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# **Financial Education - basics**

**basics**

**Dec 10, 2025**



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This book is meant to collect some notes about financial instruments and methods for financial education, and mainly focused asset allocation.

This material is part of the **basics-books project**. It is also available as a .pdf document.

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## Main goal

The ultimate goal of this material is to develop an understanding of how to manage personal savings efficiently, in line with one's own reasonable objectives.

To achieve this, some intermediate goals include:

- gaining knowledge of the **macroeconomic environment**
  - familiarizing with some of the most common **investment tools** (mainly bonds and stocks);
  - getting used to some **common-sense** and **investing principles**: minimizing certain costs when conditions are equal, risk/reward, diversification, liquidity, and other constraints/inefficiencies
  - learning **what not to do**
  - and once the poor choices have been ruled out, evaluating the reasonable options for building and managing an investment portfolio, using mainly **ETFs** as a natural choice of a (usually) liquid asset providing diversification at low cost, even for small capitals.
- 

- Introduction
  - *Summary*
  - *References*
- Macroeconomic Context for Investing
  - *Actors*
  - *Inflation*
  - *Characteristic times in economy*
  - *Policy*
- Investing Principles
  - *Introduction to principles of investing*
  - *Rebalancing*
  - *Sequence risk*
- Asset classes
  - *Introduction to asset classes*
  - *Bonds*
  - *Equity*
  - *ETFs*
- Asset allocation and Investing
  - *Introduction to investing*
  - *Modern Portfolio Theory*
  - *Capital Asset Pricing Model*

- *Merton's portfolio problem*
- *Rebalancing*
- Extra
  - *Extra and Random*
  - *Euristhics and historical correlations*
- People
  - *Resources, People and Firms*
  - *The Bull*

# **Part I**

# **Introduction**



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## **CHAPTER**

## **ONE**

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## **SUMMARY**

### **Introduction**

Financial goals; money; inflation (BC and inflation target);

### **Asset classes**

### **Asset allocation**



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**CHAPTER  
TWO**

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**REFERENCES**

Here some references to other sources, in order to reasonably organize the contents of this book

**Investment and Portfolio Management - RICE - coursera - A.Ozoguz, J.Foote**

**Global Financial Markets**

- Intro and Review of Elementary Finance Tools
- Financial system and financial assets: fixed income, equity and derivatives
- Organization of financial markets and securities trading

**Portfolio Selection and Risk Management**

- Intro and R/R: R/R trade-off
- Ptf construction and diversification
- Investor choices: utility functions, mean-variance preferences
- Optimal capital allocation and portfolio choice: mean-variance optimization (Modern Portfolio Theory)
- Equilibrium asset pricing models: CAPM, return-beta; multi-factor models (e.g. Fama-French)

**Biases and Portfolio Selection**

- Efficient Market Hypothesis (EMH), and anomalies
- Biases and realistic preferences
- Inefficient markets: equity premium, volatility puzzle (?), long-run reversal to the mean, value effect, momentum
- Investor behavior

**Investment Strategies and Portfolio Analysis**

- Performance measurement and benchmarking
- Active vs passive investing:  $R^*$  risk-adjusted return measurements: Sharpe, Sortino, Treynor'ratio, Jensens'alpha,...;comparing the  $R^*$
- Performance evaluation: style analysis and performance attribution

**Capstone: Build a Winning Investment Portfolio**

Using software for building ptf and assess its properties

- ...

## **Part II**

# **Macroeconomic Context for Investing**



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CHAPTER  
THREE

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ACTORS

**3.1 People**

**3.2 Private companies**

**3.3 Government - public**

**3.4 Banks**

**3.4.1 Central banks**

**3.4.2 Investment banks**

**3.5 Foreign regions**



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**CHAPTER  
FOUR**

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## **INFLATION**

Inflation is the **rate** at which the general level of prices for goods and services changes.

**Contents.** Definition and *inflation indices*, with examples of indices used in Italy: NIC, FOI, IPCA; *components of inflation*, with details of IPCA in Italy; *correlation with other macroeconomic quantities*; *who controls inflation*; *origin of inflation*

### **4.1 Inflation Indices (e.g. in Italy)**

Overall inflation is the weighted average of inflation on different classes of goods and services, weighted for their share of expenses.

Everyone perceives its own inflation, depending on its expenses. Different indices are usually used within an economy to track inflation for some “average individual”.

Different indices may differ on values of weights, and other “details” like the effect of discounts and public transfers.

As an example, three indices are used in Italy:

- **NIC** (Prezzi al Consumo per l'intera Collettività Nazionale), usually the general
- **FOI** (Prezzi al Consumo per Famiglie di Operai e Impiegati), usually used for contracts, pension and inflation-linked contracts, ex-tobacco and lotteries.
- **IPCA** (Indice Armonizzato dei Prezzi al Consumo, HIPC *Harmonized Index of Consumer Prices*), used for comparison and statistics in the EU

### **4.2 Weights and Price Indices of Classes of Goods and Services - Italy IPCA**

National and International Institutions for Statistics (in Italy, ISTAT) provide open-access databases collecting statistics about society and economics, including data about price.

**ISTAT.** As an example, Italian ISTAT provides data at <https://esploradati.istat.it/databrowser/#/it/dw>

All the data we need here is available under the category “Prezzi” - *Prices*. In order to reach a reasonable stability of the notebook, data have been downloaded, cleaned and stored in a folder on the repository of the project.

## 4.2.1 Inspect Data

Before producing plots, price indices and weights of level-4 categories are visually inspected. Data are usually collected in tables.

### Category Price Indices - Level-4 IPCA

### Category Weights - Level-4 IPCA

## 4.2.2 Plots

### Category weights - Level-2 IPCA

The weights assigned to IPCA (Harmonized Index of Consumer Prices) categories represent the average expenditure share of households on each category of goods and services. These weights reflect how important each category is in the consumption basket.

These weights are revised annually to account for changing consumer behavior, as one can easily realize acting on the slider of the picture below. They are the weights used in computing the overall inflation  $i$  index, as the weighted sum of inflation  $i_c$  of IPCA categories,

$$i = \sum_{c \in \text{Cat}} i_c w_c .$$

### Category Prices - Level-2 IPCA

Some categories in IPCA are subject to strong seasonal effects, meaning prices follow recurring patterns during the year.

As an example:

- Clothing and Footwear: in July–August, retailers apply seasonal discounts (saldi) in Italy and prices in IPCA do include these discounts when they are actually applied in stores, as it's shown by seasonal July/August price drops
- Fresh fruits and vegetables: prone to seasonal availability, leading to fluctuating prices.
- Travel and tourism: prices rise in summer and holidays.

Seasonality can obscure underlying inflation trends: that's why **seasonally adjusted** inflation is evaluated, see below.

```
Index(['[00] Indice generale',
       '[01] -- prodotti alimentari e bevande analcoliche',
       '[02] -- bevande alcoliche e tabacchi',
       '[03] -- abbigliamento e calzature',
       '[04] -- abitazione, acqua, elettricità, gas e altri combustibili',
       '[05] -- mobili, articoli e servizi per la casa',
       '[06] -- servizi sanitari e spese per la salute', '[07] -- trasporti',
       '[08] -- comunicazioni', '[09] -- ricreazione, spettacoli e cultura',
       '[10] -- istruzione', '[11] -- servizi ricettivi e di ristorazione',
       '[12] -- altri beni e servizi'],
      dtype='object', name='Tempo')
```

### Category Price Changes (Inflation) - Level-2 IPCA

The 12-month inflation rate (year-on-year or YoY) compares prices in a given month to the same month the year before. It's already less prone to seasonal effects than the month-to-month rate.

However, even YoY rates can exhibit seasonal patterns, especially in volatile components like food, energy, and clothing. In order to reduce volatility of inflation indices, it's possible to use:

- **Core inflation**, as a measure of inflation that excludes the most volatile items (e.g., unprocessed food, energy), in order to provide a smoothed measure of inflation trends.
- Statistical filtering, and moving averages

### Energy post-2022

Since 2022, prices in the energy and utility sectors have shown exceptional volatility. Different causes may have contributed, like geopolitical tensions (notably, the war in Ukraine), “liberalized” electricity/gas markets in Italy where price caps were adjusted or removed. Inflation in energy and electricity was also influenced by a *base effect* (e.g., very low prices in 2020–2021 followed by spikes in 2022).

Policy interventions like tax reductions and bonuses - that are not “free” -, which may or may not be reflected in consumer prices, depending on implementation.

The use of *core inflation* in 2022–2023 was arguable, as energy prices didn't just spike and revert, but was/is quite a long-term shock (war, sanctions, market and supply restructuring,...); as energy price influences many other sectors, food price rose as well, due to input cost shocks (fertilizers, transports,...) not as a result of seasonality only. Using core inflation and excluding energy and food components masked the true **cost-of living** impact on households.

### Category contributions to overall inflation - Level-2 IPCA

## 4.3 Correlations in macroeconomics with inflation

Some correlations exist<sup>1</sup> between inflation and other macroeconomics quantities.

- **Phillips Curve**: inverse relation between inflation and unemployment (in the short-run)
- **Money supply** in the long-run “*Inflation is a monetary phenomenon*”, M.Friedman.

## 4.4 Control of Inflation

Control of inflation is one of the goals of **central banks**, like the FED and the ECB.

