Homework problem set 3

Your homework must be handed in **electronically via Moodle before Wednesday September 19th, 21:00h**. This deadline is strict and late submissions are graded with a 0. The lowest of your homework grades will be dropped. You are strongly encouraged to work together on the exercises, including the homework. However, after this discussion phase, you have to write down and submit your own individual solution. Numbers alone are never sufficient, always motivate your answers.

Problem 1: (2pt)

Tests if you know how expectation and variance behave under addition and scalar multiplication.

Suppose X is a random variable with $E[X] = 5$ and $\mathrm{Var}[X] = 7$.

(a) Compute $E[(2 + X)^2]$

(b) Compute $\mathrm{Var}(4 + 3X)$.

Problem 2: (2pt)

A jar contains N Euro and M GBP coins. Coins are taken out randomly up to the first draw of a GBP coin. If each drawn coin is put back before picking a new one. What is the probability that you need at least k draws?

Problem 3: (3pt)

Tests if you can describe a RV's distribution. For this particular distribution, you also recap combinations. The last two parts are about calculating expectation and variance from a distribution directly. (Is it a problem that you need (a) for (b) and (c)?)

Assume that k balls are randomly picked from an urn containing N balls labelled from 1 to N. Let X be the largest label present in a draw.

(a) **2pt** Find an expression for the cumulative distribution function.

(b) **1pt** Now fix $N = 6$ and $k = 3$. Give $P(X)$ and calculate $E[X]$.

Problem 4: Negative binomial distribution (3pt)

Assume that a number of independent trials, each with a probability of success of p, $0 < p < 1$, are performed until q successes are registered. Let X be equal to the number of trials required, then

$$P(X = n) = \binom{n-1}{q-1} p^q (1-p)^{n-q} \qquad\qquad n = q, q+1, \ldots$$

Any RV X whose probability distribution is given by the above is said to be a *negative binomial RV* with parameters (q, p). Compute the expectation and variance of this RV with parameters (q, p).