

Theory Assignment 5 – Basic Probability, Computing and Statistics

Fall 2015, Master of Logic, University of Amsterdam

Submission deadline: Monday, October 12th, 2015, 9 a.m.

Cooperation Cooperation among students for both theory and programming exercises is strongly encouraged. However, after this discussion phase, every student writes down and submits his/her own individual solution.

Guidelines You may pick **N exercises from exercise type I**, as well as **M from exercise type II** for submission, i.e. you need to submit **a total of N+M exercises** to be able to get all points. Numbered exercises with an exclamation mark are supposed to be a bit harder and you may challenge yourself by trying to solve them.

In the directory of your private url there is folder called ‘theory_submissions’. Please upload your submission there. Your submission should be a PDF-document (use a scanner for handwritten documents!) entitled *AssignmentX_yourStudentNumber.pdf*, where X is the number of the assignment and *yourStudentNumber* is your student number. If your submission does not comply with this format, we will deduct 1 point. For each day that your submission is late, we deduct 2 points. N.B.: If multiple files are submitted for a single assignment before the deadline, the latest version will be graded.

If you have any question about the homework or if you need help, do not hesitate to contact [Thomas](#).

Exercises

Type I [N exercises: Q point per exercise]

1. Consider the outcome of (i) a flip of a fair coin, (ii) a toss of a fair four-sided die, and (iii) a toss of a fair six-sided die. Let a RV X encode (i), (ii) and (iii) and compute $H(X)$ in each case.
2. A biased coin comes up heads with a probability of $\frac{2}{3}$. Compute the entropy of the outcome of six coin flips.
3. Let X encode the sum of the roll of two fair dice. Compute $H(X)$.
4. Assume that there is an equal probability of 50% for a person in a population to be male or female. Suppose further that 20% of the males and 6% of the females are tall (height greater than some fixed threshold). Calculate the information gained by (i) learning that a male person is tall, (ii) a female person is tall, (iii) a tall person is female.

Type II [M exercises: R points per exercise]

1. Show that relative entropy does not satisfy (i) symmetry nor (ii) triangle inequality. That is, it needs not hold that $D(X \parallel Y) \neq D(Y \parallel X)$ nor $D(X \parallel Y) + D(Y \parallel Z) \geq D(X \parallel Z)$.
2. Show that $D(X \parallel Y) \geq 0$.

3. Prove that the information gained from observing n independent RVs is the sum of the information gained from their individual observation, i.e. that $H(X_1, \dots, X_n)$ is additive if $P(\bigcap_{i=1}^n X_i = x) = \prod_{i=1}^n P(X_i = x)$.