Please use the Poisson in the homework and the geometric in the exercises sheet. I think they need to see more Poissons.

# 1 Poisson EM

You observe the data set $x = \{3, 6, 8, 3, 5, 4, 7, 2, 2, 8\}$. You decide to model with a mixture model with 5 components $c_1, c_2, c_3, c_4, c_5$ which are linked to Poisson distributions with parameters $\lambda_{c_1} = 0.5, \lambda_{c_2} = 1, \lambda_{c_3} = 1.5, \lambda_{c_4} = 2, \lambda_{c_5} = 3$. Initially you assume that all the mixture components are equally likely. You also assume independence between the mixture components. Thus, your model is

$$P(X_1^{10} = x_1^{10}, Y_1^{10} = y_1^{10} | \Lambda = \lambda) = \prod_{i=1}^{10} P(Y_i = y_i) P(X_i = x_i | Y_i = y_i, \Lambda = \lambda)$$

where the $x_i$ are observed and the $y_i$ are latent. Each $Y_i$ can take on values in $\{c_1, c_2, c_3, c_4, c_5\}$.

a) Compute the marginal log-likelihood of $x$ under the initial parameter settings.

b) Perform the E-step on the data. Compute the posterior probabilities for the mixture components per data point.

c) Report the expected sufficient statistics $\mathbb{E}[c = c_i | \lambda]$ for each $1 \leq i \leq 5$.

d) Report the expected sufficient statistics $\mathbb{E}[x = v | c_i, \lambda]$ for $v \in \{2, 3, 4, 5, 6, 7, 8\}$. Use a table for this exercise!

e) Perform the M-step. Report the new parameters of the distributions $P_Y$ and $P_{X|Y_i}$ for $1 \leq i \leq 5$.

f) Compute the marginal log-likelihood of of $x$ under the new parameter settings. **Hint:** If you have done everything correctly, this value should be higher then what you got under the initial parameter settings.