*Central banks* aims at controlling inflation, matching target inflation (usually set as 2%) by means of **monetary policy**:

- interest rates (cost of money)
- non-conventional actions, like quantitative easing (QE)/tightening (QT)

<sup>1</sup> ...

A government may indirectly influence inflation with **fiscal policy**, as taxation and government spending can influence demand.

**Credibility** of targets, and actors through their actions and forward guidance may influence inflation as well: expectations influences inflation.

## 4.5 Origin of inflation

Origin of inflation?

- *short-run, medium-run*: cost-push, demand-pull, built-in (triangle model)
  - *long-run*: “inflation is always and everywhere a monetary phenomenon” M.Friedman
-

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CHAPTER  
**FIVE**

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## **CHARACTERISTIC TIMES IN ECONOMY**

**5.1 The short run**

**5.2 The medium run**

**5.3 The long run**



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**CHAPTER  
SIX**

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**POLICY**

	Monetary Policy	Fiscal Policy
Controlled by	CB	Government
Main tools	IR, Money supply	Taxes, Spending, Transfers
Speed	Usually faster	Politically slower, debated
Focus	Inflation, Illiquidity, credit	Employment, Income distribution
Independence		

## 6.1 Monetary policy

## 6.2 Fiscal policy



## **Part III**

# **Investing Principles**



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CHAPTER  
SEVEN

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## INTRODUCTION TO PRINCIPLES OF INVESTING

Investing is a core part of personal financial management—it's how individuals navigate uncertainty to meet their financial goals under real-world constraints. The most basic objective is to preserve the real value of wealth, protecting it against *inflation*; more ambitious goals include growing capital to fund retirement, education, or other life plans.

Sound investing requires understanding *return* and *risk* of available assets, and the fundamental *R/R trade off*. It also demands attention to **constraints** such as *liquidity* needs, *time horizon*, *acceptable volatility*, and *risk tolerance*. One of the main principle is *diversification* - which can reduce risk and, in some cases, enhance returns.

This section introduces the core concepts needed to build a robust investment strategy: how *compound returns* shape long-term growth, how *volatility drag* reduces expected performance, and how a clear, principle-based approaches - like *rebalancing* - may improve performance under uncertainties.

Given its set of constraints, an informed and intelligent agent, see *Portfolio construction* would take actions that try to maximise return for a given accepted risk, or minimize risk for a given desired return: this behavior can be summarized in choosing actions on a *Pareto front*, i.e. within the set of all Pareto efficient solutions.

### Sections

Section	Key Concepts
1. <i>Return</i>	
2. <i>Risk</i>	
3. <i>Risk-Return Trade-Off</i>	
4. <i>Diversification</i>	
5. <i>Portfolio Construction</i>	
6. <i>Time and Compounding</i>	Compounding and volatility drag
7. <i>Disciplined Investing</i>	PIC/PAC, rebalancing,...

## 7.1 Return

Return is the reward for investing. It can come from **capital gain** (price increase of assets bought), or **periodic cashflows**, like interest (from bonds), or dividends (from stocks). Some assets produce predictable return (either nominal, or real), other assets have less predictable returns. Any asset has some level of uncertainty, or *risk*<sup>1</sup>.

Most returns are quoted on a **per-period** basis - usually annually - and expressed as the percentage of the reward over the initial amount of the investment.

For a many-year investment, single-period returns **compound** over time.

### 7.1.1 Costs

While return are uncertain, at least to a certain level, usually costs - fees, expenses, taxes - or part of them, are certain. With equal other conditions, the intelligent investor should reduce costs (known), as higher costs reduce returns w/o changing the level of risk.

## 7.2 Risk

Risk measures uncertainty and its effects, combining probability of events and consequences of specific events. *All the assets have some systematic and some specific risks* .

Key measures (*should give info about magnitude, frequency/probability, and duration*) include:

- standard deviation or **volatility**: how much returns may deviate from their expected value),
- max loss (usually 100% can't be neglected for catastrophic although rare events), value at risk (VaR, max loss with a given probability), drawdown (maximum peak-to-trough loss)
- time-to-recover (time to recover drawdowns, in a temporal perspective)

Usually, risk metrics measure uncertainty, without discerning from positive and negative events: these metrics perceive a higher-than-expected return as a risk as well. Some metrics instead, see *Sortino ratio* in *risk-return* section, aims at quantifying only negative events as risk.

## 7.3 Risk-Return Trade Off

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**“There’s no free lunch”**

Higher expected returns usually come with higher risk.

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**...but high risk doesn’t imply high expected return**

Very stupid actions usually implies poor return with high risk. Just as an example, playing Russian roulette for fun implies an expected return worse than an alternative “do-nothing and have an ice-cream instead” scenario (at least, if your goal is not to kill yourself, and your return function does not positively weight this outcome) with higher uncertainty on the final status of your health.

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<sup>1</sup> Even the most safe assets could undergo some (really) **rare**, but usually (really) **catastrophic events**. Just as an example, it's hard to imagine what could happen even to bonds issued by the most (perceived and priced) safe government or institution, in case of its participation in a war.

Sometimes the same could happen if one plays doing trading with some random meme-stocks or shit-coins.

**Risk-adjusted return** provides an indication of the expected return per unit of risk. Common metrics are:

- **Sharpe ratio**, comparing excess return and volatility compared with a “risk-free” asset - or a benchmark

$$S := \frac{\mathbb{E}[R - R_b]}{\sqrt{\text{var}[R - R_b]}}$$

- **Sortino ratio**

$$So := \frac{\mathbb{E}[R] - T}{\text{DR}},$$

with  $T$  target return, and DR the downside deviation, i.e. the deviation w.r.t the target return evaluated only for returns  $r$  lower than the target return  $T$

$$\text{DR}^2 = \int_{r=-\infty}^T (T - r)^2 f(r) dr,$$

being  $f(r)$  the probability density function of the (continuous) random variable  $R$  representing return

## 7.4 Diversification

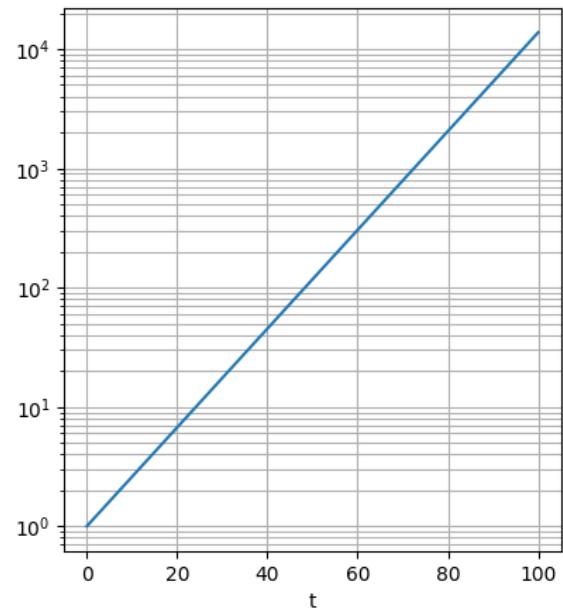
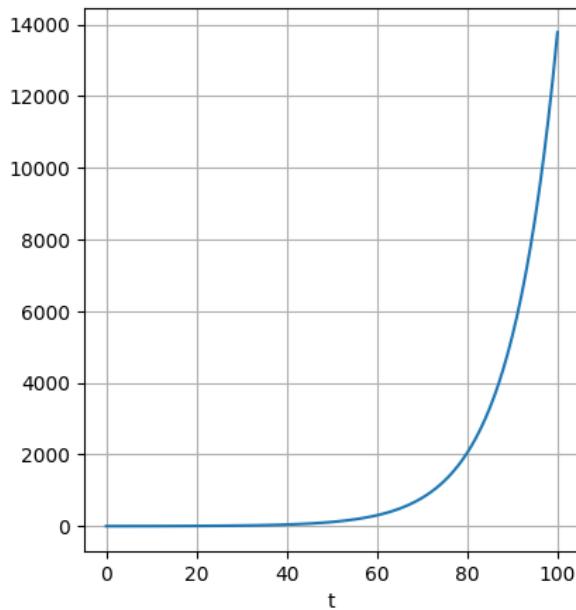
Diversification spreads risk across different investments so no single event can ruin your portfolio. Diversification works well with assets that are not - or at least they're loosely - correlated: in this case, diversification could increase return per unit of risk.

