Geometric MLE

For an i.i.d. data set $x = x_1^n$ find the MLE for the geometric distribution:

$$P(X = x) = (1 - \theta)^x \theta$$

Geometric MLE

The likelihood function is

$$L_x(\theta) = \prod_{i=1}^{n} \theta(1-\theta)^{x_i} = \theta^n (1-\theta)^{\sum_{i=1}^{n} x_i}$$

and thus the log-likelihood is

$$\mathcal{L}_x(\theta) = n \log(\theta) + \sum_{i=1}^{n} x_i \log(1-\theta) \ .$$

Geometric MLE

$$\frac{d}{d\theta}\mathcal{L}_x(\theta) = \frac{d}{d\theta} n \log(\theta) + \frac{d}{d\theta}\sum_{i=1}^{n} x_i \log(1-\theta)$$
$$= \frac{n}{\theta} - \frac{\sum_{i=1}^{n} x_i}{1-\theta}$$

Geometric MLE

Setting this to 0 gives

$$
\begin{aligned}
0 &= \frac{n}{\theta} - \frac{\sum_{i=1}^n x_i}{1-\theta} && \Leftrightarrow \\
\frac{n}{\theta} &= \frac{\sum_{i=1}^n x_i}{1-\theta} && \Leftrightarrow \\
n - n\theta &= \theta \sum_{i=1}^n x_i && \Leftrightarrow \\
\theta &= \frac{n}{n + \sum_{i=1}^n x_i}
\end{aligned}
$$