Theory Assignment 3 – Basic Probability, Computing and Statistics

Fall 2015, Master of Logic, University of Amsterdam

Submission deadline: Monday, September 21th, 2015, 9 a.m.

Cooperation   Cooperation among students for both theory and programming exercises is strongly encouraged. However, after this discussion phase, every student writes down and submits his/her own individual solution.

Guidelines   The starred exercises are relatively easy exercises for you to practice. No points are awarded for them. You may pick and choose 2 exercises from each exercise type your submission, i.e. you need to submit a total of 4 exercises to be able to get all points. Numbered exercises with an exclamation mark are supposed to be a bit harder and you may challenge yourself by trying to solve them.

In the directory of your private url there is folder called 'theory_submissions'. Please upload your submission there. Your submission should be a PDF-document (use a scanner for handwritten documents!) entitled *AssignmentX_yourStudentNumber.pdf*, where X is the number of the assignment and *yourStudentNumber* is your student number. If your submission does not comply with this format, we will deduct 1 point. For each day that your submission is late, we deduct 2 points.

If you have any question about the homework or if you need help, do not hesitate to contact Thomas.

Exercises

Type I [2 exercises: 2 points per exercise]

1. Assume that 4 balls are randomly picked one after another from an urn of 20 balls, labeled from 1 to 20, without returning them. Let X stand for the largest label selected. What is $P_X(X \geq 16)$?

2. Let X be a random variable taking any of the values 0, 5 or -5 with probabilities $P_X(X = -5) = .3, P_X(X = 0) = .3, P_X(X = 5) = .4$, respectively. What is $E_{P_X}[X^2]$?

3. Compute the probability mass function of the number of tails for 7 subsequent coin tosses.

4. A peddler is trying to sell her goods to two different clients. She estimates her first meeting to yield a profit with probability .4, and her second, independently, with probability .7. Any profit made is equally likely to be either from selling her luxury goods, which bring a profit of \$1000, her generic goods, brining in a profit of \$500. Compute the probability mass function of X, where X is the total value of her profits.

5. Let $E[X] = 5$ and $var(X) = 7$, compute

   (i) $E[(2 + X)^2]$
   (ii) $var(4 + 3X)$

6! Indicate the maximal number of people you can invite to your party so that the probability of any of them having the same birthday as you is less than $\frac{1}{2}$. Assume that birthdays are uniformly distributed and that we do not care about a persons birth year.

Type II [2 exercises: 3 points per exercise]

1. Calculate the variance of a loaded 6-sided die that has a probability of $\frac{1}{6}$ for all odd numbered sides, and $\mathbb{P}(2) = .1, \mathbb{P}(4) = .1, \mathbb{P}(6) = .3$.

2. A jar contains N Euro and M GBP coins. Coins are taken out randomly up to the first draw of a GBP coin. If each drawn coin is put back before picking a new one, what is the probability that

    (i) exactly n draws are needed?

    (ii) at least k draws are needed?

3. You make a bet with a friend to the effect that he has to pay you an amount L should an event M happen within a year. If you estimate M to happen with probability q within this period, what should you charge him to enter the bet for an expected profit of 10% percent of L?

4. Consider a group of n randomly chosen students and let $E_{i,j}$ denote the event that students i and j have the same birthday, $i \neq j$. Under the assumption that the students' birthdays are uniformly distributed throughout the same year, compute

    (i) $\mathbb{P}(E_{c,d}|E_{a,b})$

    (ii) $\mathbb{P}(E_{a,c}|E_{a,b})$

    (iii) $\mathbb{P}(E_{b,c}|E_{a,b} \cap E_{a,c})$