## 7.5 Portfolio Construction

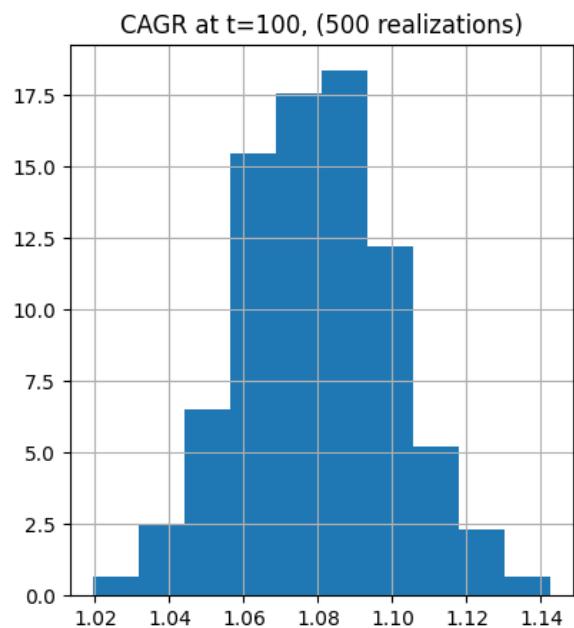
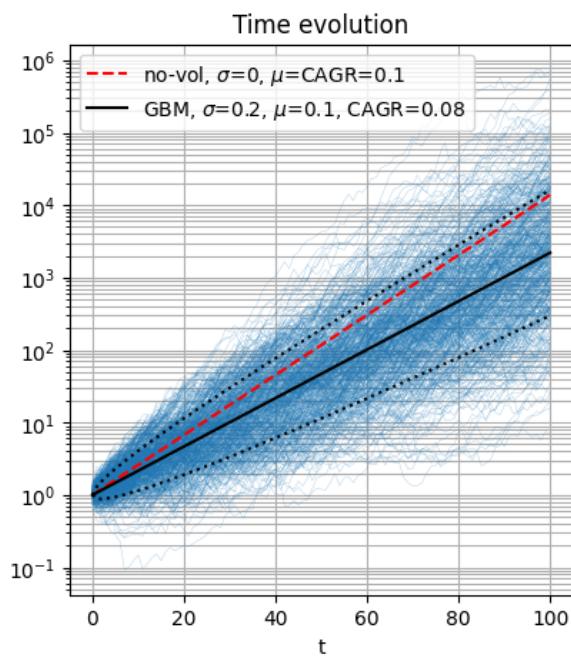
### 7.6 Time

#### 7.6.1 Compound Return

```
(Text(0.5, 0, 't'), None)
```



### Volatility Drag



todo

- “Time and risk?” Listen to *The Logic of Risk*

## 7.7 Disciplined Investing

### 7.7.1 Rebalancing

Colab Notebook, rebalancing.ipynb

**Rebalancing premium**

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CHAPTER  
**EIGHT**

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## REBALANCING

In this Notebook, two strategies on a 2-asset portfolio are discussed and compared:

- **rebalanced portfolio**, after each period
- **buy-and-hold portfolio**, without rebalancing

Effects of rebalancing and conditions for **rebalancing premium** are discussed: sometimes the expected log-return of the rebalanced portfolio may exceed the expected log-return of each single asset.

## 8.1 Libraries, parameters and useful functions

Libraries are imported and useful functions to treat conics below are defined here

### 8.1.1 Libraries

### 8.1.2 Parameters

### 8.1.3 Functions for conics

## 8.2 Rebalanced portfolio

Let the **1-period return** of the assets be normal (**todo** is this necessary? Can't one rely on central limit theorem? How long the summation must be for convergence to normal distribution, in presence of **heavy-tails** distribution? If one can't rely on central limit theorem, let use numerical methods to investigate the effect of heavy tails distributions),

$$\mathbf{r} \sim \mathcal{N}(\mu, \sigma^2) .$$

Compound return of the portfolio has **expected value**

$$\mu_p^c = \mathbb{E}[r_p^c] = \mathbf{w}^T \mu - \frac{1}{2} \mathbf{w}^T \sigma^2 \mathbf{w}$$

and **variance**

$$\sigma_{r_p^c}^2 = \mathbb{E}[(r_p^c - \mu_p^c)^2] = \dots = \mathbf{w}^T \sigma^2 \mathbf{w}$$

### 8.2.1 Shannon demon

Sometimes the expected value of the compound return of the rebalanced portfolio can be larger than the expected return of each asset class.

$$\mu_k^c = \mathbb{E}[r_k^c] = \mu_k - \frac{\sigma_k^2}{2}$$

**Example: 2-asset portfolio.** As an example, the expected value of the compound return of a 2-asset rebalanced portfolio,

$$\mathbb{E}[r_p^c] = w_1\mu_1 + w_2\mu_2 - \frac{1}{2}(w_1^2\sigma_1^2 + 2w_1w_2\rho\sigma_1\sigma_2 + w_2^2\sigma_2^2)$$

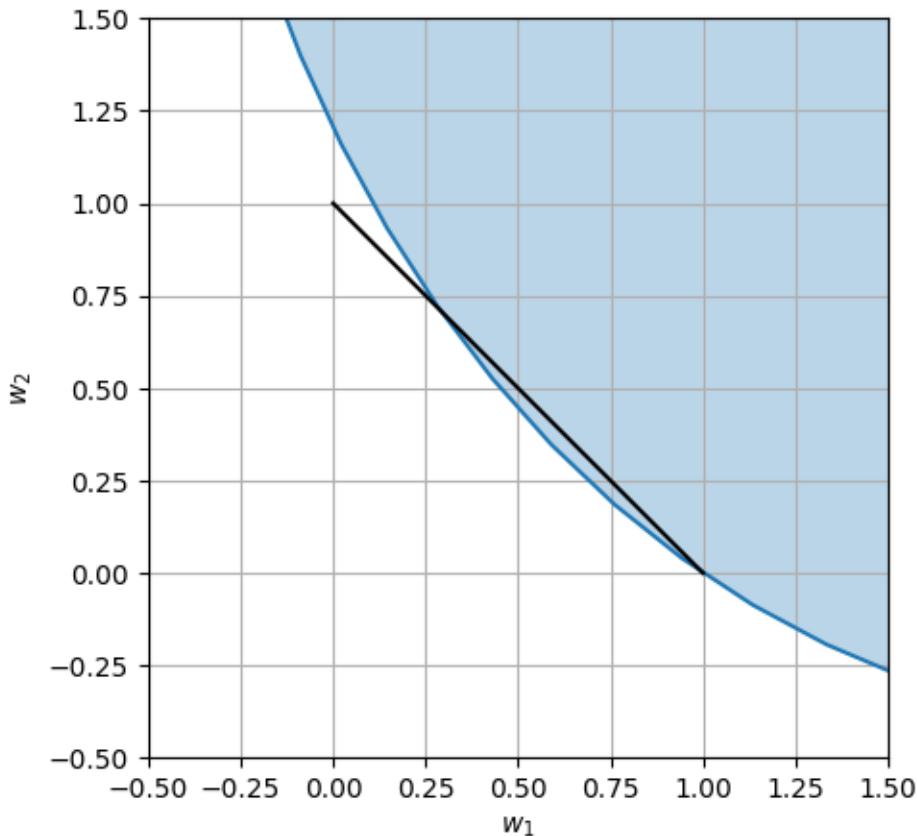
Using  $w_1, w_2$  as independent variables, for any value of the expected return, the expression of the return itself can be represented as a **conic section** in the  $w_1, w_2$ -plane. In particular,

- for  $\rho \neq 1$ , it's an **ellipse** ( $\Delta = B^2 - 4AC < 0$ ),
- for  $\rho = 1$ , it's a **parabola** ( $\Delta = 0$ )

**Portfolio allocation.** Some constraints may hold on portfolio allocation:

- fully-invested:  $w_1 + w_2 = 1$
- no short-selling:  $w_1, w_2 \geq 0$
- no leverage:  $w_1, w_2 \leq 1$

```
interactive(children=(FloatSlider(value=-0.25, description='rho', max=1.0, min=-1.0, step=0.01), FloatSlider(v...
```



This plot represents in **blue** asset allocations of the balanced 2-asset portfolio with expected value of the compound return larger than the compound return of any individual asset. **Black line** represents all the possible allocations of a fully invested portfolio,  $w_1 + w_2 = 1$  with no leverage  $w_k \leq 1$  and not short selling  $w_k \geq 0$ .

## 8.3 Comparison of portfolios: realizations of stochastic processes

In this section, rebalanced portfolio and buy-and-hold portfolio are compared. Different realizations of these two portfolio strategies are built, and used to build statistics, and discuss their properties in terms of **compound return**, **drawdowns**,...

**Note.** Here, 1-period returns are modelled as *normal random variable* so far. Anyways, it's possible (and suggested) to implement the most suited random process for modelling the return of the desired assets.

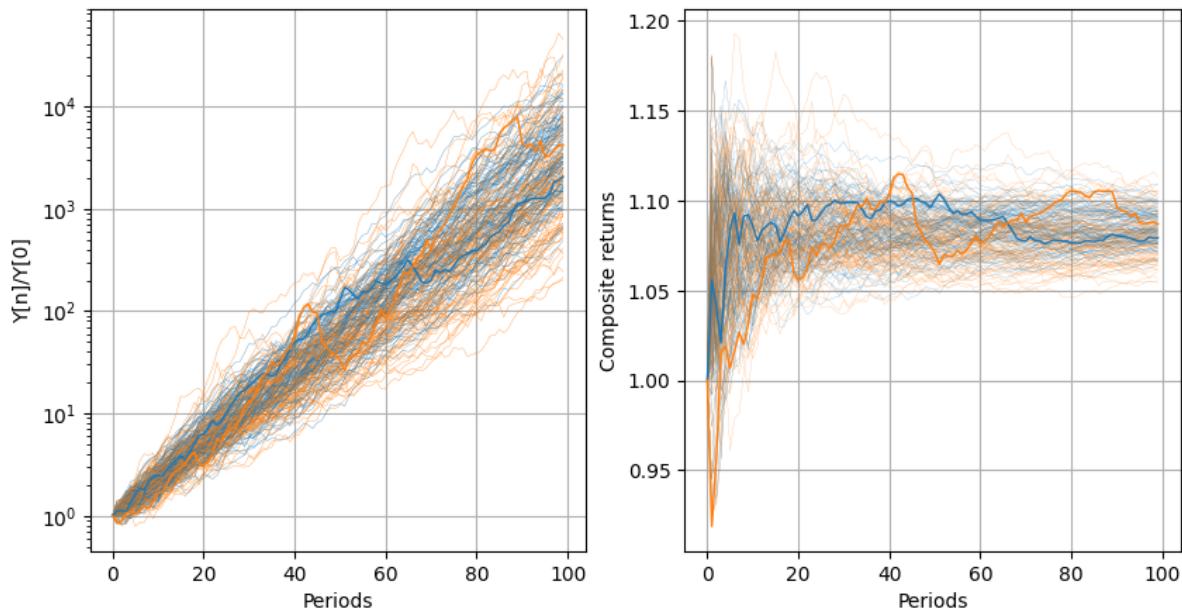
### 8.3.1 Useful functions

A useful function is introduced here to build correlated random variables with the desired expected values and covariance.

