# basics book template

basics

14 dic 2024

# Indice

If you want ot start a new basics-book, it could be a good idea to start from this template.

Please check out the Github repo of the project, basics-book project, and the landing page of the project.

CAPITOLO 1

# Special Relativity

CAPITOLO 2

# General Relativity

# CAPITOLO 3

# Statistical Physics

CAPITOLO 4

# Quantum Mechanics

## 4.1 Mathematical tools for quantum mechanics

**Definition 1 (Operator)**

**Definition 2 (Adjoint operator)**

Given an operator $\hat{A} : U \rightarrow V$, its self-adjoint $\hat{A}^* : V \rightarrow U$ is the operator s.t.

$$(\mathbf{v},\ \hat{A}\mathbf{u})_V = (\mathbf{u}, \hat{A}^*\mathbf{v})_U$$

holds for $\forall \mathbf{u} \in U,\ \mathbf{v} \in V$.

**Definition 3 (Hermitian (self-adjoint) operator)**

If $\hat{A} : U \rightarrow U$, it is a self-adjoint operator if $\hat{A}^* = \hat{A}$.

Self-adjoint operators have real eigenvalues, and orthogonal eigenvectors (at least those associated to different eigenvalues; those associated with the same eigenvalues can be used to build an orthogonal set of vectors with orthogonalization process).

## 4.2 Postulates of Quantum Mechanics

- …
- Canonical Commutation Relation (CCR) *and Canonical Anti-Commutation Relation…*

## 4.3 Non-relativistic Mechanics

### 4.3.1 Statistical Interpretation

#### Wave function

The state of a system is described by a wave function $|\Psi\rangle$

**todo**

- properties: domain, image,…
- unitary $1 = \langle\Psi|\Psi\rangle = |\Psi|^2$, for statistical interpretation of $|\Psi|^2$ as a density probability function

#### Operators and Observables

Physical **observable** quantities are represented by *Hermitian operators*. Possible outcomes of measurement are the eigenvalues of the operator

Given $\hat{A}$ and the set of its eigenvectors $\{|A_i\rangle\}_i$ (**todo** *continuous or discrete spectrum…, need to treat this difference quite in details*), with associated eigenvalues $\{a_i\}_i$

$$\hat{A}|A_i\rangle = a_i|A_i\rangle$$

$$|\Psi\rangle = |A_i\rangle\langle A_i|\Psi\rangle = |A_i\rangle\Psi_i^A$$

$$\langle A_j|\Psi\rangle = \langle A_j|A_i\rangle\langle A_i|\Psi\rangle = \Psi_j^A$$

and thus

$$\Psi_j^A = \langle A_j|\Psi\rangle$$
$$\Psi_j^{A*} = \langle\Psi|A_j\rangle$$

- identity operator $\sum_i |A_i\rangle\langle A_i| = \mathbb{I}$, since

$$\sum_i |A_i\rangle\langle A_i|\Psi\rangle = \sum_i |A_i\rangle\langle A_i|\Psi_j^A A_j\rangle = \sum_i |A_i\rangle\delta_{ij}\Psi_j^A = \sum_i |A_i\rangle\Psi_i^A = |\Psi\rangle$$

- Normalization:

$$1 = \langle\Psi|\Psi\rangle = \Psi_j^{A*}\underbrace{\langle A_j|A_i\rangle}_{\delta_{ij}}\Psi_i^A = \sum_i \left|\Psi_i^A\right|^2$$

with $|\Psi_i^A|^2$ that can be interpreted as the probability of finding the system in state $|\Psi_i^a\rangle$

- Expected value of the physical quantity in the a state $|\Psi\rangle$, with possible values $a_i$ with probability $|\Psi_i^A|^2$

$$\begin{aligned}\bar{A}_\Psi &= \sum_i a_i |\Psi_i^A|^2 = \\ &= \sum_i a_i \Psi_i^{A*} \Psi_i^A = \\ &= \sum_i a_i \langle \Psi | A_i \rangle \langle A_i | \Psi \rangle = \\ &= \langle \Psi | \left( \sum_i a_i |A_i\rangle\langle A_i| \right) |\Psi\rangle = \\ &= \langle \Psi | \hat{A} | \Psi \rangle =\end{aligned}$$

#### Space Representation

#### Momentum Representation

### 4.3.2 Schrodinger Equation

$$i\hbar \frac{d}{dt} |\Psi\rangle = \hat{H} |\Psi\rangle$$

being $\hat{H}$ the Hamiltonian operator and $|\Psi\rangle$ the wave function, as a function of time $t$ as an independent variable.

#### Stationary States

Eigenspace of the Hamiltonian operator

$$\hat{H} |\Psi_k\rangle = E_i |\Psi_k\rangle \ ,$$

with $E_k$ possible values of energy measurements. *If no eigenstates with the same eigenvalue exists, then…otherwise… Without external influence* **todo** *be more detailed!*, energy values and eigenstates of the systems are constant in time.

Thus, exapnding the state of the system $|\Psi\rangle$ over the stationary states gives $|\Psi_k\rangle$, $|\Psi\rangle = |\Psi_k\rangle c_k(t)$, and inserting in Schrodinger equation

$$i\hbar \dot{c}_k |\Psi_k\rangle = c_k E_k |\Psi_k\rangle$$

and exploiting orthogonality of eigenstates, a diagonal system for the amplitudes of stationary states ariese,

$$i\hbar \dot{c}_k = c_k E_k \ .$$

whose solution reads

$$c_k(t) = c_{k,0} \exp\left[ -i \frac{E_k}{\hbar} t \right]$$

Thus the state of the system evolves like a superposition of monochromatic waves with frequencies $\omega_k = \frac{E_k}{\hbar}$,

$$|\Psi\rangle = |\Psi_k\rangle c_k(t) = |\Psi_k\rangle c_{k,0} \exp\left[ -i \frac{E_k}{\hbar} t \right] \ .$$

#### Representations

#### Schrodinger

#### Heisenberg

#### ...

### 4.3.3 Matrix Mechanics

#### Attualization of 1925 papers

$$|\Psi\rangle = |\Psi_k\rangle c_{k,0} \exp\left[-i\frac{E_k}{\hbar}t\right]$$

$$\langle\Psi|\hat{\mathbf{X}}|\Psi\rangle$$

$$\langle\Psi|\Psi_m\rangle\langle\Psi_m|\hat{\mathbf{X}}|\Psi_n\rangle\langle\Psi_n|\Psi\rangle$$

$$X_{mn} = \langle\Psi_m|\hat{\mathbf{X}}|\Psi_n\rangle$$

$$\langle\Psi|\hat{\mathbf{P}}|\Psi\rangle$$

## 4.4 Many-body problem

Wave function with symmetries: Fermions and Bosons

# Proof Index

## definition-0



## definition-1



## definition-2