Discrete Distributions

X	1	3	5	7
$F(X \leq x)$	.5	.75	.9	1

a) What is $P(X \leq 3)$?
b) What is $P(X = 3)$?

Answers

a) 0.75
b) 0.75-0.5 $=$ 0.25

Expectation

a) Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance of losing \$5?

b) Would you pay \$5 to participate in a lottery that offers a 10% chance to win \$100 and a %90 percent chance to win nothing?

Partial Answers

This is the same calculation twice:

$$0.1 \times 95 - 0.9 \times 5 = 9.5 - 4.5 = 5$$

Memorylessness

Assume that $X \sim Geometric(p)$. Show that the geometric distribution is memoryless (or stationary), i.e. show that

$$P(X = n + k | X \geq n) = P(X = k)$$

where $n, k > 0$.

Answer

- By definition : $P(X = n + k | X \geq n) = \frac{P(X=n+k, X \geq n)}{P(X \geq n)}$
- We calculate $P(X \geq n)$ as $(1-p)^n$

$$\frac{P(X = n + k, X \geq n)}{P(X \geq n)} = \frac{P(X = n + k)}{P(X \geq n)}$$

$$= \frac{p(1-p)^{n+k}}{(1-p)^n} = p(1-p)^k$$

Variance

X	1	2	3	4	5
$P(X = x)$	.1	.2	.4	.2	.1

a) Compute the variance and standard deviation $\sigma(X)$ of X.

b) What are the variance and standard deviation of $\frac{X}{\sigma(X)}$?

Answers

a) $\mathbb{E}(X) = 3$ and thus $var(X) = .1 \times 4 + .2 \times 2 + .1 \times 4 = 1.2$. It follows that $\sigma(X) = \sqrt{var(X)} = \sqrt{1.2}$.

b) For any real RV Y we have $var\left(\frac{Y}{\sigma(Y)}\right) = \sigma(Y)^{-2}\sigma(Y)^2 = 1$. This is called normalisation.