Main Colab notebook can be found here: [https://colab.research.google.com/drive/1n5py0Zf8i3\\_jrTTk0AR7Noq2kBYwpaqx?authuser=1#scrollTo=gmbjbprjCHto](https://colab.research.google.com/drive/1n5py0Zf8i3_jrTTk0AR7Noq2kBYwpaqx?authuser=1#scrollTo=gmbjbprjCHto)

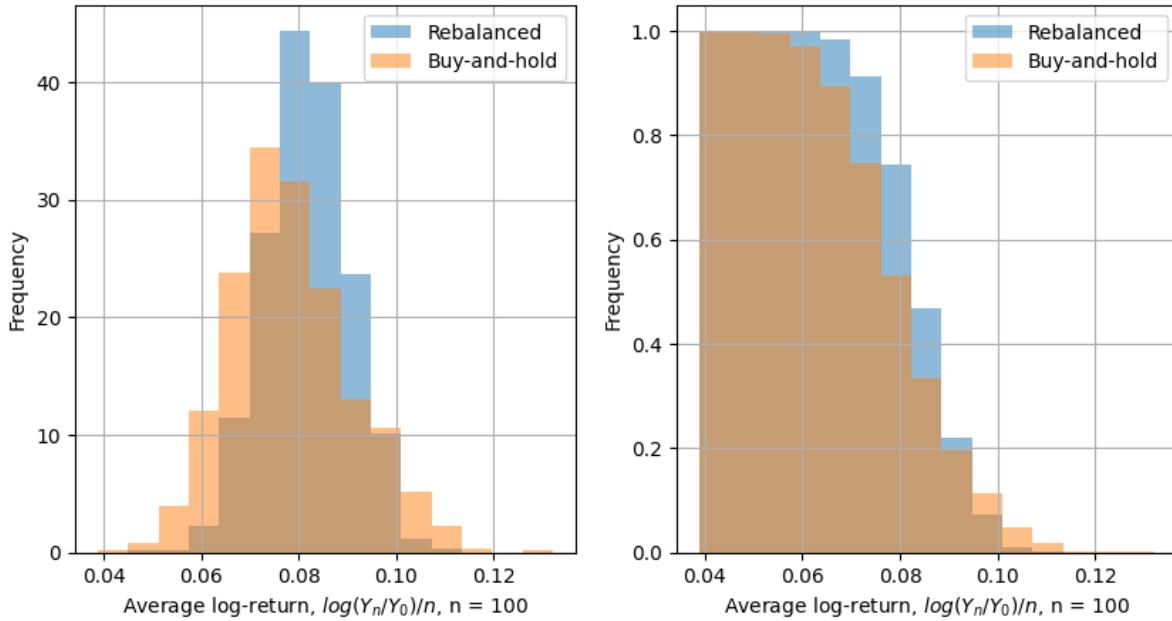
### 8.3.2 Realizations

```
[<matplotlib.lines.Line2D at 0x7f1a1ada6fa0>]
```



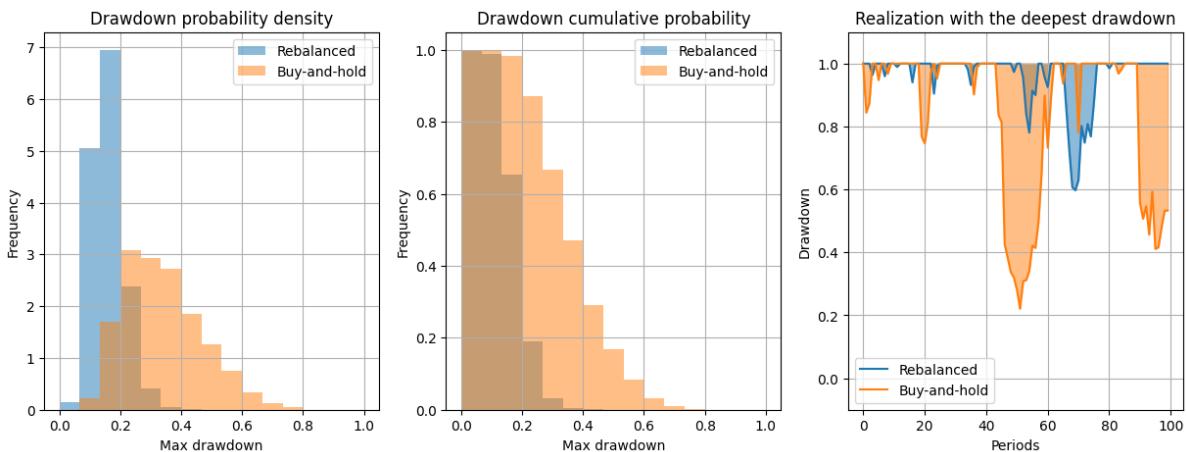
### 8.3.3 Composite return

```
Rebalanced portfolio. Compsite return
Exp. value: 0.085
Std. dev. : 0.009
Buy-and-Hold portfolio. Compsite return
Exp. value: 0.082
Std. dev. : 0.013
```



### 8.3.4 Maximum drawdown

```
Text(0.5, 1.0, 'Realization with the deepest drawdown')
```



## SEQUENCE RISK

### 9.1 Introduction

Sequence risk occurs when investment or withdrawal is distributed in time. These two scenarios may be representative of:

- Dollar Cost Averaging (**DCA**, or **PAC** in Italian for “Piano di Accumulo di Capitale”)
- **Withdrawal** during old age

Sequence in time of 1-period returns may strongly influence the composite return of a portfolio.

#### 9.1.1 Mathematical model

In a continuous-time model, sequence risk of constant-amount DCA or withdrawal can be modeled with a Geometric Brownian Motion with “drift”,

$$dX_t = \mu X_t dt + \sigma X_t dW_t + C dt ,$$

being  $C_t$  the rate of investment ( $> 0$ ) or withdrawal ( $< 0$ ),  $\mu, \sigma$  the expected value and variance of the rate of return. A discrete-time counterpart may be

$$\Delta X_{n,n+1} = (\mu_{n,n+1} + \sigma_{n,n+1} W_{n,n+1}) X_n + C_{n,n+1} ,$$

with  $\mu_{n,n+1}, \sigma_{n,n+1}$  the expected value and the variance of the 1-period return,  $W_{n,n+1}$  a unit-variance random variable representing the distribution of the returns from  $n$  to  $n + 1$ , and  $C_{n,n+1}$  the investment of withdrawal from  $n$  to  $n + 1$ .

