Theory Assignment 6 – Basic Probability, Computing and Statistics

Fall 2015, Master of Logic, University of Amsterdam

Submission deadline: Monday, October 12th, 2015, 9 a.m.

Cooperation Cooperation among students for both theory and programming exercises is strongly encouraged. However, after this discussion phase, every student writes down and submits his/her own individual solution.

Guidelines You may pick **4 exercises from exercise type I**, as well as **2 from exercise type II** for submission, i.e. you need to submit **a total of 6 exercises** to be able to get all points. Numbered exercises with an exclamation mark are supposed to be a bit harder and you may challenge yourself by trying to solve them.

In the directory of your private url there is folder called 'theory_submissions'. Please upload your submission there. Your submission should be a PDF-document (use a scanner for handwritten documents!) entitled *AssignmentX_yourStudentNumber.pdf*, where X is the number of the assignment and *yourStudentNumber* is your student number. If your submission does not comply with this format, we will deduct 1 point. For each day that your submission is late, we deduct 2 points. N.B.: If multiple files are submitted for a single assignment before the deadline, the latest version will be graded.

If you have any question about the homework or if you need help, do not hesitate to contact Thomas.

Exercises

Type I [4 exercises: 1.5 points per exercise]

1. Consider (i) a flip of a fair coin, (ii) a toss of a fair four-sided die, and (iii) a toss of a fair six-sided die. Let a RV X encode (i), (ii) and (iii) and compute $H(X)$ in each case.

2. A biased coin comes up heads with a probability of $\frac{2}{3}$. Compute the entropy of the outcome of six coin flips.

3. Let X encode the sum of the roll of two fair dice. Compute $H(X)$.

4. Assume that there is an equal probability of 50% for a person in a population to be male or female. Suppose further that 20% of the males and 6% of the females are tall (height greater than some fixed threshold). Calculate the surprisal of (i) learning that a male person is tall, (ii) a female person is tall, (iii) a tall person is female.

5. Calculate the entropy of the following: (i) pixel values whose possible values are all integers in $[0, 256]$ with uniform probability, (ii) dogs sorted by whether or not they are mammals, (iii) dogs sorted by whether they are older or not than the population's median, (iv) RV X with $P(X = 0) = \frac{1}{3}, P(X = 1) = \frac{1}{4}, P(X = 2) = \frac{1}{6}, P(X = 3) = \frac{1}{6}, P(X = 4) = \frac{1}{12}$.

6. Consider two distributions p and q over $\{a, b\}$. Let $p(a) = 1 - n$ and $q(a) = 1 - m$. Compute $D(p \parallel q)$ and $D(q \parallel p)$ for (i) $m = n$ and (ii) $n = \frac{1}{3}, m = \frac{1}{4}$.

Type II [2 exercises: 2 points per exercise]

1. Show that relative entropy does not satisfy (i) symmetry nor (ii) triangle inequality. That is, it needs not hold that $D(X \parallel Y) \neq D(Y \parallel X)$ nor $D(X \parallel Y) + D(Y \parallel Z) \geq D(X \parallel Z)$.

2. Prove that the entropy of n independent RVs is the sum of the entropy of the individual RVs, i.e. that $H(X_1, ..., X_n)$ is additive if $P(\bigcap_{i=1}^{n} X_i = x) = \Pi_{i=1}^{n} P(X_i = x)$.

* Show that $D(X \parallel Y) \geq 0$.