Theory Assignment 2 – Basic Probability, Computing and Statistics

Fall 2015, Master of Logic, University of Amsterdam

Submission deadline: Monday, September 14th, 2015, 9 a.m.

Cooperation Cooperation among students for both theory and programming exercises is strongly encouraged. However, after this discussion phase, every student writes down and submits his/her own individual solution.

Guidelines The starred exercises are relatively easy exercises for you to practice. No points are awarded for them. You may pick and choose **two exercises** for exercise type I and **one exercise** for exercise types II and III for submission, i.e. you need to submit a total of 4 exercises to be able to get all points. Numbered exercises with an exclamation mark are supposed to be a bit harder and you may challenge yourself by trying to solve them.

In the directory of your private url there is folder called 'theory_submissions'. Please upload your submission there. Your submission should be a PDF-document (use a scanner for handwritten documents!) entitled *AssignmentX_yourStudentNumber.pdf*, where X is the number of the assignment and *yourStudentNumber* is your student number. If your submission does not comply with this format, we will deduct 1 point. For each day that your submission is late, we deduct 2 points.

If you have any question about the homework or if you need help, do not hesitate to contact Thomas.

Exercises

Type I [two exercises: 1.5 points per exercise]

1. A hospital registers patients according to whether they have insurance (registered as 1 if insured and 0 if not), as well as according to their condition, rated as good, fair, or hopeless (registered as g, f, and h, respectively). Consider an experiment that consists of registering such a patient.
   - (i) Give the sample space of this experiment.
   - (ii) Let H be the event that the patient is in hopeless condition. Specify the outcomes in H.
   - (iii) Let $\mathbf{1}$ be the event that the patient is uninsured. Specify the outcomes in $\mathbf{1}$.
   - (iv) Paraphrase the event $\Omega \setminus \mathbf{1} \cup H$ and give all its outcomes.

2. Consider an experiment that consists of determining the focus of 14 students in a class as either 'logic', 'language' or 'computation', as well as their political inclination – 'left', 'center', or 'right'. How many outcomes are
   - (i) in the sample space?
   - (ii) in the event that at least one of class member is focuses on 'language'?
   - (iii) in the event that none identifies as 'right'?

3. Forty percent of the students at a certain school wear neither a wristwatch nor a glasses. 10 percent wear a wristwatch and 40 percent glasses. If one is picked randomly, what is the probability of the student is wearing

(i) a wristwatch or glasses?

(ii) a wristwatch and glasses?

4. An urn contains n white and m black balls, $n, m > 0$.

   (i) If two balls are randomly drawn, what is the probability that they are of the same color?

   (ii) If a ball is randomly drawn and then replaced before a second one is drawn, what is the probability that both drawn balls are of the same color?

Type II [one exercise: 3.5 points]

1. Let $v = \mathbb{P}(A|C)$, $w = \mathbb{P}(B|C)$ and $v \leq w$. Show that

   (i) $0 \leq \mathbb{P}(A \cap B|C) \leq v$ (a conjunction's probability is upper-bounded by its least probable conjunct).

   (ii) $b \leq \mathbb{P}(A \cup B|C) \leq 1$ (a disjunction's probability is lower-bounded by its most probable disjunct).

2. Let $v = \mathbb{P}(A)$, $w = \mathbb{P}(B)$ and $v \leq w$. Show that $a + b > 1 \rightarrow \mathbb{P}(A \cap B) > 0$.

3! An urn contains n green and m orange balls. They are drawn one at a time until a total of r, $r \leq n$, green balls have been drawn. Indicate the probability that a total of k balls are drawn.

Type III [one exercise: 3.5 points]

1. Show that $\mathbb{P}(A \cap B|C) = \mathbb{P}(A|C) + \mathbb{P}(B|C) - \mathbb{P}(A \cup B|C)$

2. For a set A, if, for some $i > 0$, $A_1, A_2, ...A_i$ are non-empty mutually exclusive subsets of A such that $\bigcup_{k=1}^{i} S_k = S$, the set $\{A_1, ..., A_i\}$ is called a partition of A. Let P_n denote a number of different partitions of $\{1, 2, ..., n\}$ such that, e.g., $P_1 = 1$ (the only possible partition of the singleton being $\{1\}$ and $P_2 = 2$ (two partitions possible; $\{\{1, 2\}, \{\{1\}, \{2\}\}\}$). Show that

   (i) $P_3 = 5$ and $P_5 = 15$

   (ii) $P_{n+1} = 1 + \sum_{k=1}^{n} \binom{n}{k} P_k$

Self-study

* Show that $\mathbb{P}(A, B) = \mathbb{P}(B, A)$.
* Show that $\mathbb{P}(\Omega \setminus A|B) + \mathbb{P}(A|B) = 1$.
* Let A and B be mutually exclusive events with $\mathbb{P}(A) = .3$ and $\mathbb{P}(B) = .5$. What is
* the probability that (a) either A or B occurs, (b) A occurs but B does not, (c) both A and B occur?