#### 9.1.2 Constant investment or withdrawal rate: analytical solution

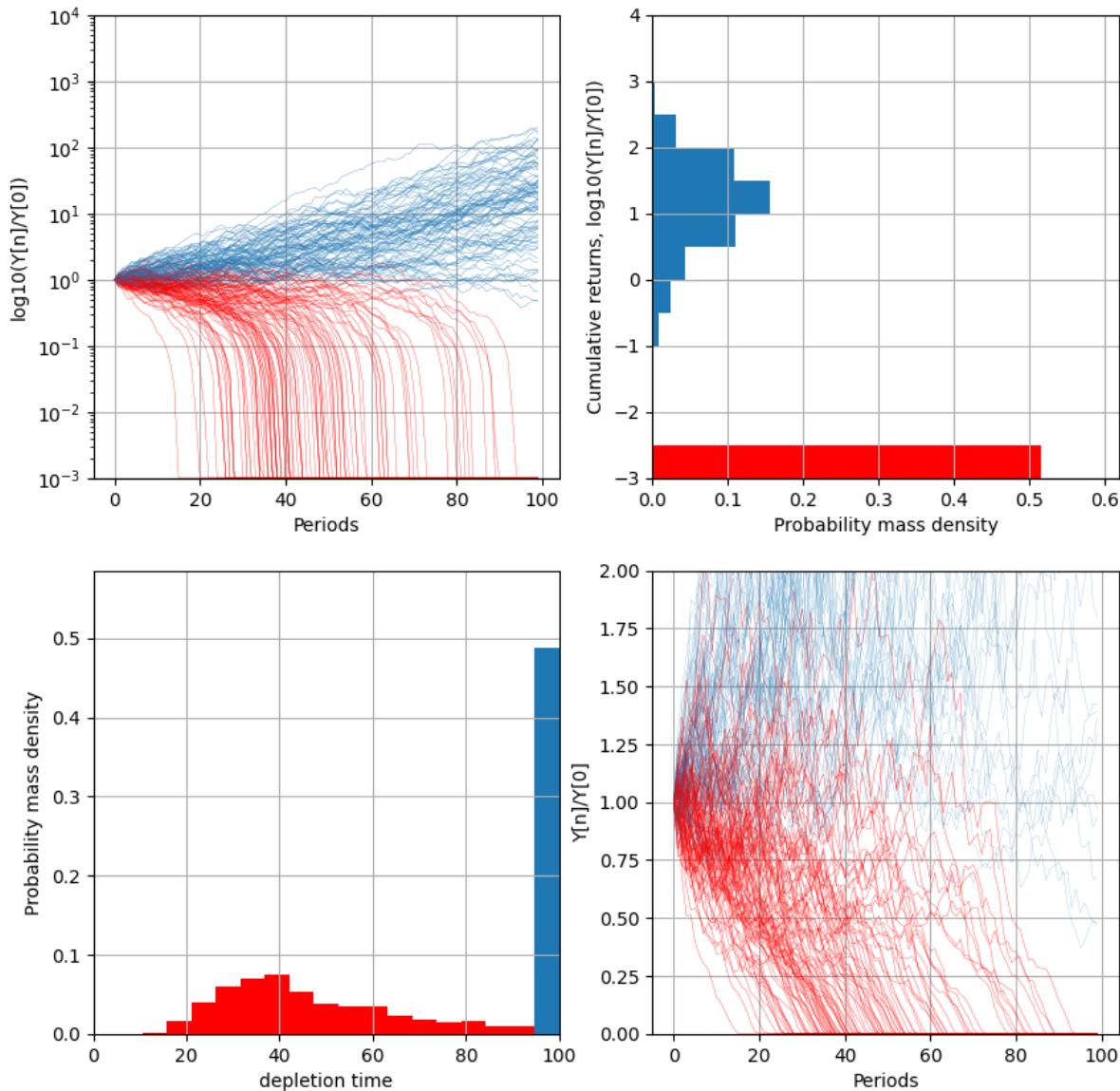
The solution of the continuous-time equation with reads

$$X_t = X_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t} + C \int_{s=0}^t e^{(\mu - \frac{\sigma^2}{2})(t-s) + \sigma(W_t - W_s)} ds$$

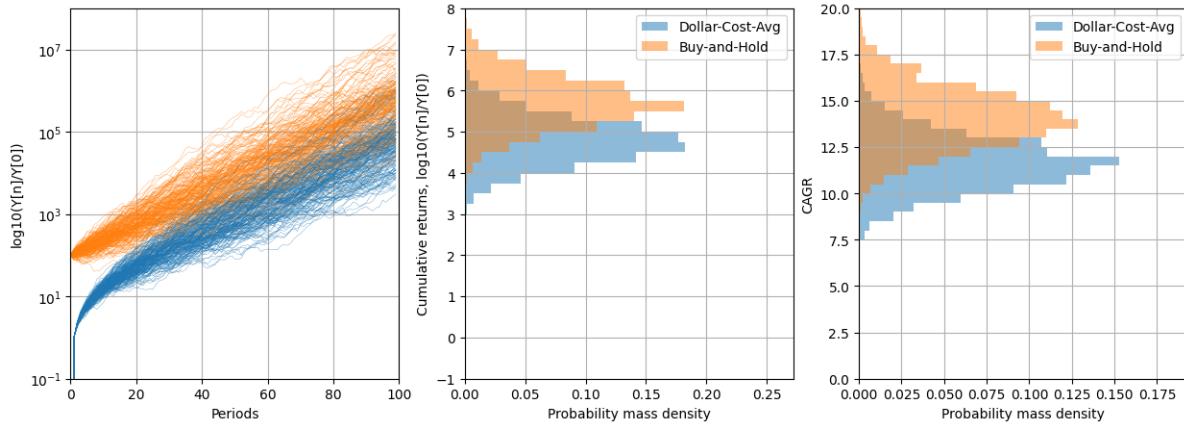
## 9.2 Realizations

### 9.2.1 Libraries and parameters

### 9.2.2 Constant Withdrawals



### 9.2.3 Dollar Cost Averaging (DCA)





## **Part IV**

# **Asset classes**



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**CHAPTER  
TEN**

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**INTRODUCTION TO ASSET CLASSES**



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CHAPTER  
ELEVEN

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BONDS

...

Here the most general expression for nominal and real **yield** are derived as a function of prices, face value of coupon, taxation and year to maturity, both in case of coupon reinvestment or not (reinvestment not always possible); a closed form solution is then derived under some assumptions, like constant (or average) rates; the effect on price and yield of credit rating and rating change, coupon, year to maturity are discussed on both examples and real-world cases.

Extra:

- definition of duration
- risks: inflation; reinvesment (at lower rates) for bonds with same maturity and different coupons
- inflation linked

## 11.1 Constant coupon bonds

### 11.1.1 W/o reinvestment

At time  $t_0$  the unit price of a bond is  $p_0$ ; investing  $Y_0$  allows to buy  $N_0 = \frac{Y_0}{p_0}$  titles; each title has the right of receiving net coupon  $C(1 - t)$ , with  $t$  taxation rate, per period (here assumed 1-year coupon range).

$$N_0 = \frac{Y_0}{p_0} = \frac{Y_0}{p_{in}} \frac{p_{in}}{p_0}$$

W/o reinvestment, the number of titles hold is constant and equal to  $N_0$ . As capital  $Y_i$  can be written as the product of unit price and number of bond in portfolio, the DCF of a bond w/o coupon reinvestment reads

$$\begin{aligned}\widetilde{DCF} &= -Y_0 + Y_N \prod_{k=1}^N (1 + r_k)^{-1} + \sum_{k=1}^N N_0 C(1 - t) \prod_{j=1}^k (1 + r_j)^{-1} \\ &= N_0 \left[ -p_0 + p_N \prod_{k=1}^N (1 + r_k)^{-1} + C(1 - t) \sum_{k=1}^N \prod_{j=1}^k (1 + r_j)^{-1} \right],\end{aligned}$$

This DCF must be corrected a CF at time  $t_N$  corresponding to tax on capital gain if  $p_N > p_0$ , discounted as

$$-N_0(p_N - p_0) t \prod_{k=1}^N (1 + r_k)^{-1} \quad (\text{only if } p_N > p_0)$$

The cumulative real return (if the discount ratio is inflation) is the ratio between the  $DCF$  and the actual value of the investment  $Y_0$ ,

$$\frac{\widetilde{DCF}}{Y_0} = -1 + \frac{p_N}{p_0} \prod_{k=1}^N (1 + r_k)^{-1} + \frac{C}{p_0} (1 - t) \sum_{k=1}^N \prod_{j=1}^k (1 + r_j)^{-1}$$

If the discount rate is constant, or the average (which average) discount rate is used, the expression of the cumulative return reads

$$\frac{\widetilde{DCF}}{Y_0} = -1 + \frac{p_N}{p_0}(1+r)^{-N} + \frac{C}{p_0}(1-t)\sum_{k=1}^N(1+r)^{-k}$$

### 11.1.2 W/ reinvestment

Time	Cashflows	$\Delta\text{Quantity}$	Quantity	DF
0	$-Y_0$	$N_0 = \frac{Y_0}{p_0}$	$N_0 = \frac{Y_0}{p_0}$	1
1	$+N_0 C(1-t)$			$(1+r_1)^{-1}$
1	$-N_0 C(1-t)$	$N_1 = \frac{N_0 C(1-t)}{p_1}$	$N_{0:1} = N_0 + N_1$	$(1+r_1)^{-1}$
2	$+N_{0:1} C(1-t)$			$(1+r_1)^{-1}(1+r_2)^{-1}$
2	$-N_{0:1} C(1-t)$	$N_2 = \frac{N_{0:1} C(1-t)}{p_2}$	$N_{0:2} = N_0 + N_1 + N_2$	$(1+r_1)^{-1}(1+r_2)^{-1}$
...				
$T-1$	$+N_{0:T-2} C(1-t)$			$\prod_{k=1}^{T-1} (1+r_k)^{-1}$
$T-1$	$-N_{0:T-2} C(1-t)$	$N_{T-1} = \frac{N_{0:T-2} C(1-t)}{p_{T-1}}$	$N_{0:T-1} = \sum_{k=0}^{T-1} N_k$	$\prod_{k=1}^{T-1} (1+r_k)^{-1}$
$T$	$+N_{0:T-1} C(1-t)$			$\prod_{k=1}^T (1+r_k)^{-1}$
$T$	$+N_{0:T-1} p_T$			$\prod_{k=1}^T (1+r_k)^{-1}$

