Board Questions

Fifth Session, Oct 3rd

1 Sufficient Statistics

You are given a data set $x = x_1^n$ of n i.i.d. geometrically distributed observations. Show that $\sum_{i=1}^{n}$ is a sufficient statistic for the geometric distribution.

2 MLE

A coin is taken from a box containing three coins, which give heads with probability $p = 1/3, 1/2$, and $2/3$. The mystery coin is tossed 80 times, resulting in 49 heads and 31 tails.

a) What is the likelihood of this data for each type on coin? Which coin gives the maximum likelihood?

b) Now suppose that we have a single coin with unknown probability p of landing heads. Find the likelihood and log likelihood functions given the same data. What is the maximum likelihood estimate for p?

3 Bayes' Rule for Parameters

A coin is taken from a box containing three coins, which give heads with probability $p = 1/3, 1/2$, and $2/3$. The mystery coin is tossed 80 times, resulting in 49 heads and 31 tails.

a) What is the likelihood of this data for each type on coin? Which coin gives the maximum likelihood?

b) Now suppose that we have a single coin with unknown probability p of landing heads. Find the likelihood and log likelihood functions given the same data. What is the maximum likelihood estimate for p?

4 Geometric MLE

For an i.i.d. data set $x = x_1^n$ find the MLE for the geometric distribution:

$$P(X = x) = (1 - \theta)^x \theta$$