Theory Assignment 2 – Basic Probability, Computing and Statistics

Fall 2015, Master of Logic, University of Amsterdam

Submission deadline: Monday, September 14th, 2015, 9 a.m.

Cooperation Cooperation among students for both theory and programming exercises is strongly encouraged. However, after this discussion phase, every student writes down and submits his/her own individual solution.

Guidelines The starred exercises are relatively easy exercises for you to practice. No points are awarded for them. You may pick and choose **two exercises** for exercise type I and **one exercise** for exercise types II and III for submission, i.e. you need to submit a total of 4 exercises to be able to get all points. Numbered exercises with an exclamation mark are supposed to be a bit harder and you may challenge yourself by trying to solve them.

In the directory of your private url there is folder called 'theory_submissions'. Please upload your submission there. Your submission should be a PDF-document (use a scanner for handwritten documents!) entitled *AssignmentX_yourStudentNumber.pdf*, where X is the number of the assignment and *yourStudentNumber* is your student number. If your submission does not comply with this format, we will deduct 1 point. For each day that your submission is late, we deduct 2 points.

If you have any question about the homework or if you need help, do not hesitate to contact Thomas.

Exercises

Type I [two exercises: 1.5 points per exercise]

1. A hospital registers patients according to whether they have insurance (registered as 1 if insured and 0 if not), as well as according to their condition, rated as good, fair, or hopeless (registered as g, f, and h, respectively). Consider an experiment that consists of registering such a patient.

   (i) Give the sample space of this experiment.

   (ii) Let H be the event that the patient is in hopeless condition. Specify the outcomes in H.

   (iii) Let U be the event that the patient is uninsured. Specify the outcomes in U.

   (iv) Paraphrase the event $(\Omega \setminus U) \cup H$ and give all its outcomes.

2. Consider an experiment that consists of determining the focus of 14 students in a class as either 'logic', 'language' or 'computation', as well as their political inclination – 'left', 'center', or 'right'. How many outcomes are

   (i) in the sample space?

   (ii) in the event that at least one of class member is focuses on 'language'?

   (iii) in the event that none identifies as 'right'?

3. Sixty percent of the students at a certain school wear neither a wristwatch nor a glasses. 20 percent wear a wristwatch and 30 percent glasses. If one is picked randomly, what is the probability of the student is wearing

(i) a wristwatch or glasses?

(ii) a wristwatch and glasses?

4. An urn contains n white and m black balls, $n, m > 0$.

   (i) If two balls are randomly drawn, what is the probability that they are of the same color?

   (ii) If a ball is randomly drawn and then replaced before a second one is drawn, what is the probability that both drawn balls are of the same color?

Type II [one exercise: 3.5 points]

1. Let $v = \mathbb{P}(A|C)$, $w = \mathbb{P}(B|C)$ and $v \leq w$. Show that

   (i) $0 \leq \mathbb{P}(A \cap B|C) \leq v$ (a conjunction's probability is upper-bounded by its least probable conjunct).

   (ii) $w \leq \mathbb{P}(A \cup B|C) \leq 1$ (a disjunction's probability is lower-bounded by its most probable disjunct).

2. Let $v = \mathbb{P}(A)$, $w = \mathbb{P}(B)$ and $v \leq w$. Show that $v + w > 1 \Rightarrow \mathbb{P}(A \cap B) > 0$.

3! An urn contains n green and m orange balls. They are drawn one at a time until a total of r, $r \leq n$, green balls have been drawn. Compute the probability that a total of k balls are drawn.

Type III [one exercise: 3.5 points]

1. Show that $\mathbb{P}(A \cap B|C) = \mathbb{P}(A|C) + \mathbb{P}(B|C) - \mathbb{P}(A \cup B|C)$

2! For a set A, if, for some $i > 0$, $A_1, A_2, ...A_i$ are non-empty mutually exclusive subsets of A such that $\bigcup_{k=1}^{i} A_k = A$, the set $\{A_1, ..., A_i\}$ is called a *partition* of A. Let $P_n \in \mathbb{N}$ be the total number of different partitions of $\{1, 2, ..., n\}$. For instance, $P_1 = 1$ (the only possible partition of the singleton being $\{1\}$ and $P_2 = 2$ (two partitions possible; $\{\{1, 2\}, \{\{1\}, \{2\}\}\}$). Show that

   (i) $P_3 = 5$ and $P_5 = 15$,

   (ii) $P_{n+1} = 1 + \sum_{k=1}^{n} \binom{n}{k} P_k$.

Self-study

* Show that $\mathbb{P}(A, B) = \mathbb{P}(B, A)$.

* Show that $\mathbb{P}(\Omega \setminus A|B) + \mathbb{P}(A|B) = 1$.

* Let A and B be mutually exclusive events with $\mathbb{P}(A) = .3$ and $\mathbb{P}(B) = .5$. What is the probability that (i) either A or B occurs, (ii) A occurs but B does not, (iii) both A and B occur?