All the cashflows from coupons are immediately reinvested so the DCF is

$$\begin{aligned} DCF &= -Y_0 + \underbrace{N_{0:T-1} (p_T + C(1-t))}_{Y_T} \underbrace{\prod_{k=1}^T (1+r_k)^{-1}}_{DF_T} = \\ &= -Y_0 + Y_T DF_T , \end{aligned}$$

with

$$\begin{aligned} N_{0:T-1} &= N_{0:T-2} + N_{T-1} = N_{0:T-2} + N_{0:T-2} \frac{C(1-t)}{p_{T-1}} = N_{0:T-2} \left[ 1 + \frac{C(1-t)}{p_{T-1}} \right] = \\ &= N_{0:T-3} \left[ 1 + \frac{C(1-t)}{p_{T-2}} \right] \left[ 1 + \frac{C(1-t)}{p_{T-1}} \right] = \\ &= \dots = \\ &= N_{0:1} \prod_{k=2}^{T-1} \left[ 1 + \frac{C(1-t)}{p_k} \right] = \\ &= N_0 \prod_{k=1}^{T-1} \left[ 1 + \frac{C(1-t)}{p_k} \right] . \end{aligned}$$

Cumulative discounted return reads

$$\begin{aligned}
 \frac{DCF}{Y_0} &= -1 + \frac{Y_T}{Y_0} DF_T = \\
 &= -1 + \frac{N_0}{N_0 p_0} \prod_{k=1}^{T-1} \left( 1 + \frac{C(1-t)}{p_k} \right) (p_T + C(t-1)) DF_T \\
 &= -1 + \frac{p_T}{p_0} \prod_{k=1}^T \left( 1 + \frac{C(1-t)}{p_k} \right) DF_T \\
 &= -1 + \frac{p_T}{p_0} \prod_{k=1}^T \left( \frac{1 + \frac{C(1-t)}{p_k}}{1 + r_k} \right).
 \end{aligned}$$

Composite discounted return is obtained, after writing the discounted cashflow as the difference between discounted cashflow at time  $t_T$  and  $t_0$ ,  $DCF = Y_T DF_T - Y_0$ ,

$$\begin{aligned}
 (1 + DCAGR)^T &= \frac{Y_T DF_T}{Y_0} = \frac{DCF}{Y_0} + 1 = \frac{p_T}{p_0} \prod_{k=1}^T \left( \frac{1 + \frac{C(1-t)}{p_k}}{1 + r_k} \right) \\
 DCAGR &= \left( \frac{p_T}{p_0} \prod_{k=1}^T \frac{1 + \frac{C(1-t)}{p_k}}{1 + r_k} \right)^{\frac{1}{T}} - 1
 \end{aligned}$$

If<sup>1</sup> price of the bond is constant throughout its whole life,  $p_k = 1, \forall k = 0 : T$ , and discount rate  $r$  is constant, the number of held bonds at time  $T - 1$  is

$$N_{0:T-1} = N_0 (1 + C(1-t))^{T-1},$$

the discounted cashflow is

$$\begin{aligned}
 DCF &= -N_0 + N_0 (1 + C(1-t))^{T-1} (1 + C(1-t)) (1 + r)^{-T} = \\
 &= N_0 \left[ -1 + \left( \frac{1 + C(1-t)}{1 + r} \right)^T \right],
 \end{aligned}$$

cumulative discounted return

$$\frac{DCF}{Y_0} = -1 + \left( \frac{1 + C(1-t)}{1 + r} \right)^T$$

and the composite discounted return reads

$$DCAGR = \frac{1 + C(1-t)}{1 + r} - 1.$$

<sup>1</sup> It's a big if. Even if credit rating and inflation are constant throughout the life of the bond, years to maturity decreases and thus - usually - the required rate decreases as well.



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CHAPTER  
TWELVE

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EQUITY

**What's equity?**

**Contents**

*Valuation methods.* Comparison and intrinsic value methods.

*Financial statements.* Introduction to financial statements of a company.

**Correlations, plots and fun-facts.**

## 12.1 Equity Valuation

### Detailed introduction

Equity valuation blends common sense, mathematics, expectations, estimation—and a bit of art. Buying shares in a company, whether directly or through a fund, means owning a (tiny) stake in a real business that produces goods and/or services and has the potential to generate earnings or free cash flows. As a shareholder, you are not just investing in market prices—you're becoming a part-owner of the enterprise. This ownership entitles you to a share of the company's profits through dividends or capital appreciation. It also comes with certain rights and responsibilities, especially during difficult periods.

When companies face financial stress or pursue growth opportunities, they may issue new shares to raise capital. This can lead to dilution, reducing the percentage ownership of existing shareholders. However, shareholders often have preemptive rights, allowing them to participate in new issuances to maintain their ownership stake. Moreover, owning equity means having a claim on the residual value of the company—what's left after all debts are paid—in both prosperous and challenging times. Understanding these dynamics is crucial to valuing equity: you're not just buying into today's performance, but into a stream of future cash flows and the complex, evolving structure of ownership.

**Sensitivity analysis** could provide an estimate of the effects of different parameters/assumptions on the final result.

**Different valuation methods** exist, and can be broadly classified in

- **comparison** approach: P/E, EV/EBITDA, or other indices used to compare companies of the same sector, marked, dimension,...<sup>1</sup>
- **intrinsic value** approach, based on **DCF**
- ...other methods for general firms (cost approach,...); valuation of financials;...

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<sup>1</sup> It's not always possible to find "equivalent" companies for the comparison...; P/E, EV/EBITDA,... whould be projected into the future to keep into account future in the value of a firm.

### 12.1.1 Comparison

### 12.1.2 Intrinsic value

- Future cash flows are estimated,
- CFs are discounted, usually for the *WACC* (Weighted Average Cost of Capital) to find the *NPV* (net present value) of the **enterprise value** *EV*
- Cash and equivalents are added to the *NPV* to find the **equity value**

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*WACC*

$$WACC = \frac{E}{V} R_e + \frac{D}{V} R_d (1 - t)$$

being  $R_e$  the **cost of equity** and  $R_d$  the **cost of debt** (maybe the easiest part to estimate accurately, since the debt structure is usually known/programmed). The factor  $(1 - t)$  usually appears as interest payments are tax-deductible.

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### Equity Risk Premium $R_e$ - Sharpe

Following W.Sharpe, equity risk premium can be estimated as

$$R_e = R_f + (R_m + R_f)\beta ,$$

being  $R_f$  the risk-free rate (usually 10Y US Treasuries), and  $R_m$  the annual return of the market/sector of the investment,  $\beta$  is a measure of risk or stock volatility of returns of the investment relative to that of the market/sector.

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## 12.2 Three Financial Statements

Financial statements are written records that illustrate the business activities and the financial performance of a company. In most cases they are audited to ensure accuracy for tax, financing, or investing purposes.

**Uses.** *Management* uses them for decision-making, budgeting and performance evaluation. *Investors* use them to assess profitability, financial health, future performance, and creditworthiness (especially *lenders*).

- **Income statement:** company performance (profit and loss) over a period. Broadly speaking:

$$\text{net earnings} = (\text{revenues} - \text{total expenses}) \times (1 - \text{tax rate}),$$

with total expenses = operative (labor + non-labor + DA) + Interest (due to debt holders), and “partial earnings” EBITDA, EBIT, EBT with trivial definition (Earnings Before: I:interest, DA: depreciation and amortization, T: tax)

- **Balance sheet:** financial position at a specific point in time, in terms of:

- assets: cash and equivalent + acc.receiv. + inventory + PPE (Plant property and equipment, subject to CapEx and depreciation,  $PPE(n) = PPE(n-1) + \text{CapEx}(n) + \text{DA}(n)$ )
- liabilities: debt + acc.pay.

– equity:

$$\begin{aligned}\text{retained earnings}(n) &= \text{retained earnings}(n-1) + \text{net earnings}(n) - \text{dividends}(n) \\ \text{shareholder equity}(n) &= \text{equity capital}(n) + \text{retained earnings}(n),\end{aligned}$$

being retained earnings( $n$ ) the **cumulative** retained earnings not distributed to shareholders.

The 2 contributions shareholder equity, total liabilities shows how the company's asset are financed: either through capital raised or retained earnings (equity), or through debt (liabilities). The **identity**

$$\text{total liabilities} + \text{shareholders equity} = \text{total asset}$$

must hold in a proper filled balance.

- **Cash flow statement** tracks the flows of cash in and out of the business over a period. Cashflows over a period modifies cash,

$$\begin{aligned}\text{closing cash}(n) &= \text{opening cash}(n) + \text{total cashflow}(n) \\ \text{opening cash}(n) &= \text{closing cash}(n-1)\end{aligned}$$

Cashflows are usually classified as:

- operating CF (DA is added back to net income, since it's not a cashflow going anywhere; it lowers income, but it's not a cashflow)
- investing CF
- financing CF

$$\begin{aligned}\text{Op.CF}(n) &= \text{net earnings}(n) + \text{DA}(n) - \Delta \text{WC}(n) \\ \text{Inv.CF}(n) &= \text{investment in PPE}(n) \\ \text{Fin.CF}(n) &= \text{issuance of debt}(n) + \text{issuance of equity}(n) - \text{dividends}(n)\end{aligned}$$

being  $\text{WC}(n) = \text{acc.rec}(n) + \text{inventory}(n) - \text{acc.pay}(n)$  the **working capital**.



