

Theory Assignment 2 – Basic Probability, Computing and Statistics

Fall 2015, Master of Logic, University of Amsterdam

Submission deadline: Monday, September 14th, 2015, 9 a.m.

Cooperation Cooperation among students for both theory and programming exercises is strongly encouraged. However, after this discussion phase, every student writes down and submits his/her own individual solution.

Guidelines The starred exercises are relatively easy exercises for you to practice. No points are awarded for them. You may pick and choose **two exercises** for exercise type I and **one exercise** for exercise types II and III for submission, i.e. you need to submit a total of 4 exercises to be able to get all points. Numbered exercises with an exclamation mark are supposed to be a bit harder and you may challenge yourself by trying to solve them.

In the directory of your private url there is folder called ‘theory_submissions’. Please upload your submission there. Your submission should be a PDF-document (use a scanner for handwritten documents!) entitled *AssignmentX_yourStudentNumber.pdf*, where X is the number of the assignment and *yourStudentNumber* is your student number. If your submission does not comply with this format, we will deduct 1 point. For each day that your submission is late, we deduct 2 points.

If you have any question about the homework or if you need help, do not hesitate to contact [Thomas](#).

Exercises

Type I [two exercises: 1.5 points per exercise]

1. A hospital registers patients according to whether they have insurance (registered as 1 if insured and 0 if not), as well as according to their condition, rated as good, fair, or hopeless (registered as g , f , and h , respectively). Consider an experiment that consists of registering such a patient.
 - (i) Give the sample space of this experiment.
 - (ii) Let H be the event that the patient is in hopeless condition. Specify the outcomes in H .
 - (iii) Let U be the event that the patient is uninsured. Specify the outcomes in U .
 - (iv) Paraphrase the event $(\Omega \setminus U) \cup H$ and give all its outcomes.
2. Consider an experiment that consists of determining the focus of 14 students in a class as either ‘logic’, ‘language’ or ‘computation’, as well as their political inclination – ‘left’, ‘center’, or ‘right’. How many outcomes are
 - (i) in the sample space?
 - (ii) in the event that at least one of class member is focuses on ‘language’?
 - (iii) in the event that none identifies as ‘right’?
3. Sixty percent of the students at a certain school wear neither a wristwatch nor a glasses. 20 percent wear a wristwatch and 30 percent glasses. If one is picked randomly, what is the probability of the student is wearing

- (i) a wristwatch or glasses?
 - (ii) a wristwatch and glasses?
4. An urn contains n white and m black balls, $n, m > 0$.
- (i) If two balls are randomly drawn, what is the probability that they are of the same color?
 - (ii) If a ball is randomly drawn and then replaced before a second one is drawn, what is the probability that both drawn balls are of the same color?

Type II [one exercise: 3.5 points]

1. Let $v = \mathbb{P}(A|C)$, $w = \mathbb{P}(B|C)$ and $v \leq w$. Show that
 - (i) $0 \leq \mathbb{P}(A \cap B|C) \leq v$ (a conjunction's probability is upper-bounded by its least probable conjunct).
 - (ii) $w \leq \mathbb{P}(A \cup B|C) \leq 1$ (a disjunction's probability is lower-bounded by its most probable disjunct).
2. Let $v = \mathbb{P}(A)$, $w = \mathbb{P}(B)$ and $v \leq w$. Show that $v + w > 1 \Rightarrow \mathbb{P}(A \cap B) > 0$.
- 3! An urn contains n green and m orange balls. They are drawn one at a time until a total of r , $r \leq n$, green balls have been drawn. Compute the probability that a total of k balls are drawn.

Type III [one exercise: 3.5 points]

1. Show that $\mathbb{P}(A \cap B|C) = \mathbb{P}(A|C) + \mathbb{P}(B|C) - \mathbb{P}(A \cup B|C)$
- 2! For a set A , if, for some $i > 0$, A_1, A_2, \dots, A_i are non-empty mutually exclusive subsets of A such that $\bigcup_{k=1}^i A_k = A$, the set $\{A_1, \dots, A_i\}$ is called a *partition* of A . Let $P_n \in \mathbb{N}$ be the total number of different partitions of $\{1, 2, \dots, n\}$. For instance, $P_1 = 1$ (the only possible partition of the singleton being $\{1\}$ and $P_2 = 2$ (two partitions possible; $\{\{1, 2\}, \{\{1\}, \{2\}\}\}$). Show that
 - (i) $P_3 = 5$ and $P_4 = 15$,
 - (ii) $P_{n+1} = 1 + \sum_{k=1}^n \binom{n}{k} P_k$.

Self-study

- * Show that $\mathbb{P}(A \cap B) = \mathbb{P}(B \cap A)$.
- * Show that $\mathbb{P}(\Omega \setminus A|B) + \mathbb{P}(A|B) = 1$.
- * Let A and B be mutually exclusive events with $\mathbb{P}(A) = .3$ and $\mathbb{P}(B) = .5$. What is the probability that
 - (i) either A or B occurs, (ii) A occurs but B does not, (iii) both A and B occur?