^{\infty} \frac{x^r}{(p+1)(p+2) \dots (p+r)} \right]$$

for $p \leq x \leq 1$ and also for $x < p$. The series is terminated when the contribution to the series is not greater than the value of ACU defined in the first arithmetic statement of the subprogram. For all other cases, the continued fraction expansion (c.f. Abramowitz and Stegun, 1965) viz.

$$I(x, p) = 1 - \frac{\exp(-x) x^p}{\Gamma(p)} \left[\frac{1}{x+} \frac{1-p}{1+} \frac{1}{x+} \frac{2-p}{1+} \frac{2}{x+} \dots \right]$$

is used. The ratio is calculated by successive iteration and the process is terminated when the relative difference between two successive iterations is not greater than ACU.

STRUCTURE

FUNCTION GAMAIN (X,P,G,IFault)

Formal parameters

<i>X</i>	Real	input: the value of the upper limit x .
<i>P</i>	Real	input: the value of the parameter p
<i>G</i>	Real	input: the natural logarithm of $\Gamma(p)$.
<i>IFault</i>	Integer	output: a fault indicator, equal to: 1 if $p \leq 0$, 2 if $x < 0$ and 0, otherwise.

Auxiliary algorithm

For the natural logarithm of $\Gamma(p)$ any standard algorithm, such as ACM Algorithm 291, may be used.

TIME

The time depends on the relative values of x and p , but on an average 550 IBM 1620 instructions are executed, using a version without logical *IF* statements.

ACCURACY

The accuracy of the result normally should not be less than the value of ACU defined as the first arithmetic statement of the subprogram. But due to truncation and rounding off errors the last digit may not be correct.

Thanks are due to the referee for suggesting improvements to the original version of the program.

REFERENCES

- ABRAMOWITZ, M. and STEGUN, I. A. (1965). *Handbook of Mathematical Functions*. New York: Dover.
- PEARSON, K. (1922). *Tables of the Incomplete Gamma Function*. London: Cambridge University Press.
- PIKE, M. C. and HILL, I. D. (1966). Algorithm 291: Logarithm of the gamma function. *Comm. A.C.M.*, 9, 684.

```

FUNCTION GAMAIN(X,P,G,IFault)
C
C      ALGORITHM AS 32 J.R.STATIST.SOC. C. (1970) VOL.19 NO.2
C
C      COMPUTES INCOMPLETE GAMMA RATIO FOR POSITIVE VALUES OF
C      ARGUMENTS X AND P. G MUST BE SUPPLIED AND SHOULD BE EQUAL TO
C      LN(GAMMA(P)).
C      IFault = 1 IF P.LE.0 ELSE 2 IF X.LT.0 ELSE 0.
C      USES SERIES EXPANSION IF P.GT.X OR X.LE.1, OTHERWISE A
C      CONTINUED FRACTION APPROXIMATION.
C
C      DIMENSION PN(6)
C
C      DEFINE ACCURACY AND INITIALIZE
C
      ACU=1.0E-8
      OFLO=1.0E30
      GIN=0.0
      IFault=0
C
C      TEST FOR ADMISSIBILITY OF ARGUMENTS
C
      IF(P.LE.0.0) IFault=1
      IF(X.LT.0.0) IFault=2
      IF(IFault.GT.0.OR.X.EQ.0.0) GO TO 50
      FACTOR=EXP(P*ALOG(X)-X-G)
      IF(X.GT.1.0.AND.X.GE.P) GO TO 30
C
C      CALCULATION BY SERIES EXPANSION
C
      GIN=1.0
      TERM=1.0
      RN=P
20  RN=RN+1.0
      TERM=TERM*X/RN
      GIN=GIN+TERM
      IF(TERM.GT.ACU) GO TO 20
      GIN=GIN*FACTOR/P
      GO TO 50

```

```

C
C      CALCULATION BY CONTINUED FRACTION
C
30  A=1.0-P
    B=A+X+1.0
    TERM=0.0
    PN(1)=1.0
    PN(2)=X
    PN(3)=X+1.0
    PN(4)=X*B
    GIN=PN(3)/PN(4)
32  A=A+1.0
    B=B+2.0
    TERM=TERM+1.0
    AN=A*TERM
    DO 33 I=1,2
33  PN(I+4)=B*PN(I+2)-AN*PN(I)
    IF(PN(6).EQ.0.0) GO TO 35
    RN=PN(5)/PN(6)
    DIF=ABS(GIN-RN)
    IF(DIF.GT.ACU) GO TO 34
    IF(DIF.LE.ACU*RN) GO TO 42
34  GIN=RN
35  DO 36 I=1,4
36  PN(I)=PN(I+2)
    IF(ABS(PN(5)).LT.OFLO) GO TO 32
    DO 41 I=1,4
41  PN(I)=PN(I)/OFLO
    GO TO 32
42  GIN=1 0-FACTOR.GIN
C
50  GAMAIN=GIN
    RETURN
    END

```

Algorithm AS 33

Calculation of Hypergeometric Sample Sizes

By F. B. LEECH

Rothamsted Experimental Station, Harpenden, Herts

LANGUAGE

ASA Standard Fortran

PURPOSE

Solution, in n , of $P = \{(N-m)!(N-n)!\}/\{N!(N-m-n)!\}$ for given values of P , N and $p = m/N$.

The program was originally written in connection with a paper on innocuity testing of inactivated foot-and-mouth disease vaccines (Anderson *et al.*, 1970). In this context, a sample of $n \text{ cm}^3$ from a vaccine batch of $N \text{ cm}^3$ is tested for the presence of residual active virus. Batches giving any reaction are rejected and the theoretical problem is to select a size of sample such that when the test is negative, the amount, if any, of active virus in the batch must be small. In this context, the theory involves