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CHAPTER  
**THIRTEEN**

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**ETFS**



## **Part V**

# **Asset allocation and Investing**



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CHAPTER  
**FOURTEEN**

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**INTRODUCTION TO INVESTING**



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CHAPTER  
**FIFTEEN**

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## MODERN PORTFOLIO THEORY

Modern portfolio theory results in a strategy of asset allocation minimizing portfolio risk - measured as volatility - for a given value of the portfolio expected return.

### Asset Modeling

A set of  $N$  assets is available. Their return (over a defined period)<sup>1</sup> is represented by a multi-dimensional random variable,  $\mathbf{X}$ , with expected value and variance

$$\begin{aligned}\bar{\mathbf{X}} &:= \mathbb{E} [\mathbf{X}] \\ \sigma^2 &:= \mathbb{E} [\Delta\mathbf{X} \Delta\mathbf{X}^T]\end{aligned}$$

with  $\Delta\mathbf{X} := \mathbf{X} - \bar{\mathbf{X}}$ .

### Asset allocation. Constraints

A portfolio, without short or leverage positions on these assets, can be represented with the set of weights (proportion) of the assets  $\mathbf{w}$ , with

$$\begin{aligned}\sum_n w_n &= 1 \\ 0 \leq w_n &\leq 1 \quad , \quad \forall n = 1 : N\end{aligned}$$

### Portfolio return

Portfolio return is

$$\mathbf{X} = \mathbf{w}^T \mathbf{X} ,$$

From linearity, its expected value reads

$$\bar{X} = \mathbb{E} [X] = \mathbb{E} [\mathbf{w}^T \mathbf{X}] = \mathbf{w}^T \bar{\mathbf{X}}$$

and its variance

$$\begin{aligned}\sigma^2 &= \mathbb{E} [(X - \bar{X})^2] = \mathbb{E} \left[ \mathbf{w}^T (\mathbf{X} - \bar{\mathbf{X}}) (\mathbf{X} - \bar{\mathbf{X}})^T \mathbf{w} \right] = \\ &= \mathbf{w}^T \mathbb{E} [\Delta\mathbf{X} \Delta\mathbf{X}^T] \mathbf{w} = \mathbf{w}^T \sigma^2 \mathbf{w}\end{aligned}$$

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<sup>1</sup> How to estimate asset return, at least in terms of expected value and variance? And how to estimate correlation of the random variables?

---

### Example 15.1 (Properties of variance matrix)

Covariance matrix is symmetric definite positive. Symmetry readily follows

$$\sigma_{ij} = \mathbb{E} [\Delta X_i \Delta X_j]$$

The matrix is definite positive as

...

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### Modern Portfolio Theory, as a constrained optimization problem

Modern portfolio theory has its own “optimal” asset allocation  $\mathbf{w}^*$  ( $\bar{\mathbf{X}}$ ) - with the desired expected return as the parameter, fixed during the optimization - as the asset allocation for which

$$\begin{aligned} & \mathbf{w}^T \bar{\mathbf{X}} = \bar{X} \\ \min_{\mathbf{w}} & \sigma^2 \quad \text{s.t.} \quad \mathbf{w}^T \mathbf{1} = 1 \\ & 0 \leq w_i \leq 1 \end{aligned}$$

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CHAPTER  
**SIXTEEN**

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## **CAPITAL ASSET PRICING MODEL**



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CHAPTER  
**SEVENTEEN**

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## **MERTON'S PORTFOLIO PROBLEM**

Merton's portfolio problem deals with the choice of optimal fraction of investment in a risky asset  $\pi_t$  and the consumption  $c_t$ .

Assuming that the wealth of a family or an individual can be invested with a fraction  $\pi_t$  in a risky part of the portfolio with expected return  $\mu$  and standard variation  $\sigma_t$ , and with a fraction  $1 - \pi_t$  in a risk-free asset with expected return  $r_t$  and zero standard deviation, the wealth  $x_t$  of a family or an individual evolves with the SDE

$$\begin{aligned} dX_t &= r_t(1 - \pi_t)X_t dt + \mu_t\pi_t X_t dt - c_t dt + \pi_t\sigma_t X_t dW_t = \\ &= r_t X_t dt + (\mu_t - r_t)\pi_t X_t dt - c_t dt + \pi_t\sigma_t X_t dW_t, \end{aligned}$$

i.e. the equation of a [geometric Brownian motion with drift](#), the same equation that can be used to discuss [sequence risk](#) in investment, especially dealing with withdrawal.

Optimization problem can be solved using [continuous-time reinforcement learning](#), see as an example [Math:Introduction to RL](#), and [Statistics:RL \(todo\)](#). Optimal solution  $\pi_t^*$ ,  $c_t^*$  is the fraction invested in the risky asset and consumption that maximises the [value function](#),

$$V(x, t) = \mathbb{E} \left[ \int_{s=t}^T e^{-\rho(s-t)} u(c_s) ds + e^{-\rho(T-t)} B(T) u(X_T) \middle| X_t = x \right],$$

i.e. the return of the choice defined as the cumulative discounted reward. Reward per unit time is the [utility function](#)  $u(c_s)$ ,  $\rho$  is a personal discount factor that weights present and future rewards, and  $B(T)$  is a bequest function.

For constant expected return and volatility of the assets, and a utility function  $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$  it's possible to find an analytical solution with optimal fraction invested in the risky asset

$$\pi^* = \frac{\mu - r}{\gamma\sigma^2}.$$

### **17.1 Recursive relation and Hamilton-Jacobi-Bellman relation**

Writing the value function  $V(X_{t+dt}, t + dt)$  and expanding as a Taylor series up to the first order in  $dt$ , it's possible to find a **Hamilton-Jacobi-Bellman** equation from a recursive relation.

$$V(X_t + dX_t, t + dt) = \mathbb{E} \left[ \int_{s=t+dt}^T e^{-\rho(s-t-dt)} u(c_s) ds + e^{-\rho(T-t-dt)} B(T) u(X_T) \middle| X_{t+dt} \right] =$$

### 17.1.1 Recursive relation

$$\begin{aligned}
 e^{-\rho t} V(X_t, t) &= \mathbb{E} \left[ \int_{s=t}^T e^{-\rho s} u(c_s) ds + e^{-\rho T} B(T) u(X_T) \middle| X_t \right] \\
 e^{-\rho(t+dt)} V(X_t + dX_t, t + dt) &= \mathbb{E} \left[ \int_{s=t+dt}^T e^{-\rho s} u(c_s) ds + e^{-\rho T} B(T) u(X_T) \middle| X_{t+dt} \right] = \\
 &= \mathbb{E} \left[ \int_{s=t}^T e^{-\rho s} u(c_s) ds + e^{-\rho T} B(T) u(X_T) \middle| X_{t+dt} \right] - \mathbb{E} \left[ \int_{s=t}^{t+dt} e^{-\rho s} u(c_s) ds \middle| X_{t+dt} \right] = \\
 e^{-\rho(t+dt)} V(X_t + dX_t, t + dt) &= e^{-\rho t} V(X_t, t) - e^{-\rho t} u(c_t) dt \\
 d(e^{-\rho t} V(X_t, t)) &= -e^{-\rho t} u(c_t) dt \\
 \rho V dt &= dV + u(c_t) dt
 \end{aligned}$$

### 17.1.2 Taylor expansion

Taylor expansion of value function  $v(t, x)$ , valued with  $x = X_t$ , reads

$$\begin{aligned}
 dv &= \partial_t v dt + \partial_x v dx + \frac{1}{2} \partial_{xx} v dx^2 + o(dt) = \\
 &= \partial_t v dt + \partial_x v dX_t + \frac{1}{2} \partial_{xx} v dX_t^2 + o(dt) = \\
 &= dt (\partial_t v + \partial_x v(r_t + (\mu_t - r_t)\pi_t)X_t - \partial_x v c_t) + \partial_x v \pi_t \sigma_t X_t dW_t + \frac{1}{2} \partial_{xx} v (\pi_t \sigma_t X_t)^2 dt + o(dt) =
 \end{aligned}$$

as  $dX_t^2 = (\pi_t \sigma_t X_t)^2 dW_t^2 + o(dt) = (\pi_t \sigma_t X_t)^2 dt + o(dt)$ . Taking expected value,

$$dV = dt \mathbb{E} \left[ \partial_t v + \partial_x v(r_t + (\mu_t - r_t)\pi_t)X_t - \partial_x v c_t + \frac{1}{2} \partial_{xx} v (\pi_t \sigma_t X_t)^2 \right].$$

### 17.1.3 Recursive relation for optimal solution

First order in  $dt$  reads

$$\rho V^* = \max_{\pi_t, c_t} \left\{ \partial_t V^* + \partial_x V^* [(\pi_t(\mu - r) + r)X_t - c_t] + \frac{1}{2} \partial_{xx} V^* (\pi_t \sigma_t X_t)^2 + u(c_t) \right\} = \max_{\pi_t, c_t} \Phi$$

### 17.1.4 Optimality condition

Zero gradient is a necessary condition for local extreme points,

$$\begin{cases} 0 = \partial_{\pi_t} \Phi(\pi_t^*, c_t^*) = (\mu - r)X_t \partial_x V^* + \partial_{xx} V^* \pi_t^* (\sigma_t X_t)^2 \\ 0 = \partial_{c_t} \Phi(\pi_t^*, c_t^*) = -\partial_x V^* + \partial_{c_t} u(c_t^*) \end{cases}$$

and thus

$$\pi_t^* = -\frac{\partial_x V^*}{\partial_{xx} V^*} \frac{\mu - r}{X_t \sigma^2}$$

**Necessary conditions on  $u()$  for the Hessian to be definite negative**

todo

**17.1.5 Example: utility function**  $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$

$$\begin{cases} \pi_t^* = -\frac{\partial_x V^*}{\partial_{xx} V^*} \frac{\mu - r}{X_t \sigma^2} \\ c_t^* = (\partial_x V^*)^{-\frac{1}{\gamma}} \end{cases}$$

**17.1.6 Example: value function**  $V^*(t, x) = f^\gamma(t) \frac{x^{1-\gamma}}{1-\gamma}$

With value function  $V^*(x, t) = \dots$  and bequest function  $B(T) = \varepsilon^T$

$$\dot{f}(t) = \nu f(t) - 1$$

with final condition  $f(T) = \varepsilon$ , and

$$\nu = \frac{1}{\gamma} \left\{ \rho - (1 - \gamma) \left( \frac{(\mu - r)^2}{2\sigma^2\gamma} + r \right) \right\}.$$

The solution of the ODE reads

$$f(t) = \begin{cases} \frac{1}{\nu} + \left( \varepsilon - \frac{1}{\nu} \right) e^{-\nu(T-t)} & , \quad \text{if } \nu \neq 0 \\ T - t + \varepsilon & , \quad \text{if } \nu = 0 \end{cases}$$

Thus

$$\begin{aligned} \pi_t^* &= \frac{\mu - r}{\gamma\sigma^2} \\ c_t^* &= \frac{X_t}{f(t)} \end{aligned}$$



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CHAPTER  
**EIGHTEEN**

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## REBALANCING

### Reasons.

- risk management:
  - adjust risk for the period of life, and risk-level
  - correct drift towards the asset with highest return, as it affects the

**Strategies.** In a passive investment strategy, rebalancing should be triggered by some rules, to be applied automatically. As an example:

- periodic rebalancing: rebalancing with constant time interval
- deviation-triggered: rebalancing when asset allocation deviation from the target allocation exceeds a prescribed threshold. E.g. approximately 10% of a 60-40 portfolio going to 65-35 (introduction of episode 152, as a summary of episode 117)
- using contributions/withdrawals

**Effects of rebalancing.** In different situations one of the following effects occurs:

- Volatility is reduced
- Risk-adjusted return improves
- Return of the portfolio is increased

Often, a rebalanced-portfolio return is larger than the weighted average of the returns of the assets of the portfolio. Shannon demon is the mathematical reason for that, *for creating return “out of thin air”*.

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**Example 18.1 (Shannon demon - on a coin flip)**

Starting with 100€, and a fair coin with 50% probability of for each outcome, either  $H$ :head or  $T$ :tail. If outcome is  $H$  you gain 50%, if outcome is  $T$  you lose 33.3%.

- If I play with all the money I have, what is the expected amount at the end of the game, for a sufficiently large number of toss?

Now, let's change the strategy: I bet only 50% of the amount I have. What's the expected amount at the end of the game?

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**Example 18.2 (Nassim Taleb, is the coin fair?)**

After 10 tosses with 10 heads, how would you bet on the next toss?

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**Example 18.3 (Kelly criterion)**

bla bla bla

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### Example 18.4 (Does rebalancing improve return, thanks to Shannon demon?)

Yes, for a portfolio with 2 assets with similar returns and low correlation. E.g.:

- S&P500 and gold (50%-50%) from 1972 to 2008 (cherry-picking?): CAGR with annual rebalancing: 10.3%, while S&P: 9.4% and gold: 8.2%.
- S&P500 and gold (50%-50%) from 1972 to 2023 (cherry-picking?): CAGR with annual rebalancing: 10.2%, while S&P: 10.5% and gold: 7.5%, but with lower volatility, lower max drawdown and a better Sharpe ratio
- S&P500 and Treasury (50%-50%) from 1972 to 2023 (cherry-picking?): CAGR with annual rebalancing: 9.3%, while S&P: 10.5% and gold: 6.4%, but with lower volatility, lower max drawdown and a better Sharpe ratio. Without rebalancing: 9.6% (higher!) as equity has much higher return and drift occurs over 50-year period.
- MSCI World and FTSE G7...

No, with 2 assets with 2 assets with very different returns. Anyways, if they have low correlation, rebalancing (may?) reduce volatility, improves risk-adjusted return, or both.

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## 18.1 Resources

- *The Bull*
  - 217. Il modo migliore per Ribilanciare il portafoglio
  - 152. La magia del ribilanciamento e il demone di Shannon
  - 117. Come ribilanciare il portafoglio (e previsioni per i prossimi 10 anni)
- *R.Arnott*. Over-rebalancing
- market sentiment about Shannon demon

## **Part VI**

### **Extra**



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CHAPTER  
**NINETEEN**

---

**EXTRA AND RANDOM**



## EURISTHICS AND HISTORICAL CORRELATIONS

### 20.1 Expected returns in the stock market

#### 20.1.1 Shiller P/E and S&P500 10-year annualised forward returns

- from Invesco, [Applied philosophy: The Shiller P/E and the S&P500 returns](#)

#### 20.1.2 Bogle Expected Return Formula

- Comment by [Ben Carlson](#), on his [website](#)



## **Part VII**

# **People**



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CHAPTER  
**TWENTYONE**

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## **RESOURCES, PEOPLE AND FIRMS**

### **21.1 Tools**

These tools have not been tested and verified here. I decline any responsibility about their use.

#### **21.1.1 Simple tools for investors**

[Simple tools for investors] (<https://www.simpletoolsforinvestors.eu/index.shtml>): **bond** monitor, calculators, tools (minus-eater, laddering,...)

#### **21.1.2 Interactive Asset Allocation - by Research Affiliates**

Expected return vs. volatility plot of different asset classes on different time horizons by *Research Affiliates*.

#### **21.1.3 Gregory Gundersen Blog**

Gregory Gundersen blog

### **21.2 People**

#### **21.2.1 Arnott, Robert**

Founder of *Research Affiliates*.

#### **21.2.2 Faber, Meb(ane)**

Co-founder and Chief Investment Officer at *Cambria Investment Management*.

### **21.2.3 Wigglesworth, Robin**

Financial Times' global finance correspondent, author of *Trillions* a book on the past, present and future of passive investing.

### **21.2.4 Zweig, Jason**

Columnist for the Wall Street Journal since 2008. A Safe Heaven for Intelligent Investor.

### **21.2.5 Ritholtz, Barry**

### **21.2.6 Maggiulli, Nick**

### **21.2.7 Carlson, Ben**

Director of Institutional Asset Management at *Ritholtz Wealth Management*. Author of the website [A Wealth of Common Sense](#)

### **21.2.8 Green, Micheal**

Foreseen “**Volmageddon**” of the 5 February 2018, the collapse of inverse ETFs or ETP: VIX, SVXY and VMIN, as a consequence of the VIC daily surge from 17 to 37 (approximately +115%, that erased daily inverse products)

*Even a moderate (~4%) equity sell-off can unleash devastating volatility swings if the short-vol market is crowded. Rebalancing demands can worsen volatility when liquidity dries up near market close.*

- *The Bull*, 192. Micheal Green: perché l'investimento passivo distorce il mercato (e come comportarci)

### **21.2.9 Yardeni, Edward**

## **21.3 Firms**

### **21.3.1 Reserach Affiliates**

Founded by *Robert Arnott*

### **21.3.2 Cambria Investment Management**

Co-founded by *Mebane Faber*

### 21.3.3 Ritholtz Wealth Management

*Barry Ritholtz, Nick Maggiulli, Ben Carlson*



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CHAPTER  
**TWENTYTWO**

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**THE BULL**

“The Bull, il tuo podcast di finanza personale”, Riccardo Spada. [Youtube channel](#)

- Jason Zweig, American financial journalist, columnist for the WSJ since 2008. [Jason Zweig - A Safe Haven for Intelligent Investor](#)
  - **224** what it takes to become an intelligent investor
- Nick Maggiulli, COO for Ritholtz Wealth Management LCC, financial blogger at [Of Dollars And Data - Act Smarter. Live Richer](#). Author of “Just Keep Buying” about the power of compounding.
  - **221** Just keep buying
- Davide Serra, founder and CEO of Algebris Investment
  - **216** The change we’re living and consequences for investors
- William Bernstein, american financial theorist and neurologist
  - **214** The 4 pillars of investing and how to manage risk and uncertainty
- Barry Ritholtz
  - **206** How **not** to invest
- Robin Wigglesworth
  - **203** the case for passive investing: navigative crises through simplicity
- Ben Carlson, Director of Institutional Asset Management at [Ritholtz Wealth Management](#). Author of the webiste [A Wealth of Common Sense](#)
  - **200** Investing is mostly a matter of common sense (and patience)
- Robert Arnott
  - **196** Fundamental investing
- Micheal Green
  - **192** Why passive investing is distorting the markets
  - [Haddad, Valentin](#), associate professor at the UCLA: how competitive is stock market? why is asset demand inelastic?
- Edward Yardeni,
  - **183** Why should we be optimistic?
- Meb Faber,
  - **180** Investing with Common Sense
- Eugene Fama, efficient market, **factor** investing, and factor ETFs

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